Clarity in mathematics instruction: The impact of teaching number sense and place value

skills on elementary school students

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Abstract

There is a concern among educators in schools with high levels of poverty that students are

lacking certain academic strategies, especially in mathematics. These students struggle to explore

data systematically and procedurally. The purpose of this pre-posttest design study is to

intentionally teach number sense skills in order to increase mathematic comprehension. Most of

the students at this school struggle with mathematics concepts-specifically number sense related

to place value. As a result, the treatment given will be repeated practice with place value and

other work with number sense. The treatment was implemented in a fifth grade classroom

containing 26 students. Findings indicated gains from pre- to post-test. The implications for

practice will be discussed.

Keywords: Mathematics Instruction; Elementary School Teachers; Feedback; Error Patterns;

Elementary School Mathematics; Comprehension; Computation; Mathematics Achievement;

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on elementary school students

Is 93 close to 90? Fifth grade students at this school often answer "no" to this type of question when the answer seems so obvious. This question, among a plethora of similar ones, is one that students at this elementary school struggle with on a regular basis. Lack of strong number sense and place value, the basis of understanding mathematics, is where the problem begins for many of the students. Thus, something must be done to fix this problem.

Clear teachers do far more than speak or lecture in an organized and easily comprehensible way, though that is not irrelevant to student understanding (Saphier, Haley-Speca, & Gower, 2008). The teacher researchers consulted the work of various researchers throughout the process of planning this action research plan. The main ideas included in the research are (1) literature on the effectiveness of clear instruction, (2) effective strategies for increasing number sense for students and (3) effective assessments for the purposes of data collection, analysis and interpretation.

The study was based on the action research plan presented by Mills (2010). The first aspect of the action research plan is literature on the effectiveness of clear instruction. There are five subheadings under this main idea of clarity of instruction which include: (a) framing the learning, (b) presenting information, (c) creating mental engagement, (d) getting inside the students' heads (cognitive empathy), and (e) consolidating and anchoring the lesson. Of the five, the teacher researchers focused most effort toward getting inside the students' heads, which will be referred to from here on as *cognitive empathy*. Cognitive empathy means the teacher is viewing the learning experience form the student's perspective and making decisions from the frame of reference (Saphier, Haley-Speca, & Gower, 2008).

One of the main concerns about teaching mathematics in an urban elementary school is captured in the following scenario. The teacher asks a student, "What is 9 plus 5 more?" and the first student answers, "9" while a second student answers, "7". Although the answers do not make sense to the teacher, there must be some thought on the part of the student as he/she answers the question. Through this action research plan, the teacher researchers were determined to find a way to *make students' thinking visible* which may assist the teacher in clearing up this misconception or confusion.

The teacher-researchers used three strategies to address the concern of math clarity in elementary classrooms: (a) questioning technique, (b) unscrambling confusions, and (c) checking for understanding.) The strategy the teachers used is a questioning technique that took place in the form of an interview. Possible questions were: How did you get that answer? How did you approach this kind of problem? Can you tell me what you did or thought about it? What did you try first? Why? What did you think this might mean? (Saphier, Haley-Speca, & Gower, 2008). By taking time to record and analyze student responses, using the "Think Link" sheet (see Appendix A), teachers hoped to be able to comprehend what lead to the student's initial "off the wall" answer.

The teachers also implemented a second strategy known as unscrambling confusions. When we detect that students are confused, the next clarity task is to find out what the students are confused about and tailor re-explanations accordingly. Teachers have many choices when they sense a student is experiencing confusion. The teacher can do nothing, re-explain, isolate the point of confusion with pinpoint questions (explained above), have a student explain his or her own current thinking or persevere and return (Saphier, Haley-Speca, & Gower, 2008). This action research plan required teachers to use each of these strategies throughout the six week

treatment of students. The teachers' main concerns were to monitor student confusion, collect

data both verbally and numerically, as well unscramble confusions students have throughout the

six weeks of treatment.

