Abstract Title Page

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Title: Improving foundational number representations through simple arithmetical training

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Abstract Body

Limit 4 pages single spaced.

Background / Context:

Description of prior research and its intellectual context.

Within the mathematics education literature there has been a focus upon fluency and its importance for academic success in mathematics. Highly accurate, quick, and relatively effortless performance at the basic level is seen as a crucial skill for performance at higher levels (Ostad, 1997; Pellegrino & Goldman, 1987; Resnick, 1983). This approach gave rise to memory-based learning activities (e.g., FASST Math, Scholastic) for encoding and efficiently retrieving basic “math facts” (e.g., fast retrieval of facts like “2+3=5” and “2+4=6” can support solving the problem “42+23”). We argue that this learning method does not lead to the development of deep understanding of numerical knowledge. Instead, the effectiveness of this approach seems to result from reducing working memory load by reducing the amount of calculation that has to be performed and by automating procedures such as “borrowing” and “carrying” (Pellegrino & Goldman, 1987; Resnick, 1983; Sweller, Mawer, & Ward, 1983). The skills being automated are not number-based, but general memory- and rule-based skills.

In contrast to this memory-based view of skill, across the cognitive neuroscience, cognitive science, and education literatures, the data converge to suggest that *analogic* numerical representations undergird the meaningful use of numeric symbolic codes and the rules for their manipulation. Analogic representations are abstract entities that preserve the quantity or size for which numbers stand. Most models of the analogic representation of numbers assume a number line analogy. In these models, each number is represented as a distribution of activation with an average or central tendency on the exact value of the number. (Dehaene, 1992; Dehaene, Dupoux, & Mehler, 1990; Gallistel & Gelman, 1992, 2000). The larger the number the more it overlaps with its neighbors, leading to reduced discriminability. This feature is consistent with the size effect: the larger the numbers involved in a problem, the more difficult is its processing (Moyer & Landauer, 1967) and the distance effect: slower and less accurate responses for comparisons of close numbers than of distant numbers; (Moyer & Landauer, 1967). Some neurological evidence also supports these models. The horizontal section of the intraparietal sulcus (hIPS) is a region associated with analogic representation and processing of numbers and appears to be specialized to represent different numerical quantities (Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene et al., 1996; Eger, Sterzer, Russ, Giaraud, & Kleinschmidt, 2003). The precision of the representation in hIPS decreases as the quantity increases (Nieder & Miller, 2004). In other words, neurons that represent small quantities are sensitive to small numerical differences (i.e., they are narrowly-tuned) while neurons that represent large quantities are more broadly tuned.

Recent findings suggest that mathematical performance can be related to the precision of the analogic representations of numbers in hIPS (Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011; Halberda, Mazzocco, & Feigenson, 2008; Peters, Slovic, Västfjäll, & Mertz, 2008). This alternative account of mathematical skills has received much less attention in the education literature and gives rise to the possibility that practice that increases the degree to which symbolic calculation is intertwined with high-quality analogic representation of quantity can also broadly enhance math fluency and skill.

Purpose / Objective / Research Question / Focus of Study:
Description of the focus of the research.

The aim of our study was to test a training program intended to fine-tune the mental representations of double-digit numbers, thus increasing the discriminability of such numbers. Our assumption was that increased fluency in math could be achieved by improving the analogic representations of numbers.

Setting:
Description of the research location.

The study was completed in the University of Pittsburgh and was a behavioral as well as imaging lab study. Only the behavioral part is reported here.

Population / Participants / Subjects:
Description of the participants in the study: who, how many, key features, or characteristics.

Forty participants (20 male) completed the experiment in return for $230 base pay plus performance bonuses for a mean total pay of $317. All participants were college students or recent graduates, 18 to 25-years-of-age with English as their first language. Participants were screened to be non-experts in math (math SAT between 600 and 700), and to not have a math-connected discipline as their major. Twenty participants (10 male) were assigned to the experimental group. The remaining participants, each matched in gender and math SAT score to an experimental participant, were assigned to the control group.

Intervention / Program / Practice:
Description of the intervention, program, or practice, including details of administration and duration.
For Track 2, this may include the development and validation of a measurement instrument.

