

**Abstract Title Page**  
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**Title:**

Understanding Treatment Effects Heterogeneities using Multi-site Regression Discontinuity Designs: Example from a “Double-dose” Algebra study in Chicago

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## **Abstract Body**

*Limit 4 pages single-spaced.*

### **Background / Context:**

“Double-dose” coursework has become increasingly popular as a strategy to improve academic outcomes of low-performing students. According to Council of Great City Schools (2009), nearly half of large urban districts today use doubled math instruction as the most common form of support for students with low academic skills. In ninth grade, it is particularly important to provide effective algebra intervention because algebra typically has the highest failure rates of all ninth-grade courses. In addition, as many states and districts now require algebra for all students in ninth grade or earlier, eliminating remedial coursework altogether, the need to assist struggling students has also increased.

Chicago Public Schools initiated a “double-dose” algebra policy in 2003, requiring students with below average skills to enroll in two periods of algebra—support algebra and regular algebra. The double-dose algebra policy was built on an earlier policy of requiring all ninth-grade students to enroll in algebra. While the “algebra-for-all” policy immersed many low-achieving students in algebra for the first time, it was less successful in raising test scores than anticipated, plausibly because many students lacked sufficient mathematical background to benefit from algebra instruction (Allensworth, Nomi, Montgomery, & Lee, 2010). In response, the double-dose algebra policy was introduced as a way to help low-achieving students catch up on skills needed for algebra coursework, while engaging all students in rigorous coursework.

An earlier study by Nomi and Allensworth (2009) showed that, using a regression discontinuity design, both the intent-to-treat  $e$  and the compliers average treatment effects were modest but highly statistically significant for students with below-average incoming math skills. The current study extends this earlier work to understand heterogeneity in the policy effects across schools as well as mediating mechanisms of the policy that may be related to treatment effect heterogeneity. We explore conditions under which schools produced the largest effects.

### **Purpose / Objective / Research Question / Focus of Study:**

To understand heterogeneity in the effects of double-dose algebra across schools, we focus on two policy implementation features and their variation across schools. The first source of variation is the degree to which schools followed the cutoff-based course assignment policy. The second source of variation is the degree to which schools segregated algebra classes on the basis of students’ prior skills in response to the policy. We ask the following research questions: 1) to what extent did double-dose algebra effects vary across schools?, and 2) how were these variations related to the ways in which schools implemented double-dose Algebra?

### **Setting:**

Chicago has the third-largest school system in the United States. The student population is about 50% African-American, 38% Latino, 9% White, and 3% Asian. Approximately 85% of students are eligible for free/reduced priced lunches.

### **Population / Participants / Subjects:**

Our sample consists of first-time ninth graders who entered Chicago high school in 2003. We excluded students with identified disabilities because their control condition (i.e., regular algebra) differed from regular algebra taken by students without disabilities. The resulting sample size is 12,916 students in 60 high schools. Table 1 shows descriptive statistics.

Among students without disabilities, 40 percent of students scored below the 50<sup>th</sup> percentile on the 8<sup>th</sup>-grade ITBS in mathematics, and the average score was 55.49. Overall 35 percent of students took double-dose algebra in 2003. Compliance with the policy was quite strong but not perfect: Among below-norm students, nearly 80 percent of students took double-dose algebra, while only 0.41 percent of above-norm students took the course. Student achievement scores come from the algebra subset of the math portion of the PLAN examination, a standardized test developed by American College Testing (ACT Inc.). The mean of algebra scores is 6.53 with the standard deviation of 2.35.

**Intervention / Program / Practice:**

The double-dose policy required first-time ninth graders with 8<sup>th</sup>-grade ITBS math scores below the national median to enroll in support algebra in addition to regular algebra. The district offered two curriculum options to double-dose algebra teachers with curricular resources and lesson plans, as well as professional development around how to use the two periods for instruction. Also, to ensure consistency between the two algebra classes, the district made the following recommendations: schools offer algebra and support algebra sequentially in a day with the same teacher, and students needing support coursework take the same algebra classes together. To follow the district’s recommendations, schools tended to create separate algebra classes—double-dose algebra for below-average students and regular algebra for above-average students. As a result, the double-dose algebra policy intensified sorting by students’ skill levels.

