Teacher Belief, Knowledge, and Practice: A Trichotomy of Mathematics Teacher Education

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March 7, 2012

Abstract

In this paper I reviewed and analyzed three important constructs- beliefs, knowledge, and practices in mathematics teacher education. I carried out literature review of teacher beliefs and practice, and beliefs and change in mathematics education in to order draw some pedagogical and research implications. I was able to draw three themes from the reviews- beliefs impact practice but within a supportive environment, teacher beliefs impact teacher change (development), and teacher knowledge is not just content knowledge, pedagogical knowledge, technological knowledge, and their possible combinations but it opens a new dimension of thinking such as pedagogical-philosophical-psychological knowledge for which we have to conceptualize an interiorized other – the epistemic student.

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Background

De facto Teaching

When I go back to memories of mathematics and mathematics education classes I attended, I can see variations in teaching and learning practices across the grade levels, time, and contexts. Teacher centered teaching and learning heavily dominated my early experiences in mathematics classes. Mathematics problem solving was fully guided by one right way approach. Once in my third grade examination, I did not care either as long division or short division, but I got the right answers by direct division for which I was punished by not giving any credit. I had no voice to speak, no reason to defend, and no way to go along except following what the teacher asked us to do. I remember those division and multiplication problems we solved but without making any sense to us. I learnt operations and procedures without getting into the meaning of what we were doing.

When I was in ninth grade, I used to go through problems, formulas, and theoretical derivations before the start of the actual lessons in the class. The mathematics teacher used to lecture and write the solutions on the blackboard. I used to recite the steps he would write next. One day, the teacher was not writing on the board, he was deeply thinking what the next step would be. I recited the next line of solution anticipating he would follow the steps. He was so furious to me. He came to me and asked me to stand up from my seat. He told me that I would be sent out of the class if I repeated the same again. After that, I never spoke ahead of a teacher anticipating what he or she would write next while solving a mathematical problem. I became a listener…and only a listener.

De Jure Learning

I learnt mathematics mostly either myself or from my friends. When I was in early elementary grades, I don't remember anything my teacher taught us specifically, but I remember the things (in mathematics) that I learnt from my friends. I was curious to learn mathematics ahead of classroom practices. That allowed me an opportunity to prepare well enough even before the start of a lesson. I used to go through problem examples and formulas. I used to discuss them with friends. My friends also knew the power of knowing ahead of the class, and they collaborated with me to teach each other and learn from each other. Even we used to conduct exams for ourselves and we used to examine each other's answer sheets and give scores to each other. I still remember how I learnt skip counting from my friends before the teacher actually taught us in the class. In many cases we learnt procedures and operations even without knowing meaning of what we were doing. But, that greatly helped us to make progress in our learning and developing our confidence in mathematics procedurally. When my friends understood the power of studying together, learning from each other dividing roles and responsibilities using "this much you teach me and that much I will teach you" formula. This helped me to make learning circle with friends right from the elementary level to until I was doing my master's degree in mathematics education. My friends were my true '*gurus*' in all of learning journey from 'skip counting' to 'operations research'.

De Novo Research

This is my third attempt to plan and develop my research agenda. The first attempt I made was on "Lived/living mathematical experiences of Nepali mathematics teacher educators". My second try was on "Pre-service teachers' lived/living mathematical experiences of quantitative reasoning and mathematical modeling through technology application in teaching and learning mathematics." My third and current endeavor is on hovering around "Teacher Beliefs, Knowledge, and Practices". Each attempt provided me a unique opportunity to review literatures, develop an understanding of the selected field, and puzzling within the conflict between my interiorized world of research and established knowledge system. Every time my research questions dragged so many issues behind it as if it will be a *messiah* in the world of mathematics education. Maybe it is a characteristic of a novice researcher. This reminds me story of '*Lord Hanuman* carrying *sanjeevani* (a herb) with a mountain' in the epic Hindu text 'The Ramayana'. *Hanuman* did not know which plant was actual *sanjeevani* because to him every plant on the mountain looked like *sanjeevani*. Metaphorically, my struggle to design a research for dissertation or pre-dissertation turned to be a vague attempt requiring enormous trimming and funneling down to a manageable size with specific problem statement, research questions, and (re)defined methodology. As a result, it refreshed my thinking, reasoning, and envisioning new domains of research in mathematics education.

Teacher Beliefs and Practices

The way a teacher practices teaching and learning mathematics in class depends upon various key factors. One of these key factors that influences one's practice is-teacher's mental schemas that constitutes a system of beliefs concerning teaching and learning mathematics (Ernest, 1989a,b). Ernest further identifies three key belief components of mathematics teachers- conception of nature of mathematics, model of the nature of mathematics teaching, and model of the process of learning mathematics. He proposes a model of relationships between beliefs and their impact on practice. This model constitutes the dynamic relationships among view of nature of mathematics, espoused and enacted models of learning mathematics, and espoused and enacted models of teaching mathematics. He argues that "mathematics teachers' beliefs have a powerful impact on the practice of teaching" (p.254). He also suggests that "values as well as beliefs and philosophies play a key role in determining the underlying images and philosophies embodied in mathematics classroom practice" (Ernest, 1991, p. 13). This paper provided me a fundamental idea of belief and practice relationship in mathematics education theoretically. Ernest (1989a,b) lacks empirical bases for the claims he makes, rather he argues his points based upon philosophical and theoretical standpoints.

Many studies (e.g., Batista, 1994; Block & Hazelip, 1995; Brown & Rose, 1995; Buzeika, 1996; Calderhead, 1996; Carpenter, Fennema, Loef, & Peterson, 1989; Day, 1996; Ernest, 1989a,b; Fang, 1996; Foss & Kleinsasser, 1996; Frykholm, 1995; Gibson, 1998; Handal, 2003; Handal & Herrington, 2003; Kagan, 1992a; Kane, Sandretto, & Heath, 2002; McGinnis & Parker, 2001; Munby, 1982, 1984; Nespor, 1987; Pajares, 1992; Perry, Howard, & Conroy, 1996; Perry, Howard, & Tracey, 1999; Prawat, 1990, 1992; Quinlan, 1999a,b; Raymond, 1993; Richardson, 1996; Rokeach, 1968; Schmidt &

Kennedy, 1990; Southwell & Khamis, 1992; Stipek, Givvin, Salmon, & Mac Gyvers, 2001; Stonewater & Opera, 1988; Thompson, 1992; Tillema, 1995; Warkentin, Bates, & Rea, 1993; Weinstein, 1990; Whitman & Morris, 1990) discuss teacher beliefs in general and beliefs toward mathematics in particular. I would like to present review of some of these scholarly works in the area of teacher beliefs in mathematics education.