The third and last strategy teachers used to determine the effectiveness of clear

instruction was checking for understanding as outlined in Saphier and colleagues, *The Skillful*

Teacher (2008). There were many ways to check for understanding through the six week

treatment. First, teachers checked for body language. Students at this school are highly

expressive which made it easy, in most cases, for teachers to perceive when they are confused

and when they understand a concept. During this action research plan, teachers frequently

checked for understanding by asking recall questions that come directly from the strategy

instruction. Lastly, teachers used a strategy called "dipstick," quite often, which meant the

teacher checked for understanding across the class as a whole.

The teacher researchers used methods such as thumbs up, white board responses, and

self-assessments in which students rated their understanding on a scale of 1-5. Together, the

strategies of cognitive empathy, unscrambling confusions, checking for understanding through

questioning, and "dipsticking" (Saphier, Haley-Speca, & Gower, 2008) provided teacher

researchers with the ability to ensure clarity in their instruction.

The next step for the teacher researchers was to implement effective strategies to increase

number sense for students. Children must be able to make sense of the various ways numbers

are used. They need to develop a sense of number that enables them to recognize relationships

between quantities; to use the operations of addition, subtraction, multiplication, and division to

obtain numerical information; to understand how the operations are related to one another; to be

able to approximate and estimate when appropriate; and to be able to apply their understandings

to problem situations. The teacher researchers implemented various strategies, learned through activities, contained within the number category of the NCTM standards. A 300s chart, placemat activity, card game and another place value game were implemented during this study. These strategies are explained further in the methods section.

Building on the research of Rovee-Collier (1995), Nuthall (1999) found that students require about four exposures to new informational knowledge to adequately integrate it into their existing knowledge base (Marzano, 2007). The text goes on to explain that students need exposure to these new concepts without having gaps in days. The teachers implement effective mathematical strategies daily in order to expose the students to the information as much as possible. Students were assessed twice a week in order to provide immediate and descriptive feedback. The teachers' goal was to assist students in finding errors in their own thinking in hope that the students will make gains during the six week treatment period.

Lastly, the final phase of learning a procedure is to develop it to the level of fluency (Marzano, 2007). Fluency in this case, means students will be able to solve problems efficiently and with mathematical precision. The teachers provided students with ample practice time in order that students develop fluency with number sense in terms of their knowledge of doubles, using operations effectively, as well as clarifying place value to the point in which a student can clearly explain his or her thinking.

The task in this action research plan is for teachers to construct effective assessments for the purposes of data collection, analysis and interpretation. In order to systematically collect data, the teachers created a daily routine for the treatment. The teachers clearly explained the routine and gave detailed, modeled instructions to ensure student comprehension (Saphier, Haley-Speca, & Gower, 2008). To assist in the data collection aspect of the action research plan,

the teacher implemented this twenty minute number sense time period at the beginning of each

math lesson. The students were comfortable with the routine as well as the fact that another

teacher might have come into the room at this time to interview them in order to collect data.

        In terms of the assessments themselves, teachers collected both quantitative and

qualitative data. The teachers used this mixed-methods design in order to triangulate data and

thus determine the effect size and percentile gain of the treatment. In terms of qualitative data,

the teacher researcher used weekly interviews to collect student responses reflecting their

number sense and knowledge.

        The teachers also collected quantitative data in various forms. In the first six weeks,

students completed a pre assessment as well as a post assessment in the form of the district

mandated Math Proficiency Assessment. The data collected was entered into Microsoft EXCEL

and analyzed. The students also took a pre assessment at the beginning of the second six weeks,

followed by the treatment and ending with a post assessment. Other assessments using

quantitative data included exit slips, quizzes and the students' test scores from the previous year.

This data will also be entered into an EXCEL spreadsheet and analyzed thoroughly.

        The question of focus is "How do I make number sense clear and accessible to students?"

More specifically, the goal is to achieve cognitive empathy, or to "get inside students' heads.' "

The teachers of this study desire to understand what students think about when asked if 26 is the

same as 62. Where do students' brains begin when processing the question and what steps do

they take to come to the final answer? During the study, students will be given regular

opportunities to refine and clarify their understanding of place value concepts and then practice

using the concepts correctly. The teacher will be using various manipulatives and activities to

discover students' areas of confusion and then help build correct knowledge of place value. The

hope or goal for the outcome of this study is that by students gaining confidence and mastery of place value, they will be more successful math students overall.