To summarize, the policy brought about simultaneous changes in course taking and classroom peer ability. Figure 1 shows that the district wide double-dose algebra enrollment was largely defined by the eligibility cutoff score. Figure 2 shows that classroom peer ability was substantially higher for students above the cut point than students below the cut point (right panel in Figure 2), while there was no such discontinuity during pre-policy years (left panel in Figure 2). Thus, the intervention consists of both double-dose algebra instruction *and* classroom peer ability, and we attempt to decompose these two effects. (Please insert Figure 1 and 2 here).

**Research Design:** Regression Discontinuity Design

**Data Collection and Analysis:**

The first analysis estimates the city-wide average intent-to-treat (ITT) effects, as well as between-school variance of these effects and school specific ITT effects. The second analysis estimates the compliers’ average effects of enrolling in double-dose algebra. Again, we are interested in estimating the city-wide compliers’ average effects, the variance of these effects, and school specific average compliers’ effects. For the compliers’ analysis, we employ an instrumental variable approach; using the cutoff score as an instrument, we identify double-dose enrollment effects among complying students. The variance and school-specific effects are estimated using random effects models, proposed by Raudenbush, Reardon, and Nomi (2011). The third analysis decomposes the effect of course taking and classroom peer ability, using school-by-cutoff score interaction terms as instruments.

*Analysis 1. ITT effects.* We estimate the ITT effects of double-dose algebra for student  $i$  in school  $j$ , using the following model;

$$Y_{ij} = \eta_j + B_j T_{ij} + \phi_{1j} X_{ij} + \phi_{2j} X_{ij}^2 + \varepsilon_{ij} \tag{1}$$

where  $\eta_j$  is a school-specific fixed intercept,  $T_{ij}$  indicates whether students scored below the cutoff ( $T=1$  if below the cutoff and  $T=0$  otherwise),  $X_{ij}$  is the ITBS percentile scores centered on the cut point (i.e., the 50<sup>th</sup> percentile),  $\phi_{1j}$  is the school-specific linear coefficient for  $X$ ,  $\phi_2$  is the curvature in the association between  $X$  and  $M$  (assumed constant across schools), and  $\varepsilon_{ij}$  is an idiosyncratic random error.<sup>1</sup> Here, we are interested in estimating the overall mean effect,  $E(B) = \beta$ , the variance in these effects across schools,  $Var(B) = \tau_\beta^2$ , as well as the school-specific effects, using the empirical Bayes method (Raudenbush & Bryk, 1985).

*Analysis 2. Compliers' average effects of enrolling double-dose algebra.* We employ an instrumental variable strategy, using the cutoff score as an implement to identify compliers' average treatment effects. We use the following two-stage-least-square models, and the first stage equation estimates double-dose algebra enrollment as an outcome;

$$M_{ij} = \eta_{mj} + \Gamma_j T_{ij} + \phi_{m1j} X_{ij} + \phi_{m2} X_{ij}^2 + \varepsilon_{mij}, \quad (2)$$

where  $M$  indicates whether students took double-dose algebra ( $M=1$  if enrolled in double-dose algebra), and the rest of the notations are defined earlier.<sup>2</sup> The overall mean effect of the policy on double-dose algebra enrollment (i.e., the city-wide average compliance rate) is  $E(T) = \gamma$ . We are also interested in the variance in these effects across schools  $Var(T) = \tau_T^2$ , and school-specific empirical Bayes estimates.

The predicted value  $\hat{M}_{ij}$  from Equation (2) captures expected double-dose algebra enrollment for student  $i$  in school  $j$ —exogenous component of double-dose algebra enrollment in school  $j$ , generated by scoring below the cut point. The second stage equation uses this expected value to estimate the outcome  $Y$ . Thus, we have;

$$Y_{ij} = \eta_{yj} + \Delta_j \hat{M}_{ij} + \phi_{y1j} X_{ij} + \phi_{y2} X_{ij}^2 + \varepsilon_{yij} \quad (3)$$

We focus our interest on the overall mean and variance of  $\Delta_j$  as well as school-specific empirical Bayes estimates of those effects. Here,  $\Delta_j$  is a combined effects of course taking and peer ability.