A training program was designed to give extensive practice with multi-digit computation. Short response windows, adaptive difficulty, and monetary incentives were applied to discourage purely symbolic strategies and to encourage the engagement of feedback-based learning mechanisms. Training consisted of 5 one-hour sessions in which Arithmetic training participants solved addition and subtraction problems. A control group was trained to type numbers, controlling for simple exposure to numbers and practice with keypad entry.
In both conditions, three levels of difficulty, consisting on the number of digits in the problems (single/double (S/D), double/double (D/D), triple/double(T/D)).
To support representational change (e.g., Raiguel, Vogels, Mysore, & Orban, 2006), the training program was also designed to encourage the engagement of feedback-based learning mechanisms: 1) the training program incorporated contingent feedback on each trial; 2) to ensure engagement in learning, monetary rewards were provided for correct answers; and 3) to maximize uncertainty about the outcomes in the current study, short presentations of the stimuli and short response windows were imposed.

Research Design:

Description of the research design.

Participants attended nine sessions over ten consecutive days, excluding Sundays. Sessions 1 and 9 were dedicated to behavioral pre- and post-tests and lasted between 1.5-2.0 hours. The pre-test started with digit entry practice (to ensure that errors would not result from the failure to locate the desired key) and included the following tasks: Math Fact Retrieval, Number Comparison, Multi-Digit Arithmetic Fluency, Dots Comparison, and Complex Math, in that order. The same tasks were given in the post-test with the addition of an Automatic Addition test. The testing tasks were selected to asses a wide range of number-related abilities (see Table 1). Sessions 2 and 8 were fMRI pre- and post-scans of one hour each (results not reported here). Sessions 3 – 7 were training sessions of one hour each.

Data Collection and Analysis:

Description of the methods for collecting and analyzing data.
For Track 2, this may include the use of existing datasets.

Multi-Digit Arithmetic Fluency: Fifteen problems of each level (S/D, D/D) and operation (addition, subtraction) without time constraints. In the post-test previously seen and previously unseen problems were compared.
Math Fact Retrieval: Forty single digit problems of three operations: addition, multiplication, and subtraction.
Number Comparison: Sixteen target numbers, selected according to ratio from standard, were compared to a standard number (18, 25, 32, and 49) in a sequential presentation.
Dots Comparison: Same as Number Comparison, but with arrays of dots.
In both comparison tasks, closer numbers are typically judged more slowly and less accurately (i.e., the distance effect). Learning should therefore mainly improve comparisons of closer numbers.
Automatic Addition: Participants estimated the size of rectangles with embedded addition problems. Trials with the larger sum occupying the larger rectangle (congruent) were compared with trials with the larger sum occupying the smaller rectangle (incongruent).
Complex Math: Sixteen SAT-like problems to be solved in 30 minutes.
Statistical analyses (usually: ANOVA) were completed with Training group, Session, and the critical independent and dependent variables within each task.

Findings / Results:

Description of the main findings with specific details.

Participants in the Arithmetic training group improved over training, with the majority passing to the second difficulty level (D/D). Some evidence was found for greater improvement on the more difficult (carry/borrow) problems. A strategy change (from Right-to-Left to Left-to-Right) was found only for the most difficult problem types (Carry-Carry).

No difference was found between seen and unseen problems in the Multi-Digit Arithmetic Fluency test, suggesting that learning was not memory-based.

Accuracy did not improve in the Math Fact Retrieval test, but the Arithmetic training group showed some improvement of reaction time in Addition. The lack of accuracy gains following training suggests that training did not further facilitate basic fact retrieval, presumably reflecting already high levels of accuracy with basic number facts in this university student population.

The interference effect, associated with Stroop-like task of which the Automatic Addition is an example, was found in accuracy rates for the Arithmetical training group but not for the Control group. This result suggests that participants in this group developed automaticity of adding double-digit numbers.

In the Numbers Comparison task, accuracy rates improved for the Arithmetic training group subjects but not for the Control group. The improvement was mainly in ratios closer to 1 (see Figure 1), suggesting fine-tuning of numerical representations. RTs of Control participants became faster but no interaction was found with Ratio, suggesting general speeding of responses that had nothing to do with the representation of numbers.

In the Dots Comparison task, accuracy was higher in the post test for Arithmetic but not for Control participants and reaction times were faster for Control but not for Arithmetic training participants. However, no interactions with Ratio were found, suggesting there was no representational change for quantities following training.

Performance in the Complex math test improved for Arithmetic training but not for Control participants.

Conclusions:

Description of conclusions, recommendations, and limitations based on findings.

