

Teaching Mathematics in the Pre-school Context

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The aim of this study is to describe what kind of knowledge base is needed when pre-school teachers work goal-oriented with children's mathematical learning. The question of this study aims to answer is: What kind of knowledge base do pre-school teachers need when they work with the object of learning in the pre-school context? Both as a theoretical and an analytical framework, we use variation theory. In all, four pre-school teachers are involved in the study. The research results indicate that it is important for pre-school teachers to have subject and pedagogical content knowledge in order to recognize children's experiences and be aware of their expressions.

Keywords: pre-school, teaching, mathematics, pre-school teacher knowledge

Introduction

The interest of mathematical learning in Scandinavian pre-schools (early childhood education) has increased during the last years. The Ministry of Education in Sweden (2006) has implemented the series of measures in order to promote children's mathematical learning. For example, mathematics is now a compulsory subject in pre-school teacher education. Furthermore, the Swedish government and municipalities have made special efforts in order to develop pre-school teachers' mathematical knowledge. Based on the PISA (Programme for International Student Assessment) results and reported studies like TIMSS (Trends in International Mathematics and Science Studies) (2007), the Ministry of Education in Sweden has reviewed the 13 years old curriculum for pre-school and clarified, for example, the goals for mathematics teaching. In the new curriculum, revised 2010, the pre-school teachers' professionalism is emphasized and the main aim for teachers in pre-school is to work with children's learning, even though care and nurturing still are important parts in early childhood education. In order to develop children's mathematical learning, the pre-school teachers, according to current research (Björklund, 2007; Doverborg & Pramling, 2001), and the new curriculum should start from children's experiences and interests and create varying learning opportunities for them. According to the curriculum, the pre-school teachers have the responsibility to design the environment that gives children opportunities to discern, express, explore and use mathematical concepts and understand the connection between concepts. Comparing similarities and differences among different objects make it possible for children to generalize and thereby build concepts (Ministry of Education and Science, 2010).

In this article, we want to stress one part of our ongoing research, where the pre-school teachers work systematically with a selected object of learning in order to promote children's learning. Learning can be seen here as a changed way to experience the object of learning (Marton, 1992) and pre-school teachers are expected

to find out necessary conditions that make this learning possible.

Theoretical Framework

In this part, we will present the theoretical framework in our study, namely, variation theory and present current research on mathematical patterning. We also want to describe previous research about teacher knowledge.

Variation Theory

As a theoretical framework in this study, we apply a theory of learning called variation theory when the pre-school teachers work with children's mathematical learning (Björklund, 2007; Marton & Booth, 2000; Marton, Runesson, & Tsui, 2004; Marton, 2005; Runesson, 2006). This theory is also used as an analytical tool when analyzing teaching in pre-school. In this theory, learning always has an object, "what" and the object is experienced and conceptualized by learners in different ways, "how". Marton et al. (2004) described three types of object of learning: the intended object of learning, the enacted object of learning and the lived object of learning.



Runesson and Marton (2002) described the lived object of learning, which denotes what the child actually learn and the intended object of learning which means the capabilities the teacher wants the children to develop. The enacted object of learning is seen from the researcher's perspective and means what is possible for the children to learn in classroom setting. It describes the space of learning in the interaction between children and teacher or among children themselves (Runesson, 2006). Variation theory derives from phenomenography research approach. There are two cornerstones in this theory. The first one is that learning always has an object, in our case, repeating patterns. The second one is that the object of learning is experienced and conceptualized by children in different ways. Variation is a primary factor in this theory and it supports children's learning. By understanding the ways children perceive something, pre-school teachers can create conditions for learning. Marton and Morris (2002) pointed out that the most powerful factor for individual learning is how the object of learning is treated in the teaching context, which aspects of the object of learning are in focus, which aspects are variants and which aspects are hold invariant. One way to explain the aspects of variant and invariant is by using an example of the concept colour. The colour is an overall concept and it is invariant. To understand that, there are different colors you should experience in order to discern, for example, the color blue (Marton & Morris, 2002). In a similar way, understanding and discerning critical aspects of the abstract concept, for instance, a mathematical concept, presupposes an experienced variation.

