Rocks: A Concrete Activity that Introduces Normal Distribution, Sampling Error, Central Limit Theorem and True Score Theory.

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Abstract

This report introduces a short, hands-on activity that addresses a key challenge in teaching quantitative methods to students who lack confidence or experience with statistical analysis. Used near the beginning of the course, this activity helps students develop an intuitive insight regarding a number of abstract concepts which are key to mastering introductory statistics. Students are introduced to true score theory, Poisson distribution, central limit theorem and directly observe the effects of sampling error. Throughout the course concepts can be related back to this experience and made less abstract and more concrete meeting the needs of a broader range of learners.

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As is often noted in the literature, methods faculty regularly face challenges in teaching elementary statistical concepts in part because they are “quintessentially abstract” (Watts, 1991). According to Chase (2000) a key aspect of statistical thinking involves understanding the abstract concepts and distinguishing them from the numerical manipulation common to many statistics courses. Watts (1991) argues that while the theory behind statistics is abstract, concrete activities in which students are actively engaged in the collection, representation, and analysis of data can help students connect the abstract concepts with real experience, thus reducing anxiety and increasing understanding. Gnanadesikan, et al. (1997, abstract) agree writing “So that students can acquire a conceptual understanding of basic statistical concepts, the orientation of the introductory statistics course must change from a lecture-and-listen format to one that engages students in active learning.” Other authors go further, arguing that hands-on activities are *necessary* for some students to gain an intuitive understanding of the underlying statistical concepts (e.g., Fisher-Giorlando, 1992). This conclusion rests in part on the fact that hands-on activities meet the needs of a broader range of student learning styles, incorporating visual, auditory, and kinesthetic activities (Meyers and Jones, 1993). However, not all hands-on activities are equal as Fisher-Giorlando (1992, p. 285) writes:

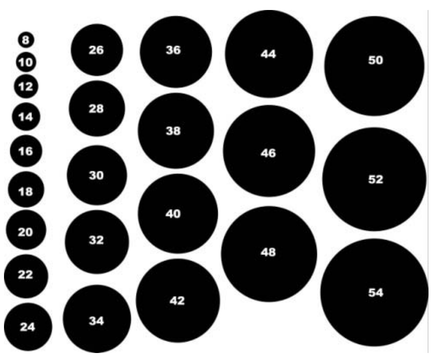
> “most of these hands-on techniques involve either using the computer or actually manipulating or collecting data in the field…. many of these students find empirical demonstrations using pseudorandom numbers unrealistic and intellectually intangible.…more concrete analogous techniques are needed for teaching research concepts in which students and teachers can manipulate objects physically.”

Gnanadesikan, et al. (1997, 10) “master teachers and great statisticians had success with hands-on activities that got people involved with statistical principles. What they observed has been borne out by recent research into the learning of statistics: activities work.” To better meet the needs of students in an introductory statistics course I created the hands-on activity described below to introduce students to the normal distribution, true score theory, sampling error and the central limit theorem.

This lesson is completed early in the term and is not concerned with mastery. The purpose is to provide some intuitive insight into concepts that will be discussed later in the course. To build a scaffold, however tentative, on which to hang ideas later in the course. In less than an hour, students are introduced to data collection and measurement error. Using their measurements, students learn how a curve develops, and experience the effects of sampling error. Throughout the course concepts can be related back to this experience and made less abstract and more concrete meeting the needs of a broader range of learners.

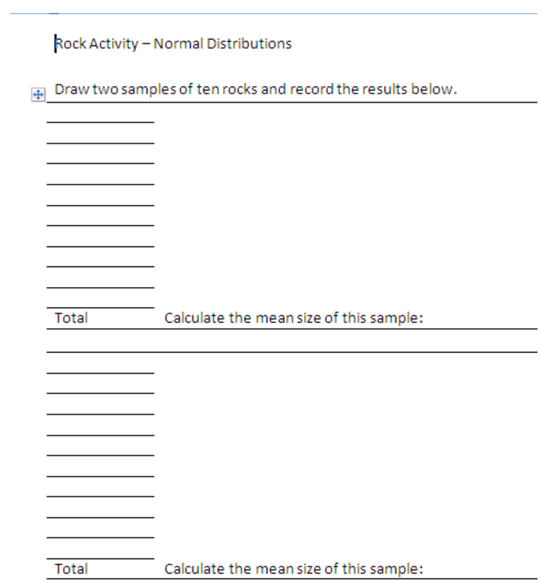
The Lesson: Rocks

To set up the lesson, obtain two gallons of medium size gravel from a landscaping supply center. This gravel provides an excellent medium for sampling and for demonstrating the normal distribution of natural elements. Two worksheets are required to measure and record the samples. The first handout is a measuring tool using circles to measure the rocks at their longest point (dia. 1).



