SKINNING THE PYTHAGOREAN CAT: A STUDY OF STRATEGY PREFERENCES OF SECONDARY MATH TEACHERS

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For my Family,
Thank you. I was formed in the crucible of your love. May I extend that circle of love and support to the world to which I seek entry – the classroom.

ABSTRACT

A series of observations in math classrooms revealed a pervasive problem of "reteaching". It matters little the subject; before the teacher can begin teaching new material, basic skills often have to be re-taught before the lesson can move forward. Perhaps some methods are more effective than others in the classroom. Consequently, this study addressed two research questions:

Q1: What teaching strategies are considered preferential by math teachers for teaching math concepts?

Q2: Do teacher demographics influence those teacher preferences?

Consequently, this study seeks to add to the existing literature regarding math teacher preferences for teaching the Pythagorean Theorem to high achieving and low achieving secondary mathematics students. Thirty mathematics teachers were asked to rate their preferences of six research-based standards with respect to teaching a specific math concept – the Pythagorean Theorem Additionally, teachers were asked to list their preferences separately for high achieving students and for low achieving students.

While there were many interesting findings, two stand out: First, the significant difference (p<.05; df=28) in preferences for effective instruction between high and low achieving students. Teachers were in agreement that Questioning strategies were the main preference for teaching the Pythagorean Theorem to high achieving students, and that Nonlinguistic (manipulatives) were preferred for Low Achieving students; second was the importance given to Brain Compatible strategies. The teachers with more experience and/or higher educational achievement exhibited a significant difference (p<.05; df=28) over those teachers with less education and experience, indicating a high preference for using Brain Compatible strategies with respect to teaching high achieving students in this specific mathematics concept.

While no one strategy has been proven effective for all classroom situations, it was of interest to this researcher that survey respondents were generally in agreement with regard to the most effective way to "skin this Pythagorean cat." This researcher hopes that further research can be done to answer the new questions that were posed as a result of this study.

Key words: Pythagorean theorem, lecture, memorization, nonlinguistic, questioning, cooperative learning, Brain Compatible, low achieving students, high achieving students, gender, teaching experience, education level, undergraduate degree, Teacher of the Year

TABLE OF CONTENTS

CHA	CHAPTER					
I	INTRODUCTION Statement of the Problem Purpose Operational Definition Limitations of the Study		1 4 4 5 5			
II	REVIEW OF THE LITERATURE Summary					
III	METHODS Survey Subjects/Population Summary		17 17 18 19			
IV	RESULTS Question One: Teacher Preferences Question Two: Differences With Respect to Demographics With Respect to Gender With Respect to Education, Experience, and Undergraduate Studies With Respect to Teacher of the Year Summary					
V	CONCLUSION AND RECOMMENDATIONS					
	WORKS CITED APPENDICES		38 42			
	APPENDIX A.	A LISTING OF TENNESSEE STATE AND CORE STANDARDS INVOLVING THE PYTHAGOREAN THEOREM				
	APPENDIX B.	SURVEY DOCUMENT	45			

LIST OF FIGURES AND TABLES

Figures and Tables			
FIGURE 1: High Achieving Students	21		
FIGURE 2: Low Achieving Students	21		
TABLE 1: Summary Table of Means by Category	23		
FIGURE 3: Gender: High Achieving Students	25		
FIGURE 4: Gender: Low Achieving Students	26		
FIGURE 5: Teaching Experience and Education Level: High Achieving Students	27		
FIGURE 6: Undergraduate Degree: Low Achieving Students	29		
FIGURE 7: Teacher of the Year: High Achieving Students	30		
FIGURE 8: Teacher of the Year: Low Achieving Students	31		

CHAPTER ONE

INTRODUCTION

Mathematics has a language of its own, a language that is often not that of the students sitting in the class. Even the teacher feels the great sucking sound made from the vacuum of the twenty or so minds perched on hunched shoulders throughout the classroom. There will be no going straight into the lesson. First there will be the introductory probes, questions intended to gauge the readiness of the audience for the material. The current lesson will require students to solve problems using the famous Pythagorean Theorem: A simple affair of a² plus b² equals c². A combination of blank looks and/or heads tucked quickly into books seek to ensure the teacher does not call on someone who has no memory whatsoever of "Pythaga-who" and his formula, much less how or when to use it. The teacher will then begin to reteach, remind, redraw, the basics of a formula that should be second nature to the sixteen-year-olds in front of her.

This writer observed this situation first hand. After four weeks of teaching eighth grade mathematics, which included lessons on the Pythagorean Theorem, she taught a classroom of eleventh and twelfth grade geometry students, who were also studying more complex problems that required the use of the Pythagorean Theorem. At the beginning of each set of lessons, the formula itself and the concept behind the formula had to be re-taught, to eighth graders, eleventh graders, and twelfth graders alike. Additionally, this author observed that the different sets of students were not just making various small mistakes; each group was making the same specific

and critical mistakes that would prevent them from finding appropriate solutions. This phenomenon was not limited to the Pythagorean Theorem. Before any new skills could be introduced to students by this writer, a brief reteaching of the basic skill sets needed for the new material had to be conducted.

Good teachers know to scaffold—to help students reach higher than they can alone and to practice low road transfer—to build upon existing knowledge and create a firm foundation for the new information being presented. Good math teachers also know that proper scaffolding and transfer of training are critical to building strong math skills. As Boaler (1993) pointed out, good teachers know that "Methods of solutions for one mathematical problem are intended, by the abstract nature of mathematics, to be generalized to other problems" (p.21). Good math teachers also know that many times the knowledge that they are hoping to build on is, for some reason or another, not in place. Reteaching becomes necessary when the basic scaffold is not there and consequently transfer of training is unlikely.

Students tend to learn to solve math problems with specificity, that is, the solution matches the problem. Changing the problem will cause the student to look for a different solution, because the student does not recognize that the same solution will fit the problem as it is differently stated (Bandura, 1977, Pajares & Miller, 1994 as cited by Ozgen & Bindak, 2011). Adding to the difficulty is the differing styles of teachers and students:

...many schools are not homogenous in their pedagogical approaches. Students may encounter one set of norms and expectations with one teacher and find a completely different set the following year. If we are concerned with students' ongoing success in mathematics, it is critical to look at identity development across as well as within specific types of classroom settings (Horn, 2008, p. 207).

When it comes to math concepts, there is definitely more than one way to "skin a cat". As a result, when the teacher asks for existing knowledge, the student does not, or cannot, make the

connection. Some possible reasons for the students' inability to make such connections may include:

- 1. The student did not learn in the first place; learned concept incorrectly;
- 2. The student cannot translate the formula to the geometric figure / Cartesian plane;
- 3. The student was taught the concept in narrow context without ability to translate to other contexts, or was taught the formula without connection to the actual parts of the figure;
- 4. There is a lack of knowledge of about mathematical functions (solving radicals, for example);
- 5. There may be an inability to solve basic algebraic equations;
- 6. The student learned the concept, but it was filed in the brain in such a way that there was an inability of the student to make a connection to the concept that is, the "right" questions were not being asked; or
- 7. Other reasons not readily apparent.

This author does not seek with this paper to resolve the "why" of reteaching. Instead, this study seeks only to lay some groundwork towards the process of identifying why students cannot easily access information that has already been acquired throughout the educational process. To that end, this study seeks to identify strategies preferred by math teachers for teaching specific math concepts. Such information might eventually be of some value as teachers seek to improve the quality of lessons and of the learning process itself.