In conclusion, the teacher researchers gained much knowledge from the literature reviews of many acclaimed researchers. The main tasks within the literature review for this action research plan were to determine (1) literature on the effectiveness of clear instruction, (2) effective strategies for increasing number sense for students and (3) effective assessments for the purposes of data collection, analysis and interpretation. Through strategy based, clear instruction, and a mixed-method design for collecting and analyzing data, the teacher researchers determined whether the treatment group had a statistically significant gain on the post assessment.

Method

Context of the study

The urban elementary school contains about 350 students. About ninety-five percent of the students qualify for free or reduced lunch, making this setting a Title I school. There are about 110 English Language Learners (ELL) students in the school from various ethnic backgrounds, about fifteen white students, and the remaining 225 are African-American students.

Participants

In the treatment class, there are 15 females and 11 males for a total of 26 students. 13 of the students are English Language Learners. Nine students are Hispanic, one is Caucasian, two are Asian and 14 are African American. All 26 students qualify for free/reduced lunch. Participant Characteristics are shown in Table 1 below.

Table 1

Characteristics of Participants (N = 26)

	n	%	M	SD
Pretest			57.62	17.75
Gender				
Male	11	42.31		
Female	15	57.69		
Race				
African-American	14	53.85		
White	1	3.85		
Other	11	42.31		
Free/Reduced Lunch	26	100.00		
ELL	13	50.00		

Instrumentation

Quantitative data was the only data collected during the first six weeks of school. The group of fifth graders took the Math Diagnostic Assessment in the beginning of the first six weeks of school. The students took the same assessment again at the end of the first six weeks of school. These two scores were to be used as the pre-assessment and post-assessment the study.

During the second six weeks, both quantitative and qualitative data was collected. At the beginning of the second six week period, students took the Math Proficiency Assessment #1 again. During the second six weeks period, all of the students in the study received the treatment of the focused practice with place value. At the end of the second six weeks period, students took

the Math Proficiency Assessment #1 one final time. The data taken from the second six weeks represented the post-test treatment data of the study.

For this research study, students engaged in place value practice everyday of the week for twenty minutes at a time. To begin the study, the teacher researcher conducted qualitative interviews with the students using the Think Link Sheet (see Appendix A). These interviews gave insight into the lack of clarity students had in mathematical thinking. Students were involved in activities or games that practice place value skills, specifically using 300 charts (see Appendix B), digit cards to compare value, manipulatives, and a placemat that required manipulating many numbers throughout the six week treatment.

The teacher researchers' plan was to teach a twenty minute structured lesson each day based on a specific number sense skill. For example, students worked on making jumps by place value within a 300 chart, or a 100 chart for ESL newcomers. The students were given a start number, such as 53, and were asked to make a jump by 5, 10, or another increment. The student then shared where they landed. Another activity partnered students and they were given a deck of digit cards. Each partner drew two cards. The students decided which card was larger and by how much. The basis of this game was to assess student knowledge of place value (determining which number is greater) as well as operations such as addition and subtraction.

A third activity required students to work with place value through the hundredths place. In this activity, one partner placed various manipulatives into a plastic divider, based on place value. The other partner would have to write the number in several ways including expanded form, standard form, and word form and presented to students as a "Place Value Game" (see Appendix C). The partners would work together to create an addition and subtraction problem with the number as well.

Through the activities described above, the teacher researchers hoped to be able to observe as well as interview students to determine the level of proficiency with number sense each student displayed. The teachers then combined their quantitative and qualitative data in order to analyze the information that had been collected. The data allowed teachers to modify instruction as well as plan future formative assessments for the students.

Design and Procedures

This study followed the pre-posttest model where one set of students was used in the study to compare results against their own results. This model was chosen because it best fit the teacher researchers' situation- one teacher was in a self-contained classroom with only one grade level of students. The teacher researchers used mixed methods in order to collect their data. To ensure the reliability of the assessment, we used the same assessment in the first six weeks. We used the MPA rather than the MDA in the second six weeks; however, both assessments are designed to assess student knowledge of number sense.