Learning should be seen holistically and not as separate parts of the world. Children experience this world based on their earlier experiences. To be able to experience and thereby learn, children need to identify some specific aspects of the object of learning. Learning is a complex process and the teacher should be aware of the critical conditions for learning (Björklund, 2007). The critical conditions are interacting parts of the whole learning process. In order to experience something, children must develop the ability to discern critical aspects of the object of learning simultaneously. How children experience the object of learning depends on the simultaneous discernment. Björklund's (2007) research concerning toddlers' encounters with mathematics in Finnish "day care" shows that reasonableness and fixed points are critical conditions in toddlers' mathematical learning. According to Björklund (2007), fixed points are something children experienced earlier and which they relate to, when they are experiencing new phenomena.

Pre-algebra in Early Childhood Education

Patterning is critical to the abstraction of mathematical ideas and the development of algebraic thinking in young children (English, 2004; Waters, 2004). Identifying patterns help children structure and generalize beyond the information they have. Although all pre-school children are engaged in pattern-related activities and recognize patterns in their everyday environment, research shows that an abstract understanding of patterns develops gradually during early childhood education.

There are many indicators that an understanding of pattern and structure is fundamental in early mathematics learning (Mulligan, Mitchelmore, & Prescott, 2006; Clements & Sarama, 2007). Despite of increased research interest in early algebra, there are still few studies about the role of young children's mathematical patterning in the development of algebraic thinking and reasoning (Kieran, 2006; Mulligan & Vergnaud, 2006; Waters, 2004). Abstracting patterns is the basis of structural knowledge and the goal of mathematics learning (Sfard, 1991; Warren, 2005). International research has an increasing attention on connections between patterns and structure and algebraic thinking in mathematics curricula (Clements & Sarama, 2007; Mulligan et al., 2006). Finding the structure of patterns is usually regarded as pre-algebraic thinking. This is an important tendency, because early algebraic understanding even influences future mathematical success (Mulligan et al., 2006).

A mathematical pattern can be defined as any predictable regularity, involving numbers or space. In every pattern, the different units are organized in some regular manner. The way a pattern organized is called its structure (Liljedahl, 2004). Research literature distinguishes among different kinds of patterns: repeating pattern, growing patterns, number patterns, geometric or pictorial patterns, etc.. For example,  is a repeating unit of a pattern containing . Growing patterns increase or decrease systematically. They represent variation of one data set, where the relationship among successive terms within the pattern can be identified (Warren, 2005). According to Warren and Cooper (2006), experiences with repeating and growing patterns can develop children's functional thinking. They also pointed out that patterns need to be presented for children in varying contexts, such as with actions, music or geometric shapes. This enhances the potential for early algebraic thinking and reasoning. For example, in English's (2004) longitudinal and cross-cultural study of young children's reasoning abilities, it was argued that patterning knowledge has an influence on the development of analogical reasoning. Children's patterning knowledge has also been to influence the ability to identify, extend and generalize patterns important to inductive reasoning.

Pre-school Teachers' Knowledge Base

Teacher knowledge is a complex concept and can be defined in different ways. In this article, we will focus on the knowledge that is needed when pre-school teachers work with children's mathematical learning. Pre-school teachers need different kinds of knowledge to design the learning environment in pre-school that gives children opportunities to develop fundamental values, skills and understanding of different aspects, for example, to develop understanding of basic properties of numbers, etc.. Some of these aspects can be seen as dimensions of a content which can be viewed as a school subject. According to the revised pre-school curriculum (Lpfö, 98, revised 2010), pre-school teachers should work goal-oriented, although play is still an important tool in children's learning.

There is an agreement among researchers that teachers need a knowledge base when they work goal-oriented. This knowledge base is defined in different ways depending on school level.

Kirova and Bhargava (2005) highlighted three areas in professional growth as critical in pre-school teachers' learning in order to guide young children's learning of mathematical concepts. Firstly, the pre-school teachers need the ability to recognize children's demonstrated understanding of mathematical concepts. Secondly, they need the ability to use mathematical language in order to guide children's progress from behavioral to representational understanding of mathematical concepts. Thirdly, they need the ability to assess children's understanding of mathematical concepts systematically. All of these three areas are needed when pre-school teachers work goal-oriented with mathematics.

Other researchers, such as Shulman (1986) and Grossman (1990), have in their classroom research argued that teacher knowledge base includes, for example, SMK (subject matter knowledge), general pedagogical knowledge, PCK (pedagogical content knowledge) and knowledge of context. SMK refers to knowledge of the contents of a subject. Teachers' SMK influences what and how they teach. PCK includes knowledge and beliefs about the purpose for teaching a subject, knowledge of students' understanding, conceptions and misconceptions, curricular knowledge and knowledge of instructional strategies and representations for teaching subjects. In fact, it is recognized that PCK forms the essential bridge between SMK and the teaching of subject matter. Grossman (1990) even pointed out that teacher knowledge must be context specific, including, for example, knowledge of the students and knowledge of the school settings.