Since there are so many concepts that must be taught within the math curriculum, a single concept thread needed to be selected for this study. The Pythagorean theorem was chosen because it is a concept thread that shows up in multiple objectives and exists as a critical component in more complex and far-reaching mathematical calculations in every branch of

mathematics (see Appendix A: A listing of Tennessee state and core standards involving the Pythagorean theorem). That it is applicable in multiple math classroom situations and for many different types of mathematical problem solving may be part of the problem of students' seeming difficulty in knowing when and how to apply it. There is definitely more than one way to skin the Pythagorean cat. The way in which the theorem is taught and the success with which its concept is retained could have long-range implications for the success of any student who is seeking to master a broad range of mathematical skills.

Statement of the Problem

A series of observations in math classrooms revealed a pervasive problem of "reteaching". It matters little the subject; before the teacher can begin teaching new material, basic skills often have to be re-taught before the lesson can move forward. Perhaps some methods are more effective than others in the classroom. However, as a novice teacher, this writer knew little of those thought to be most efficient. Consequently, this study addressed two research questions:

- Q1: What teaching strategies are considered preferential by math teachers for a specific math concept (i.e. the Pythagorean theorem)?
- Q2: Do teacher demographics influence those preferences?

This study does not seek to address the "why" of reteaching, but instead seeks to lay the groundwork for the "how", that is, to seek the opinions of secondary mathematics teachers on the strategies they prefer for teaching math concepts, specifically the concepts involved in the Pythagorean Theorem.

Purpose

The information presented in this paper and the conclusions drawn may assist in pointing researchers in additional directions for approaching the issue of reteaching that appears pervasive

within the mathematics classroom. In determining what strategies are preferred by teachers, there may be additional research that could be done to determine how those preferred strategies are impacting long-term retention of standards being taught. Such findings might further assist teachers of elementary and secondary students in identifying effective strategies to use in teaching the Pythagorean Theorem, as well as other mathematical concepts.

Operational Definition

Instructional strategy: a sequential combination of methods designed to accomplish the learner outcomes" (Clarke & Stowe, 2008, n.p.).

Limitations

The validity issues with regard to the study outlined in this chapter are: the size of the sample used for the study; the subjective nature of the preferences of the survey participants; the specific number and choices of strategies offered within the survey, and the inexperience of the researcher. While the researcher acknowledges these issues, steps were taken, within reasonable bounds, to control as many of the threats as possible.

To maximize the validity of the responses thirty math teachers were chosen for the survey, to ensure a minimum number of respondents and so that the respondents would be able to address the mathematics-specific topic. The strategies chosen were those deemed especially valid in mathematics classrooms (Marzano, Pickering, & Pollock, 2001; Knisley, 2005; Radin, 2009). While this researcher realizes it was not possible to research all of the available literature, she conducted as thorough a search as possible within the time limit and scope of the study, and believes she has provided research that is appropriate and applicable in the final study.

CHAPTER TWO

REVIEW OF THE LITERATURE

The purpose of this chapter is to outline the professional literature related to the problem identified in the previous chapter. After careful review of the available literature, it was determined that while there appear to be volumes of material on general learning methods, strategies, best practices, and innovative ideas, along with a myriad of lesson plans specific to disciplines, there appears to exist a slight deficiency in articles that discuss effective math strategies at the secondary school level with regard to specific concepts. This study seeks to add a drop to the ocean of literature and, hopefully, shed some small light on teaching strategies that are, considered preferential by secondary school math teachers for teaching strategies with regard to a specific math concept: The Pythagorean theorem.

To support the value of surveying secondary teachers with regards to the strategies they prefer for the teaching of a specific concept, literature reviewed for this study covered descriptions of and strategies for recognizing effective teachers. Also included is literature on effective teaching strategies, both in general and specific to math. In addition, literature identifying developmental or cognitive behavors that may hold connection or value to the mathematics classroom and/or mathematics instruction have also been included. This review is intended to create a pool of information to assist the reader in forming an opinion regarding the

survey results, which will be revealed in *Chapter Four*. At the conclusion of this chapter, the writer will offer a summary of the major findings of the literature.

Teachers are considered to be the most important school-related factor in student achievement (Butler, 1998; Marzano *et al.*, 2001; Horn, 2008; Radin, 2009; Harris Interactive, 2011). Further research appears to clarify the importance of teachers as deliverers of solid educational content through strategies that have been deemed "best practice" and that have been shown to be effective instruction (Marzano *et al.*, 2001; Harris Interactive, 2011).

"Students know more ways to learn than we know how to teach them" (Clarke *et al.*, 2008, n.p.). Good teachers know that they need to use a variety of strategies and "best practices" that will actively engage students in the learning process and create instructional opportunities that will allow students to apply their learning to experiences outside of education (Clarke *et al.*, 2008).

"Right now our better teachers are doing a seat-of-the-pants thing; ...they've got a feel for it" (Radin, 2009, p. 48). We can narrow [teaching knowledge] down so teachers are not "guessing so much" (Radin, 2009, p. 48). To have good teachers, one must be able to identify what defines excellence in an educator. The available literature suggests that certain forms of evaluation may be more difficult than anyone realizes. "The best school predictor of student outcomes is high-quality, effective teaching as defined by performance in the classroom" (Goldhaber, 2002; Goldhaber & Brewer, 2000; Hanushek, Kain, O'Brien, & Rivken, 2005; Write, Horn, & Sanders, 1997 as cited by Strong, Gargani, & Hacifazlioglu, 2011, p. 367).

"Effective teachers monitor student learning through the use of a variety of formal and informal assessments and offer meaningful feedback to students" (Cotton, 2000; Good & Brophy, 1997; Hattie & Timperley, 2007; Peart & Campbell, 1999 as cited by Stronge, Ward &

Grant, 2011, p. 341). The "well-designed use of formative assessment yields gains in student achievement equivalent to one or two grade levels" (Assessment Reform Group, 1999 as cited by Stronge *et al.*, 2011, p. 341). Stronge *et al.* 's (2011) study is able to follow up that conclusion with hard numbers, "The differences in student achievement in mathematics and reading for effective teachers and less effective teachers were more than 30 percentage points...[and] Top quartile [based on teacher effectiveness ratings] had fewer classroom disruptions, better classroom management skills, and better relationships with their students than did bottom-quartile teachers" (p. 349). It appears that there may be identifiable, observable characteristics of effective teachers after all.

Smoot (2010) lets the effective teachers speak for themselves. In his book, *Conversations with Great Teachers*, Smoot interviews teachers from varied backgrounds – those who teach in high school and college classrooms to classes in prisons; teachers of circus arts, ballet, acting, fencing, Zen meditation, Aztec dance, and even alligator wrestling – all of them considered excellent in their fields. The comments from Paul Karafol, a high school math teacher who has received the Edith May Sliffe Award for Distinguished High School Mathematics Teaching given by the Mathematical Association of America, seemed particularly appropriate to a study of effective math strategies:

If we taught English the way I think we teach mathematics, nobody would ever get to read a real poem or short story until they were a junior or a senior in high school, and then we'd wonder why they hate reading so much...I think you've got to present the mathematics that you think is beautiful...Now a lot of the beautiful stuff is hard...we look at students who are not at the very top, and we say, 'They can't get this...' So they end up not ever learning any mathematics that is beautiful...a lot of math is finding connections...I always look for stuff that kids use their hands with...You have to go into class that first day with the deep belief that every student is going to learn a lot of great math that year...I'm always thinking to myself, 'What will they need to know a week from now, and what do I have to do today so they're ready for that lesson?' That keeps the pace moving without leaving some of the kids behind (Smoot, 2010, pp. 24-26).