Handal (2003) discussed teachers' mathematical beliefs, and he argued that these beliefs originated from their learning experiences in schools which eventually reproduced in their classroom teaching. Handal further claimed that the teachers' beliefs may "act as a filter through which teachers make their decisions rather than just relying on their pedagogical knowledge or curriculum guidelines" (p. 47). He argues that teachers may acquire these beliefs through their experiences from their teachers. He expresses his concerns with the teacher education programs in relation to paying more attention to pedagogical knowledge, but a little or no consideration in changing their beliefs. To him, "teacher education programs are not much successful in producing teachers with beliefs consistent with curriculum innovation and research" (Kennedy, 1991, as cited in Handal, 2003, p. 49). He proclaims that our educational system is not successful to bring changes in teachers' beliefs and then in practices, and it is acting as a "vehicle to reproduce traditional *behaviorist* mathematical beliefs" (p. 50, I added the italicized part). He also discusses mathematical beliefs of pre-service teachers. He claims that many of these pre-service mathematics teachers believe mathematics as a discipline based upon rules and procedures to be memorized, there is one best way to solve any mathematical problem, and the mathematical problem solving can be dichotomized either as completely right or completely wrong. He then relates these beliefs to instructional practices. He maintains that the pedagogical beliefs and classroom practices have a dialectical relationship where it is difficult to see which affects which, but to me it is a two-way transaction between beliefs and practices. Mostly, it seems that beliefs impact practices, but there might be an impact of practices with some very powerful experiences that can affect on one's beliefs.

Stipek et al. (2001) claimed that beliefs and values carried by the prospective teachers influence their classroom practices. They argued that it is important to impact on their beliefs in order to bring changes in classroom practices. They studied teachers' beliefs about mathematics teaching and learning and the links between their beliefs and practices. They involved twenty-one elementary teachers in Los Angeles County in California as participants for the study. They also considered the students in the classes they were teaching as the research participants who were present at the beginning and the end of the school year. They utilized student assessments, teacher belief survey, and classroom observations, videotapes of classroom practices as the sources of data. They found that there was an association between teacher's beliefs and their classroom practices in the predicted direction. They claimed that "more traditional beliefs were associated with more traditional practices" (p. 223). They further clarified that "teachers who held more traditional beliefs also gave students relatively less autonomy and maintained a social context in which mistakes were something to be avoided" (p. 223). This study clearly shows interrelationship between teacher beliefs and their classroom practices. I think, in a perfect autonomous environment, teacher beliefs and classroom practices mutually affect each other in a dynamic way.

Yates (2006) conducted a study on one hundred and twenty seven primary teachers using survey tool to measure beliefs about mathematics and teaching and learning of mathematics, and experiences of curriculum reforms. This was a purely quantitative study that utilized statistical measure of central tendency, correlational and factor analyses. Yates further reported that "teachers' beliefs about the nature of mathematics were not statistically significantly related to their beliefs about the teaching and learning of mathematics" (n.p.). He claimed that "teachers' beliefs were not related to their age, qualifications or length of mathematics teaching experience, suggesting that their beliefs had probably been formed through an apprenticeship of observation from their own experiences as students in mathematics classrooms" (Fang, 1996 as cited in Yates, 2006, n.p.). This only reports a partial truth in relation to how teacher beliefs are formed, and how they impact their practices in teaching. The quantitative analysis of beliefs and practices may produce a result that may represent a context, which actually does not exist. Averaging teaching contexts itself is a big flaw in the game of numbers and equations.

Perkkila (2003) studied six primary school teachers' mathematics beliefs and teaching practices utilizing a Likert-scale belief questionnaire that included statements concerning teachers' beliefs and conceptions about mathematics, learning and teaching mathematics, and use of mathematics textbooks. Perkkila adopted Ernest's (1989) categories of views toward mathematics- the instrumentalist view, the Platonist view, and problem solving view- for analyses of teacher's beliefs toward mathematics, learning mathematics, and teaching mathematics. She found that the teachers' teaching practices were influenced by their past experiences of learning mathematics in schools. She further argued teachers' beliefs about the content of mathematics were strongly linked to teaching practice than their beliefs about mathematics teaching and learning. She suggested that "teacher education program should pay more attention to students' own thinking and reflection" (Perkkila, 2003, p. 7). The idea of teacher beliefs within the box of the instrumentalist view, the Platonist view, and problem solving view does not include beliefs of all teachers exhaustively. I think we cannot categorize every teacher's beliefs within these trichotomies. There may be some continuum of such belief system within which we can place one's belief either as more instrumentalist and less of problem solving or vice versa.

Sometimes there can be cases that institutional demand, teachers' ability to cope with situation, time, and resource constraints, and test requirements may produce some degree of inconsistency in one's beliefs toward pedagogy and actual classroom practices (Brosnan, Edwards, & Erickson, 1996; Grant, 1984; Kessler, 1985; Taylor, 1990; Van Zoest, Jones, & Thompson, 1994). Op't Eynde, De Corte, and Verschaffel (2002) present a model of students' beliefs in relation to learning mathematics and problem solving. They claim that mostly research literatures in beliefs toward mathematics learning and problem solving fall into four categories:

(i) Beliefs about the nature of mathematics, mathematics learning, and problem solving
(ii) Beliefs about self in the context of mathematics learning and problem solving i.e. motivational beliefs
(iii) Beliefs about mathematics teaching and social context of mathematics learning, and problem solving
(iv) Epistemological Beliefs, i.e. beliefs about the nature of knowledge and the process of knowing

I think these categories of beliefs are related to the subject matter and process, self-confidence, context (class or community), and philosophical standpoint. Such categorization of beliefs certainly helps to structure the study of beliefs, but yet it is not exhaustive system of categorization.

Op't Eynde, De Corte, and Verschaffel (2002) claim that there is not a joint effort to develop a comprehensive categorization of all students' mathematics related beliefs. They also discuss the very basic nature and foundation of beliefs- psychologically held considerations about the world that are accepted to be true; uncritical acceptance of what we see or hear as product of social life; belief and knowledge operate together; central beliefs are stronger than the peripherals; driving force behind a change in belief is not primarily logical in nature but rather psychological; beliefs serve ego-enhancement, self-protective, and personal and social control purposes. They defined students' mathematics related beliefs as: " students' mathematics-related beliefs are implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context" (p. 27). They clarify beliefs about mathematics education that include beliefs about mathematics as a subject, mathematics learning and problem solving, and mathematics teaching. Beliefs about the self include self-efficacy beliefs, control beliefs, task-value beliefs, and goal-oriented beliefs. Then, they discuss beliefs about social contexts that include social norms in the class (the roles and function of the teacher and students), and socio-mathematical norms in the class. Defining belief or belief system itself is an attempt to dig a hole into the system. To me, definition may help in understanding some aspects of beliefs, but belief as subjective conceptions that one holds to be true is a big flaw. There is no way to judge whether one's belief is true and other's belief is false, but it relates to what one values with respect to others. I think valuing something or some ideas does not mean that we contend it to be true.