*Analysis 3. Mediating effects.* Schools varied considerably in terms of the extent to which they enrolled students in their assigned coursework as well as the degree to which schools sorted algebra classes based on students' prior skills. By taking advantage of this variation, we use school-by-cutoff interactions as instruments, generating 60 instruments to identify the effects of the two mediators among compliers. Here, we estimate the first-state equation (2) for  $M$  and the parallel equation using classroom peer ability  $C$  as an outcome. We then use both  $\hat{M}_{ij}$  and  $\hat{C}_{ij}$  in equation (3). Note that we are not able to estimate school-specific effects of course-taking and class composition given one instrument per school.

**Findings / Results: ITT results.** The overall average ITT effect is 0.35 ( $P < 0.001$ ), equivalent to a standardized effect size of 0.15. The effect varied significantly across schools; the variance of the average ITT is 0.16 ( $p < 0.01$ ). Figure 3 shows the distribution of the school-specific ITT effects

<sup>1</sup> Nomi and Allensworth (2009) provide specification checks on the quadratic function; the authors used pre-policy cohorts to specify underlying relationships between the assignment variable  $X$  and the outcome  $Y$ .

<sup>2</sup> Note that the subscript "m" is added on the parameters and error term of Equation (2) to distinguish these from the corresponding parameters of Equation (1)

*Results on compliers average treatment effects.* The result of the first-stage equation (2) showed that the city-wide compliance rates were high; the average effects of the cut point on double-dose algebra enrollment were 72%. Compliance rates also varied substantially across schools; some schools enrolled less than 50% of students in their assigned course, while many schools had relatively high compliance rates of 80% or higher. Figure 4 shows the distribution of the school-specific compliance rates (please insert Figure 4 here).

We then estimated the compliers' average effect of enrolling in double-dose algebra on their test scores. The city-wide average compliers' effect was 0.50 ( $p < 0.001$ ) with the effect size of 0.21. The variance of these effects was .39 ( $p < 0.05$ ). In addition, Figure 5 presents the relationships between school-specific ITT effects and school-specific compliance rates. The figure shows that schools with similar compliance rates have substantial variability in the ITT effects, suggesting that the compliers' average effects also vary among schools with similar compliance rates.<sup>3</sup>

We also compared differences in the ITT effects and compliers' average treatment effects among schools that differed in the degree to which they sorted algebra classes on the basis of students' initial incoming skills. On average, the effect of the cut point on peer ability was -0.24, and importantly, the degree of sorting also differed across schools. Table 2 showed that both ITT effects and compliers' average treatment effects were weaker for schools with greater sorting than schools with less intense sorting.

*Results on mediating effects of double-dose algebra coursework and peer ability:* The results of two-stage least squares showed that the effect of double-dose algebra enrollment was 0.69 (effect size=0.3) and the effect of peer ability was 0.81 (effect size=0.4). An additional analysis showed that the school-specific effects of the cut point on course enrollment and peer ability were not highly correlated ( $r = -0.27$ ), see Figure 6. These findings together suggest that schools are those that implemented double-dose algebra without creating sorted algebra classes.

## **Conclusions:**

It is critical to provide effective algebra intervention for students with low math skills, particularly in urban districts where many students enter high schools with math skills well below grade level. A prior study by Nomi and Allensworth (2009) showed that double-dose algebra was successful in raising test scores for students with below-average math skills. We extended this earlier work to understand heterogeneity in the treatment effects across schools. We showed that the effects of double-dose algebra varied considerably across schools, even among schools with similar levels of compliance with the course assignment policy.

An unintended consequence of this policy was that schools tended to create separate algebra classes for below- and above-average students. As a result, the policy intensified skill-based sorting. Importantly, the degree of sorting also varied by schools; gaps in classroom peer ability between below- and above-average students were larger in some schools than others, and these variations seemed to be related to the effects of double-dose algebra. We showed that enrolling in double-dose algebra would benefit learning, but having lower-ability classmates would hurt learning. Thus, the overall effects of double-dose algebra seemed to be weaker for schools with more intense sorting than schools that used less intense sorting.