S. I. Pramling and N. Pramling's (2008) research about teaching in Swedish pre-schools claimed that the pre-school teachers have the responsibility to create conditions for children's learning and they discussed what kind of knowledge the pre-school teachers need when they create different opportunities for children's learning. They stressed that what they call subject didactics for young children is a research area under development. They mean that subject didactics can be seen as a link between different contents (subjects) and the educational activities, i.e., how to create conditions for children's learning about a specific content like mathematics. This view corresponds to Shulman's (1986) and Grossman's (1990) ideas concerning the connection between SMK and PCK.

Issues concerning teaching subject in pre-school are highlighted in many countries. For example, in New Zealand, Hedges and Cullen (2005) meant that pre-school teachers are expected to have knowledge about both the subjects they will teach and knowledge of the context where teaching is conducted. Teaching in pre-school should be based on communication and requires a relationship among pre-school teacher, children and content. The pre-school teacher should be able to plan the content of teaching and be flexible when necessary, reflect on their own choices and critically analyze and evaluate them.

In studies conducted by Lee and Ginsburg (2007) are shown that pre-school teachers introduce mathematics for four year old children in different ways. It can be done through children's play, exploration, discovery learning and problem-solving. According to Lee and Ginsburg (2007), pre-school teachers need to understand how children learn (the processes of learning) as well as what they learn (the content of learning). The authors also highlighted that teachers need to become familiar with a range of developmentally instructional strategies and the tools for assessment which support children's mathematical learning, such as problem-solving, communicating, reasoning and representations.

Hedges and Cullen (2005) seemed to agree with Lee and Ginsburg, when they argued that purposeful teaching and learning occur when pre-school teachers' subject knowledge contributes to suitable pedagogical strategies used in children's everyday pre-school context. Pre-school teachers' subject knowledge is important but as a pre-school teacher, you have to know what experiences the children have and what kinds of didactic

and pedagogical strategies you can use.

To sum up, all these researchers pointed out that “pre-school teachers’ knowledge base” seems to include the following elements: ability to plan for pedagogical goals, ability to justify content of teaching, ability to reflect on choices and ability to analyze and evaluate pedagogical goals critically. Thus, pre-school teachers have to be flexible and use different strategies and methods that create conditions for learning depending on children’s earlier experiences. Knowledge of the content, knowledge of the learning processes, didactic and pedagogical knowledge and of course subject knowledge are all aspects of knowledge that pre-school teachers need when they are working with children’s mathematical learning.

The Purpose of the Article

The aim of this study is to describe what kind of knowledge base is needed when pre-school teachers work with children’s mathematical learning. The study is based on the idea of variation theory (Marton, 2005; Marton et al., 2004; S. I. Pramling & N. Pramling, 2008). By reflecting on pre-school teachers’ conceptions and teaching experiences with the object of learning, we seek an answer to the following overall question: What kind of knowledge base do pre-school teachers need when they work with the object of learning in the pre-school context? In order to answer the question, we use following two sub-questions:

- (1) How do pre-school teacher work goal-oriented with the object of learning?
- (2) What conceptions do pre-school teachers express about their goal-oriented work?

Method and Design of the Study

The study includes four pre-school teachers from three pre-schools in the same community in Sweden and 28 four-year-old pre-school children. The pre-school teachers have voluntarily chosen to participate in this study which started in January, 2010. Three of the pre-school teachers are experienced teachers, i.e., more than ten years experience. The pre-schools are multicultural, where most of the children have Swedish as a second language.

The methods used for collecting the empirical data are video recordings and interviews. In this article, we have used variation theory as an analytical tool when analyzing transcribed observations from the video recordings concerning pre-school teachers’ goal-oriented work/teaching with mathematics in pre-school. The interviews concerning the pre-school teachers’ conceptions of their goal-oriented work were first transcribed and then analyzed and categorized by the researchers to the categories of description, inspired by previous research about teacher knowledge (Shulman, 1986; Grossman, 1990).

