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Title:

Playing Linear Number Board Games Improves Children's Mathematical Knowledge

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Abstract Body

Background/context:

Among the most serious educational challenges facing the United States is the large discrepancy in academic performance between children from different economic backgrounds. Children from impoverished backgrounds achieve at a lower level than other students throughout the course of schooling (e.g., Alexander and Entwisle, 1988; Geary, 1994; 2006). One reason is that children from low-income backgrounds start school with far less academic knowledge than peers from more affluent families; substantial differences are present even before children start kindergarten.

Although these differences in preschoolers' knowledge are present in many subjects, they appear to be especially substantial in knowledge of mathematics (Case, Griffin, & Kelly, 1999).Kindergartners' performance on tests of mathematical knowledge is predictive of mathematical achievement in third, fifth, and eighth grade, and even in high school (Duncan et al., 2007; Stevenson & Newman, 1986). For example, in six longitudinal studies reviewed by Duncan et al. (2007), the standardized beta coefficients relating early and later mathematical knowledge were more than twice as large as the coefficients relating early and later reading proficiency, control of attention, and socio-emotional competence. These findings led us to analyze the sources of differences in young children's numerical knowledge and to use the analyses to generate methods for helping low-income preschoolers increase that knowledge.

Increases with age and experience in reliance on linear representations of numerical magnitudes seem to play a central role in the development of mathematical knowledge. Both changes in representations of numerical magnitudes and the relation of these changes to the growth of mathematical knowledge have been illustrated in research on number line estimation. The number line estimation task that has been used in this research involves presenting lines with a number at each end (e.g., 0 and 100) and no other numbers or marks in-between; the goal is to estimate the location on the number line of a third number (e. g., "Where would 74 go?") This task is particularly revealing because it transparently reflects the ratio characteristics of the number system. Just as 80 is twice as large as 40, the distance of the estimated position of 80 from 0 should be twice as great as the distance of the estimated position of 40 from 0. More generally, estimated magnitude (y) should increase linearly with actual magnitude (x), with a slope of 1.00, as in the equation $y = x$. Such estimates reflect a linear representation of numerical magnitudes.

Early in development, however, children's estimates often do not increase linearly with numerical magnitude. Many preschoolers, including ones who can count perfectly from 1-10, do not even understand the rank order of the numbers' magnitudes. This poor understanding of the rank order of numerical magnitudes is evident in these preschoolers' inaccurate numerical magnitude comparison (Ramani & Siegler, 2008; Whyte & Bull, 2008), number line estimation (Siegler & Ramani, 2008; Whyte & Bull, 2008), and performance on several other measures of understanding of numerical magnitudes (Condry & Spelke, 2008; LeCorre & Carey, 2007). Even after children understand the rank order of numbers, they often rely on logarithmic rather than linear representations of numerical magnitudes through fourth grade; between kindergarten and fourth grade, linearity of estimates is correlated with success on a variety of estimation tasks, numerical categorization, arithmetic, and mathematics achievement test scores (Booth & Siegler, 2006; 2008; Siegler & Booth, 2004).

These findings raise the question: What types of experiences lead children to first represent the magnitudes of verbally stated or written numerals as increasing linearly? Counting experience during the preschool period probably contributes, but such experience appears insufficient to create the linear representations. Children often count perfectly in a numerical range well before they generate linear representations of numerical magnitudes in that range (Le Corre, et al., 2006, Ramani & Siegler, 2008).

If counting experience is insufficient to yield linearly increasing magnitude representations, what other numerical experiences might contribute? One common activity that seems ideally designed for producing such representations is playing linear, number board games -- that is, board games with linearly arranged, consecutively numbered, equal-size spaces (e.g., *Chutes and Ladders*.) Such board games provide multiple cues to both the order of numbers and the numbers' magnitudes. The greater the number in a square, the greater: a) the number of discrete movements of the token that the child has made, b) the number of number names that the child has spoken, c) the number of number names that the child has heard, d) the distance that the child has moved the token, e) the endpoint of the token's travel, and f) the amount of time that has passed since the game began. The linear relations between numerical magnitudes and these kinesthetic, auditory, visuo-spatial, and temporal cues provide a broadly based, multi-modal, embodied foundation for a linear representation of numerical magnitudes.