The results show an advantage of the training program over control training. The selective advantages of the Arithmetic training were widespread and included gains on SAT-like complex math questions, automatic addition, math facts retrieval, and representational change of numbers. This pattern of results suggested that when engaged in a training program that emphasizes fast processing of double-digit numbers, participants enhanced a variety of skills and representations that together comprise the basis for skilled numerical processing. Rather than merely increasing the ability to retrieve math facts from memory, our training program also increased automatic arithmetical proficiency and the precision of representations of double-digit numbers.

Appendices

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Appendix A. References

References are to be in APA version 6 format.

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Appendix B. Tables and Figures

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Test	Intended purpose
Multi-Digit Arithmetic Fluency	Tests whether advance in training was due to memorizing the solutions of trained problems or due to general improvement in the skill of arithmetical problem solving.
Math Fact Retrieval	Tests whether advantage of training was due to strengthening of memory traces for single-digits math facts.
Automatic Addition	Tests the degree of proficiency of adding two double-digit numbers.
Number Comparison	Tests the effect of the training on the analogic representations of (symbolic) numbers.
Dots Comparison	Tests the effect of the training on the analogic representations of non-symbolic quantities.
Complex Math	Tests the effect of arithmetic training on solving complex math problems.

Table 1: The transfer tests used in experiments 1 and 2 and their intended purposes.

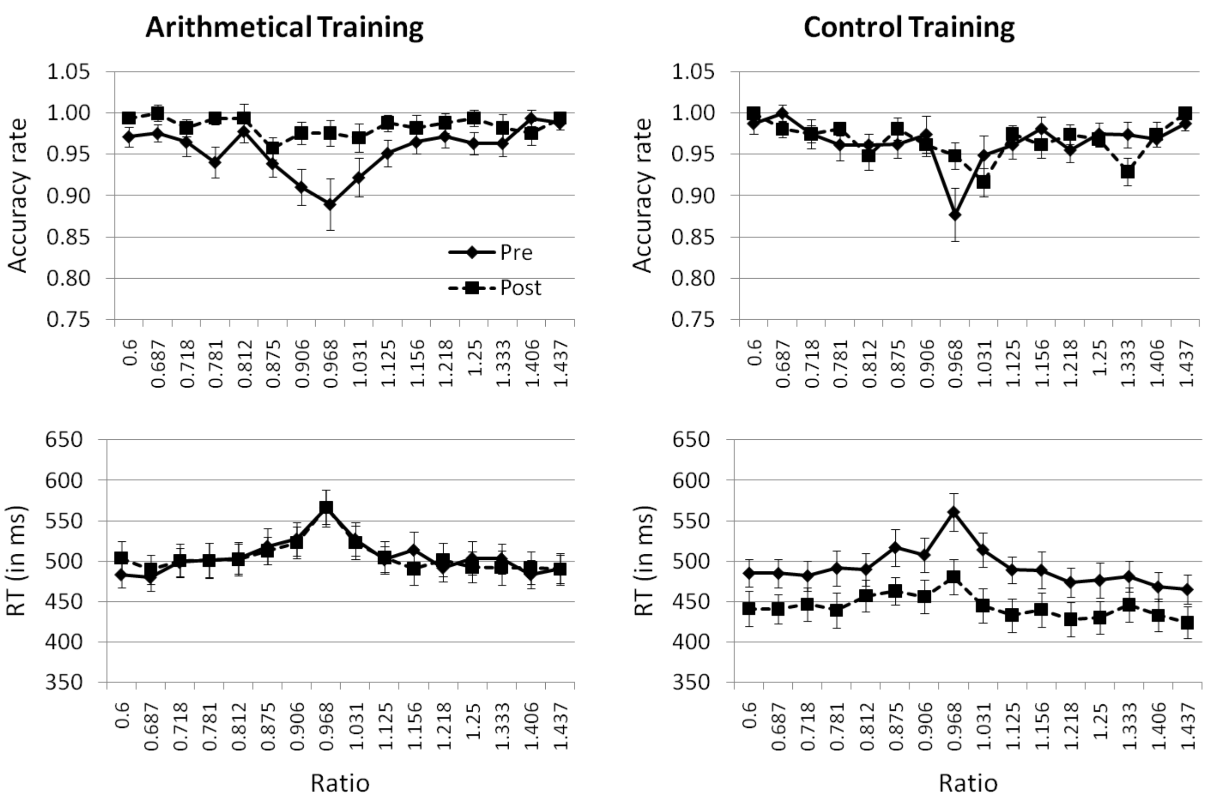

Figure 1: Accuracy rates and RT as a function of session (pre – solid line, post – dashed line) and ratio, for each group, in the Numbers Comparison task.