There is also literature that posits the idea of actual teaching (the curriculum) as being of more importance than the teacher. It is the act of teaching itself and the constant adaptation and improvement of the lessons that have the greatest potential for making a difference in the classroom (Hiebert & Morris, 2012). Examples of high-impact instructional routines are "...posing problems, providing explanations, responding to student thinking, and leading class discussions" (Ball *et al.*, 2009, Grossman & McDonald, 2008 as cited by Hiebert *et al.*, 2012, p. 93). Even "novices" can learn the routines of good teaching because the good routines occur often. It is the lessons themselves and their continued improvement that will drive the classroom (Hiebert *et al.*, 2012).

Strategic planning and large scale reform (Schmoker, 2011) is what does not work, and a look should be taken at the "simple, proven, and affordable structures" that would allow teachers to work as a "learning community". A learning community is "...teams of teachers who, through short-term trial and error, have found more effective ways to teach certain math applications, reading comprehension skills, difficult physics concepts, or elements of persuasive writing" (Schmoker, 2011, n.p.).

"Instead of trying to 'reform' a school or system, we should be creating the conditions for teams of teachers to continuously achieve...short-term wins in specific instructional areas" (Schmoker, 2011, n.p.). Collaborative effort between teachers in modifying and improving lessons is the real path to school reform, "It is a rare school that has established regular times for teachers to create, test, and refine their lessons and strategies together" (Schmoker, 2011, n.p.). An instrumental part of developing good lessons is in setting and sharing goals. Education progress will come when "The wisdom of the profession becomes its standard practice" (Hiebert *et al.*, 2012, p. 95).

With all the focus on the quality of teachers and lessons, one could almost forget the student component and the accompanying literature on behavioral development. This paper is not the forum for examining behavioral development in any depth, but there are three developmental behaviors that appear to be relevant to an analysis of teaching strategies and so deserve some brief mention. These are self-efficacy, the student's ability to feel "safe" in the classroom, and Vygotsky's process of concept formation.

Self-efficacy is belief in oneself: "People fear and tend to avoid threatening situations they believe exceed their coping skills, whereas they get involved in activities and behave assuredly when they judge themselves capable of handling situations that would otherwise be intimidating" (Bandura, 1977, p. 194). For students, this means that "The strength of [students' beliefs] in their own effectiveness is likely to affect whether they will even try to cope with given situations" (Bandura, 1977, p. 192).

Turkish researchers Ozgen *et al.* (2011) use the term math literacy (ML) to describe "... competence in the mathematical content, process and situations faced" (p. 1085). Their research echoed the lessons of Bandera, "...self-efficacy beliefs affect the students' academic successes" (Chen, 2003; Pajares *et al.*, 1994; Usher, 2009 as cited by Ozgen *et al.*, 2011, p. 1086).

Students' self-efficacy plays a role in their confidence or lack of confidence to become involved in acquiring new activities and concepts in the classroom (Bandura, 1986; Schunk, 2009; Zimmerman, 2000 as cited by Ozgen *et al.*, 2011). However, teachers should also realize that self-efficacy beliefs vary from student to student, are not stable, and can be changed. Using the right educational strategies will allow teachers to affect their students' levels of self-efficacy (Ozgen *et al.*, 2011). A core curriculum that is rigorous, when joined with good teachers who

support students' learning, will assist in providing "identities of mathematical competence" for students (Horn, 2008). This identity is sometimes difficult:

...students suffer because of the image of mathematics – 'an arcane body of knowledge, containing immutable, eternal truths...can be discovered only by "gifted", "sagacious" [sic] minds. (Adda, 1989 as cited by Boaler, 1993, p. 18).

However, forming such an identity is critical for students who hope to perform well in mathematics (Horn, 2008).

The second element of behavioral development applicable to student learning is the recognition by educators that students must feel safe in order to function, and that learning will not take place without that (Wolff, 2006; Butler, 1998; Radin, 2009). "When strong feelings of anger, love, concern, fear, hate, excitement, jealously, sadness, etc. are active, our abilities to problem solve and think critically are diminished" (Gibbs, 1995, as cited by Butler, 1998, p. 37). "When a student feels threatened in any way, not even the most creative teaching will be stored in long- or short-term memory... For learning to occur, all real and perceived threats between students and between teacher and student, in every environment in the school...must be eliminated and replaced with a sense of trust" (Butler, 1998, pp. 37-39). It is important for teachers to remember how important emotion is to the process of learning,

...for survival purposes, our brains are hard-wired to pay attention to and remember those experiences with an emotional component...However, emotional responses can have the opposite effect if situations contain elements that a person perceives to be threatening... Under these conditions, emotion is dominant over cognition and the rational/thinking part of the brain is less efficient (Wolfe, 2006, p. 13).

One teacher (Bennett, 2010) was so concerned about creating a non-threatening atmosphere that he chose not to ask his students questions about the lessons in case they would be embarrased by not having the answer. He had to learn that he could achieve the same affect by not being as "judgmental" in order to ask those questions that were vital for checking understanding.

The third notion of behavioral development is that of Vygotsky's (1996) "process of concept formation". Specifically, concept formation applies to the idea that students learn by adding new ideas onto existing knowledge, or scaffolding. Scaffolding is also related to brain-based education (Boaler, 1993; Butler, 1998; Wilson, 1999; Otero, 2006; Wolfe, 2006, Ding, Reay, Lee, & Bao, 2009; McCosker & Diezmann, 2009). Some researchers are of the opinion that every teacher ought to have training in how the brain stores information, in order to be able to identify those strategies and lessons that will allow them to teach the way the brain works (Boaler, 1993, Otero, 2006; Ding *et al.*, 2009). The ability of a teacher to identify a student's existing knowledge and be able to use that information to design appropriate and effective instructional strategies is critical, and has "far-reaching implications" for students' development (Otero, 2006; Ding *et al.*, 2009). Vygotsky's theory of concept formation is a means of obtaining this important skill (Otero, 2006).

It is important to combine school based learning with that of the student's own experiences (Boaler, 1993). "School children recognize that school mathematics is not a part of the world outside of school" (Maier, 1991, as cited by Boaler, 1993, p.19). "Students are more likely to make meaning and gain understanding when they link new information to prior knowledge, relate facts to big ideas, ask essential questions, and apply learning in a new context" (Clarke *et al.*, 2008, n.p.). Increasing student achievement requires an active participation by the students in the learning process and multiple instructional opportunities in which to apply their learning to their own personal experiences outside of the classroom (Clarke *et al.*, 2008).

Effective instructional strategies are those that are research-based and there must be practical perspectives on their use (Marzano *et al.*, 2001). Effective strategies are defined as those practices which have a measurable effect on student achievement for all students in all

subjects across all grade levels (Marzano *et al.*, 2001). Marzano *et al.* (2001) have identified nine criteria, listed in the order of their effect (from highest to lowest): Identifying similarities and differences; summarizing and note taking; reinforcing effort and providing recognition; homework and practice; nonlinguistic representation; cooperative learning; setting objectives and providing feedback; generating and testing hypotheses; and cues, questions, and advance organizers.

Using effective strategies means making mathematics real to students. Class projects are excellent ways to connect the applicability of math skills to the real world. A Chattanooga, TN, furniture maker partnered with a local seventh-grade class to build math skills in an effort to "get them to focus". They decided to design and build a desk. The builder brought them rough lumber to show them from the beginning what would be involved. The students designed, measured for, and planned the desk. The furniture maker said that he approached the job just the same as he would for any of his professional clients. (Hardy, 2012).