The measuring tool features a series of circles starting with a diameter of 8 mm with each subsequent circle increased in size by two millimeters to a maximum of 54 mm. The advantages of using the circles to measure the rocks are that it is cheap, quick and easy, and allows a sufficient degree of accuracy in measuring each rock at its longest point. This is also a good opportunity to remind students of the "True-Score Theory" (all measurements consist of a true score $\pm$ error) as the circles increase by two millimeters and the rocks are irregular, both leading to estimates that contain a degree of inaccuracy or error. To reinforce this aspect of the lesson, select a rock whose length falls between two circles and pass it around for students to measure. Once a number of students have measured the rock, have them report the measurements and discuss the ways in which we use measuring tools and the implicit or explicit rules we apply when ambiguity occurs (e.g., go to the next highest/lowest number). At the end of the lesson students will be prepared to learn more about the True Score Theory's distribution of error.

Using physical objects rather than the "pseudorandom" objects in textbook exercises, or computer simulations provides a better reflection of the real work using statistics. Real data is messy. For example, respondents pay more or less attention to their survey ratings, operational definitions often involve some judgment, inter-rater reliability in observational data is rarely 100%. If instead of giving students exact measurements in textbooks and simulations, they are required to measure and assign the values, they get a fairly realistic experience involving both actual measurements and one way error is introduced. This approach is more analogous to actual measurement in the field than textbook exercises which offer arbitrary figures that reinforce student's misperception regarding the meaning/certainty of data. It also allows the student the opportunity to generate data as opposed to simply copying numbers from the text or having a random number generator fill in the data on the computer. This results in greater ownership and engagement.



Rock Activity – Normal Distributions

Draw two samples of ten rocks and record the results below.

Total Calculate the mean size of this sample:

Total Calculate the mean size of this sample:

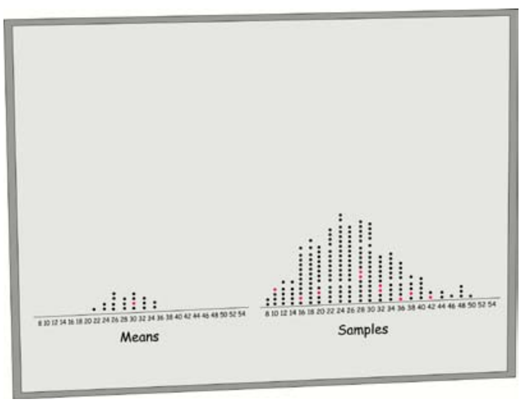
In addition to the measuring tool, each student also needs a simple worksheet that provides a way to record the length of each rock and then to calculate the mean length for the sample (dia. 2). Depending on the number of students in the class, each student can record one or more samples with the handout modified appropriately. Approximately 150-250 samples are best. Using samples of ten simplifies the mean calculations. Each student should also have a paper cup in which they can collect their sample.

The Lesson

Before beginning the lesson, the instructor introduces the sampling process explaining that many common things from potato chips to concrete aggregate require sampling to ensure that the overall batch meets specifications. Two horizontal number lines are then drawn on the white/chalkboard side by side. Both lines are marked off in two millimeter increments to match the measurements of the circle tool. To begin the activity, the two gallons of rocks are poured into a five gallon bucket. The bucket is shaken to thoroughly mix the rocks. (It is generally a good idea to wash the rocks to avoid a dust cloud when mixing the rocks.) The students take a handful of rocks and count out ten into their cup.

The students measure their rocks individually and then calculate the mean for the sample. The individual measurements are used to create a dot plot on one number line, while the means are graphed onto the other.

