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*The Aftermath of Accelerating  
Algebra: Evidence from a  
District Policy Initiative*

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## **The Aftermath of Accelerating Algebra: Evidence from a District Policy Initiative**

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### **Abstract**

In 2002/03, the Charlotte-Mecklenburg Schools initiated a broad program of accelerating entry into algebra coursework. The proportion of moderately-performing students taking 8<sup>th</sup> grade algebra increased from less than half to nearly 90%, then reverted to baseline levels, in the span of just six age cohorts. We use this policy-induced variation to infer the impact of accelerated entry into algebra on student performance in math courses as students progress through high school. Students affected by the acceleration initiative scored significantly lower on end-of-course tests in Algebra I, and were either no more likely or significantly less likely to pass standard follow-up courses, Geometry and Algebra II, on a college-preparatory timetable. We also find that the district assigned teachers with weaker qualifications to Algebra I classes in the first year of the acceleration, but this reduction in teacher quality accounts for only a small portion of the overall effect.

## Introduction

In 2008, the California State Board of Education voted to require all students to enroll in Algebra by 8<sup>th</sup> grade.<sup>1</sup> This policy initiative, yet to be actually implemented, represents the culmination of a decades-long movement toward offering algebra instruction before the traditional high school years.<sup>2</sup> Nationally, the proportion of eighth-grade students enrolled in algebra doubled between 1988 and 2007 (Perie, Moran and Lutkus, 2005; Walston and McCarroll 2010), reaching rates over 50% in three states and the District of Columbia.<sup>3</sup> The movement to offer algebra instruction before high school has been inspired in large part by correlational research documenting significant differences in later-life outcomes between those students who enroll in algebra by 8<sup>th</sup> grade and those who do not.

Correlation need not imply causation, and it is unclear whether accelerated algebra enrollment yields positive or negative effects (Loveless, 2008). This paper provides a quasi-experimental estimate of the causal impact of accelerating the introduction of algebra coursework. We analyze a policy initiative introduced in one of the nation's best-performing large school districts, Charlotte-Mecklenburg Schools (CMS), in 2002/03.<sup>4</sup> This initiative led students at many points in the initial math achievement distribution to take Algebra I earlier than they would have at baseline. After maintaining the acceleration policy for two years, the district reversed course, reverting almost entirely to its previous placement pattern. We use the across-cohort variation in placement patterns created by these abrupt

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<sup>1</sup> This vote came at the urging of then-governor Arnold Schwarzenegger, who referred to algebra as “the key that unlocks the world of science, innovation, engineering, and technology. See “California to Require Algebra Taught in 8<sup>th</sup> Grade,” *USA Today*, July 11, 2008; Eddy Ramirez, “8<sup>th</sup>-Grade Algebra Requirement in California Gets Sidelined,” *U.S. News and World Report*, December 29, 2008.

<sup>2</sup> In this paper, we use the term algebra to refer generically to a content area in mathematics, Algebra to refer to a course focusing on this content area, and Algebra I to refer to the course traditionally taken at the beginning of a college-preparatory math sequence in North Carolina public schools. We similarly distinguish between Geometry courses and the content area known as geometry.

<sup>3</sup> In 2007, early algebra-taking rates exceeded 50% in California, Maryland, Utah, and the District of Columbia (Loveless, 2008).

<sup>4</sup> Charlotte-Mecklenburg ranked first among the set of large American school districts with district-specific reports from the National Assessment of Educational Progress (NAEP) in terms of 4<sup>th</sup> grade mathematics scores for all test administrations between 2003 and 2009.

shifts in policy to infer the impact of acceleration, by comparing students with similar initial math achievement who were subjected to different placement policies solely on the basis of their age cohort.

We examine whether acceleration increased the likelihood that students would stay on track to pass three college-preparatory math courses – Algebra I, Geometry, and Algebra II – within six years of beginning seventh grade. Students who do so also meet the North Carolina State Board of Education’s minimum standards for a college-preparatory course of study.<sup>5</sup> We use standardized end-of-course tests designed by the state to assess performance in each course, rather than the grade assigned by the course’s instructor.<sup>6</sup>

Our results indicate that Charlotte-Mecklenburg’s acceleration initiative worsened the Algebra I test scores of affected students and reduced their likelihood of progressing through a college-preparatory curriculum. Moderately-performing students who were accelerated into Algebra I in 8<sup>th</sup> grade scored one-third of a standard deviation worse on the state end-of-course exam, were 18 percentage points less likely to pass Geometry by the end of 11<sup>th</sup> grade, and were 11 percentage points less likely to pass Algebra II by the end of 12<sup>th</sup> grade, compared to otherwise similar students in birth cohorts that were not subjected to the policy. Lower-achieving students who were accelerated into taking the course in 9<sup>th</sup> grade also exhibited significant declines in all outcomes considered. By contrast, higher-performing students who were accelerated into Algebra I in 7<sup>th</sup> grade, despite receiving lower test scores on the Algebra I test, showed no ill effects on subsequent course completion.

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<sup>5</sup> The State Board of Education permits a student to substitute a more advanced mathematics course – one using Algebra II as a prerequisite – for Geometry, or an alternative course sequence labeled Integrated Math I, II, and III in the state’s official curriculum guide. In practice, the full Integrated Math sequence was not offered by any school in CMS during the period of study. Note also that admission to the 16-campus University of North Carolina system for most of the cohorts in our study required additional coursework beyond Algebra II. Thus completion of the three-course sequence was neither necessary nor sufficient for college admission. Nonetheless, failure to pass Algebra II effectively guaranteed that a student would not meet state standards for college-readiness.

<sup>6</sup> The state mandates that at least of the course grade in one of these courses be based on the end-of-course score. See GreatSchools, “Testing in North Carolina,” <http://www.greatschools.org/students/local-facts-resources/435-testing-in-NC.gs>, 1/11/12.

The contrast between these results and prior correlational research reflects the severe selection bias plaguing those previous studies. It is undeniable that students who take algebra early tend to do better in subsequent math courses, but this correlation arises because it is usually the best students who are selected to take algebra early. Once this selection bias is eliminated, the remaining causal effect of accelerating the conventional first course of algebra into earlier grades, in the absence of other changes in the math curriculum, is for most students decidedly harmful. We caution that our results apply to the impact of varying the timing of the conventional first course in algebra, holding math instruction in the early grades constant. It is quite possible that more systematic intervention to transform the math curriculum at earlier ages to promote readiness for algebra by 8<sup>th</sup> grade could well prove beneficial.

### **Origins of the Algebra Acceleration Movement**

As suggested by the brief history sketched above, accelerating algebra instruction into middle school has been widely espoused as a strategy for improving the mathematics achievement and college-readiness of American high school students. Nationwide, the proportion of 13-year-olds enrolled in algebra courses rose from 16% in 1988 to 29% in 2004 (Perie, Moran, and Lutkus, 2005). Among students in the nationally representative Early Childhood Longitudinal Survey Kindergarten cohort, just over one-third were enrolled in either algebra or a more advanced math course in 2006/07, when most of the cohort was in 8<sup>th</sup> grade (Walston and McCarroll, 2010). As noted above, this national average obscures very high rates of 8<sup>th</sup> grade algebra-taking in some jurisdictions.

This movement has been supported in part by assigning a causal interpretation to correlational research. Eighth grade students enrolled in algebra outscore their counterparts on 8<sup>th</sup> grade standardized math tests (Walston and McCarroll, 2010). By the time they reach 12<sup>th</sup> grade, early algebra-takers have completed more years of advanced math and attain higher scores on 12<sup>th</sup> grade



math assessments (Smith, 1996). Additional research has documented higher achievement outcomes among students who enroll in algebra at any point in their secondary school career (Dossey et al., 1988; Gamoran and Hannigan, 2000). Ma (2005a; 2005b) reports that the improvement in math skills associated with enrollment in 8<sup>th</sup> grade algebra is strongest for the lowest-achieving students – particularly those below the 65<sup>th</sup> percentile of the 7<sup>th</sup> grade math distribution. To date, no study has attempted to address concerns regarding selection into accelerated algebra on the basis of unobserved characteristics.<sup>7</sup>

Indeed, doubts about the reliability of previous studies have provoked a backlash against accelerating algebra into middle school. Opponents of accelerated algebra argue that too many students enter the course unprepared for advanced work and may in fact fall behind their peers enrolled in less rigorous coursework. Loveless (2008) documents the poor math performance of some students enrolled in the course by 8<sup>th</sup> grade, and he notes the inattention to the problem of selection in prior work justifying the push to offer algebra in middle school. The Loveless report itself, however, provides no evidence on the causal question of whether early placement in algebra promotes or retards mathematics achievement. The poorly-performing students he cites may have performed just as poorly in a more traditional 8<sup>th</sup> grade math course. An empirical assessment of the effects of accelerating the first algebra course requires observation of a counterfactual scenario: otherwise identical students who take algebra on a traditional schedule.

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<sup>7</sup> Ma (2005b), for example, reports that only 4% of students below the 65<sup>th</sup> percentile of the 7<sup>th</sup> grade math distribution are placed in algebra by 8<sup>th</sup> grade.

## Conceptual Framework

### *Algebra timing, mathematics skills, and labor productivity*

From an economic perspective, algebra skills can be valued for two basic reasons. First, algebra skills may contribute directly to labor productivity.<sup>8</sup> Second, algebra skills might serve as inputs into the production of higher-order mathematical knowledge, which in turn may have an independent effect on productivity. The notion that algebra is a “gateway” derives from this second interpretation. These two effects on productivity can be summarized in this expression:

$$(1) y = y(a(t_a), h(a, t_h)),$$

where  $y$  is a measure of productivity,  $a$  is a measure of algebra skill,  $h$  is a measure of higher-order mathematical skill, and  $t_a$  and  $t_h$  measure the amount of time devoted to the study of algebra and higher-order topics, respectively. All three functions in equation (1) are presumed to be nondecreasing in their arguments. If students are expected to complete their human capital investment by a specific age, the case for accelerating entry into algebra is clear: initiating algebra earlier allows more time for instruction in both algebra and higher-order topics, thereby unambiguously increasing productivity.

Things get more complicated when we introduce the possibility that both algebra and higher-order math skills rely on the degree to which students have mastered lower-order topics in mathematics. Consider the formulation:

$$(2) y = y(l(t_l), a(l, t_a), h(a, l, t_h))$$

where  $l$  and  $t_l$  represent lower-order mathematical skill and the time devoted to learning these skills, respectively. While we did not introduce an explicit time constraint in our initial formulation, it makes sense here to assume a fixed amount of time available between school entry and the end of human capital investment. In this formulation, the opportunity cost of accelerating introduction to algebra is clear. Indeed, the question of algebra timing reduces to a matter of how much time to allocate to

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<sup>8</sup> Beyond improving labor productivity and earnings, math skills may also increase utility by promoting better consumption decisions by boundedly-rational agents (Benjamin, Brown, and Shapiro, 2006).

lower-order subjects. The belief that students enter algebra too late is equivalent to an argument that too much time is devoted to lower-order subject matter.