Effective curriculum also demands that teachers address all three types of mathematical knowledge: conceptual, procedural and declarative (Miller, Stringfellow, Kaffar, Ferreira, & Manci, 2011). Lessons that address *conceptual knowledge* will deepen students' understanding of mathematical operations and the relationships and connections among those operations. *Procedural knowledge* will come as students gain the ability to solve mathematical problems step-by-step to ultimately achieve accurate solutions. *Declarative knowledge* includes the basic mathematical facts that support the procedural knowledge required to move on to higher level skills. Evidenced-based practices that support these skills include explicit instruction using manipulative devices (nonlinguistic representations, models) along with the mathematical computations being presented (Miller *et al.*, 2011). Nonlinguistic representation takes an

entirely new turn when Pythagoras, toga and all, strides into the classroom and begins declaring, "One: The universe is one. We are all part of its unity, from the grains of sand to the sun and moon, from the ants to the people to the monsters of the oceans" (Shirley, 2000, p. 652).

Dramatization as a form of nonlinguistic representation will make the presentation more interesting and informative...students should be encouraged to creatively act out the lives of the men and women of mathematics (Shirley, 2000).

Students must be active learners (Perels, Dignath, & Schmitz, 2009). With all of the information available and the emphasis on being life-long learners, the development of self-regulated learning should become an important aim of educators. Self-regulation involves goal setting, doing the activity required to achieve the goal, evaluating (self-reflection) the result of the effort, and drawing conclusions from the activity for further learning (*i.e.*, learning from mistakes) (Perels *et al.*, 2009). This type of inquiry-based activity is consistently cited as an important component of state standards for student learning: Inquiry-based learning is that learning in which "...students critically and systematically engage in examining, interpreting, and analyzing questions regarding the world around them, and then communicate their findings, providing convincing arguments for their conclusions" (Marshall, Lotter, Smart, & Sirbu, 2011, p. 307). In addition to actively involving students, inquiry-based or "constructivist" learning theories "favor instructional strategies such as inquiry over didactic instruction because... learning can only occur if students are given the time and the means to develop their own understandings" (Marshall *et al.*, 2011, p. 307).

The learning model that seems to hold promise for mathematics instruction is Kolb's learning model (Evans, *et al.*, 1998 as cited by Knisley, 2005). In the Kolb model a student's learning style is determined by whether the student prefers the concrete or is more comfortable

with abstract ideas, and if they are more comfortable with active experimentation over reflective observation. These learning styles translate into the mathematical learning styles of allegory, integration, analysis and synthesis (Knisley, 2005). *Allegorizers* seek to solve problems by looking for similar approaches in previous examples. *Integrators* are the students searching for examples in the text. *Analyzers* desire logical, step-by-step solutions, and *synthesizers* are the students who use what they have already learned to develop their own strategies to solving problems (Knisley, 2005). Knisley (2005) has observed that the styles of problem solving defined by Kolb's method appear to be the most commonly used, however, students have varied the style used based on the problem at hand. When students cannot apply one of the styles successfully, they tend to resort to heuristic reasoning, that is, in Knisley's (2005) opinion, "...knowledge without understanding... an arbitrary and unreliable approach to problem solving [that] must surely be responsible for much of the "math anxiety" that so often plagues students in introductory courses"(n.p.).

In the Kolb model, the roles for the teacher adjust to the teaching task at hand: Allegorization where the teacher is storyteller, integration with the teacher as guide, analysis where the teacher is expert, or synthesis with the teacher acting as coach. The Kolb model already makes an effective tool for many teachers who may not be aware that they are even using the model. For example, when a hypotenuse is measured with a ruler to corroborate the use of the Pythagorean theorem, the result is a student who will see "the measurements and the theorem produce the same result...[and] will use the theorem independent of any measurements" (Knisley, 2005, n.p.).

Summary

In conclusion, good teachers and good lesson planning appear to be critical to educational endeavors. While good teachers are desired, it can be difficult to identify good teaching through observation. There may be value in paying attention to the curriculum and its improvement instead of just trying to find good teachers. While many teachers often know intuitively what works and what does not, the best strategies should be backed by research. Teachers should constantly seek to improve their lessons and determine what works through reflection, sharing with other teachers, and continual modification. Lesson plans should actively involve students, scaffolding upon the students' personal experiences and knowledge and provoking students to question and examine results. While there is no magic bullet, no instructional strategy that works equally well in all situations, it is possible that there are some instructional strategies that, used consistently, may be more effective than others at communicating certain math concepts.

CHAPTER THREE

METHODS

The purpose of this chapter is to outline the methods that were used to collect data to address the questions identified in *Chapter One*. To study the questions, thirty math teachers from three rural counties in two states were asked to rate their preferences for teaching a specific math concept – the Pythagorean Theorem. Six research-based standards were identified for the teachers to rate, and a Likert scale of 1 to 7 was used to indicate least to most effective for each standard. Additionally, the survey was split into two sections. The teachers were asked to rate their preferences separately: High achieving (HA) students (those making a B or better), and low achieving (LA) students (C or below). The teachers' demographic information was collected to determine how selected teacher demographics might impact those preferences. At the conclusion of this chapter, the writer will offer a brief summary of her thoughts on these methods.

Survey

Data collection was done via a written survey (see *Appendix B*). The survey was designed to determine mathematics teachers' preferences with respect to strategies for teaching the Pythagorean Theorem to both HA and LA high school students. The surveys were anonymous and were distributed by unrelated third parties to control bias.

To determine preferential strategies, i.e., how students are currently being taught to "skin" this particular "cat", teachers were asked to rate their preferences of six teaching strategies for two categories: HA students (B or better) and LA students (C or below). Teachers used a Likert scale of 1 to 7, "less effective" to "most effective" to rate their preferences.

Of the six strategies, five were developed from a list of strategies identified by Marzano *et al.* (2001) as effective based on research and best practice and one was included because of the many studies showing a strong connection to scaffolding and the increasing availability of information on how brains learn (Wolfe, 2006; Radin, 2009). The first five strategies chosen were Lecture, Memorization, Questioning, Cooperative Learning, and Nonlinguistic (manipulatives). The sixth strategy was Brain Compatible (Radin, 2009). Teacher demographics – years of experience, current grade level, highest level of education, undergraduate major, and Teacher of the Year awards – were collected to determine whether those demographics would impact teacher preferences.

Subjects/Population

To create a sufficiently diverse sample choice, a range of math teachers from multiple schools within three districts – one outside of the state and two within the state – were chosen for the sampling. Since the originally observed phenomena were in schools with a primarily rural student population, the teachers surveyed were also chosen from school districts with primarily rural populations. The three school districts where the surveys were distributed were the Fannin County School District, in Georgia; and the Polk County School District and the Bradley County School District in Tennessee. There were a total of thirty responses, with approximately one third of the responses coming from each of the three school systems. All responses were voluntary.

The demographic information indicated a diversified selection of survey respondents. Out of thirty respondents, eight were males and twenty-two were females, with one-third of those being recipients of Teacher of the Year. While all respondents were teachers at the secondary level, there was a wide range of teaching experience – three years or less to more than 26 years. Education levels ranged from bachelor to doctorate, and undergraduate majors were varied; about two-thirds of the respondents had earned their undergraduate degree in mathematics.

Summary

In conclusion, the primary methodology was a teacher survey, which offered a list of six strategies for teaching the Pythagorean Theorem. Secondary math teachers from rural school districts were selected to participate in the survey and were asked to identify their preferred strategies for teaching the identified standard to HA and LA students. Demographic information was obtained for each survey participant to determine how those demographics impacted the study results. The respondents' answers were collected and analyzed to determine the answers to the questions posed in *Chapter One*. The results of the survey appear in *Chapter Four*. A discussion by this researcher of the findings and her conclusions are presented in *Chapter Five*.