Furinghetti and Pehkonen (2002) used nine characterizations of beliefs to discuss agreements and disagreements among 18 panelists of mathematics educators in order to clarify their understanding of beliefs. These nine characterizations included- certain types of judgment about a set of objects; individual's subjective knowledge; individual conceptions referred to general mental structures; assumptions about the presence of entities and different worlds; incontrovertible personal realities held by individual; one's stable subjective knowledge including feelings; individual's empathies and feelings; conscious or subconscious preferences concerning the discipline of mathematics; and stable, long-lasting learned predisposition to respond to certain things in a certain way. They found that some of the disagreements among the panelists were related to the term incontrovertible, stable, conception, and relation between beliefs and knowledge. Their agreements aligned to the origin of beliefs (context, personal identity, and affective), and effect of beliefs on individual's behavior and reaction. They suggest that "contextualization and goal-orientation make the characterization an efficient one" (p. 53). They also highlight the different fields that beliefs concern- mathematics, classroom norms, and an individual's personality. But they keep these concerns open with other different possibilities to be included that "may have consequences both in the individuals' cognitive and affective domains" (p. 53). They claim that "the boundary between affective factors and beliefs is often fuzzy" (McLead, 1992 as cited in Furenghetti & Pehkonen, 2002, p. 53). They also point to the characterization of beliefs as conscious and unconscious beliefs that have a significant impact in one's expectation, participation, and contribution in the mathematics classroom. I agree that beliefs impact one's cognitive and affective domains, and consequently the effect goes to practices.

Leder and Forgasz (2002) discuss commonly used definition of beliefs, ways of measuring beliefs in general and in mathematics education in particular. They argue that there is no common consensus to define belief, however, they indicate that "beliefs, attitudes, and values are intrinsically related" (p. 96). It is worth of discussing the summary of methods for measuring attitude/beliefs. The authors bring attitudes and beliefs together for this purpose. They list the ten different methods of measuring beliefs (and also attitudes): Thurstone's equal-appearing interval scales, Likert-scales, semantic differential scales, Guttman scaling, projective techniques, checklist/inventories, physiological measures, repertory grid techniques, interviews, and observations. Among these methods Likert-scales, interviews, and observations are commonly used methods for measuring beliefs. They also list beliefs in mathematics education research among which "integrated system of personalized assumptions about the nature of mathematics, of students, about learning, and about teaching" (Artzt, 1999 as cited in Leder and Forgasz, 2002) seems to be more relevant to this study. Artzt's (1999) study was based on the theme that "teachers' knowledge, beliefs, and goals influence their instructional practice" (Leder and Forgasz, 2002, p. 101). Although their claim is not a new one, but their argument make more sense in the way they connect beliefs with knowledge and practice.

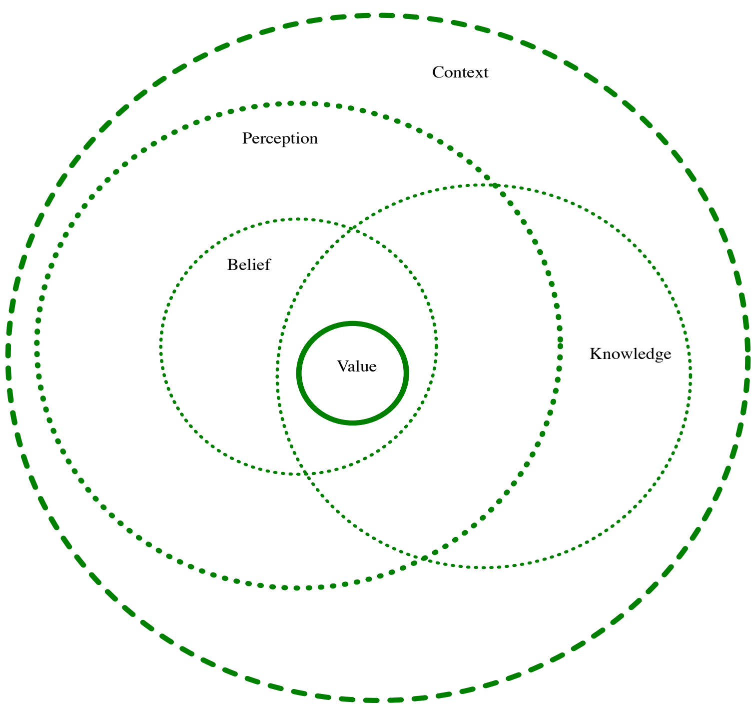

Fig. 1: Interrelationship between belief, knowledge, perception, and value

These studies (as discussed above) indicate complex relationship among beliefs, knowledge, values, perceptions, and practices. These identities form a total system of life world within which an individual behaves in a certain way in certain situation. If I imagine these constructs in a Venn diagram, then belief, knowledge, and perception partially overlap over each other forming a common region at the core constituting personal value (Hatfield, 2012, personal communication). The partial overlapping regions constitute sub-constructs with a complexity of one's affective, cognitive, social, and cultural constructs. These overlaps are very fluid regions except the core that may change over one's experiences and contexts representing qualitative changes in those constructs.

Teacher Beliefs and Change

I already discussed interrelation of belief, knowledge, perception, and values. It seems that these constructs are constitutive of each other and impact each other. Therefore, study of change of beliefs, knowledge, and practice also constitutes a progressive relationship in which they all stand on each other within a context. I am trying to review literatures to see how these construct play a role for change in one's practice in general and how belief changes action in particular.

Wilson and Cooney (2002) report impacts of teachers' beliefs on their ability and tendency to change. They assert that "when the emphasis of research shifts toward a sense-making perspective, boundary lines between knowing and believing become blurred as we seek to understand the phenomena of teacher change and what drives that change" (p. 131). They emphasize that "teacher change consists not solely of changes in classroom behavior, but of conceptualizing in a relativistic way the very act of teaching" (p. 132). They compare the dualistic orientation toward mathematics teaching (emphasizing product without accompanying meaning) with relativistic view of mathematics teaching (emphasizing context and process). They argue that the notion of teacher change in relation to their beliefs and practices attends to their journey from a dualistic orientation to a relativistic one. This change "attends to context, including basing instruction on what students' know, then teaching becomes a matter of being adaptive rather than a matter of using a particular sequence of instructional strategies" (p. 132). They claim that the characterization of reform-oriented teacher is rooted in the ability of the individual to doubt, to reflect, and to reconstruct. Wilson and Cooney (2002) emphasize that:

> Teacher education and mathematics teaching in general become a matter of focusing on reflection and on the inclusion of doubt in order to promote attention to context. This opens a new vista for creating situations in teacher education in which teachers can develop a reflective posture toward their teaching *and a reflexive posture toward their envisioning of future teaching*. (p. 132, I added the italicized part)

To me, the notion of change from dualistic perspective to relativistic perspective seems promising for teacher change. But this change is always dependent on the context within which teachers have to function. In a rigid dualistic environment, a teacher rarely gets an opportunity to bring ideas of relativistic perspective for change to occur in classroom practices. This reminds me a story of a lady teacher in Nepal. She was teaching mathematics in a school. She was motivated by constructivist approach of teaching and learning and she wanted bring change in her classroom practices by engaging students in constructive group activities. The principal was a dualistic oriented person who saw her not teaching (lecturing) but letting students talk to each other making the classroom environment noisy and less controlled. He called him in his office and warned her for not controlling the class and not teaching herself. The school context did not allow her to practice as a relativist/constructivist teacher. This is how context (environment) constraints or supports teacher change.