Equation (2) implies that the optimal allocation of time across mathematical topics depends on a number of relationships: the relative importance of lower-order skills in the production of higher-order skills, the marginal impact of time on skill acquisition, and the relative importance of various types of mathematical skill on productivity. A proposal to reallocate time away from lower-order skills makes the most sense if lower-order skills are relatively unimportant in the production of algebra and higher-order skills, and if lower-order skills are similarly unimportant determinants of productivity.

#### *The opportunity cost of acceleration*

What kinds of topics are short-changed when algebra is accelerated? To get an idea, Table 1 describes the key competencies that North Carolina's standard course of study establishes for several pre-algebra courses, ranging from 7<sup>th</sup> Grade Math to Introductory Math, the course prescribed for students who do not take Algebra I upon entry into high school.<sup>9</sup>

The similarity in course objectives across 7<sup>th</sup> and 8<sup>th</sup> grade math, and the high school introductory math course, suggests the possibility of diminishing returns in lower-order mathematics instruction. The objectives of 8<sup>th</sup> grade math and Introductory Math are nearly identical, suggesting that the high school course largely repeats subject matter for students who did not master it the first time around. Furthermore, the distinctions between 7<sup>th</sup> and 8<sup>th</sup> grade math objectives are minor; eighth graders, for example, are expected to perform computations with irrational numbers whereas in seventh grade computation with rational numbers is sufficient.<sup>10</sup>

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<sup>9</sup> These competencies form the basis for standardized End-of-Grade tests in mathematics conducted since the early 1990s.

<sup>10</sup> A rational number is one that can be expressed as the ratio of two integers.

Although a perusal of these stated objectives suggests that pre-algebra courses are incremental if not redundant, it is possible that many students need repeated exposure to this subject matter. It is interesting to note, furthermore, that each of the middle-grades math courses includes significant attention to geometry. Computation of volume and surface area is a key component of the 7<sup>th</sup> grade curriculum, and the Pythagorean theorem is mentioned specifically in the 8<sup>th</sup> grade curriculum. Both topics appear in the high school Introductory Math course, and both relate directly to subjects covered in the state's official Geometry curriculum, which focuses in part on right triangles, problems involving surface area and volume, and elementary proof-writing.

Algebra I acceleration is not the only curricular reform designed to improve mathematics achievement. California's Math A and New York's Stretch Regents curriculum are examples of reforms that target the quality of pre-algebra instruction rather than the timing of algebra coursetaking (White 1995; White et al. 1996; Gamoran et al. 1997).<sup>11</sup> Although evidence on the effectiveness of these programs is inconclusive (White et al 1996; Gamoran et al, 1997), these alternatives may offer promising avenues to improve achievement in the event that accelerating algebra is judged not to be worth the cost of forgone pre-algebra instruction.

The larger question of which math subjects have the strongest effects on productivity is beyond the scope of our empirical analysis, although it certainly bears heavily on the question of optimal time allocation. In one study pertinent to this issue, Rose and Betts (2004) analyze transcript data from the High School and Beyond dataset, based on straightforward methods to address concerns about self-selection into higher-order courses. This study suggests that the labor market return to higher-order coursework is greater than the return to coursework at the level of introductory algebra or geometry.

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<sup>11</sup> Math A is a high school curriculum used in certain districts used to transition lower achieving students to a college-preparatory algebra and geometry curriculum. The Stretch Regents program permits students to take New York State's rigorous Regents curriculum at a slower pace. See Gamoran (1997) for further description.

## Data and Methodology

### *Setting*

Our analysis makes use of data on students enrolled in the Charlotte-Mecklenburg Schools (CMS), provided by the North Carolina Education Research Data Center. During the period of our analysis, CMS was the largest school district in North Carolina, and one of the 25 largest in the United States, serving over 100,000 students. The district is racially diverse; in 2002/03, the first year of implementation for the Algebra acceleration program we study, 44% of all students were black, 8% were Hispanic, and 4% Asian. About 40% of the district's students participated in the federal free and reduced price lunch program, slightly above the state average.

Charlotte-Mecklenburg has a strong reputation for mathematics performance. The district's fourth grade students ranked first among 18 major school districts in the 2009 National Assessment of Educational Progress (NAEP) math assessment. It was the only district in this group with 4<sup>th</sup> grade math scores significantly higher than the national average. To put this high performance in context, however, it should be noted that, because it covers both suburban and urban neighborhoods, CMS has a larger share of middle class students than do most large school districts.<sup>12</sup>

Beginning around 2002/03, CMS adopted an unusually aggressive policy to accelerate placement of middle and high school students in Algebra I.<sup>13</sup> The district not only broke from its past patterns of course-taking but also diverged dramatically from policies followed by most other districts in North Carolina. By all appearances, there were two precipitating factors that accounted for Charlotte-Mecklenburg's aggressive approach. First, the state of North Carolina had increased from three to four the number of math courses required for admission to the University of North Carolina system. Second,

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<sup>12</sup> Ranked by performance of students eligible for federal school lunch subsidies, CMS placed 4<sup>th</sup> among the 18 districts. Nonetheless, CMS presents a case of algebra acceleration in a large urban district with relatively strong math performance.

<sup>13</sup> Although we have found no written statement of Charlotte-Mecklenburg's policy, its existence and influence have been substantiated by contemporaneous reporting and the recollection of administrators who worked in the system at the time of implementation.

the district's then superintendent strongly believed as a matter of pedagogy that algebra should be offered to many, if not most, students in middle school, rather than waiting until they are in high school. Later described as "a bear on getting middle school kids in eighth grade to learn Algebra I," this superintendent announced at the beginning of the 2001/02 year that his goal would be to increase to 60% the portion of students in the district who were proficient in Algebra I by the end of eighth grade, as indicated by scoring at level 3 or above on the state's end-of-course test.<sup>14</sup>

Several other policy changes transpired in CMS during the period of our study. The districted ceased busing students to desegregate schools in 2002, and implemented a public school choice plan, incorporating a lottery system for oversubscribed schools the same year (Hastings, Kane, and Staiger 2005, 2006a, 2006b; Hastings et al. 2007; Deming et al. 2011; Vigdor 2011). These changes may have led to systematic declines in instructional quality for African-American and other disadvantaged students (Jackson 2009). We detail below several strategies for addressing potential confounding effects. Most importantly, we obtain very similar results when analyzing a similarly-timed initiative in North Carolina's third-largest school district.

Figure 1 summarizes information on algebra-taking patterns in CMS for six age cohorts included in this study. It is based on a longitudinal sample of students described in more detail below.<sup>15</sup> For each

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<sup>14</sup> *Educate!*, September 16, 2001, p. 5. As evidence of the superintendent's focus on increasing the number of middle school students taking algebra, one informant described how he ordered middle school principals to overhaul schedules after the school year had commenced in order to increase the number of middle school students in algebra classes. In an interview after he stepped down as CMS superintendent, Eric Smith stated, "The middle school math piece was the gatekeeper and it is the gatekeeper. It's the definition of what the rest of the child's life is going to look like academically, not just through high school but into college and beyond. If they make it into algebra one, the likelihood of getting into the AP class and being successful on the SAT and having a vision of going on to college is dramatically enhanced. And so our pressure to make sure that kids were given that kind of access to upper level math in middle school was a critical component of our overall district strategy."<sup>14</sup> <http://www.pbs.org/makingschoolswork/dwr/nc/smith.html>, 4/5/11.

<sup>15</sup> Note that all analyses reported in this paper "undo" effects of grade retention by comparing students only to those in their entering cohort. To be precise, therefore, our analyses study not the impact of taking Algebra I by 8<sup>th</sup> grade, but the effect of taking the course within two years of beginning seventh grade.

student, we record the year in which he or she first takes North Carolina's end-of-course test in Algebra I.<sup>16</sup>

The initial cohort enrolled in 7th grade for the first time in 1999/2000, three years prior to the algebra acceleration initiative. At this time, rates of algebra-taking by 8<sup>th</sup> grade were high relative to the national average for high-performing students, but low for low-performing students. Ninety-seven percent of CMS students in the top quintile of the statewide 6<sup>th</sup> grade math score distribution were enrolled in Algebra I by 8<sup>th</sup> grade, compared to 75% of top quintile 8<sup>th</sup> graders nationwide, as recorded in the 2009 NAEP assessment (Walston and McCarroll 2010). By contrast, only 2% of CMS students in the lowest 6<sup>th</sup> grade math quintile had enrolled in Algebra I by 8<sup>th</sup> grade, compared to 13% in the national NAEP data.

The cohort entering 8<sup>th</sup> grade in 2001/02, just two years later, experienced a very different pattern. For this later cohort, the rate of early algebra-taking shifted dramatically at lower points in the distribution. For students around the median, the likelihood of taking algebra by 8<sup>th</sup> grade increased from 47% to 86%. For students in the second-lowest quintile, the rate increased from 14% to 65%. Even in the lowest quintile of the 6<sup>th</sup> grade math distribution, the rate of Algebra I taking rose to 18%.<sup>17</sup>

Just two years after the push to accelerate algebra started, however, the district reversed course. By the time the cohort that entered 7<sup>th</sup> grade in 2004/05 had reached middle school, assignment patterns were nearly back to those for the 1999/2000 cohort, except in the top two quintiles, where a modest amount of acceleration remained in place. This rapid reversal of the acceleration policy

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<sup>16</sup> More precisely, we present the year in which a student first appears as a data point in the Algebra I EOC test file. A small number of students appear in the dataset but do not have a valid test score. These students are excluded from analyses using test scores as a dependent variable below, but are included in analyses of subsequent course-taking.

<sup>17</sup> Our data are derived from end-of-course test records, which may not accurately measure the number of students assigned to take Algebra I in a given year. Students may withdraw from the course in advance of test administration, for example. There is some evidence that the rate of withdrawal rose in 2002/03 along with the rate of course-taking. In that year, an administrative count of course enrollment in Algebra I for CMS enumerates over 900 students for whom we have no test score record. In most other years, the discrepancy between the two sources of enrollment data is small. We discuss potential implications of this pattern below.

provides us with the first means of distinguishing acceleration effects from the effects of resegregation and school choice.

Figure 2 shows that the acceleration policy involved more than pushing students into 8<sup>th</sup> grade algebra. For certain students, the likelihood of taking Algebra I by 7<sup>th</sup> grade also increased substantially over time. In the 1999/2000 cohort, just under half of top quintile students, 12% of second quintile students, and 2% of middle quintile students took Algebra I as 7<sup>th</sup> graders. In the top quintile, the rate of 7<sup>th</sup> grade Algebra I enrollment rose monotonically, reaching 75% by 2005. In the second quintile, the 7<sup>th</sup> grade Algebra I-taking rate rose to nearly 40% in 2004 before retreating somewhat.