CHAPTER FOUR

RESULTS

The purpose of this chapter is to present the results of the methods described in the previous chapter. These results will be organized around the two fundamental questions addressed in *Chapter One*: What teaching strategies are preferred by math teachers for teaching math concepts? Do teacher demographics influence those teacher preferences?

The teaching strategies considered preferential for teaching math concepts and the influence of teacher demographics on those preferences will be explored in the following two sections: *Teacher Preferences for Teaching Math Concepts* and *Demographic Effects on Teacher Preferences*. In each case, a graph of the results has been included to facilitate a discussion of the major findings in the data. At the conclusion of this chapter, the writer will offer a brief summary of her general observations relative to the two questions cited above.

Question One: Teacher Preferences

Teachers surveyed were in agreement that Questioning strategies were the main preference for teaching the Pythagorean Theorem to high achieving students (see *Figure 1*).

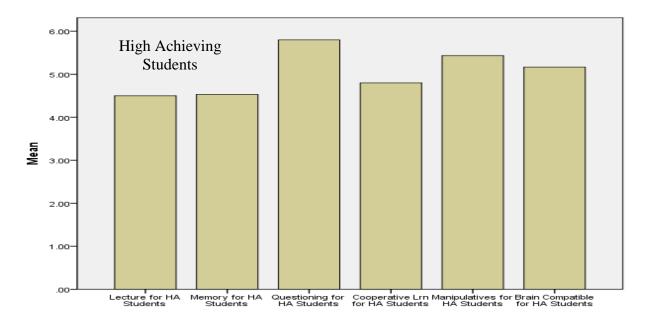


Figure 1

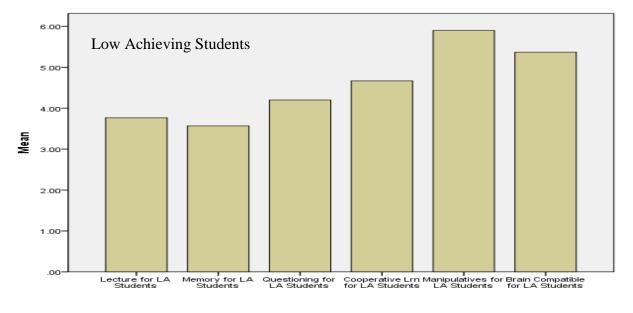


Figure 2

The teachers surveyed were also in agreement as to the best strategy for teaching low achieving students, but the preference was for Manipulatives (Nonlinguistic) (see *Figure 2*). This discernible difference (p<05; df=28) suggests that there is possibly a difference in the way teachers see and teach students of differing abilities. The idea that certain strategies are more

effective with different students, instead of the idea that one strategy is an effective way to teach a specific topic, could make a difference in the strategies teachers choose when presenting specific math topics to their students. One side note of these findings was also interesting:

While Questioning was considered the most preferred strategy for HA students, it was less preferred, over half way down the list, as a strategy for LA students. This finding was shared overall among the teachers responding.

The differences in teachers' preferences of Questioning vs. Nonlinguistic (manipulatives) strategies between the two groups of students were specific to teaching the Pythagorean Theorem. It is possible that the results could be due to that specificity. A similar survey for a different mathematics concept might possibly result in a different set of choices on the part of the survey respondents.

Question Two: Differences with Respect to Demographics

The Summary Table of Means by Category (*Table 1*, p. 24) has been highlighted to illustrate differences in teacher preferences across demographic groups. Demographics did seem to influence teacher preferences, mainly for HA students, although there were some exceptions for two categories with respect to teaching LA students.

Data from the surveys were tabulated and analyzed with the Mann-Whitney U test, also known as the Mann-Whitney-Wilcoxon (MWW) or Wilcoxon rank-sum test instead of the normally used parametric T score. The MWW is a non-parametric test that is valid for assessing whether one of two samples of independent observations tends to have larger values than the other (when used for equal sample sizes) and is specific to comparing sample pairs. It is considered to be more "robust" than the parametric T score when used with small samples of independent observations, especially when the levels of data are nominal, as in this survey.

	Category	Effectiveness Rating (1-7) of Instructional Strategies for HIGH ABILITY Students					Effectiveness Rating (1-7) of Instructional Strategies for LOW ABILITY Students						
Demographic		Lecture	Memorization	Questioning	Cooperative Learning	Manipulatives (Non- Linguistic)	Brain Compatible	Lecture	Memorization	Questioning	Cooperative Learning	Manipulatives (Non- Linguistic)	Brain Compatible
	Male	3.63	3.88	5.38	4.13	4.63	4.88	3.38	3.13	4.00	4.88	5.25	5.00
Gender	(n=8) Female												
C 5 u .c.	(n=22)	4.82	4.77	5.95	5.05	5.73	5.27	3.91	3.73	4.27	4.59	6.14	5.50
	(n=30)	4.50	4.53	5.80	4.80	5.43	5.17	3.77	3.57	4.20	4.67	5.90	5.37
Teaching	1-3 yrs (n=8) 4-15 yrs	4.50	5.00	5.88	5.13	5.13	5.00	3.88	3.00	4.13	4.63	5.50	5.00
Experience	(n=11) 16+ yrs	4.73	4.64	5.64	5.00	5.73	4.45	4.00	4.18	3.91	4.91	5.91	5.50
	(n=16)	4.27	4.09	5.91	4.36	5.43	6.00	3.45	3.36	4.55	4.45	6.18	5.37
	BA (n=8)	4.78	5.00	5.78	5.81	4.78	5.50	4.25	3.20	4.00	5.30	5.40	5.80
Education	MA (n=16)	4.50	4.50	5.78	4.85	5.67	5.67	3.60	3.70	4.00	4.74	5.90	4.90
Level	EDS + (n=6)	4.25	4.00	5.75	4.00	5.67	6.25	3.50	3.60	5.00	4.00	6.40	5.9
Undergraduate	Math (n=18)	4.40	4.50	6.00	5.00	5.50	5.25	3.98	3.50	4.75	4.70	6.00	5.2
Major	Other (n=12)	4.60	4.50	5.50	4.50	5.25	5.33	3.50	3.45	3.50	4.75	5.60	5.5
	1	ı				1		1	1	1	Т	Т	ı
Teacher of the	Yes (n=9)	4.11	3.78	5.78	4.78	5.67	5.44	3.22	3.67	4.78	4.56	6.22	5.3
Year	No (n=21)	4.67	4.86	5.81	4.81	5.33	5.05	4.00	3.52	3.95	4.71	5.76	5.3

The second analysis of variance used is the Kruskal-Wallis (KW) one way analysis of variance by ranks. Also a non-parametric method, it is useful for comparing more than two samples that are independent or not related. The KW assumes an identically-shaped and scaled

distribution (as parametric tests) for each group, except for any difference in medians. When the KW leads to significant results, as in the sampling for this survey, then the assurance is that at least one of the samples is different from the other samples. While this test does not identify where the differences occur, or how many differences actually occur, it is useful for extending the MWW test to three or more groups.

The Friedman test (FT), another non-parametrical statistical model could have been used, as it is valuable and similar to the KW, but was not available to this researcher at this time. It might be useful when attempting to replicate the data found here to use the FT as an alternate assessment method of comparing the current data to any subsequent data.