Recent studies have demonstrated the usefulness of this analysis for improving the numerical knowledge of preschoolers from low-income backgrounds (Ramani & Siegler, 2008; Siegler & Ramani, 2008). Playing a linear numerical board game with squares numbered from 1 on the left end to 10 on the right for four 15-20 minute sessions yielded the predicted improvements in preschoolers' proficiency on four numerical tasks: numerical magnitude comparison, number line estimation, counting, and numeral identification. The gains remained evident nine weeks after the posttest (Ramani & Siegler, 2008). Peers who played an identical game, except for the squares varying in color rather than number, did not improve on any of the tasks. Moreover, amount of board game experience outside the laboratory context correlated positively with proficiency on all four tasks, and children from middle-income backgrounds reported playing board games (but not video games) at home and at the homes of friends and relatives more often than preschoolers from low-income families. Together, these findings strongly suggest that differences among individuals and socio-economic groups in experience playing board games contribute to differences in early numerical knowledge.

Purpose/objective/research question/focus of study:

The present study focused on two main goals. One was to test the *representational mapping hypothesis*: The greater the transparency of the mapping between physical materials and desired internal representations, the greater the learning of the desired internal representation. The implication of the representational mapping hypothesis in the present context is that if the desired internal representation of numerical magnitudes is a linear number line, then playing the number game with a linear board should promote greater learning of numerical magnitudes than playing the identical game with a circular board.

A second major goal of this study was to test the prediction that forming a linear representation of numerical magnitudes should improve young children's ability to learn answers to arithmetic problems. Linear representations of numerical magnitudes seem likely to help children learn arithmetic because such representations maintain equal subjective spacing throughout the entire range of numbers, thus facilitating discrimination among answers to different problems. Consistent with this perspective, linearity of number line estimates is positively correlated with arithmetic proficiency among first through fourth graders (Booth & Siegler, 2006; 2008). Thus, playing the linear number board game was expected to produce greater subsequent ability to recall the answers to arithmetic problems following instruction in them; it also was expected to produce errors on the arithmetic problems that were closer to the correct answer.

Setting:

The research was conducted at seven Head Start centers and two child care centers in Pittsburgh, Pennsylvania. Children were presented the materials in unoccupied rooms when possible and in the hallways outside of classrooms when that was the only available space.

Population/Participants/Subjects:

Participants were 88 preschoolers (56% female), ranging in age from 4 years 0 months to 5 years 5 months (*M* = 4 years 8 months, *SD* = .47). Among them, 34% were African-American, 61% Caucasian, and 5% Other (Asian, Hispanic, Biracial, or Unknown). The children were recruited from seven Head Start classrooms and two childcare centers, all of which served families with very low incomes. The families from the Head Start classrooms met the income requirements for Head Start established by the Federal government for 2007 (e.g. for a family of three, annual income below $17,170). Almost all of the other families (96%) received government subsidies for childcare expenses.

Intervention/Program/Practice:

All of the preschoolers met individually with an experimenter for five 15-20 minute sessions within a three-week period. Sessions were held in either their classroom or an unoccupied room nearby. Each experimenter met with the same children for all sessions in the study, and with approximately equal numbers of children in each of the three conditions.

Linear board game condition. A board 52 cm wide and 24 cm high was used in the linear board game condition. The name of the game, "The Great Race," was printed at the top of the board. Below the name were 10 equal size, different colored squares arranged in a horizontal array. Each square contained one number, with the numerical magnitudes increasing from left to right. The word "Start" was immediately left of the "1" square; the word "End" was immediately right of the "10" square.

The game also included a spinner with a "1" half and a "2" half, as well as a "bear token' and a "rabbit token." The child chose the bear or the rabbit token before each session to represent his or her progress on the board; the experimenter took the remaining token. Due to children almost always choosing to go first, and that being a substantial advantage in this game, children won most games.

At the beginning of each session, the experimenter told the child that they would take turns spinning the spinner and that whoever reached the end first would win. The preschoolers then were told that on each turn, the player who spun the spinner would move her or his token the number of spaces indicated on the spinner. The experimenter also told the child to say the numbers on the spaces through which the token moved. Thus, children who were on the square with a 3 and spun a 2 would say, "4, 5" as they moved.

If a child erred or could not name the numbers, the experimenter correctly named them and then had the child repeat the numbers while moving the token. Preschoolers played the game approximately 20 times, with each game lasting about three minutes. Children were not told explicitly "that's right" or "that's wrong," but the correction procedures provided implicit feedback.

Circular board game condition. The only difference between the linear and circular board game conditions was the board itself. There were two circular boards, each divided into 12 wedges. Both were 38 inches high and 41 inches wide. Ten of the wedges, those located approximately at the locations of 2:00 through 10:00 on an analog clock, included the numbers 1-10 ordered consecutively. On one board, the numbers increased clockwise; on the other, the numbers increased counterclockwise.