For students in the lowest two quintiles of 6<sup>th</sup> grade math test scores, the acceleration policy had its biggest effect in an increased propensity to take Algebra I by 9<sup>th</sup> grade. Figure 3 shows a peak among students entering 7<sup>th</sup> grade in 2000/01, who would have entered 9<sup>th</sup> grade in 2002/03 under normal rates of academic progress. Over 70% of lowest-quintile students in this cohort had taken Algebra I by 9<sup>th</sup> grade. But by the time the 2004/05 cohort came through, just over a third of students in the bottom quintile were getting this treatment. . Similar fluctuations occurred in the fourth and middle quintiles.

### *Data and Sample Selection*

Our data are derived from North Carolina Education Research Data Center longitudinal records on students who entered 7<sup>th</sup> grade in the Charlotte-Mecklenburg district between 1999/2000 and 2004/05 and were observed in Algebra I EOC test score files for that district.<sup>18</sup> In most cases, we also restricted the sample to students with valid scores on the state's standardized 6<sup>th</sup> grade mathematics

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<sup>18</sup> This restriction allows us to focus exclusively on the question of whether accelerating students into algebra yields benefits, rather than the broader question of whether algebra instruction itself is beneficial. Results obtained with a broader sample of students are qualitatively similar to those reported here.



assessment.<sup>19</sup> We tracked progress through college-preparatory math courses using the state’s end-of-course (EOC) examinations in Algebra I, Geometry, and Algebra II. Our ultimate sample consists of 36,790 students across six cohorts.<sup>20</sup>

### *Identification Strategy*

Our estimation strategy takes advantage of the significant policy changes that took place in Charlotte-Mecklenburg over just a few years. We exploit these changes to estimate local average treatment effects for taking Algebra I by the time students reach a certain grade. We begin by examining the effect of taking the course by 8<sup>th</sup> grade, and we later look at the effects of accelerating Algebra I into 7<sup>th</sup> grade or 9<sup>th</sup> for students at different points in the initial achievement distribution. The estimated treatment effects are “local” to that set of students subjected to differing treatment status across cohorts within our six-cohort sample. For example, our estimate of the effect of taking Algebra I by 8<sup>th</sup> grade applies primarily to students in the middle of the initial test score distribution; students at the top of the distribution virtually always take Algebra I by 8<sup>th</sup> grade, while those at the bottom rarely do.<sup>21</sup>

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<sup>19</sup> We made exceptions to this restriction for the first cohort, wherein problems involving NCERDC identification codes greatly reduced matching rates from seventh backwards to sixth grade. For students with missing 6<sup>th</sup> grade information in this cohort, we assign them to an initial test score decile on the basis of their 7<sup>th</sup> grade assessment. We have estimated each of our regression specifications excluding these students; results are not significantly affected by their exclusion.

<sup>20</sup> Some of the students included in our sample may exit the dataset because they leave the CMS system, to attend a different public district, a private or charter school. If such students complete Geometry or Algebra II coursework, we will incorrectly code them in our analysis. Due to differences in student ID coding between CMS and other North Carolina districts, we are unable to satisfactorily track students who transfer to a different district or to a charter school. Moreover, given data limitations it is impossible for us to distinguish a student who attrits from one who persists without taking EOC exams. This poses a problem for our analysis only to the extent to which transfer behavior correlates with algebra acceleration, conditional on decile and cohort effects. If parents respond to the decline in mathematics performance associated with algebra acceleration by switching to a different school district, we may in fact overstate our results. Note that we are similarly unable to identify students who drop out of school; since students cannot pass EOC exams after dropping out, however, they are not miscoded.

<sup>21</sup> Our results may have some bearing on the most prominent algebra policy debate, regarding California’s initiative to require 8<sup>th</sup> grade algebra. Note, however, that this initiative is most relevant for the bottom 40% of the math ability distribution in that state, since the rate of 8<sup>th</sup> grade algebra-taking is already close to 60%. Our estimate of the effect of 8<sup>th</sup> grade algebra acceleration pertains more directly to students towards the middle of the ability distribution. Our results on accelerating low-performing students into 9<sup>th</sup> grade algebra may provide some additional insight as to the effects of acceleration in that subset of the population.

Our basic estimation strategy is a version of differences-in-differences: we compare the outcomes of students stratified by initial ability level, as measured by 6<sup>th</sup> grade math scores, across cohorts. In order to implement this strategy in a manner that produces local average treatment effects, we use instrumental variable estimators. The outcome equation is of the form:

$$(4) y_{idc} = \alpha_c + \alpha_d + \beta T_{idc} + \varepsilon_{idc}$$

where  $y_{idc}$  is the outcome of interest for student  $i$  belonging to initial achievement decile  $d$  in cohort  $c$ ,  $\alpha_c$  and  $\alpha_d$  are cohort and decile fixed effects,  $T_{idc}$  is an indicator for whether the student received the treatment – in this case, taking Algebra I by a certain point in their career – and  $\varepsilon_{idc}$  is an independent and identically distributed error term. Cohort fixed effects account for policy changes or other contemporaneous effects that apply to all students in a cohort, while decile fixed effects account for broad differences in outcome trajectories for students with differing initial ability. The use of decile effects rather than a linear control for test score allows us to account for potentially nonlinear effects of initial ability on later outcomes.

Prior work in this literature has often estimated single equations along the lines of (4), arguing that controls for prior achievement adequately correct for unobserved determinants of the outcome that also correlate with the treatment indicator, implying that  $\beta$  is an unbiased estimate of the true treatment effect. To assess this argument, we present OLS estimates of equation (4) for comparison with our preferred IV results below.

In our preferred specifications, we address the endogeneity of assignment to an accelerated algebra class by estimating the first stage equation:

$$(5) T_{idc} = \gamma_c + \gamma_d + \sum_{c=1}^6 \sum_{d=1}^{10} \delta_{dc} + \eta_{idc}$$

where  $\gamma_c$  and  $\gamma_d$  are cohort and decile fixed effects, the  $\delta_{dc}$  terms are cohort-by-decile fixed effects, and  $\eta_{idc}$  is a second error term. Predicted values of equation (5) are then used in place of actual treatment

status in equation (4). Effectively, the estimation strategy associates across-cohort-and-decile variation in the propensity to take Algebra I by a certain grade level with across-cohort-and-decile variation in the outcome of interest. We attribute a positive (negative) effect to acceleration if students subjected to a higher probability of earlier algebra than others in the same initial ability decile in another cohort exhibit superior (inferior) performance in Algebra I and subsequent math courses. Because the identifying variation in algebra timing is at the cohort-by-decile level, we cluster standard errors at that level.

In principle, we would like to estimate the impact of early progression to Algebra I on performance in that course and subsequent math topics. This goal is complicated by the fact that many students who enroll in Algebra I do not complete subsequent math coursework. Thus, any effort to estimate the impact of Algebra I timing on performance in Geometry or Algebra II must contend with a sample selection problem: we can observe performance in subsequent math courses only for those who actually take and complete those subsequent courses. If acceleration is beneficial, for example, one effect might be to increase the proportion of marginal students who take follow-up courses, possibly leading to a decline in average performance in those courses.

Table 2 illustrates the potential severity of the selection problem, by tracing the progress of students in two cohorts. Consider first the cohort of CMS students who entered 7<sup>th</sup> grade in 1999/2000 and took Algebra I for the first time the following year. Because this cohort arrived before the district's accelerated algebra push, only 28.1% of them took Algebra I in 8<sup>th</sup> grade. About 88% of them passed the EOC test in the subject, and over four-fifths of them progressed immediately to Geometry the next year. Most of the non-progressing students retook Algebra I as 9<sup>th</sup> graders. About three-quarters of the 8<sup>th</sup> grade Algebra I takers in the first cohort took the Algebra II EOC two years later, and 82% took the Algebra II EOC by the time they would ordinarily have graduated from high school.

In contrast, among students entering 7<sup>th</sup> grade in 2002/03, the first year of the acceleration initiative, a much higher share, almost half, took Algebra I in 8<sup>th</sup> grade. The weaker average quality of

this group shows up in lower pass rates. Only 68.7% of them proceeded to Geometry the following year, only 61.4% completed the three-course sequence by the end of 10<sup>th</sup> grade, and just 73.6% finished the sequence by the time they would ordinarily have graduated.<sup>22</sup> This worsening record of course completion for the accelerated cohort, presumably caused by the exit of many lower-performing students, would leave a comparatively strong group of students to take subsequent courses. The likely result, therefore, would be a positive bias on estimates of the effect of acceleration on Geometry or Algebra II test scores. .

Rather than attempt to estimate a selection correction model, or use any other procedure to account for the selection of marginal students out of the sample of subsequent course-takers, we redefine our outcome variables such that they are observed for all students, whether they enroll in a follow-up course or not. Specifically, we analyze whether students attain a passing grade on a mathematics end-of-course test soon enough to keep them on track to complete Algebra II within six years of beginning seventh grade.<sup>23</sup> Students who never take a course are coded as not having passed that course.

Because both the outcome and treatment variables are binary, the most appropriate means of simultaneously estimating equations (4) and (5) is by bivariate probit. In such a case, both  $y_{idc}$  and  $T_{idc}$  can be thought of as latent variables, with the observed binary outcome dependent on whether the latent variable exceeds a particular value. For ease of interpretation, we also present two-stage least-

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<sup>22</sup> The attrition problem illustrated in Table 2 is even more severe among students who take Algebra I at a later point in time. For students first taking the Algebra I EOC as ninth graders in 2002/03, 66% proceed to take the Geometry EOC the following year, and 54% take the Algebra II EOC the year after that. Interestingly, this progress is excessive in relation to the group's pass rate on the initial Algebra I exam, which is only 48%. These summary statistics clearly associate the acceleration policy with lower course passage and progression rates. Such a pattern could conceivably be explained entirely by selection patterns, however. Our IV procedure promises to directly compare the performance of marginal students assigned to different courses.

<sup>23</sup> Our definition of a passing grade on the EOC test is based on the proficiency standard in place for most of the years in our sample, which was roughly equal to the 20<sup>th</sup> percentile of the statewide distribution. In 2007, the state adopted stricter grading standards on the EOC, placing the passing threshold closer to the 40<sup>th</sup> percentile of the statewide distribution. By using a uniform standard based on a specific point in the distribution, we assume that there is no meaningful change in the statewide distribution of Algebra I test scores over time. In alternative specifications, we also analyzed the propensity to pass mathematics courses within a fixed number of years after first taking Algebra I. Results do not vary substantively across specifications.

squares results. In the case of passing Algebra I, the fact that we have restricted our sample to students who took Algebra I at some point in their career permits us to use the actual test score as a dependent variable.

## Results

We present the results of three related interventions – accelerating certain students into Algebra I in 7<sup>th</sup>, 8<sup>th</sup>, or 9<sup>th</sup> grade – on Algebra I test scores and indicators for whether students pass Algebra I, Geometry, or Algebra II on a timetable that will permit them to complete the sequence by the time they would ordinarily graduate from high school. To set the stage, Table 3 presents the results of simple OLS specifications examining the basic relationship between Algebra I timing and the outcomes above. These estimates should not be interpreted as causal effects, even though they include indicators that restrict comparison to students in the same decile of the 6<sup>th</sup> grade math test distribution. Even conditional on decile, assignment to earlier algebra in the cross-section is likely to be correlated with unobserved determinants of math achievement.