Wilson and Cooney (2002) analyzed reports of different studies focusing teachers' beliefs and change (e.g., Even, 1999; Even & Torish, 1995; Lloyd, 1999; Lloyd & Frykholm, 2000; Lloyd & Wilson, 1998; Sowder, Armstrong, Lamon, Simon, Sowder, and Thompson, 1998; Wood & Sellers, 1997). They identified and addressed three key themes- reflection, teachers' abilities to attend students' understanding, and content versus pedagogy. They claimed that those studies illustrated the relationship between teachers' beliefs to their thoughts and actions. They found that in those studies "teacher reflection about student understanding is particularly powerful in terms of helping teachers connect their instruction to the work students are doing" (p. 143). Finally, they analyzed teachers' focus on teaching and learning process in relation to content and pedagogy. They found some inconsistencies between pedagogical beliefs and actual classroom practices in a report by Lloyd and Wilson (1998). As I said earlier, the inconsistencies might occur due to unfavorable school environment, lack of parents' support, and lack of resources.

Hart (2002) reported a four-year follow-up study of teachers' beliefs after participating in a teacher enhancement project. This study examined the beliefs teachers hold about their own change process. Hart claims that "a critical component of developing this body of knowledge (*about teacher change*) is teacher's voice" (p. 167). Hart (2002) argues that:

> If teachers' experience change in their beliefs about teaching and learning in a way that is consistent with the philosophy of a particular model of change, then it is imperative not only to the philosophy of a particular model of change, then it is imperative not only to examine the nature of that change and describe and identify factors that facilitate change. In order to develop effective teacher education programs, we must not only identify the presence of change, but teachers' beliefs about their change. (p. 167)

In this study, Hart (2002) utilized a sixteen-item survey listing factors that may have impacted teacher change. She used several generic factors- such as reform movement and innovative curricular materials in the survey. She mailed the survey to fifty-three teachers and received thirty-three back. Besides the survey, she conducted interviews to a sample of teachers in order to confirm and expand the survey data. She asked ten teachers "if they saw themselves having a before and after with respect to the reform mathematics education, and if so, to describe how their teaching had changed" (p. 168). Ninety percent of the respondents to the survey identified three items- colleagues, modeling of strategies, and collaboration- as being very helpful in supporting their change. The interviews identified an open communication, questioning strategies, multiple ways of problem solving, collegiality and collaboration, modeling strategies, reflective practice and debriefing, being in the student position as important factors that lead the teachers feel change in teaching. Since this study utilized multiple sources of data (both quantitative survey and qualitative interviews), the findings seem to be more justifiable and reliable compared to other studies with only one kind of tool.

Chapman (2002) examined two secondary mathematics teachers' beliefs in the context of teaching in high schools "changing their practice on their own from a predominantly teacher-centered perspective to a more student-centered perspective" (p. 177). He characterized the mathematics teachers into three categories- those who change their teaching on their own, those who change their teaching with external support, and those who do not change their teaching in spite of involvement in professional development programs. He is not sure of the underlying pattern of response to change, but attributes the nature of relationship between thought and action as a significant contributor to it. Chapman further claims that "less attention has been given to belief structure, and particularly, its possible relationship to change or growth of the teaching of the experienced teachers" (p. 179). His study focused on four experienced high school mathematics teachers from different schools. He utilized a humanistic approach framed in phenomenology for data collection and analysis. He used interviews, role-play, and class observations as tools for data collection. "The interviews focused on paradigmatic and narrative accounts of the teachers' past, present, and possible future teaching behaviors, and their thinking in relation to word problems, problem solving, and mathematics in general" (p. 180). He discussed how two teachers went through various stages of development (e.g., realization that there was a difference between teaching mathematics and doing mathematics, going through dilemma and tensions, problem solving as a game, making connections, reflecting upon one's own thinking, relaxing of creative engagement). He expressed one dilemma that is "knowing what alternative experiences one should be exposed to in order to influence all of the appropriate beliefs" (p. 192). I think narrative accounts of lived/living mathematical experiences of the participating teachers certainly strengthened this study. Class observation and role-play method complemented the interview narratives increasing the trustworthiness of the data.

Philippou and Christou (2002) examined efficacy beliefs of 157 primary teachers in Cyprus in relation to teaching mathematics. They used five-point Likert-scale with twenty-eight items. They measured the personal teacher efficacy through five dimensions: external and internal interpretation of control, the mathematics teaching anxiety, the mathematics teaching enjoyment, the school climate, and the efficacy beliefs of the pre-service mathematics teachers. They also interviewed eighteen of the participants. The interviews focused on teaching mathematics, pre-service teachers' concerns, and their evaluations of the pre-service teacher education program they attended. They reported "a high level of teacher self-confidence in teaching mathematics, even though they were not successful to control pupils' learning" (p. 221). The interview result showed that there was a high degree of positive personal self-efficacy beliefs, and ability to help non-motivated students, though there was low degree of success in managing the school climate and appreciating the pre-service program.

These studies (as discussed above) clearly identified the interrelationship between beliefs and change mediated through teacher development/education programs. Beliefs certainly impacts change, but there should be an intervening part that is knowledge. If one believes that action research in classroom enhances teacher development leading to teacher change, he or she should have the necessary knowledge of planning, organizing, and evaluating his or her action research. Therefore, a progressive relationship among belief, knowledge, and practice leads to change depending upon context or environment Belief, knowledge, and practice play a significant role in the process of teacher change (conceptually pulling the vertices of teacher change away) that becomes easier in the flexible school environment compared to rigid and structured environment. Flexible context or environment in schools allow teachers to bring in flexible plans that emphasize creative actions, reflections, generative activities expanding their horizons of development with loose, dashed, boundary (Fig. 2b). Inflexible context or environment may limit teachers' generative and reflective activities and the horizons of teacher change within the structured, solid, boundary (Fig. 2a).