The procedure followed in the circular board condition was identical to that in the linear board condition. Half of the children in this condition played the clockwise version of the game (n = 15), and the other half played the counterclockwise version (n = 14). Similar to the linear board game condition, preschoolers played the circular board games approximately 20 times, with each game lasting about three minutes.

Research Design:

The study was a randomized field trial in which children were randomly assigned to the linear board game or circular board game condition.

Data Collection and Analysis:

In the pretest at the beginning of Session 1 and the posttest at the end of Session 4, children were presented a pretest and a posttest on several tasks.

Number line estimation. Children were presented 18 sheets of paper, one at a time. On each sheet was a 25 cm line, with "0" just below the left end, and "10" just below the right end. A number from 1-9 inclusive was printed approximately 2 cm above the center of the line, with each number printed on 2 of the 18 sheets. All numbers from 1-9 were presented once before any number was presented twice; the nine numbers were ordered randomly both times. Children were told that they would be playing a game in which they needed to mark the location of a number on a line. On each trial, after asking the child to identify the number at the top of the page (and helping if needed), the experimenter asked, "If this is where 0 goes (pointing) and this is where 10 goes (pointing), where does N go?"

Numerical magnitude comparison. Children were presented a 20-page booklet, each page displaying two numbers between 1 and 9 inclusive, and asked to choose the bigger number. The experimenter first presented 2 warm-up problems with feedback, followed by 18 experimental problems without feedback. The 18 experimental problems were a randomly chosen half of the 36 possible pairs. On the warm-up problems, the experimenter pointed to each number and asked (e.g.), "John (Jane) had one cookie and Andy (Sarah) had six cookies. Which is more: one cookie or six cookies?" On the two warm-up problems, the experimenter corrected any errors that were made (e.g., "Actually, six cookies is more than one"), and repeated the problems until the child answered them correctly. On the 18 experimental problems, half of the children within each condition were presented a given pair in one order (e.g., "Is 6 bigger than 3") and half in the opposite order ("Is 3 bigger than 6.")

Arithmetic problems and training. The arithmetic pretest was composed of four addition problems, presented in the order: 2+1, 2+2, 4+2, and 2+3. Children were asked, "Suppose you have N oranges and I give you M more; how many oranges would you have then?" As on the other pretest and posttest tasks, no feedback was given.

At the beginning of Session 5, children received training on the first two arithmetic problems that they had answered incorrectly on the pretest. The training involved presenting the two problems and their answers three times in alternating order. For example, children who erred on all four problems on the pretest were presented 2+1 and 2+2 in the first cycle of Session 5, 2+2 and 2+1 in the second cycle, and 2+1 and 2+2 in the third cycle. The problems were presented in the same "oranges" context as on the pretest. Children needed to answer each problem within 5 s; if they failed to do this, they were prompted to answer. On each trial, after children stated their answer, they were asked to explain how they obtained that answer. Then, they were given feedback and told the right answer. For example, on 2+2 they were told either "That's right; 2+2 is 4" or "No, 2+2 is 4." The children's explanations indicated that on almost all trials (96%), they retrieved the answer from memory or guessed. After the third cycle of feedback problems, children received the addition posttest, in which they were presented the same four problems in the same order as on the pretest and asked to state the answer.

Findings/Results:

The data were consistent with both hypotheses. On the number line estimation task, linearity, slope, and accuracy were all greater on the posttest for those who played the game with the linear board than for those who played it with the circular board. Mean percent variance accounted for by the best fitting linear function for each child increased significantly from 14% to 39% among children who played the linear board game; it increased non-significantly from 15% to 21% among those who played the circular game. Differences between groups were not significant on the pretest but were significant on the posttest.

On the magnitude comparison task, percent absolute error improved from pretest to posttest among children who played the game with the linear board (29% to 21%) and also among those who played the game with the circular board (29% to 26%). The results did not differ on the pretest, but children who played the linear game were more accurate on the posttest.

Even more striking was the learning to learn effect in arithmetic: Children who earlier had played the linear board game learned more from subsequent practice and feedback on addition problems than children who earlier had played the circular board game (45% versus 30% correct posttest responses). Their errors were also closer to the correct answer than were the errors of children in the circular board game condition.

Conclusions:

The linear number board game used in the present study has several advantages that recommend it for widespread use. It involves little if any expense; a board could easily be drawn on a piece of paper or cardboard, small household objects could be used as tokens, and a spinner or even a coin could be used to determine the number of spaces on each move. The game also requires little if any instruction for parents or preschool teachers who might want to use it. Even overworked parents could fit in a game or two before bedtime, given that each game takes about three minutes. And the game does not require much total investment of time to produce large gains; it produced its effects in the present and previous studies after roughly one hour of play.

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Appendix A. References

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