Consistent with earlier studies, our OLS specifications associate earlier placement in Algebra I with better outcomes. Students who complete Algebra I by 8<sup>th</sup> grade score two-tenths of a standard deviation better on the end-of-course test and are significantly more likely to attain passing scores on higher-level math exams on a college-preparatory schedule. The probability of completing the college preparatory sequence, equal to about 50% in our entire sample, is 15 percentage points higher among students who complete Algebra I by 8<sup>th</sup> grade, conditional on 6<sup>th</sup> grade math test decile. Interpreted naively, the apparent advantage associated with early access to algebra is equivalent to the predicted impact of raising a student's 6<sup>th</sup> grade math test score between one and two deciles in the distribution. To reiterate our previous discussion, however, these OLS estimates, like many prior estimates in the literature, are potentially contaminated by selection bias.

### *The impact of 8<sup>th</sup> grade algebra on moderately-performing students*

We focus primarily on the effects of offering Algebra I in 8<sup>th</sup> grade to moderately-performing students, after which we discuss the estimated effects of the other acceleration patterns in CMS. Table 4 shows instrumental variable estimates of the impact of taking Algebra I by 8<sup>th</sup> grade.<sup>24</sup> These estimates include two-stage least squares results for all four outcome variables as well as bivariate probit results for the three binary outcomes.<sup>25</sup> Each model controls for 6<sup>th</sup> grade test score decile and cohort fixed effects, and uses a set of cohort-by-decile fixed effects as instruments. The instruments rely on across-cohort variation in the probability of students in a given decile taking Algebra I by a specific point in time. First stage results uniformly indicate a sufficient amount of variation to assuage potential concerns about weak instruments. In the first stage of the Algebra I test score regression, for example, the *F*-statistic on excluded instruments is 46.0 with a *p*-value less than 0.0001.

In three cases out of four, the results contrast starkly with the basic patterns revealed in our OLS analysis and previous research. Accelerated students score 36% of a standard deviation lower on their Algebra I end-of-course tests and are significantly less likely to pass courses in Geometry and Algebra II on a college-preparatory schedule.<sup>26</sup> Two-stage least squares estimates indicate that accelerated students are 18 percentage points less likely to pass the Geometry EOC test within five years of beginning 7<sup>th</sup> grade, and 11 percentage points less likely to pass Algebra II within six years. Bivariate

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<sup>24</sup> Technically, the dependent variable measures whether a student has taken the Algebra I EOC exam within two years after beginning 7<sup>th</sup> grade.

<sup>25</sup> Note that the sample size for the passing equations is slightly larger than that for the Algebra I test score equations. The analysis includes roughly 250 students – out of a base of over 32,000 – who appear in the EOC test roster for Algebra I but have a missing test score. We include this small set of students in the course passing specifications, coding them as not having passed Algebra I. Results are unaffected by the exclusion of these students. In addition to these 250 students with missing scores, an additional set of students are coded as exempt from testing, and these are systematically excluded from all of our analyses.

<sup>26</sup> In additional specifications, we examined the effect of algebra acceleration on the 8<sup>th</sup> grade end-of-grade mathematics test, which is administered to all 8<sup>th</sup> grade students regardless of course enrollment. We found no significant effects, suggesting that any gain to 8<sup>th</sup> graders from enrolling in Algebra I are offset by weaker mastery of non-algebraic subjects covered on the EOG test.

probit results indicate similarly large effects: a student with a 50% chance of passing Geometry by the time he or she completes high school at baseline is estimated to have only a 39% chance of completion if accelerated into Algebra I in 8<sup>th</sup> grade.<sup>27</sup>

The two-stage least squares estimate of the impact of acceleration on passing Algebra I is negative, but small in magnitude and statistically insignificant. Given that the acceleration apparently reduces students' Algebra I test scores, the lack of an effect on passing the course (within four years of beginning 7<sup>th</sup> grade) suggests that students retake the test and pass it the second time around. This pattern is consistent with the basic information in Table 2, which indicates that retaking Algebra I became more commonplace in the wake of the acceleration initiative. In sum, these results show that cohorts exposed to Algebra I on the accelerated schedule implemented by Charlotte-Mecklenburg did worse in subsequent math courses. This finding is consistent with the hypothesis that, by taking Algebra I earlier, they ended up having insufficient grounding in pre-algebraic math.

#### *The impact of accelerating Algebra I for high and low achievers*

Mirroring Table 4, Tables 5 and 6 examine the impact of accelerating high-performing students into Algebra I in 7<sup>th</sup> grade and low-performing students into Algebra I in 9<sup>th</sup> grade, respectively. Table 5 omits results for specifications examining whether students pass Algebra I by 10<sup>th</sup> grade; the success rate among students subjected to acceleration into Algebra I in 7<sup>th</sup> grade is sufficiently high that there is almost no variation in the outcome available to analyze.<sup>28</sup>

Accelerated placement into Algebra I in 7<sup>th</sup> grade was applied primarily to students in the top two quintiles of the 6<sup>th</sup> grade math distribution. Results in Table 5 indicate that the students receiving

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<sup>27</sup> Note that the overall probability of passing Geometry for CMS students in our sample is 48.5%. Summary statistics for all dependent variables appears in Appendix Table A1.

<sup>28</sup> These specifications, available on request, indicate a significant negative effect of acceleration on the propensity to pass Algebra I by 10<sup>th</sup> grade. We infer that this effect reflects particularly negative effects on middle-decile students placed in 7<sup>th</sup> grade Algebra.

this accelerated treatment experienced a decline in Algebra I EOC test scores comparable to the declines experienced by their counterparts accelerated into Algebra I in 8<sup>th</sup> grade. Point estimates in course completion specifications are uniformly negative, but at most are one-third the magnitude of their counterparts in Table 4 and statistically insignificant. The case for offering Algebra I to high-achieving students in 7<sup>th</sup> grade thus appears to be stronger than the case for offering the course to moderate-performers in 8<sup>th</sup> grade. While we are unable to observe tangible benefits of acceleration here, bear in mind that the rate of Algebra II completion among high-performing students is very high at baseline. Moreover, acceleration into 7<sup>th</sup> grade may increase the likelihood of completing higher-level coursework beyond Algebra II, which we cannot directly observe in our data.

At the other end of the spectrum, students accelerated into Algebra I in 9<sup>th</sup> grade, drawn primarily from the lowest two deciles of the 6<sup>th</sup> grade math test distribution, show strong signs of negative impact. In this group, acceleration is associated with a full standard deviation decline in Algebra I EOC scores and significant reductions in the likelihood of passing Algebra I or any subsequent course. Two-stage least squares results indicate that accelerated students experience an astounding 46 percentage point drop in the likelihood of passing Algebra I by 10<sup>th</sup> grade. The bivariate probit coefficient is somewhat more modest in size, indicating that a student with a 50% chance of passing Algebra I at baseline has only a 29% chance upon acceleration. A student with a 50% chance of passing Geometry by 11<sup>th</sup> grade at baseline is estimated to have a 26% chance if accelerated. The likelihood of passing Algebra II by 12<sup>th</sup> grade is similarly diminished.

Virtually all CMS students affected by the acceleration initiative exhibited poorer math performance in the year of acceleration. Students with high initial achievement appear to have suffered only modest subsequent adverse effects, if any. Lower-performing students, even those placed in Algebra I in the 9<sup>th</sup> grade rather than the 10<sup>th</sup>, appear to have been harmed more severely. The time path of the policy, showing the district reversed acceleration for low- and moderately-performing



students but maintained it at the high end, suggests that the district may well have correctly perceived the pattern of effects.

## Robustness checks

These results are derived from a fairly strong identification strategy. We exploit variation in cohort exposure to acceleration across deciles, as well as the differential actions taken by CMS after 2004 – maintaining acceleration for some types of students but reversing course for others. As noted above, however, Charlotte-Mecklenburg undertook other significant policy changes at the same time as the algebra initiative. Replacing its former practice of busing for racial balance with a school choice plan could potentially have affected the math performance of some students.

To assess this threat to validity, we perform both verification and falsification tests. For verification, we evaluate an Algebra I placement policy change undertaken by another major North Carolina school district during the time period covered by our analysis. In these verification tests, we use a two-stage estimator identical to that employed in the analysis of patterns in CMS.

As a falsification test, we examine test score patterns in districts that did *not* appear to adopt any significant policy change over this time period, to see if the patterns observed in Charlotte-Mecklenburg show up, which should not be the case if our estimates for CMS are indeed the result of the district’s algebra policy. Here we use a two-sample instrumental variable estimator.<sup>29</sup> The first stage is estimated using data from CMS. In the second stage of this procedure, using data from one of three other districts, we replace information regarding a student’s Algebra I placement with a variable measuring the likelihood that a student in the same state test score decile and cohort would be placed in Algebra I by a certain point in time *had that student been enrolled in CMS*. This application of two-

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<sup>29</sup> Since the bivariate probit model requires observations to be identical in both equations, our falsification tests exclusively use two-sample-two-stage least squares estimators (Inoue and Solon, 2010). Standard errors are computed using the method of Murphy and Topel (1985).

sample instrumental variables analysis is nonstandard, in the sense that we do not expect the procedure to produce significant results. In theory, the instrumental variable is irrelevant in the second sample, and therefore predicted values based on the instruments should not be correlated with outcomes.

*Verification test: Guilford County*

The Guilford County school system is the state's third largest, serving the cities of Greensboro and High Point as well as surrounding areas. Figure 4 shows that Guilford pursued a policy of acceleration on a similar timetable to CMS. A student's likelihood of completing Algebra I by 8<sup>th</sup> grade increased substantially between the 2001 and 2002 cohorts. Guilford's acceleration was actually more dramatic than that in CMS. Lowest-quintile students in the 2004 cohort were placed in Algebra I in 8<sup>th</sup> grade at a rate above 40%, nearly twice the maximum rate observed for that quintile in CMS. Rates of Algebra I placement by 8<sup>th</sup> grade peaked at 80% in the next-lowest quintile, and in the middle quintile exceeded 90%. In contrast with CMS, which had reverted to baseline by the time the 2005 cohort entered 7<sup>th</sup> grade, Guilford's acceleration is still quite apparent in this last cohort.

Table 7 shows the results of two-stage least-squares and bivariate probit estimates of the effect of 8<sup>th</sup> grade Algebra I acceleration in Guilford County. Results here are similar to those in CMS in most cases. The estimated impact of acceleration on Algebra I EOC test scores is statistically significant, negative, and nearly identical to the point estimate obtained in CMS. Two-stage least-squares estimates suggest that acceleration raised the likelihood of passing Algebra I by 10<sup>th</sup> grade, though the bivariate probit coefficient is insignificant and smaller than that obtained in the CMS case. Effects on passing Geometry and Algebra II are uniformly negative and significant, with point estimates comparable to those in CMS.