As a result of this study, significant differences were noted in several areas. Differences between men and women were most significant (p<.01; df=28) regarding preferences with respect to Lecture (see Figure 3) and were less significant (p<.1; df=28) but worth noting with regard to Memorization (see Figure 4). These differences will be discussed in the section titled Difference with Respect to Gender. There were significant differences (p<.05; df=28) with respect to Brain Compatible preferences between teachers with more experience and education over those who did not as much experience and/or education (see Figure 5). In addition, those teachers who had undergraduate degrees in mathematics had differences in preferences with regard to Questioning for LA students that differed significantly (p<.05; df=28) from those teachers whose undergraduate degrees were not in mathematics (see Figure 6). These differences will be covered in the section titled Differences with Respect to Education, Teaching Experience, and Undergraduate Studies. Interestingly, Teacher of the Year recipients' strategy preferences used with HA students in regard to memorization strategies (low effectiveness) and for LA students with regard to Questioning strategies (higher effectiveness) were significantly at

odds (p<.05; df=28) with those of teachers who had not been awarded Teacher of the Year. Those differences will be discussed in the section titled *Difference with Respect to Teacher of the Year*.

With Respect to Gender

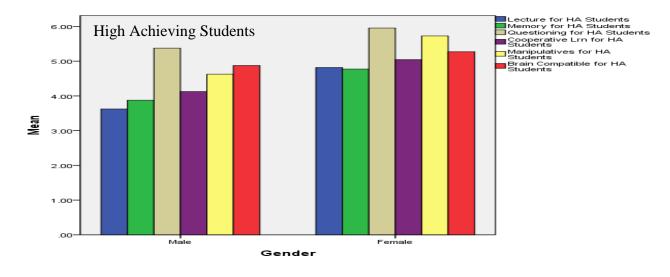


Figure 3

When evaluating for gender, it appears there were differences in preferences for Nonlinguistic (manipulatives) and Lecture (p<.01, df=28) when teaching HA students. Males appeared to rate Nonlinguistic strategies as the third preferential strategy and Lecture at the bottom of their strategy preferences for teaching the Pythagorean Theorem. Females, on the other hand, gave Nonlinguistic strategies a high second place as the preferred teaching strategy, and considered Memory as the least preferred strategy for working with HA students on the Pythagorean Theorem. For females, Lecture was preferred to Memory as a strategy for teaching the Pythagorean Theorem to HA students.

The difference from males to females was the most significant difference in the study and may simply be a difference in the way males and females communicate. It is interesting to note that men dominated the teaching profession for many centuries, and lecture was the primary

method of instruction during those years. It may be that the results reflect some cultural change in the male teachers' opinions of the value of lecture as a teaching method and may be an appropriate subject for further study.

It is of note that both males and females indicated that Questioning was the preferred strategy for HA students, and the two groups only began to show significant differences with respect to strategies that were considered less effective, that is, the lesser preferred strategies. This may indicate that these differences, while significant, might not impact the teaching of the specific concept as strongly as they would have had this been a difference in the most preferred strategy. As a result, these differences may not be as critical as some other results in this study when considering further research.

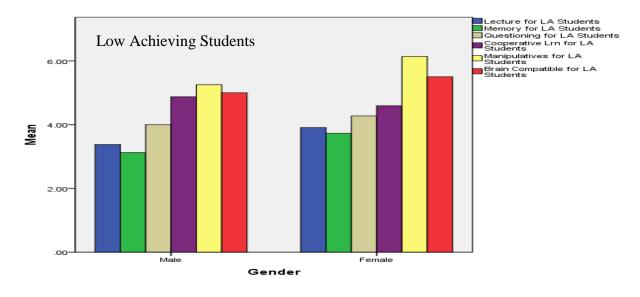


Figure 4

Analyzing for gender did not produce any difference in the preferences of teachers with respect to LA students (See *Figure 4*). The level of preference differed slightly between males and females, but overall, the order of preference remained the same for both groups:

Nonlinguistic (manipulatives) remained the preferred strategy, with Memorization the least preferred.

With Respect to Education, Experience, and Undergraduate Discipline

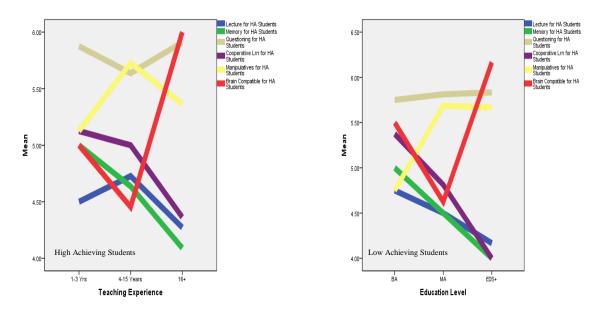


Figure 5

Evaluation of preferences based on teacher experience indicated significant differences (*p*<.05; *df*=28) with respect to Brain Compatible strategies (*Figure 5: Teaching Experience*). The teachers with more experience preferred Brain Compatible over all of the other strategies listed for teaching the Pythagorean Theorem to HA students. Teachers with 4-15 years of experience rated Brain Compatible strategies at the bottom – least preferred – while teachers with 1-3 years of experience rated Brain Compatible strategies next to last. It may be that those teachers with more teaching experience could have intuitively noticed more about how children learn, and since what they have observed is in line with what is now being learned about Brain Compatible strategies, they prefer Brain Compatible related strategies. Those teachers with less experience may have been exposed to Brain Compatible theories, but while they see them as preferable, they lack the experience to identify them as the most preferable, or the teachers surveyed may not have been familiar with research regarding Brain Compatible strategies.

When considering the opinions of respondents by educational level, there was also a significant difference (*p*<.05; *df*=28) regarding the opinion of effectiveness of Brain Compatible strategies for HA students (*Figure 5: Education Level*). Respondents who had earned doctoral degrees or above considered Brain Compatible to be the most preferred strategy for HA students. Respondents who had earned their master's degree considered Brain Compatible strategies to be fourth on their list of the most preferred strategy, while those respondents who had not progressed beyond a bachelor's degree placed brain-compatible learning second behind Questioning as the preferred strategy for teaching the concept of the Pythagorean Theorem to HA students.

The difference of preferences with respect to teachers' education levels was quite unexpected. Those teachers who have earned doctoral degrees and those teachers who have earned just their bachelor degrees appear to share, to some degree, a preference for Brain Compatible strategies. It could be that those teachers who have earned doctoral degrees have a greater understanding of how the brain works, and have identified Brain Compatible strategies as preferred strategies for student learning. The teachers who have not progressed beyond bachelor's degrees may be newer to the teaching profession, and thus could have more information about Brain Compatible strategies because it is a more recent addition to the teaching arsenal (Wolfe, 2006). Another possibility is, again, a lack of information on the part of teachers with master's and bachelor's degrees about the Brain Compatible strategies. More, specific questions about the reasons for preferring Brain Compatible strategies would need to be gathered before a definite understanding of why these two diverse groups appear to be in alignment with respect to this strategy.