Understanding the nature and meaning of normal distributions is central to many statistical concepts. By having the students engage in a hands-on process of sampling, measuring, and representing their data concretely and collectively, their understanding of normal distributions grows organically as part of the activity. The distribution of the collective sample grows increasingly “normal” in shape as the cumulative sample grows in size (assuming a normal distribution of the population of rocks used in the activity). What makes the rocks exercise so powerful is the collective activity of entering the samples on the board and watching how the distribution evolves. This helps students translate the abstractions inherent in curves into a solid conceptual understanding of what those displays represent.



In recording the means, a more satisfactory visual result can be obtained if students round up or down to multiples of 2 mm so that the dots are aligned directly over one of the measurements on number line. For example, a mean score of 20.7 mm would be represented by a dot over the 20 mm mark on the number line. This allows the small number of samples to be represented in a way that illustrates the normal distribution of means and the central-limit theorem. One important goal of statistics instruction is to help students develop an “intuitive grasp of the relationship between a parent population and the distribution of a sample mean” Hagtvedt et al. (2007). The central-limit theorem states that if random samples are selected from a parent population that has a “sensible” distribution, then the distribution of the sample means will approximate a normal distribution (Colwell & Gillet, 1994). As the students mark their sample means on the dot plot, the distribution of means quickly begins to resemble a normal distribution providing an excellent opportunity to discuss sampling error and probabilistic estimates of sample representativeness of the population parameter.

As students are putting the dots into the dot plot, one or two are asked to use a different color marker or chalk. This facilitates the discussion about sampling error. It is a simple matter to demonstrate, both pictorially with the odd colored marks and numerically from the range of means students calculate on their worksheets, how a sample mean could differ from other samples or from the mean generated by the collective sample. If the size of the class permits, students can stop by a computer station and add their sample data into a statistical program such as SPSS to allow for the calculation of the collective sample mean and standard deviation.

Once the dotplots are complete, and the data has been entered on a common computer worksheet, the mean and standard deviation are calculated and plotted on the board. The combination of the dots graphed by the students representing the individual rocks and the mean/SD makes the meaning of the Empirical Rule easy to grasp in a concrete way.

Conclusion

The importance of deeper conceptual understanding is essential as delMas, Garfield & Chance (1999) write, "in order to develop better statistical reasoning, students need to first construct a deeper understanding of fundamental concepts." This exercise illustrates the underlying concept of a normal curve in a concrete way that, for my students, results in a more I ntuitive understanding than that achieved with computer simulations or text books. It also demonstrates the idea that any individual sample may vary from the population parameter; but, given that the parent population distribution is reasonably normal, that those variations over many samples form a normal distribution which allows probabilistic statements regarding the likelihood that a sample reflects the population parameter. Students see the effects of the increasing size of the collective sample on the development of a somewhat normal distribution as they watch their classmates add their samples to the dot plot on the board. The use of a different color for a select number of samples reinforces these lessons. As an introduction to many of the statistical concepts, this is a compact, easy and effective method.

Chance, B. (2000). Components of Statistical Thinking and Implications for Instruction and Assessment. *Paper Presented at Annual Meeting of the American Educational Research Association April 12th, 2000*. New Orleans.

Colwell, D.J. & Gillet, J.R. (1994). Illustrating the Central Limit Theorem. *Teaching Statistics, Vol. 16*(3), 38.

delMas, R. C., Garfield, J., and Chance, B. L. (1999), "A Model of Classroom Research in Action: Developing Simulation Activities to Improve Students' Statistical Reasoning" *Journal of Statistics Education* [Online], 7(3) www.amstat.org/publications/jse/secure/v7n3/delmas.cfm.

Fisher-Giorlando, M. (1992). Sampling in a Suitcase: Multistage Cluster Sampling Made Easy. *Teaching Sociology*, *Vol. 20*(4), 285-287.

Gnanadesikan, Sheaffer, Watkins and Witmer (1997), "An Activity-Based Statistics Course," *Journal of Statistics Education*, [Online], 5(2).

Hagtvedt, R., Jones, G., & Jones, K. (2007). Pedagogical Simulation of Sampling Distributions and the Central Limit Theorem. *Teaching Statistics: An International Journal for Teachers of Pupils Aged Up to 19, v29*(3), 94-97.

Meyers, C. & Jones, T. (1993). *Promoting Active Learning: Strategies for the College Classroom.* San Francisco, CA: Jossey-Bass.

Watts, D. G. (1991). Why Is Introductory Statistics Difficult to Learn? And What Can We Do to Make It Easier? *The American Statistician, v 45*(4), 290-291.