The next step of our work is to examine what school characteristics may be related to treatment heterogeneity and whether the effects may also vary by different student subgroups.

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<sup>3</sup> Note that school-specific compliers' average effects can be derived by dividing ITT effects by compliance rates for each school.

## Appendices

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### Appendix A. References

Allensworth, E., Nomi, T., Montgomery, N., & Lee, V. E. (2010). College preparatory curriculum for all: Academic consequences of requiring Algebra and English I for ninth graders in Chicago. *Education Evaluation and Policy Analysis*, 31(4), 367-391.

Council of the Great City Schools (2009). Urban Indicator, Retrieved from [http://www.cgcs.org/Pubs/Urban\\_Indicator09.pdf](http://www.cgcs.org/Pubs/Urban_Indicator09.pdf)

Nomi, T., & Allensworth, E. (2009). “Double-dose” algebra as an alternative strategy to remediation: Effects on students’ academic outcomes, *Journal of Research on Educational Effectiveness*, 2, 111-148

Raudenbush, S. W., Reardon, S., & Nomi, T. (2011). *Statistical Analysis for Multi-site Trials Using Instrumental Variables*. Unpublished manuscript, under review.

## Appendix B. Tables and Figures

Table 1. Descriptive statistics

Variables	Mean	SD
Scoring below the cutoff	0.40	0.49
Taking double-dose algebra	0.35	0.48
ITBS math percentile	55.49	22.09
Algebra scores	6.53	2.35

Table 2. The effect of double-dose algebra on algebra scores by the degree of sorting

	All schools			Schools with at least 50% compliance		
	Degree of sorting			Degree of sorting		
	Low	Average	High	Low	Average	High
ITT	0.49	0.35	0.25	0.43	0.35	0.25
Compliers' effects	0.69	0.53	0.32	0.63	0.53	0.32
School N	19	19	22	13	19	22