For the last years, there has been a lot of research based on variation theory at several school levels (Marton, 2005; Holmqvist, 2006; Runesson, 2006), although not so much in pre-school. However, there are some research in this area even at this level in Scandinavian, for example, Björklund (2007), Holmqvist, Tullgren, and Brante (2009) and S. I. Pramling and N. Pramling (2008). These studies are mainly focused on children’s ways to discern critical aspects of the object of learning and what kinds of learning possibilities are created by pre-school teachers. In our research, these perspectives are important although our aim is also to describe what kind of knowledge base is needed when pre-school teachers work goal-oriented with children’s mathematical learning.

Goal-Oriented Work in Pre-school

The research design in the study is based on variation theory. The pre-school teacher’s works goal-oriented with the object of learning, i.e., a repeating pattern. In order to make it possible for children to

discern and explore repeating patterns, pre-school teachers create the situations where children worked with circles in different colors and sizes. The geometrical form, i.e., circle is in this case invariant but both the sizes and the colors of circles are varied.

In this paper, we are going to describe pre-school teachers' goal-oriented work when they use critically variation theory as a theoretical base. They choose an object of learning (repeating pattern) and design a learning situation about repeating patterns with circles. The teachers present to children, in their own pre-school, how to create repeating patterns with circles (e.g., every second circle is large and every second circle is small). Children work in pairs together with their teacher and one of the researcher's video records the activities. After that, the pre-school teachers analyze the video recordings together with a researcher. We will also present when one pre-school teacher work with the object of learning in a play situation.

Results

The research questions deal with what knowledge-based pre-school teachers need when they work goal-oriented with object of learning (repeating pattern) and four years old children. In this part, we are going to describe the results of goal-oriented work in the pre-school context. Firstly, we will present two ordinary and plain episodes and describe how a pre-school teacher works with the object of learning. The following episodes include a planned situation and a play situation.

In the first video episode, the pre-school teacher (Monica) is sitting between two children, one boy (Emerett) and one girl (Yasmine), at a table (see Figure 1). The pre-school teacher has black and white circles in different sizes in her hand and puts a white circle on the table and says,

(1) Monica: A circle (smiles)?

(2) Yasmine: Yes (smiles).



Figure 1. Monica creates a repeating pattern with circles.

Both children seem to be interested and curious, the learning condition is good. Monica has an intention and ambition to draw children's attention to the planned learning object when she tells the children in turn to look.

(3) Monica: I will make a pattern (smiles).

(4) Yasmine: Mm (smiles)!

(5) Monica: (looks at the girl) You will watch.

(6) Monica: (looks at the boy and says his name) Emerett, you will watch (the boy leans back and nods).

Monica strives to create conditions for learning by telling them to watch and she has both children's attention as she continues,

(8) Monica: White circle (puts a white circle in front of her at the table).

(9) Monica: Big (points to the circle).

(10) Yasmine: Big (lean forward).

(11) Monica: Yes.

Yasmine is interested and repeats the pre-school teacher's word. Emerett is still looking, still interested and doing what the pre-school teacher tells him to do, namely, watch. At a first look, it seems that he is passive and not interested, because he is just sitting and watching. After several closer analysis of the video episode, we interpret that Emerett is interacting, while he is just doing what he was told to do, and he waits for the next move in the turn-taking. The two children have different approach in the interaction, but still they are interacting. Turn-taking in formal pre-school settings (like this one) includes, for example, non-verbal expressions, especially when it comes to the youngest children. Monica interacts with Yasmine by confirming her actions. Yasmine continues to repeat the forms in the pattern that Monica creates on the table.

(12) Yasmine: Small, big.

One of Monica's intentions is to interact with both children and she strives to make the repeating pattern possible for the children to discern by asking for their opinions. In the next line (13), she is going to say "little", but she reflects in and on her action, on what she is going to say and ask for the children's perceptions instead. By doing that, she can learn about children's experiences.

(13) Monica: Yes, which one is little (puts a small circle next to the big circle)?

(14) Yasmine: This one (the girl leans forward and points) the small circle.

Yasmine interacts verbally and bodily and Emerett is also interacting while he is watching. He is doing what the pre-school teacher told him to do. One important part of interaction is that children's ways of perceiving objects differ from each other, from the teacher and from the researcher. Such differences are a consequence of the interaction. From our perspective, the issue is to describe and understand the space of differences that occur in these learning situations. Monica wants to have knowledge of Emerett's perceptions and she moves the circles towards him and asks a question:

(16) Monica: Emerett, which one is small?

(18) Emerett: Yasmine (says the name of the girl).

One of Monica's intentions with the design of the activity is that Emerett with the help of Yasmine's "pattern experiences" should have the possibility to discern the pattern. In the interaction between Monica and the children, the girl has more experiences on this area and acts as a peer educator.