Generally speaking, then, the Guilford results lend support to the conclusion that accelerating moderately-performing students into Algebra I in 8<sup>th</sup> leads to persistent negative effects on mathematics

performance. The Guilford results particularly assuage the concern that the CMS patterns might reflect the impact of the nearly-simultaneous cessation of racial busing and move toward school choice.

### *Falsification tests*

A proper falsification tests looks for (spurious) evidence of treatment effects in a sample that was not exposed to the treatment. In this context, we examine the relative performance of students in the 6<sup>th</sup> grade test score deciles and cohorts that would have been subjected to acceleration had they enrolled in CMS, but who attended different districts.

To ensure this is a valid test, we must first verify that students in other districts were not in fact exposed to the acceleration policy, or to any other simultaneous initiative affecting the same deciles in the same cohorts. Table 8 shows that this is in fact a debatable point in at least one of the three cases considered here. The results depicted here are derived from individual-level probit equations of the form:

$$(6) T_{idc} = \gamma_d + \gamma_c + \beta \hat{T}_{dc}^{CMS} + \eta_{idc}$$

where  $T_{idc}$  is an indicator for whether student  $i$  in cohort  $c$  and decile  $d$  completed Algebra I by the end of 8<sup>th</sup> grade, and  $\hat{T}_{dc}^{CMS}$  is the treatment rate for students in the same cohort and decile in CMS.<sup>30</sup> If there were no relationship between placement patterns in student  $i$ 's district and those in CMS, we would expect the coefficient  $\beta$  to be indistinguishable from zero.

Table 8 presents estimates of  $\beta$  using three alternate school districts. In one case, Winston-Salem/Forsyth (henceforth WSF), the estimated coefficient is statistically significant at the 5% level. In Wake County, which is now the state's largest district, the coefficient is negative with a  $t$ -statistic of -1.7, indicating that the cohort/decile cells subjected to acceleration in CMS were subjected to *deceleration* in

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<sup>30</sup> Standard errors in these equations are estimated using the Huber-White method for clustering at the cohort/decile level.

Wake. In Cumberland county – the state’s fifth-largest district, serving the Fayetteville area – the estimate of  $\beta$  is in fact larger in absolute value than in Wake, but owing to the district’s smaller size it is not statistically significant.

As a result of this evidence, we are not fully confident that any of these alternate counties serve as valid falsification tests.<sup>31</sup> Nonetheless, in Table 9 we report the results of the proposed two-sample procedure for all three counties.

The first column of results examines Algebra I test score patterns in the three alternate districts. The significant negative effects recorded in CMS and Guilford County are not present here. The Wake County coefficient is in fact statistically significant and opposite in sign to the CMS result – consistent with the observation above that Wake restricted access to Algebra I in the cohort-decile cells where CMS expanded access. The WSF and Cumberland point estimates are both less than one-fourth the magnitude of the CMS coefficient. These findings support our conclusion that accelerated students score significantly worse on the Algebra I EOC exam than observationally similar counterparts who were not accelerated.

The remaining columns check the course passage outcomes in the falsification districts. As we found no significant effect of acceleration on the likelihood of passing Algebra I by 10<sup>th</sup> grade in CMS, it is perhaps unsurprising that we find no such effect in the falsification districts either. In the Geometry and Algebra II specifications, we fail to replicate the pattern of significant negative effects observed in CMS and Guilford County, with one exception. In Wake County, students in cohort/decile cells subjected to acceleration in CMS exhibit significantly lower rates of passing Geometry by 11<sup>th</sup> grade. In further investigation, we attribute this result to inexplicably poor performance among Wake Geometry students in a single cohort. As the Forsyth and Cumberland coefficients are both less than half the CMS point estimate, we do not consider the Wake result a particular cause for concern.

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<sup>31</sup> Estimating equation (6) for Guilford County, which by comparison of Figures 1 and 4 appear to have pursued similar acceleration policies, yields a positive coefficient.

### **Investigating an alternative mechanism: instruction quality**

As noted above, one explanation for our finding that early exposure to Algebra I was detrimental is that the acceleration caused students to miss important pre-algebra course material. Another explanation is that the district's need for additional capacity in Algebra I caused it to sacrifice instruction quality. In the first year of the acceleration initiative, the district needed to offer instruction to an unusually large group of students – the last un-accelerated cohort and the first accelerated cohort. Between 2001/02 and 2002/03, the number of CMS students taking the Algebra I EOC exam increased from under 9,000 to over 11,000. It is important to distinguish between the two alternative explanations for the exposed cohorts' observed poor performance – insufficient pre-algebra grounding or decline in instruction quality. The course timing explanation would imply that a permanent shift to accelerated algebra would generate the same types of results we observe in Charlotte-Mecklenburg's brief policy experiment. In contrast, if the negative impacts are the result of a temporary fall in instruction quality, the apparent cost of acceleration would be confined to the phase-in period.<sup>32</sup>

The increased demand for Algebra I instruction could have affected the quality of instruction in many respects. Administrators could respond by boosting class sizes, by assigning less-qualified teachers to the course, or by reallocating highly-qualified instructors away from the subjects they would otherwise teach. Table 10, which tracks the number and qualifications of Algebra I teachers in CMS over time, shows that administrators avoided the first type of response. Between 2002 and 2003, the number of Algebra I teachers increased by roughly 25%, and the number of sections taught per teacher

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<sup>32</sup> Although the transition to the accelerated steady-state could have been accomplished in a single year, in practice enrollments persisted at an elevated level for several years. This reflects the increased rate of Algebra I retaking occasioned by the drop in performance documented above. The post-acceleration steady state might therefore result in a permanently higher level of Algebra I enrollment.

increased by 16%. There is no increase in class size, however. In fact, the mean class size for Algebra I was slightly smaller in 2003 than it was in 2002.<sup>33</sup>

Table 10 also shows a noticeable decline in teacher quality, as proxied by teacher qualifications from 2002 to 2003. The average experience of Algebra I teachers, weighted by enrollment in sections taught, declined from 10.8 years to 8.8 years in 2003. Nearly one-third of Algebra I students were taught by a teacher with less than three years' experience in 2003, up from under a quarter the year before. Licensure test score information, which is available only for a subsample of teachers, indicates a decline in credentials as well, both on general and subject-specific tests.

Table 11 shows the time allocation of teachers who taught at least one Algebra I section in 2003 and who were also tracked in the state's personnel system in the prior year. In the acceleration year, instructors of Algebra I spent less than half of their time teaching that specific course. The remainder of math teaching time was divided among both less- and more-advanced courses, ranging from pre-algebra to courses beyond Algebra II. A comparison with teaching patterns in the prior year reveals that teachers responsible for increasing the district's Algebra I capacity did so primarily by teaching fewer sections of pre-algebra, as well as teaching fewer other subjects including language arts and science. The proportion of time these teachers devoted to pre-algebra declined dramatically, whereas the proportion of time they devoted to higher-level subjects held steady or increased. Presuming that administrators tend to assign more qualified math teachers to higher-level courses, this pattern supports the general impression that the acceleration was accomplished by shifting less-qualified teachers into Algebra I.

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<sup>33</sup> Of course, the effect of class size on student learning in secondary schools is uncertain. Experimental evidence drawn from the early grades suggests that the beneficial effects of small class sizes dissipate rapidly as students age (Krueger, 1999). On the other hand, survey data indicates that math teachers in secondary schools adopt different practices in smaller classes (Betts and Shkolnik, 1999). There has been at least one experimental study of the impact of class size on performance in high school algebra, but the results were statistically inconclusive (Jensen, 1930).

Could this substitution of less-qualified teachers explain the entire acceleration effect? Recall that students assigned to novice teachers have been repeatedly shown to exhibit poorer test score performance than their peers assigned to veterans (Boyd et al. 2008; Clotfelter, Ladd and Vigdor 2007, 2010; Rivkin, Hanushek, and Kain 2005). Suppose that the novice-veteran differential was 15% of a standard deviation – an estimate at the very high end of the distribution observed in recent studies. Exposing 8.5% of students to novices would then yield a prediction that test scores would decline by just over 1% of a standard deviation – a tiny fraction of the test score effects reported in Tables 4, 5, and 6 above. Additional effects might accrue to the extent that teacher experience levels decline marginally at other points in the distribution; most estimates in the literature suggest that the returns to experience beyond the first few years are relatively small, however.

Many studies of the effect of teachers on student test scores conclude that teacher quality is not adequately reflected in any observed credential. These studies typically infer quality on the basis of “value-added” scores, derived from teacher fixed effects in longitudinal models of student achievement growth.<sup>34</sup> Some of these studies report that the difference between a high-performing and low-performing teacher might be as high as a full student-level standard deviation (Rivkin, Hanushek, and Kain, 2005; Rockoff 2004).

To assess the hypothesis that the adverse acceleration effects represent a decline in teacher “value-added,” note that our point estimates indicate effect sizes of up to one standard deviation in some cases. Such an effect could be accomplished only if teachers of non-accelerated students were drawn exclusively from the top tail of the distribution, and teachers of accelerated students from the bottom tail. The data presented in Tables 10 and 11 indicate that 72% of Algebra I sections offered in

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<sup>34</sup> We are unable to consistently compute “value-added” scores for the Algebra teachers in our sample for a number of reasons. As indicated above, a substantial number of Algebra teachers have no prior experience. As indicated in Table 11, Algebra teachers spend no more than one-third of their time teaching that course, and their performance in other courses is difficult or impossible to assess with test scores. Assessment of performance as a Geometry instructor is complicated by selection into the course; assessment of performance as a middle school math instructor is rendered impossible by the absence of student-teacher links in the North Carolina administrative data for middle school classrooms.

the acceleration year were taught by a set of individuals who also led 62% of such sections in the prior year. This discussion implies that the reduction in teacher quality required to explain the estimated adverse acceleration effect is far too large to be plausible.<sup>35</sup>

Thus, although we find strong evidence that CMS accommodated the surge in Algebra I enrollment associated with the 2003 acceleration by calling upon teachers with weaker credentials, the implied reduction in teacher quality is far too small to explain away the entire negative effect of acceleration on Algebra I test scores. Hence, we interpret our findings in light of the conceptual model presented above, namely that accelerating students into algebra is undesirable for many students because it shortens the time for them to master the skills they need to succeed in algebra and in subsequent math courses.