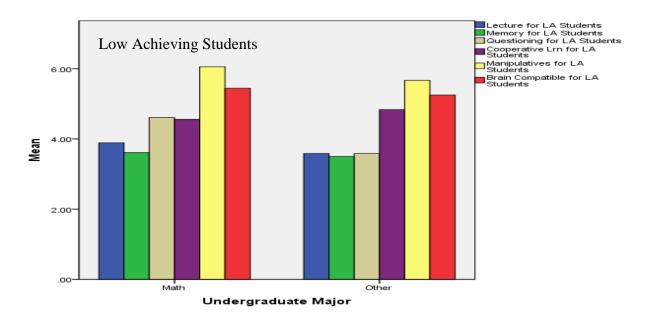


Figure 6

Up to this point, the focus on significant differences has rested with HA students. However, with regard to LA students, there was a significant difference (p<.05; df=28) noted when the respondents' opinions were grouped by undergraduate major (Figure 6). While Questioning was considered the most preferred strategy for teaching the concept of the Pythagorean theorem to HA students, and Nonlinguistic (manipulatives) was considered to be the most preferred strategy for LA students, those teachers with an undergraduate degree in mathematics rated Questioning for LA students much higher (third overall) than those without a mathematics based undergraduate degree (Questioning ranked fourth). Since Questioning has such strong support for initiating higher level thinking in students (Hughes, 1974; Butler, 1998; Marzano et al., 2001; Wolfe, 2006; Bellido, Ramos, & Wayland, 2007; Clarke et al., 2008; McCosker et al., 2009; Bennett, 2010; Schneider, Rittle-Johnson, & Star, 2011), it would seem that more study might need to be done to understand why math teachers apparently feel this strategy deserves preference, and teachers without mathematics undergranduate degrees apparently do not feel it is as effective with respect to teaching LA students.

With Respect to Teacher of the Year

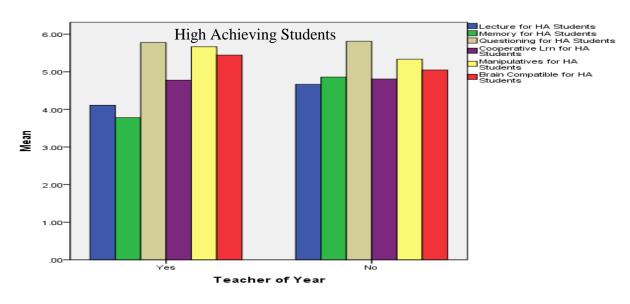
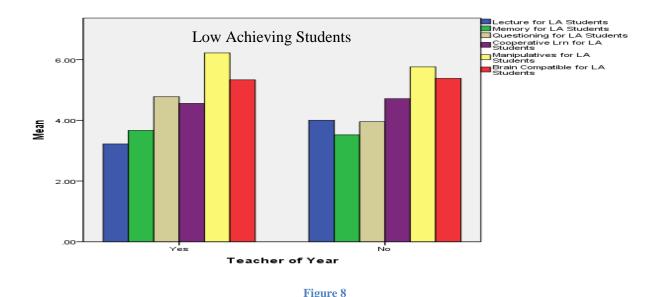


Figure 7

The preferences of those teachers who had been awarded Teacher of the Year showed a significant difference (p<.05; df=28) with respect to the preference for Memorization (Figure 7). The use of Memorization for teaching the Pythagorean Theorem to HA students was the least preferred strategy of this group. Those who had not been awarded Teacher of the Year rated Lecture as the least preferred strategy for the same student category. While these differences deal with the lesser preferred strategies, it is interesting that Lecture would be seen to possibly have more value for HA students than simple Memorization. It would be interesting to see further studies on how Teacher of the Year recipients use Lecture with respect to teaching HA students.



Teacher of the Year recipients also indicated a greater preference for Questioning strategies with LA students (*Figure 7*). Questioning ranked third in effectiveness for Teacher of the Year recipients when teaching LA students, while those who had not been awarded Teacher of the Year rated it slightly below Lecture for the same group. When considering best practice and the abilities of teachers who have been awarded Teacher of the Year, perhaps there should be some attention paid to this preference for LA students, since the value of questioning with respect to higher level thinking as been clearly established (Bellido, Ramos, & Wayland, 2007).

Summary

In conclusion, as a group, secondary math teachers appeared to share a preference for Questioning as an effective strategy with regards to teaching the Pythagorean Theorem to HA students. There also appeared to be agreement among the general population of survey respondents showing a preference for Nonlinguistic (manipulatives) strategies when teaching the concept of the Pythagorean Theorem to LA high school students.

Within categories by demographics, with regard to LA students, there was a similar common preference of Nonlinguistic (manipulatives) as the strategy considered most effective.

The reasons for such agreement may well be that teachers, experiences with students of differing abilities may direct their choices regarding the difference in strategy preferences for those groups. Alternatively, teachers may see students differently with respect to grades, which may cause them to consider different strategies with different students

The reasons for the significant differences in results when evaluated by demographics are at best a guess without more information. With regard to gender differences, males and females do communicate differently, which might have been reflected in their preference ratings.

Experienced teachers and those with more education may just be more experienced with how students learn, and may have developed a preference for Brain Compatible strategies as a result of that experience. Teacher of the Year recipients may be more adept at using best practice strategies, and are possibly more able to identify those strategies that are more and less preferable in specific teaching situations. It is hoped that further studies could help to identify reasons for the agreement and disagreement within groups, and might possibly help to determine if the difference between the groups is truly due to teacher experience or some other factor that was not controlled for in the study. Additionally such research may be valuable in raising new study and research possibilities that could help to shed some light on the pervasive problem of reteaching in the mathematics classroom.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

The purpose of this chapter is to reflect on each aspect of this investigation and draw reasoned conclusions from the data. Based on these conclusions, the writer will make what appear to be appropriate recommendations that address each conclusion. Briefly, this chapter will include the writer's thoughts on the problem as originally identified in *Chapter One*; the coverage of the professional literature in *Chapter Two*; the methods that generated the data on which these conclusions are based; alternative explanations for the findings in *Chapter Four*; and finally a conclusion and recommendation on the overall issue at hand.

Regarding the problem of reteaching, the writer concluded that it was an extremely broad and complex problem. The questions that were asked would only assist in defining a small portion of one issue that it was hoped might help shed some light on the larger problem. To further address the larger issue of reteaching, the writer recommends that subsequent investigations might help to identify preferred strategies for other high school mathematics concepts. This would help to develop a collection of information on how students are being taught mathematics. It is hoped that more information on effective and preferred strategies from mathematics teachers might possibly shed more light on the question of reteaching.

With respect to the problem presented in *Chapter One*, this study sought to answer two questions: What teaching strategies are considered preferential by math teachers for teaching math concepts? Do teacher demographics influence those teacher preferences? By way of an answer, this writer has presented the results of a survey of math teacher preferences for strategies used to teach the Pythagorean Theorem to HA and LA students at the secondary level. It is hoped that this effort has added new and relevant information to the literature regarding teacher preferences. There are a number of ways in which future studies of this material could be improved upon, from the makeup of the survey materials to the selection of the population of teachers taking the survey.

In this writer's opinion, the professional literature was heavy in material on general learning methods, strategies, best practices, and innovative ideas, as well as lesson plans specific to disciplines. This writer found, however, a slight deficiency in articles that discuss effective math strategies at the secondary school level with regard to specific mathematics concepts. To address this concern, the writer recommends that more research be done to determine those strategies that are currently considered preferential by secondary teachers in the other mathematics specific concepts not covered in this study.

Upon reflection, the writer has concluded that the methods used to generate the data in this study may possibly be flawed. The sample of teachers selected, while appropriate to simulate the existing population, was from a rural population, and might not appropriately reflect preferences in a larger urban community. Consequently, the writer would recommend that this issue be addressed by expanding and replicating the survey in non-rural school districts, with a larger population of teachers. In addition, the writer concluded the survey instrument could have included a more complete list of teaching strategies, to address the possibility that teachers

surveyed were using strategies that were not reflected on the survey instrument. This writer recommends that subsequent investigations also consider including specific definitions for each strategy and consider including an opportunity on the survey for respondents to list preferred strategies that were not reflected in the original survey choices.