(19) Monica: Yasmine, show once more which one is little.

(20) Yasmine: That one (the girl points to the small circle).

(21) Monica: Yes, it is small.

(22) Monica: Big circle, round (follows the circle shape with her finger at the same time as she talks).

(23) Emerett: Emerett (says his own name and looks at Monica).

Emerett has discerned one of the pre-school teacher's intentions. Now, he is verbal and with just saying his name and he wants to show Yasmine and Monica that he knows what to do. Now, it is his turn in the turn-taking process. Monica recognizes and pays attention to Emerett's interest and willingness to participate in communication about the learning object. She is not sure if Emerett knows the circle concept, even though circle is usually the first geometric shape children discern in their toddler life. Monica gives each child an amount with colored circles in different sizes. Yasmine immediately starts to make repeating patterns of each second large red and each second small white circle. Emerett is still watching. The pre-school teacher encourages Emerett to look at the girl and then she says,

(26) Monica: You can also build a pattern of circles.

The boy takes one red big circle and says,

Emerett: Red.

Monica: Yes, it is a red circle.

Then, Emerett takes a small red circle and puts it closer to the big red circle and says,

(29) Emerett: Red.

(30) Monica: Yes, you have one big red circle and one small red circle.

Emerett continues with the other circles and puts them together in coloured pairs. Yasmine continues making repeating pattern with coloured circles (see Figure 2).



Figure 2. The children create different patterns.

In the next video episode, we meet Emerett when he is playing with cars and Monica strives to reach Emerett's interest by playing with him. She has tried to show him the repeating pattern of the parked cars and now they have moved down the cars below the toy garage. Monica wants Emerett to discern a repeating pattern among the cars. She uses color as variant aspects, because she knows that Emerett have interest in different

colors (see Figure 3).

(5) Monica: We should have every second (the teacher points to the first red car) red, blue, red ... (then she is pointing to the empty space between two cars but the boy interrupts and points to the first red car).

(6) Emerett: No, this one is purpur.



Figure 3. Different experiences.

According to Monica, the first car is dark red, but for Emerett, it is purpur. Monica listens to him and agrees that the first car can be purpur and Emerett continues to repeat the colors.

(18) Emerett: Blue, red, blue, red ... (the teacher points to each car and the boy says the color).

(19) Monica: And now? (the teacher is pointing to the empty space after the last red car).

(20) Emerett: That one I park and ... and there I park and there I park a green (he takes a green car).

(21) Monica: No, no, no just red and blue just red and blue (the boy sets back the green car).

(22) Emerett: Just red and blue?

(23) Monica: Yes, red and blue (the boy parks a blue car between two red cars).

(26) Emerett: This one is purpur (points to the first car).

The episodes above illustrate, when pre-school teachers work goal-oriented with the object of learning, small pieces of different experiences are important for learning of the intended object, like the repeating pattern and different color experiences.

The second research question aims to give an answer to what kinds of conceptions do pre-school teachers have about their goal-oriented work. From the pre-school teacher's expressions about their goal-oriented work, we describe following four categories: recognizing children's experiences, listening to the children's expressions, awareness of content and capturing everyday situations. A more careful description of the four categories as follows.

Recognizing Children's Experiences

To recognize children's experiences in pre-school is an important part of the pre-school teacher's work with learning, because children's previous experiences constitute the base for children's learning. In this study,

when the object of learning is to discern repeating pattern pre-school teachers must have not also knowledge of children's previous pattern experiences, but also knowledge of children's different ways to express their experiences. Some of these children do not use their verbal language, which is the reason why the pre-school teachers have to study their actions. If the pre-school teacher can see what kinds of different patterns children create in their everyday life and to what children pay their attention in the planned learning situations, the pre-school teacher can recognize children's previous experiences.

It is important to recognize children's experiences of the object of learning (what) in order to know how to create conditions for learning. One of the pre-school teachers means that it is their responsibility to recognize children's earlier experiences.

... it is up to us (the pre-school teachers) to recognize children's earlier experiences and continue to build on it all the time (I 1)¹.

She also expresses that she has recognized that children have a lot of various experiences concerning patterns and give an example from a situation where children expressed their experiences about patterns on their stockings.

... it was a lot of variation (referring to children's various experiences of patterns concerning stockings), it was different colors, patterns, ... so it was really rich of variations (I 1).