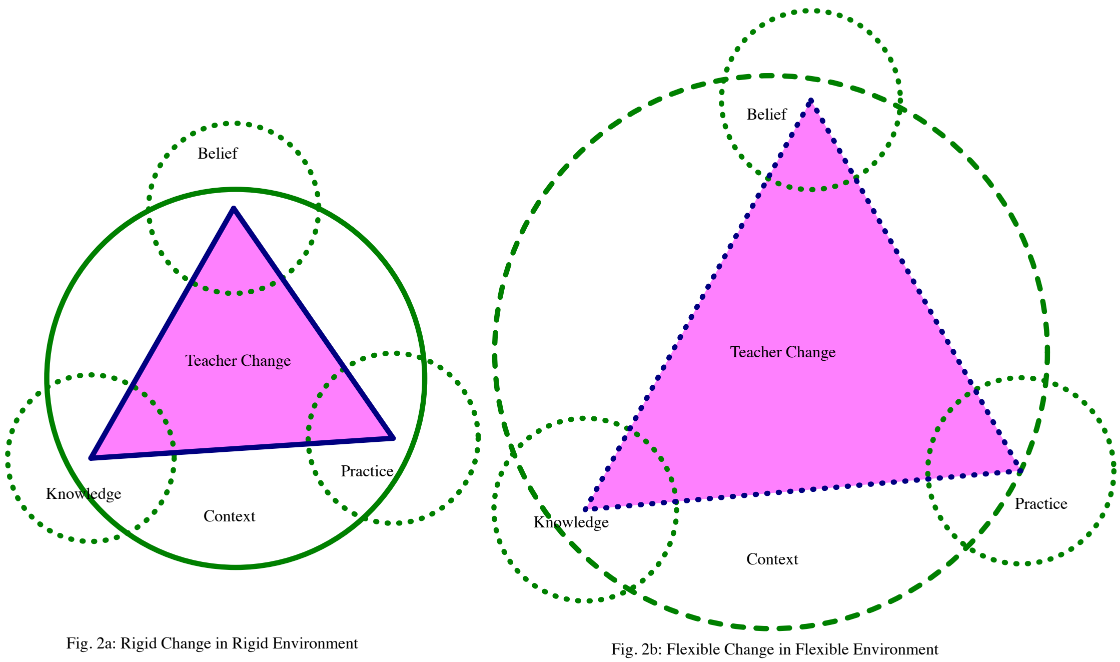

Fig. 2: Impact of environment on teacher change

Teacher Knowledge

Several studies have been conducted on teacher knowledge of content, teaching, and learning (e.g., Ball & Bass, 2003; Ball, Lubienski, & Mewborn, 2001; Calderhead, 1996; Elbaz, 1991; Ethell, 1997; Grossman, 1990; Grossman, Wilson, & Shulman, 1989; Gudmundsdottir, 1991; Hativa, 2000; Hill & Ball, 2004; Hill, Ball, & Schilling, 2004; Johnston, 1992; Lowery, 2002; Marks, 1990; Mewborn, 2001; Omrod & Cole, 1996; Shulman, 1986, 1987; van Driel, Verloop, van Werven, & Dekkers, 1997). These studies on knowledge of contents, students, pedagogy, and classroom context have laid a foundation for teacher knowledge. In this context, Mewborn (2001) states that "the issue of what types of knowledge are essential for teaching mathematics in elementary school has been the subject of numerous studies. Also she reminds us Dewey's claim that knowledge for teaching and knowledge for doing in a discipline are different from each other.

Mewborn characterizes the literature on teachers' knowledge into five major research genres- the studies conducted in the 1960s, 1970s, 1980s, 1990s, and 2000s. "Earlier researches were mostly quantitative studies that sought a connection between teachers' knowledge and students' achievement" (p. 29) and these researcher could not establish a strong relationship between teachers knowledge and students' achievement. Later studies in 1970s and 1980s attempted to characterize the strengths and weaknesses in teachers' knowledge in particular content areas, such as fractions and geometry. The studies in 1990s and 2000s attempted to utilize qualitative analysis of complexity of relationship of knowledge and teaching practice. Mewborn (2001) claims that "many elementary teachers do in fact lack a conceptual understanding of the mathematics they are expected to teach" (p. 30). The literatures do not adequately report that these teachers had the opportunity to learn mathematics conceptually somewhere in their teacher preparation programs (Mewborn, 2001). She reports teachers' misconceptions or lack of conceptual understanding the in the areas of quotitive division, generate a word problem for a whole number divided by a fraction, ratio and fraction, and area and perimeter as a few examples. She claims that "although these teachers possessed some knowledge of mathematics, they lacked adequate knowledge of mathematics as a discipline and/or pedagogical content knowledge to enable them to teach mathematics in ways consistent with current reform efforts" (p. 31). Now I would like to discuss teacher knowledge in mathematics education under the categories of content knowledge (CK), pedagogical knowledge (PK), and pedagogical content knowledge (PCK).

Content Knowledge (CK)

Can one teach mathematics without content knowledge? Can one teach mathematics having content knowledge only? I think it is not possible to even to imagine one teaching mathematics if he or she does not have content knowledge. If I have to teach content (lets say the addition of improper fractions), but I do not have content knowledge (knowledge of improper fraction), then how can I teach kids that content (improper fraction)? However, next question is more thoughtful in terms of teaching mathematics when one has content knowledge only. If I understand what teaching means and what activities teaching mathematics demands, together with content knowledge, then teaching mathematics becomes comfortable, joyful, and meaningful. Then, this means, having content knowledge is necessary but not a sufficient for teaching mathematics. Therefore, content knowledge can be necessary, but not sufficient condition for addition of improper fractions (or any topic in mathematics).

Ball, Thames, and Phelps (2008) report new progress on the nature of content knowledge for teaching. They mention that a central contribution of Shulman and his collaborators was to reframe the analysis of teacher knowledge in a way that focuses the role of content in teaching. They further emphasizes that subject matter was little more than context. "Next contribution of Shulman and his colleagues was to represent content understanding as a special kind of technical knowledge key to the profession of teaching" (Ball et al., 2008, p. 390). More specifically, content knowledge is the knowledge about the subject and subject matter, for example, mathematics and its structure (Shulman, 1986).

Shulman (1986) points at the items on the 1875 California Teachers Examination that dealt with subject matter alone (almost). Only about 50 points out of 1000 possible points were given over to the 10-item subtest on Theory and Practice of Teaching. This shows that the teachers examination in those days were fully content driven. Those examinations covered written arithmetic, mental arithmetic, and algebra among many other areas (outside mathematics). There was no geometry in the test, but there was industrial drawing and may be it covered aspects of geometry. Whatever the area (subject) of the test was, “ninety to ninety-five percent of the test was on the content, the subject matter to be taught or at least on the knowledge base assumed to be needed by teachers, whether or not it was taught directly” (p. 5).

Shulman (1986) contends that the emphasis on the subject matter to be taught stands in sharp distinction to the emergent policies of the 1980’s with respect to the evaluation of teachers. He further mentions that “policy makers read the research on teaching literature and find it replete with references to direct instruction, time on task, wait time, ordered turns, lower-order questions, and the like” (p.5). “They find little or no references to the subject matter, so the resulting standards or mandates lack any reference to content dimensions of teaching” (p.5). “Similarly, even in the research community the importance of content has been forgotten” (p. 5) and Shulman says this as a missing paradigm in teacher preparation during the period. He further points to the spirit of 1870s as pedagogy essentially ignored and in the 1980s the content was conspicuously absent. He questions, “Has there been a cleavage between the two? Has it always been asserted that one either knows content or pedagogy is secondary and unimportant, or that one knows pedagogy and is not held accountable for the content?” (p. 6).

For Shulman (1986), Bloom’s cognitive taxonomy, Gagne’s varieties of learning, Schwab’s distinction between substance and syntactic structures of knowledge, and Peter’s notions (that parallel Schwab’s) are different ways to represent content knowledge. Shulman argues that content knowledge involves going beyond knowledge of the proofs or concepts of a domain.

> “It requires understanding the structures of the subject matter that include both the substantive and syntactic structures. The substantive structures are the variety of ways in which the basic concepts and principles of the discipline are organized to incorporate its facts. The syntactic structure of a discipline is the set of ways in which truth or falsehood, validity or invalidity, are established. (Shulman, 1986, p. 9)

Goldschmidt and Phelps (2007) argue that teacher educators and others concerned with teacher excellence and preparation have recognized the importance of teachers having an essential mastery of content. They further mention that concern over the subject matter preparations is incorporated in state and federal policies. They refer to No Child Left Behind (NCLB) Act of 2001 requiring teachers pass the state test demonstrating subject matter knowledge in mathematics (including other subjects). This mandates that teachers need to understand content as it is taught and learned in mathematics classes.