While there were many interesting findings as a result of this study, there are several findings that stand out to this researcher. One is the differences in the preferences for effective instruction between HA and LA students. Overall, teachers preferred Questioning for HA students when teaching the Pythagorean Theorem. This would seem to make sense, since the ability to ask questions has been identified as an important factor in pushing students to engage in higher-level thinking and to be able to understand concepts in a meaningful way (Hughes, 1974; Butler, 1998; Marzano et al., 2001; Wolfe, 2006; Bellido, Ramos, & Wayland, 2007; Clarke et al., 2008; McCosker et al., 2009; Bennett, 2010; Schneider, Rittle-Johnson, & Star, 2011). Making time in the class period for good questioning strategies, i.e., questions that can be answered with more than just a yes or no, or questions that require the student to explain, clarify, or defend his or her answer, may possibly deserve a second look from educators hoping to improve students' understanding of the material. With respect to LA students, the survey respondents were in agreement that Nonlinguistics (manipulatives) was the preferential strategy for LA students. If Questioning is so powerful, why isn't it the preferred strategy to assist LA students in improving their higher level thinking skills? More study may be necessary to explain why the survey respondents indicated a difference in strategies with respect to teaching the two achievement groups. Also, using the Pythagorean Theorem may have been sufficiently narrow in scope, and identifying preferences for a different strategy might have resulted in different results.

The second item of interest was the importance given to Brain Compatible strategies. Why did the teachers with more experience and/or higher educational achievement exhibit a significant difference in their preference for this strategy? Similar to this was the finding of the difference in preferences between Teacher of the Year recipients and those who had not received such an award. Teacher of the Year recipients are generally honored by their colleagues for the excellence and skills that they bring to their teaching experience. Teacher of the Year recipients were in agreement that memorization, defined on the survey as "presentation/ memorization of formulas, theorems; homework" was the least effective strategy. Knisley (2005) described it as the least challenging, heuristic form of teaching. Having students memorize without understanding and then practice the same problems over and over does not appear to be an effective method of teaching mathematics. It may be that Teacher of the Year recipients understand this and may possibly use such information to inform their strategy preferences.

More research may be needed in evaluating the preferences of experienced teachers with regard to what does and does not work in the secondary classroom.

This study is by no means conclusive, and in fact, has raised some new questions that the writer hopes will be answered by further study: Why is Questioning, a strategy that is considered so effective in education, not considered preferential for teaching LA students by the general population of educators surveyed, and yet it is given a higher preference for that group by Teacher of the Year recipients? Why do educators with more teaching experience and higher levels of education prefer Brain Compatible strategies for teaching specific math concepts? What do these experienced and qualified teachers possibly know that the general population may not?

While no one strategy has been proven effective for all classroom situations, it was of interest to this researcher that survey respondents were generally in agreement with regard to the most effective ways to "skin the Pythagorean cat." This researcher hopes that further research can be done to answer the new questions that were posed as a result of this study and to somehow help students to recognize and utilize all the math skills at their disposal no matter in what shape the "cat" may appear.

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WORKS CITED

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APPENDICES

APPENDIX A A PARTIAL LISTING OF TENNESSEE STATE AND CORE STANDARDS INVOLVINGTHE PYTHAGOREAN THEOREM

APPENDIX A A PARTIAL LISTING OF TENNESSEE STATE AND CORE STANDARDS INVOLVING THE PYTHAGOREAN THEOREM

Algebra I

Assess

CLE 3102.4.1, 3102.4.2 Use the Pythagorean Theorem to find the missing measure in a right triangle including those from contextual situations.

CLE 3102.4.3 Understand horizontal/vertical distance in a coordinate system as absolute value of the difference between coordinates; develop the distance formula for a coordinate plane using the Pythagorean Theorem

SPI 3102.4.2 Solve contextual problems using the Pythagorean Theorem

Algebra II

Assess

CLE 3103.4.5, 3103.4.6 Know and be able to use the fundamental trigonometric identities, including the Pythagorean identities, reciprocal identities, sum of sine and cosine, and odd and even identities.

SPI 3103.4.3 Describe and articulate the characteristics and parameters of parent trigonometric functions to solve contextual problems.

Geometry

Assess

CLE 3108.4.10 Develop the tools of right triangle trigonometry in the contextual applications, including the Pythagorean Theorem, Law of Sines and Law of Cosines

CLE 3108.4.20 Prove key basic theorems in geometry (i.e., Pythagorean Theorem, the sum of the angles of a triangle is 180 degrees, characteristics of quadrilaterals, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length)

CLE 3108.4.42 Use geometric mean to solve problems involving relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.

CLE 3108.4.43 Apply the Pythagorean Theorem and its converse to triangles to solve mathematical and contextual problems in two- or three-dimensional situations.

CLE 3108.4.44 Identify and use Pythagorean triples in right triangles to find lengths of an unknown side in two- or three-dimensional situations.

CLE 3108.4.45 Use the converse of the Pythagorean Theorem to classify a triangle by its angles (right, acute, or obtuse)

SPI 3108.4.14 Use properties of right triangles to solve problems (such as involving the relationship formed when the altitude to the hypotenuse of a right triangle is drawn)

APPENDIX B SURVEY DOCUMENT

What Works in Math Education

Please provide the following information about yourself: Gender: Male Female	Years of Teaching Experience: 1- 3	4-15 16-25
26+		
Current Grade Level Taught: 6-8; 9-1	2	
Highest Level of Education: Bachelors	; Masters; Ed. Spec;	Doctorate
Undergraduate Major:		
Have you ever been a "Teacher of the Year" (b	ouilding level or higher)? Yes No	

DIRECTIONS: As a beginning math teacher, I value your opinion. Please **RATE the effectiveness** from **1** (*Less Effective*) **to 7** (*Most Effective*) of each instructional strategy by **CIRCLING one number in each line** *for both Higher Ability* (B's or better in Math) and *Lower Ability* (C's or below in Math) math students.

Math Standard: CLE 3108.4.43: Apply the Pythagorean Theorem and its converse to triangles to solve mathematical and contextual problems in two or three dimensional situations. Student Age: 16

HIGHER ability math students (generally B's or better)	INSTRUCTIONAL STRATEGY	EFFECTIVENESS						
		Less Effe	ctive		Effective			Most
	LECTURE: note taking, structured vocabulary,	1	2	3	4	5	6	7
	MEMORIZATION: presentation/memorization of formulas, theorems; homework	1	2	3	4	5	6	7
	QUESTIONING: question/pause/encourage higher level thinking responses	1	2	3	4	5	6	7
	COOPERATIVE LEARNING: student grouping, pair and share, team projects	1	2	3	4	5	6	7
	NON-LINGUISTIC: manipulatives, pictures, diagrams, physical representations,	1	2	3	4	5	6	7
	BRAIN COMPATIBLE: 15-20 min. instruction "chunks," walk and talk learning	1	2	3	4	5	6	7

	INSTRUCTIONAL STRATEGY	EFFECTIVENESS						
LOWER ability math students (generally C's or below)		Less Effe	ctive		=			Most
		Effective						
	LECTURE: note taking, structured vocabulary,	1	2	3	4	5	6	7
	MEMORIZATION: presentation/memorization of formulas, theorems; homework	1	2	3	4	5	6	7
	QUESTIONING: question/pause/encourage higher level thinking responses	1	2	3	4	5	6	7
	COOPERATIVE LEARNING: student grouping, pair and share, team projects	1	2	3	4	5	6	7
	NON-LINGUISTIC: manipulatives, pictures, diagrams, physical representations,	1	2	3	4	5	6	7
	BRAIN COMPATIBLE: 15-20 min. instruction "chunks," walk and talk learning	1	2	3	4	5	6	7