## Conclusion

Algebra is often described as a “gateway” to higher-level mathematics. Because of the largely hierarchical nature of mathematics instruction, however, the gateway label could equally well be applied to a range of pre-algebra courses, geometry, or any other math subject in the hierarchy. Moreover, the strong positive correlation between the timing of Algebra and later outcomes has been incorrectly interpreted as implying that failure of students to take the course before high school adversely affects their subsequent ability to enroll in the higher level math courses needed for college. That interpretation is incorrect because selection problems make it inappropriate to conclude that the correlation reflects a causal relationship. Our empirical evidence, based on a clear policy intervention affecting nearly the entire distribution of students in one of the nation’s largest school districts avoids

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<sup>35</sup> Suppose that the set of “new” Algebra I teachers were drawn entirely from the bottom tail of the value-added distribution, with scores of -0.5. Suppose further that the teachers who cease teaching Algebra I after 2002 were drawn exclusively from the top tail of the value-added distribution, with scores of 0.5. Assuming the average quality of teachers leading Algebra I sections in both 2002 and 2003 remained the same, the anticipated effect on Algebra I test scores would be -0.23 standard deviations, smaller than the observed test score effect in 8<sup>th</sup> grade and less than one-fourth the estimated effect in 7<sup>th</sup> and 9<sup>th</sup> grades.



the selection bias, and shows that early administration of Algebra I – *when not preceded by a longer-run strategy to accelerate the math curriculum* – is actually harmful for success in math.

Our results imply, for example, that California’s proposal to increase the proportion of students taking introductory algebra in 8<sup>th</sup> grade from 59% to 100%, absent any wholesale reform in pre-algebra math courses, would worsen rather than improve the college-readiness of affected students. Our results also cast doubt on assignment practices in school districts such as the District of Columbia, in which 4<sup>th</sup> grade math performance is significantly worse than in CMS, according to NAEP assessments, yet 8<sup>th</sup> grade algebra placement is the norm.

We must be a bit more cautious, however, in evaluating the impact of the past expansion of 8<sup>th</sup> grade algebra enrollment from one-sixth to one-third of the nation’s students over the past few decades. Presumably, the students affected by this expansion were drawn largely from the top two quintiles of the math achievement distribution. As Figure 1 shows, our identifying variation comes almost entirely from students at lower points in the achievement distribution. Assessing the impact of placing higher-achieving students in algebra in 8<sup>th</sup> grade would require observing policy variation within that group.

The optimal rate of 8<sup>th</sup> grade algebra-taking is undoubtedly greater than zero. Indeed, our results indicate that the increase in Algebra I taking among 7<sup>th</sup> graders in CMS has had no significant adverse long-term effects. Our evidence also suggests that the optimal rate of 8<sup>th</sup> grade algebra-taking, in a population equivalent to that in CMS, is at or below the observed baseline rate around 50%.

More generally, this evaluation illustrates the hazards of basing policy initiatives on simple correlational evidence, without first taking steps to assess the validity of causal interpretation.

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Table 1: North Carolina Standard Course of Study Competency Goals (2003)

Course	Competency Goals
7 <sup>th</sup> Grade Math	<p>Understand and compute with rational numbers.</p> <p>Understand and use measurement involving two- and three-dimensional figures.</p> <p>Understand and use properties and relationships in geometry.</p> <p>Understand and use graphs and data analysis.</p> <p>Demonstrate an understanding of linear relations and fundamental algebraic concepts.</p>
8 <sup>th</sup> Grade Math	<p>Understand and compute with real numbers.</p> <p>Understand and use measurement concepts.</p> <p>Understand and use properties and relationships in geometry.</p> <p>Understand and use graphs and data analysis.</p> <p>Understand and use linear relations and functions.</p>
Introductory Mathematics (High School pre-Algebra)	<p>Understand and compute with real numbers.</p> <p>Use properties and relationships in geometry and measurement concepts to solve problems.</p> <p>Understand and use graphs and data analysis.</p> <p>Understand and use linear relations and functions.</p>
Algebra I	<p>Perform operations with numbers and expressions (exponents, polynomials).</p> <p>Describe geometric figures in the coordinate plane.</p> <p>Collect, organize, and interpret data with matrices and linear models.</p> <p>Use relations and functions to solve problems.</p>

Source: North Carolina, *NC Standard Course of Study*, 2003.

<http://www.ncpublicschools.org/curriculum/mathematics/scos/2003/k-8/index>, 1/12/12.

Table 2: Progression of math courses for two CMS cohorts

	1999/2000 cohort (n=7,179)	2002/03 cohort (n=8,076)
Proportion of cohort taking Algebra I in 7 <sup>th</sup> grade	11.0%	16.2%
Proportion of cohort taking Algebra I in 8 <sup>th</sup> grade	28.9	47.8
Conditional on taking Algebra I in 8 <sup>th</sup> grade:		
Proportion passing Algebra I EOC test in 8 <sup>th</sup> grade	87.5	80.5
Proportion enrolled in Geometry in 9 <sup>th</sup> grade	81.8	68.7
Proportion passing Geometry EOC in 9 <sup>th</sup> grade	65.5	45.7
Proportion enrolled in Algebra II in 10 <sup>th</sup> grade	74.0	61.4
Proportion passing Algebra II EOC in 10 <sup>th</sup> grade	63.7	47.6
Proportion enrolled in Algebra II by 12 <sup>th</sup> grade	82.4	73.6
Note: Cohorts are defined by the year in which they first enter 7 <sup>th</sup> grade. For purposes of analysis in this paper, grade-repeating students are re-assigned to their original cohort.		

Table 3: Correlates of Math Success Measures: OLS Estimates

Independent variable	Algebra I Test Scores	Pass Algebra I by 10 <sup>th</sup> grade	Pass Geometry by 11 <sup>th</sup> grade	Pass Algebra II by 12 <sup>th</sup> grade
Enrolled in Algebra I by 8 <sup>th</sup> Grade	0.197*** (0.031)	0.130*** (0.012)	0.104*** (0.011)	0.154*** (0.009)
Year entered 7 <sup>th</sup> grade (2000 omitted)				
2001	0.132*** (0.027)	0.039*** (0.011)	0.010 (0.016)	0.022* (0.010)
2002	0.047 (0.027)	0.015 (0.012)	-0.047** (0.016)	-0.033** (0.012)
2003	0.011 (0.025)	0.023 (0.012)	-0.045** (0.016)	-0.055*** (0.013)
2004	0.036 (0.029)	0.054*** (0.011)	-0.038* (0.016)	-0.031** (0.010)
2005	0.188*** (0.043)	0.103*** (0.022)	0.005 (0.017)	0.012 (0.013)
6 <sup>th</sup> grade math test score decile (lowest omitted)				
Second lowest	0.227*** (0.039)	0.155*** (0.019)	0.040** (0.015)	0.064*** (0.011)
Third lowest	0.403*** (0.043)	0.267*** (0.021)	0.096*** (0.017)	0.136*** (0.015)
Fourth lowest	0.617*** (0.047)	0.397*** (0.015)	0.180*** (0.023)	0.223*** (0.014)
Fifth lowest	0.796*** (0.035)	0.462*** (0.015)	0.298*** (0.018)	0.300*** (0.015)
Sixth lowest	0.998*** (0.035)	0.511*** (0.011)	0.411*** (0.015)	0.390*** (0.014)
Seventh lowest	1.227*** (0.043)	0.545*** (0.012)	0.560*** (0.019)	0.461*** (0.014)
Eighth lowest	1.510*** (0.046)	0.566*** (0.017)	0.674*** (0.018)	0.552*** (0.013)
Ninth lowest	1.828*** (0.046)	0.577*** (0.018)	0.750*** (0.019)	0.596*** (0.016)
Highest	2.445*** (0.051)	0.574*** (0.019)	0.813*** (0.018)	0.644*** (0.016)
<i>N</i>	36,308	36,790	36,790	36,790
Adjusted <i>R</i> <sup>2</sup>	0.608	0.343	0.431	0.304

Note: Standard errors, corrected for clustering at the decile-cohort level, in parentheses. Algebra I test score is taken from the student's first test administration. Course passage is defined as passing the state's standardized end-of-course test in that subject. Grade-retained students are kept with their original cohort.

\*\*\* denotes a coefficient significant at the 0.1% level, \*\* the 1% level, \* the 5% level.

Table 4: Instrumental Variable Estimates of the Impact of Acceleration into Algebra I in 8<sup>th</sup> Grade

Independent variable	Algebra I Test Score	Pass Algebra I by 10 <sup>th</sup> grade	Pass Geometry by 11 <sup>th</sup> grade	Pass Algebra II by 12 <sup>th</sup> grade			
	2SLS	2SLS	BP	2SLS	BP	2SLS	BP
Enrolled in Algebra I by 8 <sup>th</sup> Grade	-0.364*** (0.094)	-0.027 (0.047)	0.154 (0.117)	-0.184*** (0.030)	-0.493*** (0.097)	-0.111*** (0.023)	-0.273*** (0.069)
<i>N</i>	36,308	36,790	36,790	36,790	36,790	36,790	36,790
Adjusted <i>R</i> <sup>2</sup>	0.576	0.326		0.394		0.272	

Note: Standard errors, corrected for clustering at the decile-cohort level, in parentheses. Algebra I test score is taken from the student's first test administration. Course passage is defined as passing the state's standardized end-of-course test in that subject. Grade-retained students are kept with their original cohort. Sample is restricted to those students observed as seventh graders who take Algebra I at some point over the next five years. All models control for 6<sup>th</sup> grade math test score decile and cohort fixed effects, and instrument for Algebra I enrollment by 8<sup>th</sup> grade using a set of decile-by-cohort indicators. Columns headed "2SLS" are estimated by two-stage least squares; columns headed "BP" are estimated by bivariate probit.

\*\*\* denotes a coefficient significant at the 0.1% level, \*\* the 1% level, \* the 5% level.



Table 5: Instrumental Variable Estimates of the Impact of Acceleration into Algebra I in 7<sup>th</sup> Grade

Independent variable	Algebra I Test Score	Pass Geometry by 11 <sup>th</sup> grade		Pass Algebra II by 12 <sup>th</sup> grade	
	2SLS	2SLS	BP	2SLS	BP
Enrolled in Algebra I by 7 <sup>th</sup> Grade	-0.392* (0.184)	-0.067 (0.058)	-0.163 (0.215)	-0.032 (0.039)	-0.064 (0.129)
<i>N</i>	36,308	36,790	36,790	36,790	36,790
Adjusted <i>R</i> <sup>2</sup>	0.603	0.423		0.292	

Note: Standard errors, corrected for clustering at the decile-cohort level, in parentheses. Algebra I test score is taken from the student's first test administration. Course passage is defined as passing the state's standardized end-of-course test in that subject. Grade-retained students are kept with their original cohort. Sample is restricted to those students observed as seventh graders who take Algebra I at some point over the next five years. All models control for 6<sup>th</sup> grade math test score decile and cohort fixed effects, and instrument for Algebra I enrollment by 8<sup>th</sup> grade using a set of decile-by-cohort indicators. Columns headed "2SLS" are estimated by two-stage least squares; columns headed "BP" are estimated by bivariate probit.

\*\*\* denotes a coefficient significant at the 0.1% level, \*\* the 1% level, \* the 5% level.

Table 6: Instrumental Variable Estimates of the Impact of Acceleration into Algebra I in 9<sup>th</sup> Grade

Independent variable	Algebra I Test Score	Pass Algebra I by 10 <sup>th</sup> grade	Pass Geometry by 11 <sup>th</sup> grade	Pass Algebra II by 12 <sup>th</sup> grade			
	2SLS	2SLS	BP	2SLS	BP	2SLS	BP
Enrolled in Algebra I by 9 <sup>th</sup> Grade	-1.016*** (0.280)	-0.463*** (0.164)	-0.551* (0.260)	-0.267** (0.084)	-0.647*** (0.195)	-0.169** (0.065)	-0.478** (0.172)
<i>N</i>	36,308	36,790	36,790	36,790	36,790	36,790	36,790
Adjusted <i>R</i> <sup>2</sup>	0.549	0.236		0.401		0.276	

Note: Standard errors, corrected for clustering at the decile-cohort level, in parentheses. Algebra I test score is taken from the student's first test administration. Course passage is defined as passing the state's standardized end-of-course test in that subject. Grade-retained students are kept with their original cohort. Sample is restricted to those students observed as seventh graders who take Algebra I at some point over the next five years. All models control for 6<sup>th</sup> grade math test score decile and cohort fixed effects, and instrument for Algebra I enrollment by 8<sup>th</sup> grade using a set of decile-by-cohort indicators. Columns headed "2SLS" are estimated by two-stage least squares; columns headed "BP" are estimated by bivariate probit.

\*\*\* denotes a coefficient significant at the 0.1% level, \*\* the 1% level, \* the 5% level.

Table 7: Verification Test using District with Similar Acceleration Policy (Guilford Co.)

Independent variable	Algebra I Test Score	Pass Algebra I by 10 <sup>th</sup> grade	Pass Geometry by 11 <sup>th</sup> grade	Pass Algebra II by 12 <sup>th</sup> grade			
	2SLS	2SLS	BP	2SLS	BP	2SLS	BP
Enrolled in Algebra I by 8 <sup>th</sup> Grade	-0.353*** (0.065)	0.052* (0.020)	0.075 (0.187)	-0.090** (0.027)	-0.403*** (0.110)	-0.101*** (0.025)	-0.347*** (0.100)
<i>N</i>	23,937	24,171	24,171	24,171	24,171	24,171	24,171
Adjusted <i>R</i> <sup>2</sup>	0.599	0.291		0.431		0.278	

Note: Standard errors, corrected for clustering at the decile-cohort level in 2SLS specifications, in parentheses. Bivariate probit models estimated with clustered standard errors failed to converge; conventional standard errors are reported in those specifications. Bivariate probit models drop observations in cells lacking variation in either outcome variable. Algebra I test score is taken from the student's first test administration. Course passage is defined as passing the state's standardized end-of-course test in that subject. Grade-retained students are kept with their original cohort. Sample is restricted to those students observed as seventh graders who take Algebra I at some point over the next five years. All models control for 6<sup>th</sup> grade math test score decile and cohort fixed effects, and instrument for Algebra I enrollment by 8<sup>th</sup> grade using a set of decile-by-cohort indicators. Columns headed "2SLS" are estimated by two-stage least squares; columns headed "BP" are estimated by bivariate probit.

\*\*\* denotes a coefficient significant at the 0.1% level, \*\* the 1% level, \* the 5% level.

Table 8: Assessing the Validity of Falsification Tests

Independent variable	Dependent variable: Enrollment in Algebra I by 8 <sup>th</sup> grade		
	Wake County (Raleigh)	Forsyth County (Winston-Salem)	Cumberland County (Fayetteville)
Proportion of CMS students in same cohort/decile who take Algebra I by 8 <sup>th</sup> grade	-0.284 (0.166)	0.702* (0.323)	-0.391 (0.414)
<i>N</i>	34,610	14,930	14,754

Note: Equations are estimated by probit and include cohort and decile fixed effects. Standard errors, corrected for clustering at the cohort/decile level, in parentheses.  
\* denotes a coefficient significant at the 5% level.

Table 9: Falsification Tests using Three Alternate Districts

Coefficient on 8 <sup>th</sup> grade Algebra I- taking rate in same decile/cohort, CMS in:	Dependent Variable			
	Algebra I test score	Pass Algebra I by 10 <sup>th</sup> grade	Pass Geometry by 11 <sup>th</sup> grade	Pass Algebra II by 12 <sup>th</sup> grade
Wake County	0.097* (0.046)	0.012 (0.015)	-0.127*** (0.027)	-0.040 (0.028)
Forsyth County	-0.081 (0.068)	-0.032 (0.033)	-0.044 (0.039)	-0.080 (0.043)
Cumberland County	-0.078 (0.067)	0.017 (0.035)	-0.020 (0.043)	0.018 (0.047)

Note: Standard errors in parentheses have been computed using the Murhpy-Topel (1985) method, as applied to two-sample two-stage least squares by Inoue and Solon (2010). All equations estimated by TS2SLS.

\*\*\* denotes a coefficient significant at the 0.1% level, \*\* the 1% level, \* the 5% level.

Table 10: Algebra Teacher Characteristics by School Year, Charlotte-Mecklenburg Schools

	1999/2000	2000/01	2001/02	2002/03	2003/04	2004/05
Number of Unique Teachers	183	222	198	249	228	228
Number of Sections per Teacher	2.038	1.905	2.051	2.378	2.232	2.031
Number of Students per Teacher	43.71	40.68	43.90	49.01	47.84	43.36
Enrollment-weighted mean characteristics						
Years of Experience	11.23	10.56	10.82	8.768	9.895	10.52
2 or Fewer Years' Experience	20.99%	26.85%	23.10%	31.57%	24.91%	27.14%
General Licensure Scores	0.217	0.183	0.138	0.097	0.217	0.100
<i>Number of Teachers with General Scores</i>	<i>165</i>	<i>192</i>	<i>171</i>	<i>214</i>	<i>195</i>	<i>203</i>
Math Licensure Scores	0.639	0.603	0.539	0.453	0.417	0.333
<i>Number of Teachers with Math Scores</i>	<i>33</i>	<i>42</i>	<i>35</i>	<i>58</i>	<i>48</i>	<i>42</i>

Note: Licensure test scores are standardized to have mean zero and standard deviation one for teachers taking the same test in the same year.

Table 11: Teacher Time Allocation in Charlotte-Mecklenburg Schools, 2001/02-2002/03

Subject Areas	2002/03		2001/02	
	Teacher Sections	Percentage	Teacher Sections	Percentage
Mathematics	961	79.1%	838	72.9%
Pre-Algebra & Lower Level	198	16.3%	393	34.2%
Algebra I	428	35.2%	251	21.8%
Geometry	66	5.4%	58	5.0%
Algebra II & Higher Level	79	6.5%	62	5.4%
Other Mathematics	190	15.6%	74	6.4%
Language	163	13.4%	201	17.5%
Science	34	2.8%	48	4.2%
Social Studies	26	2.1%	31	2.7%
Other Subjects	31	2.5%	31	2.7%
Total Observations	1215	100%	1149	100%

Note: Sample consists of teachers assigned to at least one section of Algebra I in 2002/03 who also appear in CMS course assignment records for 2001/02. “Other Mathematics” includes Technical Math I & II, Discrete Math, Integrated Math I & II, and Special Topics in Mathematics. “Other Subjects” includes computer science, health and physical education, vocational education, non-classroom activities (such as SAT preparation) and miscellaneous.

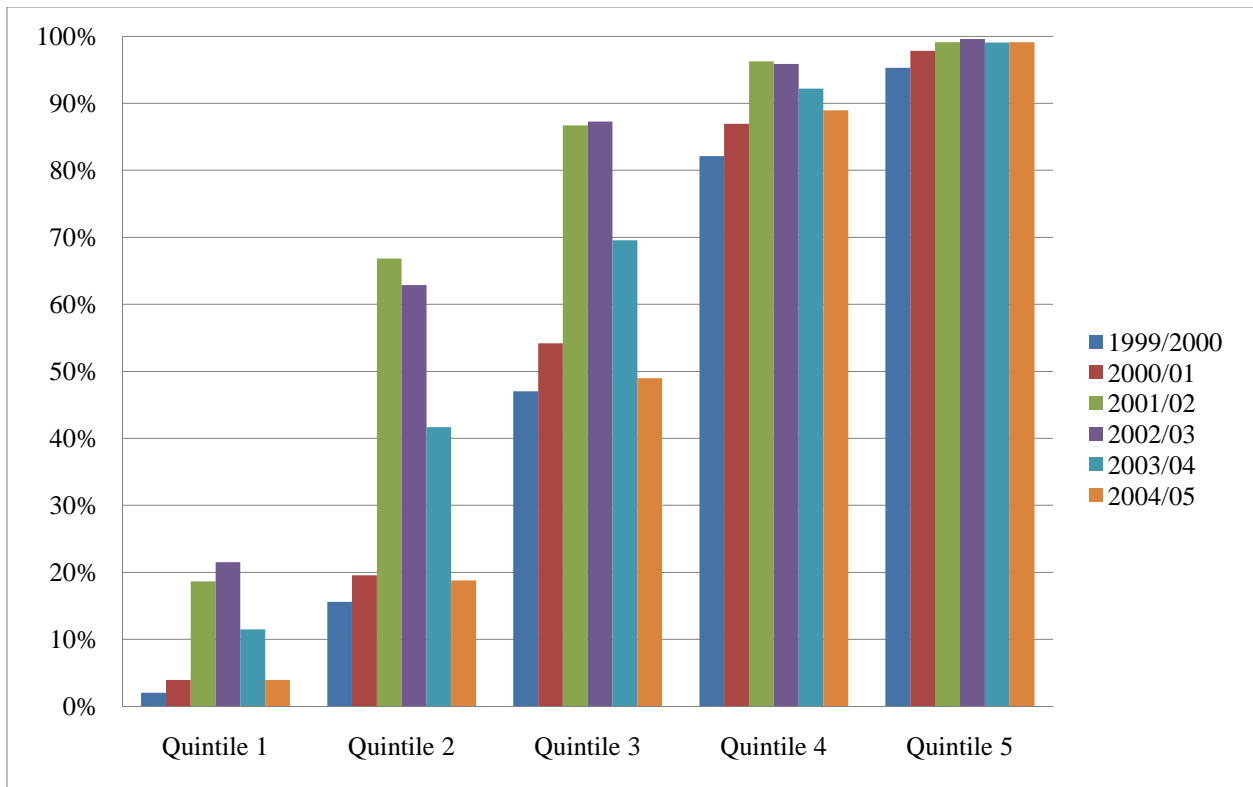


Figure 1: Probability of taking Algebra I by 8<sup>th</sup> grade, by 6<sup>th</sup> grade math test score quintile and year entering 7<sup>th</sup> grade, Charlotte-Mecklenburg Schools.



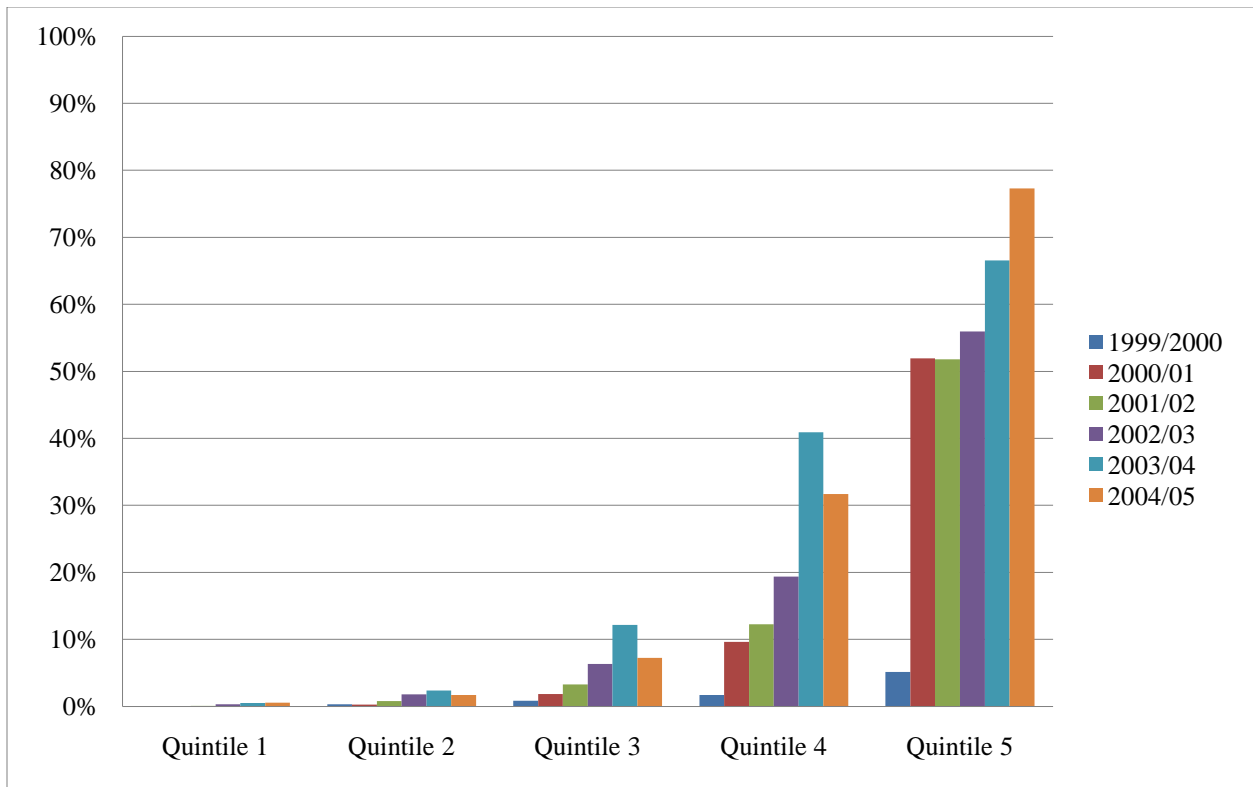


Figure 2: Probability of taking Algebra I by 7<sup>th</sup> grade, by 6<sup>th</sup> grade math test score quintile and year entering 7<sup>th</sup> grade, Charlotte-Mecklenburg Schools.

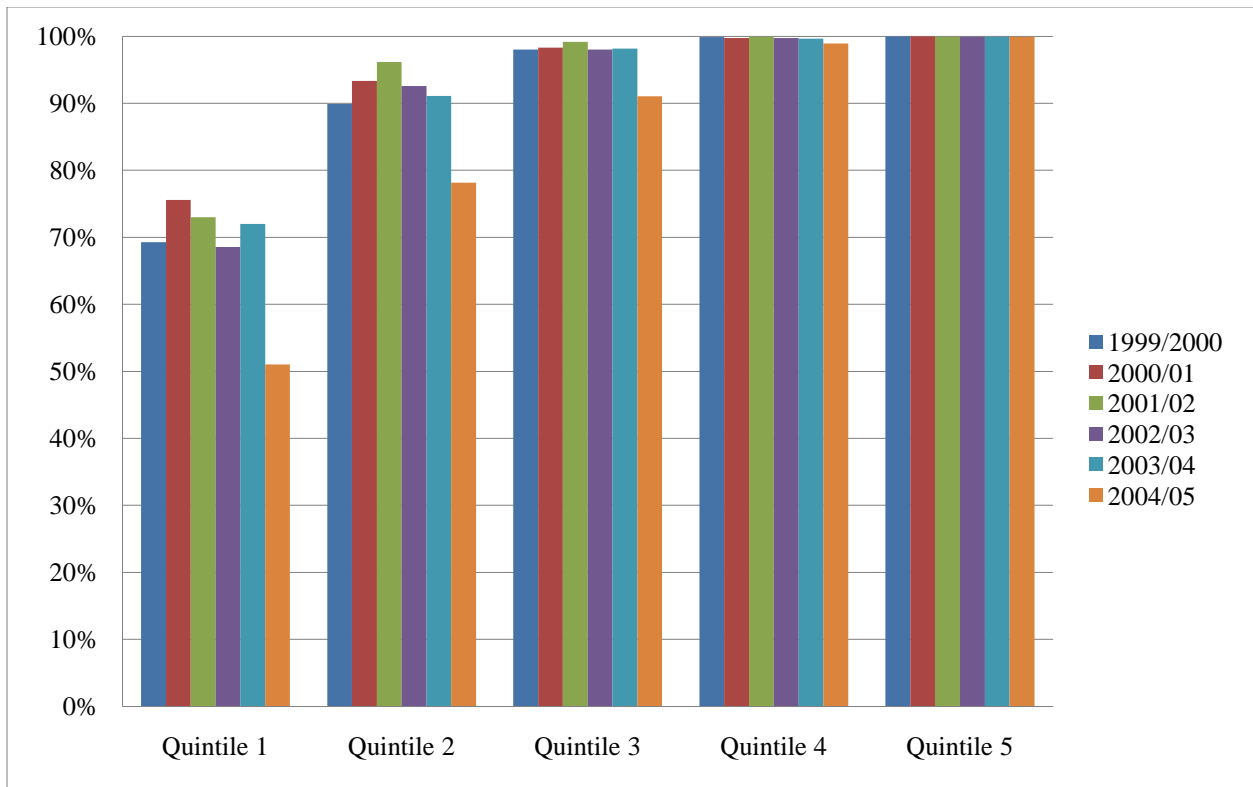


Figure 3: Probability of taking Algebra I by 9<sup>th</sup> grade, by 6<sup>th</sup> grade math test score quintile and year entering 7<sup>th</sup> grade, Charlotte-Mecklenburg Schools.

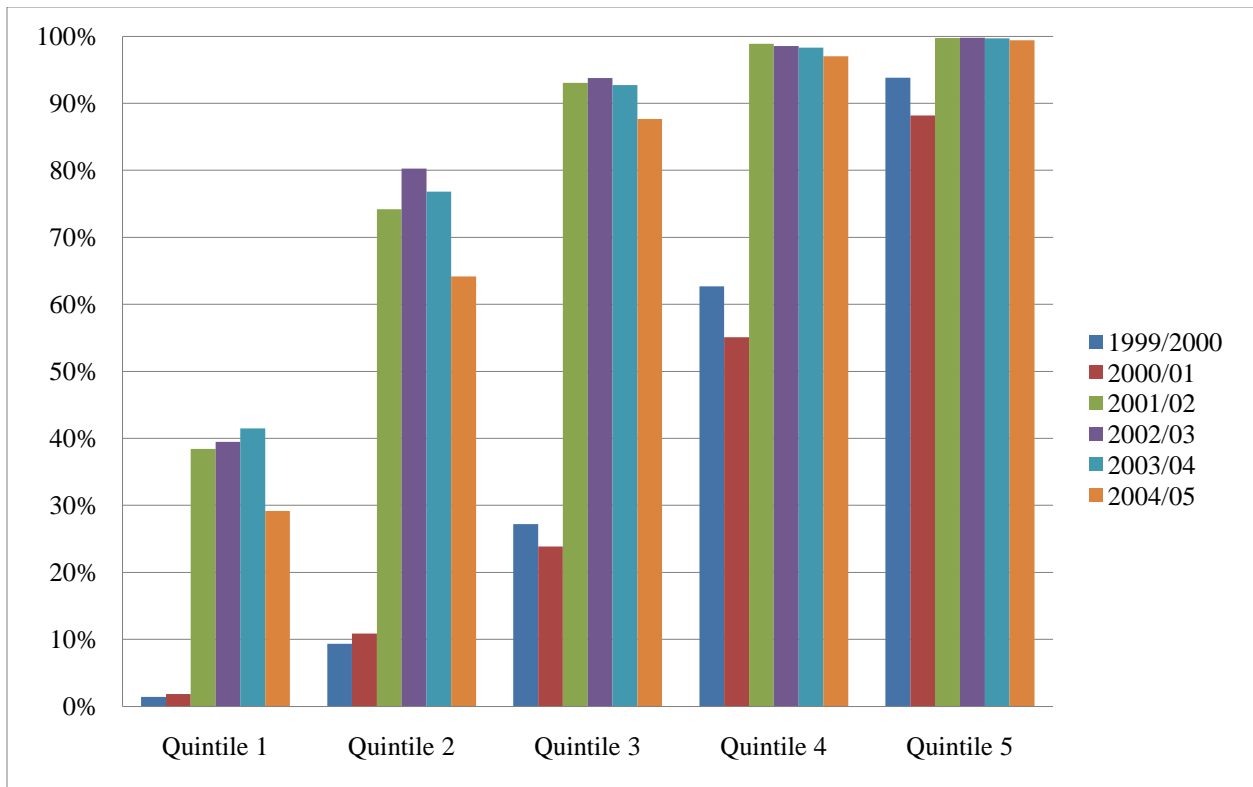


Figure 4: Probability of taking Algebra I by 8<sup>th</sup> grade, by 6<sup>th</sup> grade math test score quintile and year entering 7<sup>th</sup> grade, Guilford County Schools.

Table A1: Summary Statistics for Dependent Variables

School District	Algebra I test scores	Pass Algebra I by 10 <sup>th</sup> grade	Pass Geometry by 11 <sup>th</sup> grade	Pass Algebra II by 12 <sup>th</sup> grade
CMS	-0.121 (1.050)	72.7%	48.5%	49.5%
Wake County	0.583 (0.953)	87.3%	68.4%	66.1%
Guilford County	-0.212 (1.033)	76.0%	49.8%	52.2%
Forsyth County	0.022 (1.021)	74.2%	52.1%	50.8%
Cumberland County	-0.074 (0.910)	70.2%	45.5%	44.8%

Note: In each district, sample is restricted to those students observed consistently for a period of 6 years beginning in 7<sup>th</sup> grade, and who take Algebra I at some point during this period. Mean and standard deviation reported for test scores, sample proportion for all other variables.