To recognize children's experiences, the pre-school teachers need knowledge of children's different experiences and that also includes the knowledge of the cultural background. The children in this study are multilingual and they are developing language which includes body language, mother tongue, Swedish and mathematical language. The pre-school teachers mean that when they work with multilingual children, they must recognize children's experiences and talk about every basic concept to ensure that children understand what they are talking about. One of them gives one example,

... but then they have the language against them ... if I say classify, they look at me, classify (?) what is classify? ... when we have talked about it when we have classified they say: Now, I have classified (I 1).

The pre-school teachers point out that they have to recognize children's conceptual understanding very carefully.

Listening to the Children's Expressions

In order to be responsive to children and their thoughts about contents, the pre-school teacher needs to be present with the children and listen to children's different expressions. To listen is to respond to children's questions and to be an active listener who strives to get knowledge of children's expressions, and the pre-school teacher needs to be aware of children's different "pattern" experiences. The pre-school teachers in the study mean that they have to meet children's expressions in different situations. Sometimes, they follow the child's intention in everyday activities and sometimes, the pre-school teachers themselves take the initiative to the learning situation.

... in some way I think like this, there is not one way (to work goal-oriented with children's learning) ... because when I sit with a child for a while ... and I have thought things out it has been felt like we thought together in any way, the child and I, and we have shared thoughts and I have been the one that shows (M 2).

¹ The participants are identified by the first letter in their names and by the number of the category.

She expresses that as a teacher, she needs to get knowledge of children's experiences and sometimes the best way is to sit and talk together with just one or two children, although the most common way for the pre-school teachers' in this study to get knowledge of children's experiences is to be with children in their daily pre-school activities.

Awareness of Content

When pre-school teachers work goal-oriented with a specific object they have to be aware of the content of subject according to this study. This awareness includes subject knowledge and how children express their experiences of the content. They also mean that when they work with a specific subject their own awareness of the content has increased. The pre-school teachers express the importance to be present and listen to children's expressions in relation to a specific content. To be aware of the content includes having knowledge of the content in relation to children's different experiences. From the interviews, the following example shows that awareness is important.

You can be aware of it (children's earlier experiences) and just help them a little (for example, to discern) (I 3).

The pre-school teachers in this study are also aware of the importance of giving children opportunities to develop rich language. During the study, they have been aware of the need to connect mathematical language to the social everyday language. One teacher expresses her opinion in this way,

but the children must have a rich variation of words (the relation between mathematical and the social everyday language) (I 1).

One pre-school teacher in the interview also expresses her own learning and awareness in relation to children's experiences and expressions about a mathematical content in the following examples,

It is probably my awareness that has increased (I 3).

Earlier, I probably thought that if I only use the (mathematical) words, they will understand them at the end (I 3).

... but I have not been aware of that it was mathematics all the time and how we can discern patterns and now we look at patterns on sweatshirts and we do not look for pattern with butterflies, now we look at if there are patterns on the wings of butterflies (I 3).

The pre-school teacher talks about awareness and of a rising awareness in relation to her own knowledge, both in relation to mathematical knowledge and knowledge of children's conceptual experiences. All pre-school teachers in this study express that they did not had knowledge of children's different conceptual understanding before they started this study. They often have thought that the children knew more than they did, and the pre-school teachers mean that it is in the every day events that they have to be aware of the children's expressions about the contents.

Capturing Everyday Situations

There are no lessons in pre-school and the pre-school teachers mean that it is important to capture everyday situations in order to make the specific learning object of visible for the children. In the pre-school curriculum, the intentions for pre-school are that early childhood education shall give children opportunities to develop their learning in play and the learning environment in Swedish pre-schools should provide learning opportunities both inside and outside of the pre-school. According to variation theory, learning is holistic and children experience different phenomenon based on their earlier experiences. Therefore, in order to promote

learning, there is a need for the pre-school teacher's to capture situations where children's experiences of patterns are not only visible but also to create situations where children can explore a variation of patterns. Following examples show the pre-school teachers' opinions how to capture situations for learning in the daily pre-school activities.

I can see the great value to explore this with the children in their situations, then of course, we (the teachers) can also design and create, but based on the situations the child is active in and see opportunities to show and see if they can follow the logic (M 3).

... children automatically play with pair bonding and sorting, without thinking on it, but now I am there and puts into words (mathematical concepts) what they do (I 3)

We think that children understand more concepts than they really can, it is in the everyday life when they play with pail and shovel that I with variation can help them to discern (I 4).