Figure 1. Double-dose algebra enrollment rate by math percentile scores

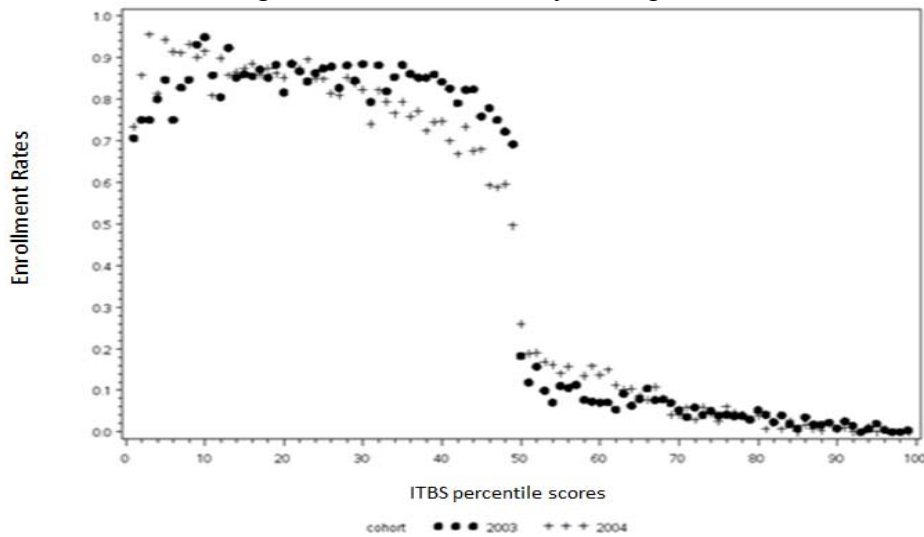


Figure 2. Classroom average skill levels by math percentile scores

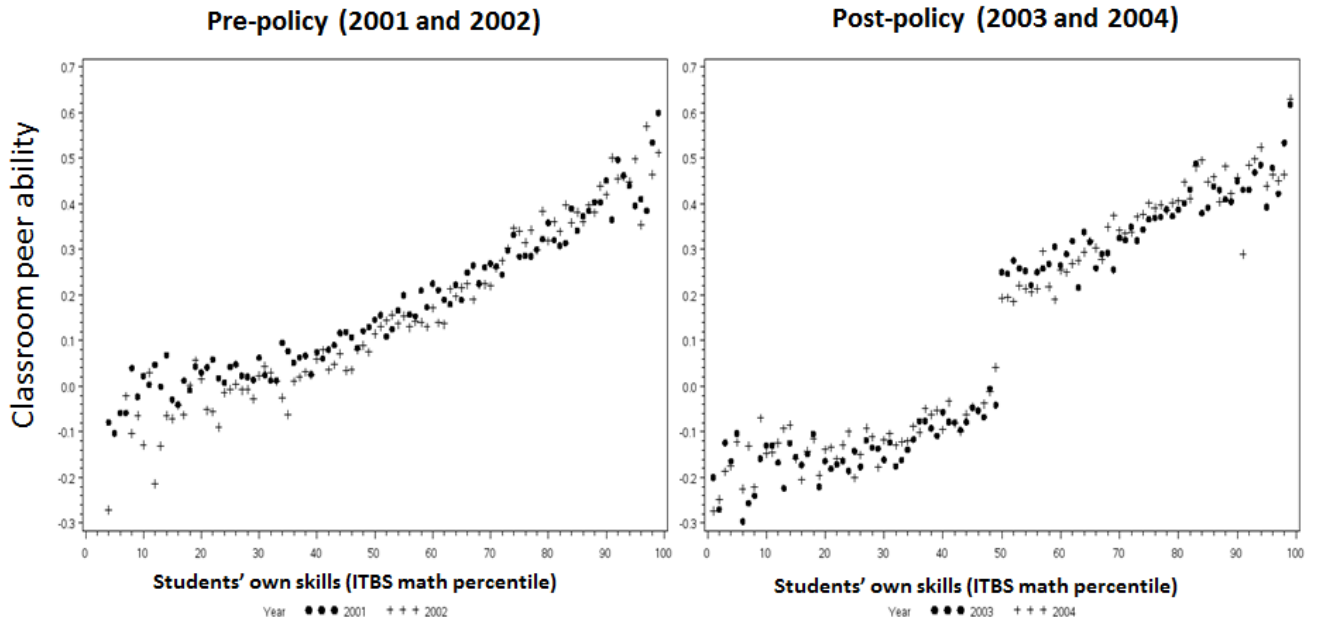


Figure 3. Distribution of ITT effects.

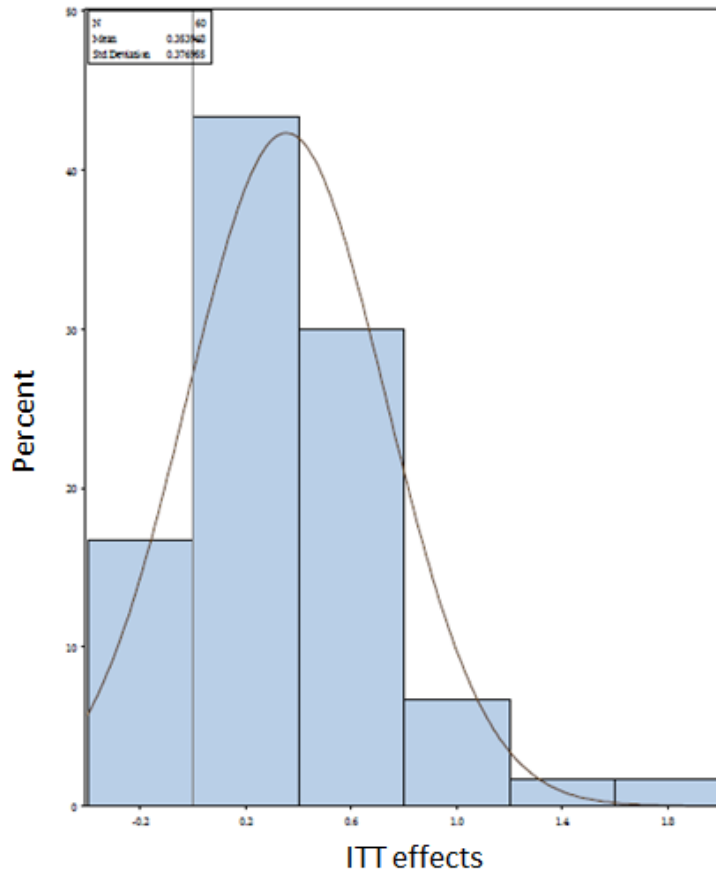




Figure 4. Distribution of the effect of cutoff on double-dose algebra enrollment.