Shulman (1986)’s analysis of focus on contents in 1870s and on pedagogy on 1980s shows a total shift of paradigm in a century, but still we are grappling with the question of what is a proper balance. Now the NCLB Act (2001) focuses importance of content knowledge together with pedagogical knowledge for teaching in schools. National Council of Teachers of Mathematics (2000), in its Principles and Standards for School Mathematics, has explicitly identified five core strands as content standards from grade kindergarten through grade twelve. These strands are (i) number and operations, (ii) algebra, (iii) geometry, (iv) measurement, and (v) data analysis and probability. NCTM (2000) states six principles- equity, curriculum, teaching, learning, assessment, and technology- for school mathematics. These principles emphasized content of mathematics to be appropriately challenging that encourage students to learn increasingly more sophisticated mathematical ideas and help teachers to be able to draw on their knowledge with flexibility of their teaching.

Pedagogical Knowledge (PK)

As I already discussed from Shulman (1986) that the pendulum bob has swung from the reign of contents in 1870s to reign of pedagogy in 1980s. Focus on pedagogy had been realized during the beginning of the reform period and the result was that there was excessive focus on pedagogical knowledge. Vistro-Yu (2005) states-

> Pedagogical knowledge in mathematics is that kind of knowledge that a teacher uses to deal with the everyday task of teaching and relating to students in the classroom. It is that kind of knowledge that teachers hope to improve when they say they want to become better teachers because they realize that this is where they draw all the "tricks" that they can muster to make their students' learning experiences valuable. (p. 2)

National Council of Teachers of Mathematics (2000) has stated five process standards: problem solving, communication, connections, reasoning and proof, and representation. To me these process standards are an indication toward an emphasis on pedagogical knowledge of teachers. Mathematics teachers should be prepared to teach contents and process standards are guidelines about what pedagogy they should have in order to teach mathematics the way they are expected to teach. Problem solving standard states that solving a problem is not only goal of learning mathematics, but also a major meaning of doing so. Standards for reasoning and proof states that mathematical meaning and proof offer a powerful way of developing and expressing insights about a wide range of phenomena. The standard further says that by exploring phenomena, justifying outcomes, and using mathematical assumptions in all content areas and with different anticipations of sophistication at all grade levels, students have to understand that mathematics makes sense. The communication standard emphasizes that through communication, ideas become the object of reflection, refinement, discussion, and amendment. The connection standard states that mathematics is not a collection of separate strands or standards, even though it is often portioned and presented in this manner. This standard shows NCTM's view that mathematics is integrated field of study. Finally, representation standard states that mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, numbers, and letter symbols, spreadsheet displays, and so on. The document further clarifies that the ways in which mathematical ideas are represented is fundamental to how people understand and use those ideas. These process standards implicitly clarify the importance of pedagogical knowledge for mathematics teachers though it does not speak the word 'pedagogy' specifically in those standards. May the word 'pedagogy' is more political and NCTM wants teachers to be autonomous in pedagogical choice or 'pedagogy' reflects personal philosophical and political stand in which NCTM wants to be neutral.

Pedagogical Content Knowledge (PCK)

Shulman (1986) states that pedagogical content knowledge includes the most useful forms of depiction of ideas, the most powerful associations, illustrations, explanations, and proofs in a word, the most useful ways of demonstrating and formulating the subject that make it logical to others. He further states:

> Pedagogical content knowledge also includes an understanding of what makes the learning specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1986 as cited in Ball et al., 2008, p. 392)

PCK has three components: knowledge of content, knowledge of curriculum, and knowledge of teaching (An, Kulm, & Wu, 2004; Turnuklu & Yesildere, 2007). According to An, Kulm and Wu (2004), knowledge of teaching is the core component of pedagogical content knowledge. “PCK includes useful representations, unifying ideas, clarifying examples and counter examples, helpful analogies, important relationships, and connections among mathematical ideas” (Grouws & Schultz, 1996, p. 443). Mathematical content knowledge and pedagogical content knowledge are integral parts of effective mathematics instruction and construction of mathematical concepts in students’ mind (Shulman, 1986; Turnuklu & Yesildere, 2007). PCK is associated with the way in which teachers communicate their subject matter knowledge to their PK that enriches in pedagogical thinking, reasoning, and thoughtful constructs of mathematics (Cochran, DeRuiter, & King, 1993; Kahan, Cooper, & Bethea, 2003).

In their study, Turnuklu and Yesildere (2007) found that having a profound understanding of mathematical knowledge was an essential, but not adequate to teach mathematics. They pointed out the relationship between knowledge of mathematics and knowledge of teaching mathematics. They claimed that all mathematics teachers should be educated both from the viewpoint of mathematical knowledge and pedagogical content knowledge. Strawhecker (2005) reported that the teacher and the quality of teacher preparation influence student achievement and attitudes toward mathematics. She concluded, from her study, that the field experience and other aspects of mathematics teacher preparation influence pre-service teachers’ pedagogical content knowledge for mathematics.

I think PCK is a conglomeration of mathematical knowledge, mathematical content knowledge, pedagogical knowledge, and knowledge of pedagogical relationships. Here, pedagogical relationship, to me, is a central part in PCK. According van Manen (1994) pedagogical relationships are of three types: personal, intentional, and interpretive. If a teacher has mathematical, content, and pedagogical knowledge, then he or she may or may not have pedagogical sensibilities in terms of his or her relationship to students. At a personal level, the relationship between students and teacher helps to understand each other. This is very important for a teacher to be pedagogically caring. I agree that a teacher has intentional relation with students in terms of helping them develop their full potential to grow intellectually (e.g., mathematically and linguistically). At interpretive level of pedagogical relationship, a teacher should be able to interpret and understand the present situation and experiences of the child and anticipate the moments when the child in fully becomes able to participate in the culture (van Manen, 1994). Therefore, to me, PCK is greater than just combination of CK and PK because in these relationships whole is always greater than just mechanical sum of parts.

Relationship among CK, PK, and PCK

We saw in Shulman (1986) that mathematics education in 1880s was almost content driven, and after a century it was pedagogy driven in 1980s. I think now we are somewhere in between these two continuums. National Council of Teachers of Mathematics (2000) has tried to balance the content and pedagogy (process) by clearly stating five content standards and five process standards, though it does not clearly mention about pedagogy.

Ball (2000) identifies two aspects central to practice and actual work of teaching: the capacity to deconstruct one's own knowledge into less refined and concluding form, where critical components are comprehensible and discernible. Ball mentions that knowing for teaching requires a transcendence of the tacit understanding that characterizes and is sufficient for personal knowledge and performance. Certainly it goes to pedagogy of caring (van Manen, 1994) and making sense of students' errors or appreciate their unconventionally expressed insight (Ball, 2000) requires this remarkable capacity to unpack one's own highly compressed understanding. Then content knowledge, pedagogical knowledge, and pedagogical content knowledge become an amalgamation of teachers' knowledge. Also, they complement each other, may be, though this knowledge may not be mutually exclusive and exhaustive. I think, there is not a clear boundary among CK, PK, and PCK. The boundary is blurred, and it is permeable to each of the knowledge (CK, PK, and PCK).