In order to analyze the data, the teacher researchers used Microsoft EXCEL, specifically the data analysis tools. A dependent sample t test and independent sample t test was run to assist in analyzing the data. The teacher researchers also determined the mean of the students' scores, as well as the standard deviation of those scores. They examined the data to establish if it showed statistical significance and then computed an effect size to determine a percentile gain.

To ensure fidelity of the study, a school visit was conducted by three professionals from JCPS, the Gheens Institute for Innovation, as well as Spaulding University. The rubric used by the observing team (see Appendix D) gives a brief description of the activities used in this study. The teachers also discussed sharing the school visit rubric with the rest of the teachers in our school community as a school-wide effort to teach place value and number sense intentionally.

Findings

According to the data, the mean of the pretest scores was 57.62 and the mean of the post test scores was 61.19. Thus, there was an overall increase in assessment scores by 3.57 points. When analyzing the mean scores based on the demographics it was found that males scored an average of 61.36 on the pre test and 62.00 on the post test, which shows an increase of 0.64 points. Female students scored a mean score of 54.87 on the pre test and a score of 60.60 on the post test, showing an increase of 5.73 points. Females raised their performance significantly more than males in the class of study.

In another analysis of the data, African American students earned a mean score of 61.86 on the pre test and 64.93 on the post test, with an increase of 3.07 points. Students classified as "other" in Table 1 consisted of Asian and Hispanic students. These students earned a mean score of 50.64 on the pre test and a 55.91 on the post test, showing an increase of 5.27 points. Of this analysis, the "other" students had the largest increase in points from the pre test to the post test. The final demographic analysis was performed to analyze the student scores of students who are considered ELL (English Language Learners) or receive Free/Reduced Lunch. ELL students earned a mean score of 51.15 points on the pre test and a mean score of 55.69 on the post test, earning them an increase of 4.53 points. As 100% of the students in this class receive Free/Reduced lunch, the mean scores for this sub group are the same as that of the mean scores for the entire class.

The teachers used Microsoft Excel to run a dependent sample t test in order to determine if the study's results proved to be statistically significant. The t-value obtained was 1.47, thus the results of the study did not prove to be statistically significant ($p > .05$). As result, although positive gains were observed, the degree of gains did not reach statistically significant levels.
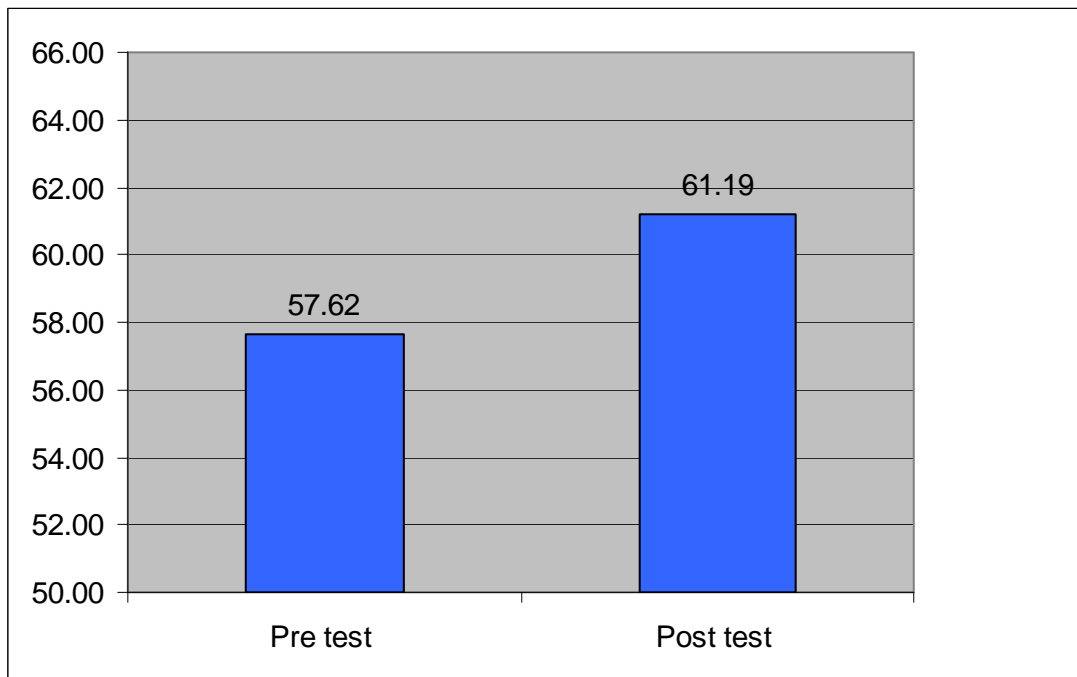
Figure 1. Achievement gains from pre test to post test during the treatment period.