In the flow of events in the pre-school context, it is not easy to follow each child's expressions. Therefore, the pre-school teachers can see the possibilities with focus on the object of learning and the children's experiences. They also express the importance of being aware of the children's perspectives and the problem when the teachers are content-driven, i.e., when they work goal-oriented.

To sum up, the results of how pre-school teacher work goal-oriented with the object of learning, the intended object of learning, the lived object of learning and the enacted object of learning show various ways of discern and experience the learning object (in this case repeating pattern). The results also show pre-school teachers' conceptions about their goal-oriented work: recognizing children's experiences, listening to the children's expressions, awareness of content and capturing everyday situations. All these conceptions are crucial knowledge for the pre-school teachers in teaching situations.

Discussions and Conclusions

The overall aim of this study is to explore and describe what kind of knowledge base is needed when pre-school teachers work goal-oriented with children's mathematical learning by using variation theory. In this section, firstly, we discuss the results of our two research questions. Secondly, we talk about possible implications of this study for preschool teachers' knowledge base and some conclusions and suggestions for future research.

Our first question concerns how pre-school teachers work goal-oriented. The results from the two video episodes in this study show that the pre-school teacher's intention (intended object of learning) is to make it possible for the children to discern the repeating pattern. The pre-school teacher (called Monica) has knowledge of the children's earlier experiences, and in the first video episode with two children (Emerett and Yasmine), she listens to each child's expressions in relation to the content. Monica uses variation when she makes a repeating pattern of black and white circles and when Yasmine builds a pattern of red and white circles. The pattern unit with big and small circles is kept constant and the colours vary. Monica shows how a geometric repeating pattern with every second big and every second small circle might look like. Critical features of this pattern are the repetition of big and small circles and Monica has an intention to reach the children's attention by saying "big circle, little circle..." each time she adds a new circle to the circle pattern. She also draws the circle form with her finger when she is saying "big circle, small circle..." in order to clarify the circular shape. Monica points out similarities (circle shape) and differences (size) in relation to the critical aspects. During the

analysis of video recordings, it becomes clear to all participants that Monica in the first video episode should use the word “pattern” more often, in order to illuminate the comprehensive picture of a pattern. The pre-school teachers discuss that children need to discern different kinds of patterns. Monica’s intention is also that the children simultaneously should discern several critical aspects of the pattern, i.e., repetition, circle shape, circle size and circle color and thereby discern the pattern as a whole.

Lived objects of learning for Emerett are similarities and differences, when it comes to the circles’ size and colour and the relationship between parts and whole. Monica makes it possible for both children to discern the circles, the sizes and the colour, but it is only Yasmine who discerns the intended repeating pattern. Emerett creates own patterns in coloured pair. Björklund’s (2007) research shows that the critical aspects called fixed points have importance in toddler’s learning process. The results of the video episode indicate that Emerett relates to his earlier experiences and the fixed points for him are color and shape, similarities and differences between shape and size. He can also discern that the circles of the same color belong together, even though they have different sizes. Yasmine’s lived object of learning and Monica’s intended object of learning are close to each other. Yasmine has a lot of experiences making patterns in pre-school activities. She discerns the parts, like the shape, size, color, the repetition and repeating pattern as a whole.

Concerning to the enacted object of learning, Monica makes it possible for the girl to discern different aspects of the pattern simultaneously. Yasmine discerns the whole, i.e., the pattern, and the repeating parts, i.e., circles, sizes and colours. From her earlier experiences, she can create own varying patterns. Emerett discerns the circles, sizes and colours simultaneously and creates own pattern, though he started to match circles in colours and sizes related to his fixed points, but not a repeating pattern.

Monica uses her didactic, pedagogical knowledge and knowledge of children’s understandings and experiences of patterns when she organizes learning situation, i.e., Emerett with little “pattern experiences” together with Yasmine with a lot of “pattern experiences”. Monica’s intention is that Yasmine’s experiences would help Emerett to discern the circle pattern.

The results from the second video episodes show that the intended object of learning is to make it possible for Emerett to discern a repeating pattern among the cars and to use his play with colored cars as a starting point. Monica’s intention is that the boy should discern a repeating pattern where every second car is red and every second car is blue, i.e., the whole pattern and the parts where the colors are variant aspects. Emerett’s lived object of learning is that the cars have different colors and he does not discern the repeating pattern at first because from his point of view, the first car is purple and not red. The enacted object of learning is that Emerett finally understands Monica’s intention with cars parked every second red and every second blue car.