Turnuklu and Yesildere (2007) claimed that having a profound understanding of mathematics is necessary, but not sufficient to teach mathematics. They further stress that it is not possible to teach mathematics without having pedagogical knowledge as well. Their claim clearly shows an integral relationship between content knowledge and pedagogical knowledge. Shulman (1987) argues that the actions of both policy makers and teacher educators in the past have been consistent with the formulation that teaching requires basic skills, content knowledge, and general pedagogical skills. He further says that teaching is trivialized, their complexity ignored, and its demands diminished. "Teachers themselves have difficulty in articulating what they know and how they know it" (p. 5). Shulman proposes a model of pedagogical reasoning and action that consists of comprehension, transformation, representation, selection, adaptation, instruction, evaluation, reflection, and new comprehensions. I think the model well combines content knowledge, pedagogical knowledge, and pedagogical content knowledge. "A proper understanding of the knowledge base of teaching, the sources for that knowledge, and the complexities of the pedagogical process will make the emergence of such teachers more likely" (Shulman 1987, p. 20). This is immensely powerful notion of combining the three aspects (CK, PK, and PCK) for a model of pedagogical excellence that should be or can be a basis for reforms in mathematics education.

Technology: A New Dimension in Teacher Education

Technology has influenced all sectors of life: individual and social, private and public, and almost all kinds of professions. In this context, today's teachers have a challenge to know about technological tools so that they can effectively use them in classroom teaching of mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) in its technology theme states that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (n.p.). NCTM also states its position in relation to the role of technology in teaching and learning mathematics that clearly highlights the significance of technology tools in effective teaching and learning of mathematics. Today's teachers need to align their mathematics lessons to the Common Core Standards (CCS), which demands application of technology in teaching and learning mathematics.

Researchers (e.g., Hatfield, 1984; Heid, 2005; Olive, 2011) discuss the importance of technology in mathematics education. These researchers claim that pre-service teachers' awareness, knowledge, and experiences of different technological tools and their skill to integrate technological tools into teaching and learning of mathematics can have a significant impact on students' learning of mathematics. Pre-service or in-service teachers' beliefs, attitudes and values toward technology in teaching and learning mathematics may impact these generative utilities of technological tools. Various studies (e.g., McGinnis et al., 1996; Tharp et al., 1997) discussed pre-service/in-service teachers' belief toward the use of technology in teaching mathematics. Olive (2011) discusses research about the use of technology and its effect on learning and teaching outcomes. Investigating how the different uses of technology affect learning and teaching outcomes should also be a focus of the research on learning and teaching with technology, as well as a concern of the design research (p. 82). It is obvious that there is a high expectation of technology integration in mathematics education.

Carlson et al. (2002) argue that in a technology based age of information, complex systems are fundamental objects of study in their own right – not only because such systems are important objects, that impact lives of ordinary people, but also because (mis)understandings about them underlie great many beliefs, principles, policies, and processes that need to be understood for informed citizenship and ethical behavior in complex societies. They further clarify that "kind of 'systemic understandings' that are highlighted in the context of complex systems also are involved in the development of many other types of concepts, skills, principles, beliefs, attitudes, and problem solving processes that are not as obviously systemic in nature" (p. 46). I think technology tools and applications are critical to resolve issues of complexity in the systemic environment. These tools and applications also help understand very dynamic and chaotic nature of teaching, learning, and teaching learning of mathematics. I say these activities are dynamic because both teachers and learners move from one level of understanding to the next level each time they go through mathematical pedagogical activities mediated through artifacts and technology. Also, they are chaotic in the sense that it is sometimes very complex and even impossible to predict what will happen next. A small (simple) incident in teaching learning (often overlooked) can create a tremendous impact on one's vision of life and life-world.

Lampert and Ball (1998) bring the issues of pre-service elementary mathematics teacher education programs in two state universities. They connect their experiences as teachers, teacher educators, and then as teacher education researchers. They experimented impact of technology (multimedia environment) in elementary mathematics teacher education in two universities. They attempted to construct the kind of teaching and learning widely promoted as "teaching for understanding" by bringing their classroom teaching into the arena of formal, university-based *teacher education* that was the subject of intense critique at the time. They engaged pre-service mathematics teachers in a technological environment (mainly computer and videos) designing a different kind of opportunities for teachers to learn different kind of things about pedagogical practices bringing their experiences as teachers, their awareness of the ineffectiveness of teacher preparation, their understanding of practice, and their appreciation of perspective teachers as learners. They came to understand that their own teaching provided a context for their thinking about learning to do that kind of teaching. They studied dilemmas that they faced regularly and examined the issues of the teacher's role and of the social complexities of group-work in classrooms, and they thought what that meant for teacher education.

Nordin et al. (2010) investigated the pedagogical usability of a digital module prototype that integrated dynamic geometry software, Geometer's Sketchpad (GSP) in mathematics teaching. Their pedagogical application criteria included student control, student actions, objective-oriented, function, value added, enthusiasm, knowledge value, flexibility, and response. They found that the sample digital modules met the necessities of the pedagogical usability criteria, enabled integration of GSP in mathematics teaching. They also suggested a study on the application of GSP in mathematics teaching in order to stimulate higher order thinking skills among high school students.

Hoong (2003) investigated how far-reaching use is of GSP in the secondary schools, and how the tool is applied by teachers in their geometry lessons. He used angle properties of a polygon, angle properties of points/lines, angle properties relating circle, congruency, coordinate geometry, geometrical constructions, locus, mensuration, properties of special quadrilaterals, properties of triangles, Pythagoras theorem, similarity, symmetry, transformations, trigonometric rules and formulas, trigonometrical graphs, vectors, and others in order to see the relevancy in relation to textbook, teaching, and learning. He found that 80% of teachers used the GSP in teaching of mathematics in different topics in geometry. Most of them were found to use the GSP for teaching transformations compared to other areas of geometry. Stigler and Hiebert (1999) argue that "one other approach to understanding the difficulties of integrating information technology (IT) in the classroom stems from seeing teaching as a complex *cultural activity"* (p. 97) and this complexity of culture sometimes becomes a barrier for change in teaching and learning. There are many literatures (e.g., Bennett, 1994, 1995; Boehm, 1997; Olive, 1998 to name a few) about the use of the GSP in teaching and learning of geometry in schools and higher education. The dynamic nature of the GSP has revolutionized the application of this tool in mathematics classrooms.

Leatham (2002) focuses on pre-service mathematics teachers' beliefs and transformation in their beliefs and practices of teaching and learning mathematics with technology through mathematics methods course. Using qualitative grounded theory constant comparative method to analyze and discuss four cases on beliefs about mathematics, beliefs about teaching mathematics, beliefs about learning mathematics, and beliefs about teaching with technology, he found that the pre-service teachers' beliefs about teaching with technology were directly linked to their beliefs about learning with technology. He identified the nature of mathematics, mathematics teaching, learning, and technology from the study of these cases. Leatham reports Ben's (one of the participant's) beliefs toward mathematics as- mathematics is not just set of rules, but it is more than that. He (Ben) believes problem-solving tools in terms of logical thinking and reasoning. He also believes that "applying the logic of mathematical problem solving to real-world situations where decisions need to be made is a powerful use of the tools in his mathematical toolbox. Ben strongly believes on conceptual understanding of mathematics even though he was more confident with procedures. In relation Ben's beliefs toward mathematics teaching, Leatham (2002) further states that teacher's role to create an environment that motivates students to learn is a very powerful one. He could utilize varieties of teaching approaches being flexible with the classroom context in general and individual student's need in particular. Ben had both positive attitude and good knowledge of using Geometer's Sketchpad in geometry class. However, he did not believe that any of his high school experiences (e.g., Calculus) constituted teaching with technology. Leatham claims that Ben's involvement in teacher education program allowed him to understand different roles of technology- for example, facilitating exploration, and visualization.