Aside from quantitative data collected and analyzed, the teacher researchers also collected qualitative data to gain insight into cognitive empathy. The data was collected from the Think Link interview discussed previously. When given the question, "What number is made up of 3 ones, 5 tens and 7 hundreds?" we were interested to hear one student report, "357". This answer shows the student does not understand place value. The student simply heard the numbers 3, 5 and 7 then put them together in the order they were presented. A second student was asked to subtract 378-290. The student recorded the first step as "8-0=0 on the Think Link interview sheet. This type of data was collected from the students and used to design some of the twenty minute intentional place value/number sense lessons during the treatment.

Discussion

The purpose of this study was to conduct a, mixed method study about a mathematics clarity treatment over six weeks that specifically and intentionally taught place value and number sense skills to a class of 26 at risk students in a fifth grade class. The teacher researchers collected both qualitative and quantitative data at the beginning and end of the treatment to track student progress. As stated in the findings section, the data showed the overall class average improving by 3.57 points. This data point suggested to the teacher researchers that although this brief six week treatment was not statistically significant, it did show growth in student achievement.

As previously stated, students of high poverty struggle with numeracy concepts. This fact proved to be true in this study. Despite socio-economic barriers, however, the teacher researchers found that students in high poverty can make progress in the area of numeracy and this manifested itself within the data. The focus of the treatment on these concepts in an intentional, daily way proved to support students and their understandings as the data shows that the mean score increased from the pretest to the posttest. Intentionality of applying the treatment daily for a set amount of time was a crucial piece for the increase in student understandings and raising of scores.

While the study and its collected data did not show statistical significance, the teacher researchers found the treatment to be beneficial for students. As a result, the treatment will be continued with students in this classroom of study. Certain aspects may be modified, such as the time given to the treatment each day or the number of days the treatment is given in a week. It could also be possible that certain students begin receiving more time (or less time) than other students with the treatment based on their specific needs in the area of number sense.

Implications for Teacher Practice

Typically, in an at risk school not meeting Adequate Yearly Progress, students struggle with number sense and the teachers strive to find various ways to reach each student and increase achievement. This study was the perfect case for this idea. The teacher-researchers identified the main problem or limitation for students in this fifth grade class which is a good representation of the school in general. The teachers came up with various place value/number sense activities and incorporated an intentional twenty minute session into daily lesson planning.

This design proved to be good practice for teachers to follow in the future, far past the six week treatment period discussed in this study. Through the collection of weekly data from activities such as 100/300s charts, a teacher created place value game, digit cards, and the placemat, we were able to see a slight increase in scores through the six week treatment period in nearly every student. This study allowed the students to "play" with numbers and make connections to patterns they had not previously been made transparent to them.

The teacher-researchers also collected qualitative data from students through interviews. The qualitative data collected was beneficial for the teacher, because our main priority was to make student thinking clear, or find cognitive empathy, as mentioned in the literature review. Previously, the teacher-researchers mentioned fielding "off the wall" answers from our students. Through the Think Link interviews, the teacher-researchers were able to sift through the answers and really discuss with students what they were thinking when they approached a posed problem.

By listening to the students defend their reasoning, it was clear to the teachers the students had little idea of how numbers "work" and the value of those numbers. Through careful analysis of both the qualitative and quantitative data, the teachers were able to see some misconceptions cleared and basic number knowledge gained. The teachers discussed continuing

the treatment through the end of the school year and using the same assessment to determine any

gains in student achievement.