The results show that when pre-school teachers work goal-oriented with variation theory as a theoretical base, they require knowledge of subject (repeating pattern) and knowledge of theory (variation theory). They need to find out critical conditions of the learning object and how to vary them. The pre-school teachers also need didactic knowledge to know which strategies will support children’s different learning and knowledge of children’s experiences, interest and different ways of expressing themselves. For example, in Emerett’s play with cars, simple aspects like different color experiences had influence on his learning opportunities.

Our second question concerns what conceptions the pre-school teachers express about their goal-oriented work. The results from the analysis show four categories of description: recognizing children’s experiences, listening to the children’s expressions, awareness of content and capturing everyday situations as crucial knowledge base for the pre-school teacher in teaching situations. The first category, recognizing children’s

experiences, is crucial when pre-school teachers are working with the object of learning, because the object of learning is experienced and conceptualized by children in different ways (Marton et al., 2004). The second category, listening to children's expressions, maybe sounds obviously but pre-school children express themselves not only with words, their actions are also expressions. Sometimes, it can be difficult for the pre-school teacher to understand what children express and that is why it is important to really listen to children's expressions. The third category, awareness of content, is also an important aspect when teachers work with a subject. Children's early learning will depend on pre-school teachers' awareness of the importance of patterns and structure. When the object of learning is repeating patterns, they have to be aware of the repeating pattern and in what ways children create patterns. Finally, the fourth category, capturing everyday situations, is probably the most difficult category in goal-oriented work. When the object of learning is repeating pattern, there is the structure in pattern that ought to be possible for children to discern. In Scandinavian, pre-schools children do not have lessons and a common way to work with children's learning is to be with children and capture learning situations in the daily life. To capture a learning situation and to listen carefully what children express are important in early childhood education. It is in these situations that the pre-school teachers' can challenge, expand and clarify the situation to be meaningful and instructive. When the content relates to children's interest in the daily activities, an increased focus on content learning is possible.

When pre-school teachers work systematically goal-oriented, as they do in this study, subject knowledge (knowledge of patterns) and theoretical knowledge (knowledge of variation theory) are together with the other pedagogical knowledge important aspects of a teacher knowledge base, although we notice that Monica and her colleagues do not have deeply knowledge of pattern and knowledge of variation theory. They do not know that pattern is fundamental to mathematical learning and they do not know that patterning knowledge has an influence on the development of analogical reasoning. They have knowledge about variation theory superficially, but the systematically goal-oriented work is not tradition in Swedish pre-schools.

Different kinds of knowledge components are important and obvious for teachers who teach pupils, but it is clear that they also are important for teachers in early childhood education when they work systematically towards the curriculum goals. The aim of this study is to discuss what kind of knowledge a pre-school teacher need in the goal-oriented work. We have described how the object of learning was planned and carried out by the pre-school teachers in two video episodes and what categories the data analysis shows. They need knowledge about the purpose for teaching a subject (repeating pattern), knowledge of children's experiences, conceptions and understanding, knowledge of variation theory and what strategies they can use in order to promote children's learning. These different aspects of knowledge can be essential parts of PCK for pre-school teachers (Grossman, 1990; Schulman, 1986). Other important aspects or dimensions of pre-school teachers' knowledge are: recognizing children's different experiences, listening to children's expressions, awareness of content and capturing everyday situations. These dimensions of knowledge can be strategies not only to obtain knowledge of children's understanding, but also to challenge and make it possible for children to develop pattern knowledge.

In conclusion, the pre-school teachers in this study needed theoretical support from the researchers both in relation to repeating pattern (subject) and theory (variation theory). However, as previously discussed, they are not used to work systematically goal-oriented, but when they do it they need different kind of knowledge components. Working with variation theory as in this study the pre-school teachers have to find out what the

critical conditions for the object of learning are in different play situations together with the children. That means that they must have deep knowledge of the theory (almost lived theory) and when they see and capture a learning situation, they can make some content invariant and vary other aspects in order to make the object of learning visible for the children. All this must be in regard to children's experiences and interest and that mean deep knowledge of content. We have presented the results and what kind of knowledge base we suggest pre-school teachers need when they work goal-oriented with a subject like mathematical patterns and variation theory. The knowledge base can assist them to promote children's conceptual learning, but pre-school teachers' needs education to develop the knowledge. This is a small study and the research concerning this area will continue.

Finally, our suggested results implicate that pre-school teacher education needs to include more mathematics for early childhood education and education on theories like variation theory that promote mathematical learning.

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