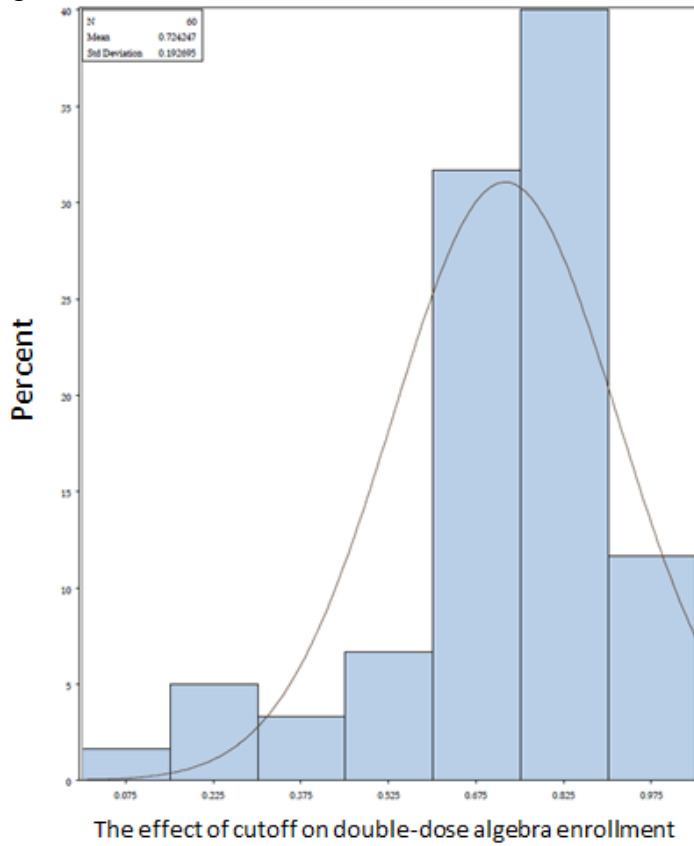


Figure 5. The effect of the cutoff on algebra scores (ITT effects, vertical axis) and the effect of the cutoff on algebra enrolment (horizontal axis)

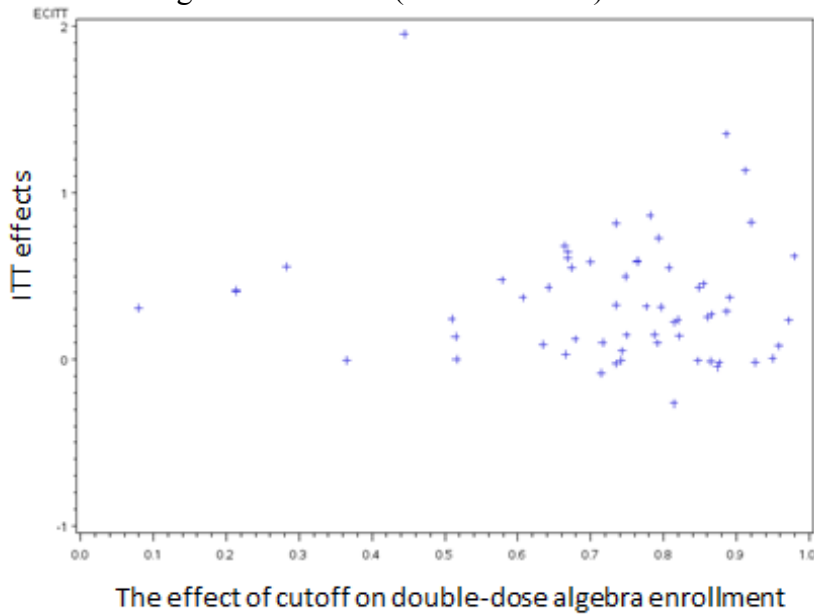


Figure 6. The effect of cutoff score on double-dose algebra enrollment and classroom peer ability