The relationship between content knowledge, pedagogical knowledge, and technological knowledge form new epistemological ground- pedagogical content knowledge, pedagogical technological knowledge, and technological content knowledge. These overlap further to create a zone of technological-pedagogical-content knowledge (TPCK) (Koehler, 2011, Fig. 3). The complexity of relationship of beliefs toward mathematics content (CK), mathematics teaching and learning (PK), and teaching and learning with technology (TPCK) may have a potential impact on pre-service teachers' practices of teaching mathematics. This study seeks to understand how the beliefs and practices of the pre-service secondary mathematics teachers influence each other while moving from methods and geometry class to residential practice. Figure 1 shows the conceptual model of teacher knowledge with complex relationship among CK, PK, TK, PCK, TPK, TCK, and TPCK with a context. Mishra and Koehler (2006) agree that "in practical terms, this means that apart from looking at each of these components in isolation, we also need to look at them in pairs: pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK), and all three taken together as technological pedagogical content knowledge (TPCK) *altogether constituting teacher knowledge*" (p. 1026, I added the italicized part).

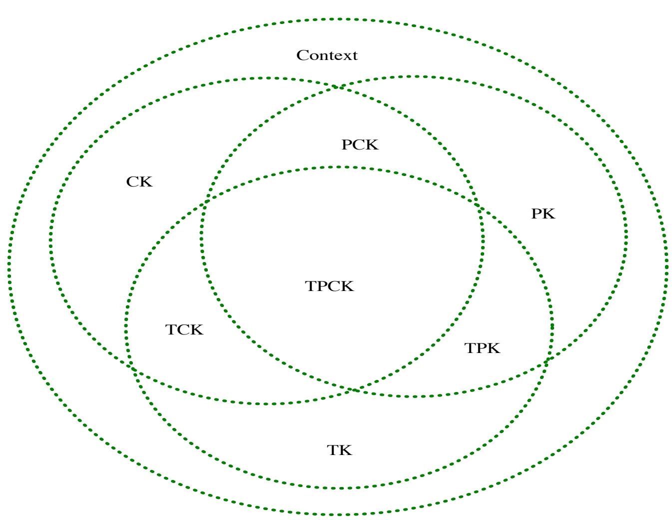

Fig. 3: Conceptual Model of Teacher knowledge (Adapted from http://tpack.org/)

New Directions and Future

Within the background of 'de facto teaching', 'de jure learning', and 'de novo research' I am motivated to study an issue that makes sense to me as a 'to be teacher educator' of mathematics education. Then, current area of study "Beliefs, Knowledge, and Practices in Mathematics Education" makes me feel that I am in the right shooting spot, but still it sounds a big field to me. There are many studies that focus on one aspect, either beliefs or knowledge or practices or at most combination of two of them. Until now, I am planning to carry all the three together to give a complete sense of connecting beliefs with knowledge and practices of pre-service teachers. From the review of literatures I came know that belief, knowledge, and practice are the three important constructs that affect each other, and also overall teaching and learning in the classrooms. "Each informs and is informed by others" (Askew et al., 1997, p. 21). Askew et al. further mention that teacher's effectiveness is greatly affected by the interrelationship of these constructs. Askew et al. (1997) argue that:

> Our starting point for understanding effective teachers is a model of *teachers' classroom practices* informed by two complementary aspects: a set of *beliefs*, and a collection of knowledge (including subject knowledge) and understandings that teachers have about numeracy and its teaching which we refer to as *pedagogic content knowledge*. (p. 21)

In this context, today's teachers have a challenge to know about technological tools so that they can efficiently use them in classroom teaching of mathematics. They are expected to be familiar with many of these tools, and they should be competent users of some of these technological tools in teaching mathematics. The National Council of Teachers of Mathematics (NCTM, 2000) in their Principles and Standards for School Mathematics included technology among the six over arching themes. Technology theme states that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (n.p.). NCTM also states its position in relation to the role of technology in teaching and learning mathematics that clearly highlights the significance of technology tools in effective teaching and learning of mathematics not only for increasing proficiency in mathematics, but also for access to mathematics.

> Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology. Effective teachers maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When technology is used strategically, it can provide access to mathematics for all students. (NCTM, 2008, n.p.)

The federal, state, and local educational institutions have highly emphasized the integration of technology into teaching and learning mathematics. Schools are trying to introduce technology in mathematics and science classes for effective teaching and learning. Then, teacher preparation programs in colleges of education have the responsibility to prepare teachers who can integrate technology in an efficient way to teach mathematics (Abbitt & Klett, 2007). Abbitt and Klett (2007) further point out notable variability in nature and practices of technology integration in mathematics teacher education programs across institutions of higher education. Pre-service mathematics teachers' knowledge and experiences of technology tools and how to integrate technology into teaching and learning of mathematics can have a significant impact on students' learning of mathematics.

Past studies on beliefs and practices of pre-service teachers toward mathematics teaching and learning with technology focused static analysis. There is scarcity of studies that analyzed the dynamism of beliefs, knowledge, and practices over the learning experiences from content and methods class to residential practices. This study intends not only to understand pre-service teachers' knowledge and beliefs toward the nature of mathematics, teaching, and learning in conjunction with technological tools, but it also examines the student teachers' practices of using technological tools in their teaching and learning mathematics considering dynamism of those beliefs, knowledge, and practices as the central aspects of their experiences.

The agenda of Wyoming Institute for the Study and Development of Mathematical Education (WISDOM^e) to conduct research in the three principal identities- Quantitative Reasoning and Mathematical Modeling (QRaMM), Technological Tools and Applications in Mathematics Education (TTAME), and Developing Investigations in Mathematical Experiences (DIME) has generated a new hope in the field of research in mathematics education. The WISDOM^e planning conference in September 8-10, 2010 provided me a unique opportunity to know about these fields to some extent. This virtual (web-based) institute aims to "stimulate and support collaborative research interactions among a global alliance of participating researchers" (Hatfield, 2010, p. 7). In this context Steffe's (2010) abstraction of epistemic students makes a perfect sense. "Epistemic students are interiorized others- the dynamic organizations of schemes of actions and operations in one's mental life" (Steffe, 2010, p. 22). Recognizing and conceptualizing that interiorized other is or should be the goal of studying beliefs, knowledge, practices, and teacher change (development) in mathematics education. I/we have a long way to go, and this is possible through collaborative efforts personally and institutionally. These new directions have certainly heightened my hope to draw a new line within the field of mathematics education in general and "beliefs, knowledge, and practices" in particular in the changed context of "De Facto Teaching, De Jure Learning, and De Nomo Research".

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