Limitations of the Study

   The teacher researchers found three main limitations of the study. The first was the fact

that the MPA could not be accessed for the pre-test, thus the MDA data had to be used in the first

six week grading period. The MPA is a district assessment that is only accessible during a

certain window of time and was not available on the pre-test date. Although the two assessments

for pre and post tests were not identical, the number sense strand is the focus of both the MDA

and MPA.

   The second limitation was the fact that the study or treatment period was only six weeks

long. The teacher researchers had to introduce the students to a daily routine of practicing place

value and number sense as well as practice how each type of data collection point was to be

completed.

   With any class of elementary students, but especially at risk populations, students need

time to practice skills multiple times. In this six week study, the students had few opportunities

to practice each skill which may have affected their score on the post test. The class in the study

is made up of 26 students of which 13 are students who speak English as a second language or

ESL; the study may have been affected by the lack of English vocabulary let alone math

vocabulary these 13 students know and –with only six weeks, there was not enough time to fully

go in-depth with the treatment.

   A third and final limitation for not detecting statistically significant findings was the fact

that the sample size was small. The limited sample size did not provide enough statistical power

to detect statistically significant findings with the dependent-sample t-test.

Conclusion

The teacher researchers in this study had to ask themselves one simple yet complicated question: Can all students learn? Many would find excuses such as poverty, family situation, and other outside of school factors and claim that some students simply cannot learn as well as students in other situations. The teachers in this study identified a school wide mathematical issue that seemed to stem from students' lack of number sense and chose to develop an action research plan to tackle the problem. We believe this was an important study to conduct, because mathematical number sense is a skill that all students need to acquire and it was especially important for this group of fifth graders as they prepare for the transition into middle school.

The topic of clarity in mathematics instruction was very important, because our students harbor many misconceptions and it is necessary to obtain cognitive empathy to determine what goes on inside their heads as they think through mathematical problems. Although this study did not report statistically significance due to limited implementation time and small sample size, the teacher researchers will continue to combat student misconceptions and intentionally teach number sense and place value skills to our students. This is an important area of work for any teacher working in Title I environments in the subject area of mathematics.

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Appendix A- Think Link Sheet

Equation	Word Problem
Model	Explanation

Appendix B- 100/300 Chart
100/300s Chart Directions:

1. One group member will choose a number on the chart.

    3.   Each group member will take turns "making a jump" on the chart by following the
          directions below.

    •   Make a jump to show **10 more**.

    •   Make a jump to show **25 less**.

    •   Make a jump to show a **multiple of this number**.

    •   Make a jump to the **nearest 10**.

    •   Make a jump to the **nearest odd number**.

    •   Make a jump to show **5 more**.

    •   Make a jump to show **2 times this number** (if possible).

    •   Make a jump to show **½ this number** (if possible).

    3.   When you are finished, allow the next group member to select a start number. Follow
          the same directions above to make your jumps!

Appendix C- Place Value Game

(Place Value Game)

Name: _____ Date: _____

1. Write the number in word form.
   _____
   _____

2. Write the number in standard form.
   _____

3. Write the number in expanded form.
   _____

4. What is 10 less than this number? _____

5. What is 100 more than this number? _____

6. Below, solve an addition and subtraction problem that equal this number.

Addition:	Subtraction:

Appendix D-Classroom Visit Rubric

Clarity in Mathematics Instruction

Place value activities:

Activity	Present in today's lesson?	Comments
1. Manipulatives Students use 100 blocks, strips and singles to create numbers		
2. Place Value Chart Students use post it notes to create numbers through the hundred millions		
3. Card Game Students use MI2 digit cards to decide whose number is larger/smaller by comparing cards		
4. 300s chart jumps Students use a 300s chart to make jumps of 5, 10, 25, 100, etc.		
5. Place value game Students use a plastic dish and tokens to create various numbers with a partner		
6. Multiplication break apart strategy Students break apart two and three digit numbers and multiply by place value		
7. Number talk/Placement Students use a placemat to manipulate a 3 digit number in various ways using place value		

Other comments: