

**Abstract Title Page**  
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**Title:** Effects of Cognitive Strategy Interventions on Word Problem Solving  
and Working Memory in Children with Math Disabilities

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## **Abstract Body**

*Limit 4 pages single spaced.*

### **Background / Context:**

*Description of prior research and its intellectual context.*

Although current categories of learning disabilities include as specific disabilities calculation and mathematical problem solving [see IDEA reauthorization, 2004, Sec. 300.8(c)(10) ], the majority of research focuses on calculation disabilities. Previous studies have shown, however, that deficits in word problem solving difficulties are persistent across the elementary school years even when calculation and reading skills are in the normal achievement range (Swanson et al., 2008). Previous research shows that growth in working memory (WM) is related to growth in word problem solving accuracy for children with math disabilities (MD e.g., Swanson, 2006, Swanson et al., 2008). However, the research is unclear as to the interventions that compensate for WM limitations in children with MD that in turn positively influence performance on problem solving measures.

One instructional approach considered in this study that may compensate for WM limitations in children with MD is to implement strategies that direct such children's attention to relevant components of word problems in the context of increases in irrelevant information (i.e., interference). That rationale for this approach is two-fold. First, several studies have found that verbal strategy instructions (e.g., Montague, 2008; Montague, Warger, & Morgan, 2000; Xin, 2008) as well as visual-spatial strategies (e.g., Kolloffel, Eysink, de Jong, & Wilhelm, 2009; van Garden, 2007) enhance children's math performance relative to control conditions (see Baker, Gersten, & Lee, 2002; Gersten et al., 2009; Swanson, 2009 for reviews). However, it is unclear from this research as to whether strategies children help children with MD compensate for their WM limitations. Second, strategy training may help children with MD focus their attention on relevant information. Several studies have suggested that a "key" mechanism that underlies WM performance is "controlled attention;" that is an individual's ability to access and process relevant information in the context of interfering information (Engle et al., 1999; Unsworth, 2010; Unsworth & Engle, 2007). Difficulties in controlled attention have been found to underlie some of the cognitive deficits experienced by children with MD (e.g., Passolunghi, Cornoldi, & De Liberto, 2001; Passolunghi & Siegel, 2001). Therefore, it is worth investigating whether strategy training helps children to attend to relevant information in word problems (i.e., key components of word problems) than in turn compensate for WM demands. Although strategy training is not predicted to modify WM per se (which is considered a stable construct, although see Swanson, 1992), strategy training may help increase the retrievability of information for children constrained by WM demands. In the present study we assessed whether strategy instruction that teaches children to focus on the relevant propositional structures of word problems improves solution accuracy. The relevant propositions were related to accessing numerical, relational, and question information, as well as accessing the appropriate operations and algorithms for obtaining a solution (Mayer & Hegarty, 1996; Swanson, Cooney & Brock, 1993). Of particular interest was to examine whether some strategies are more effective than others when compensating for individual differences in WM as well as when the demands on controlled attention (i.e., WM) increase across instructional sessions. The increases in WM demands were achieved by increasing the number of irrelevant propositions within word problems during instruction. The irrelevant propositions were simple in the early stages of training (one or two irrelevant sentences) and were increased at fixed levels across later intervention sessions (three to eight irrelevant sentences).

## **Purpose / Objective / Research Question / Focus of Study:**

*Description of the focus of the research.*

The purpose of this intervention study was to determine whether children with MD improve in problem solving ability as a function of strategy intervention that emphasizes relevant components of word problems. Children with MD and without MD were randomly assigned to one of three treatment groups: verbal strategies, visual strategies or a combination of both verbal and visual strategies. This study addressed three questions: (1) Do cognitive strategies that direct MD children's attention to relevant components of word problems in the context of distracting or irrelevant information enhance mathematical problem solving accuracy when compared to control conditions? (2) Are visual-spatial strategies in isolation or when combined with verbal strategies more effective than verbal strategies in isolation for children with MD? and (3) Do specific cognitive strategies and increases in WM load (number of irrelevant sentences in word problems) play an independent or interactive role in facilitating solution accuracy and transfer?

## **Setting:**

Children in the treatment conditions were provided instruction in small groups during math time and were instructed in a class or library in the same school setting. All children in the study participated with their peers in their home rooms on tasks and activities related to the district wide math school curriculum.

## **Population / Participants / Subjects:**

*Description of the participants in the study: who, how many, key features, or characteristics.*

One hundred and twenty (120) children from grades 2 and 3 in a Southern California public school district participated in this study. Of the 120 children selected, fifty-five were females and sixty-five were males. Ethnic representation of the sample was 63 Anglo 17 Hispanic, 8 African American, 8 Asian, and 24 mixed and/or other (e.g. Anglo and Hispanic, Native American). The mean SES of the sample was primarily low SES to middle SES based on free lunch participation (85% of sample received free lunch). Because the focus of this study was on children word problem solving difficulties, our criteria for defining MD was a score between the 25<sup>th</sup> and 90<sup>th</sup> percentile on a measure of Fluid intelligence (Raven Colored Progressive Matrices Test) and a score below the 25<sup>th</sup> percentile (below a standard score of 90 or scale score of 8) on standardized word problem solving math tests. The story problem subtests from the Test of Math Ability (TOMA, Brown, Cronin, & McIntire, 1994) and Key Math (Connolly, 1998) were both used to identify children below the 25<sup>th</sup> percentile (scale score of 8). This procedure separated the sample into 71 children with MD (32 females) and 49 children (39 females) without MD. Performance on standardized measures of word problem solving accuracy for the MD sample was below the 25<sup>th</sup> percentile (scale score at or below 8, standard score below 90), whereas their norm-referenced scores on calculation, reading comprehension, fluid intelligence and digit naming speed were above the 35<sup>th</sup> percentile. No significant differences emerged between children with and without MD as a function of grade, ethnicity, gender or chronological age.

## **Intervention / Program / Practice:**

Each experimental treatment condition included 20 scripted lessons administered over 8 weeks. Each lesson was 30 minutes in duration and was administered three times a week in small groups of 4 to 5 children. Lesson administration was done by one of six tutors (doctoral students). Children were presented with individual booklets at the beginning of the lesson, and

all responses were recorded in the booklet. Each lesson within the booklet consisted of four phases: warm-up, instruction, guided practice, and independent practice. The warm-up phase included two parts: calculation of problems that required participants to provide the missing numbers ( $9+2=x$ ,  $x+1=6$ ;  $x-5=1$ ) and a set of puzzles based on problems using geometric shapes. This activity took approximately 3 to 5 minutes to complete. The instruction phase lasted approximately 5 minutes. At the beginning of each lesson, the strategies and/or rule cards were either read to the children (e.g., to find the whole, you need to add the parts) or reviewed. Depending on the treatment condition, children were taught the instructional intervention (Verbal Strategy, Visual Strategy, or Verbal Strategy + Visual Strategy). The steps for the Verbal Strategy-only approach included: find the question and underline it, circle the numbers, put a square around the key word, cross out information not needed, decide on what needs to be done (add/subtract/or both) and solve it. For the Visual Strategy-only condition students were taught how to use 2 types of diagrams. The first one represented how parts made up a whole. The second type of diagram represented how quantities were compared. The diagram consisted of 2 empty boxes, one bigger and the other smaller, at which students were to fill in the correct numbers representing the quantities. An equation with a question mark was presented. The question mark acted as a placeholder for the missing number provided in the box. Finally, for the combined Verbal + Visual Strategy condition, an additional step (diagram) was added to the 6 Verbal Strategy steps described above. This step included directing students to fill in the diagram with given numbers and identify the missing numbers in the corresponding slots in the boxes. The third phase, *guided practice*, lasted 10 minutes and involved students working on three practice problems. Tutor feedback was provided on the application of steps and strategies to each of these three problems. In this phase, students also reviewed problems from the examples from the instructional phase. The tutor assisted students with finding the correct operation, identifying the key words, and providing corrective feedback on the solution. The fourth phase, *independent practice*, lasted 10 minutes and required students to independently answer another set of three word problems without feedback. If the student finished the independent practice tasks before 10 minutes were over, they were presented with a puzzle to complete. Student responses were recorded for each session to assess the application of the intervention and problem solving accuracy.

### **Research Design:**

Children were randomly assigned to either a control group ( $N=26$ ) or to one of three treatment conditions: Verbal Strategies ( $N=40$ ), Verbal + Visual Strategies ( $N=34$ ), and Visual Strategies-only ( $N=20$ ). The verbal treatment conditions drew strategy steps and activities based on the work of Montague (2007; 2008), Fuchs et al. (2003, 2004) and Jitendra et al. (1998), whereas the visual strategy intervention drew upon the work of Van Garderen and Montague (2007) and related studies using diagrams from the Singapore curriculum (e.g., Kolloffell et al., 2009; Looi & Lim, 2009; Ng & Lee, 2009). Chi-square test at pretest indicated that no significant differences emerged among the 4 treatment conditions as a function of risk status for MD,  $\chi^2$  ( $df=3$ ,  $N=120$ )=1.34,  $p > .05$ , grade,  $\chi^2$  ( $df=3$ ,  $N=120$ )=7.55,  $p > .05$ , ethnicity,  $\chi^2$  ( $df=12$ ,  $N=120$ )=19.81,  $p > .05$ , or gender,  $\chi^2$  ( $df=3$ ,  $N=120$ )=1.22,  $p > .10$ . To control for the possible impact of classroom teacher, each participating classroom included students in the different strategy conditions. To control for the impact of the graduate student tutors who implemented the interventions, all tutors were randomly rotated across days of the week and across treatment conditions, so that no one intervention group received instruction from the same

graduate tutor each time (i.e. "tutor 1" might give Strategy A in the morning timeslot on Monday, but then "tutor 2" presented the next Strategy A lesson to the same students during that timeslot on Wednesday). During the lesson sessions, tutors were randomly observed by an independent observer (a post doctoral student, a non tutoring graduate student, and/or the project director). The observers independently filled out evaluation forms covering all segments of the lesson intervention. Observations of each tutor occurred for 6 sessions randomly distributed across instructional sessions. Interrater agreement was calculated on all observations and exceeded 90% across all observed categories.

### **Data Collection and Analysis:**

A battery of tests measuring problem solving processes and accuracy, calculation, problem solving processes, and working memory at pretest and post-test. Moderator variables included measures of reading comprehension, estimation, unitization, naming speed, and inhibition (random generation of numbers). Because the data reflected treatments of students nested within classrooms, the primary analysis included a hierarchical linear model (HLM, Bryk & Raudenbush, 2002) that analyzed treatment effects nested within classrooms. The between-child variance components,  $\tau^2_{0} = \text{Var}(U_{0j})$  and  $\tau^2_{01} = \text{Var}(U_{01j})$  reflected individual differences in level of performance as a function of treatments embedded within classrooms. The simple conditional model was expressed as:  $y_{ij} = \beta_0 + \beta_{01}(\text{Verbal Strategy training}) + \beta_{02}(\text{Verbal+Visual-Strategy training}) + \beta_{03}(\text{Visual Strategy-only training}) + \beta_{04}(\text{control}) + \beta_{05}(\text{pretest}) + U_{0j} + U_{01j} + R_{ij}$  where  $y_{ij}$  is the dependent variable (e.g., post-test performance) measured at post-test,  $\beta_0$  is the grand mean,  $\beta_{01}$  to  $\beta_{04}$  are the treatment conditions, after controlling for pretest differences ( $\beta_{05}$ ). Several conditional models were compared testing the goodness of fit. The four treatment conditions were entered as binary variables (e.g., strategy-only + 1, other conditions 0) that by default allowed comparisons for three treatments with the control treatment. The same random effect and the residual as included in the unconditional model were included in the conditional models.

### **Findings / Results:**

The primary findings were: (a) both children with and without MD significantly improved word problem solving accuracy at post-test relative to other conditions, (b) pretest/post-test changes in correctly identifying relevant/irrelevant components mediated the effects of treatment on solution accuracy, (c) transfer (tasks independent of training) occurred on CBM of text word problems, verbal WM and calculation, and (d) individual differences in WM capacity interacted with treatment conditions when predicting overall solution accuracy.

### **Conclusions:**

For both math ability groups, both verbal+visual and visual-spatial strategies-alone significantly improved post-test problem solving accuracy. The visual strategy-alone treatment improved performance on measures of computation, timed quizzes, and verbal working memory span measures when compared to other conditions. Strategy conditions interacted with WM capacity (measured at pretest) but not to increases in WM load across intervention sessions when predicting post-test solution accuracy. The results indicate that specific strategy training (visual strategies) and general process training (gradual increases in WM load) significantly improved solution accuracy when compared to control conditions.

## Appendices

### Appendix A. References

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**Appendix B. Tables and Figures**  
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