

Understanding the Teaching and Learning of Fractions: A South African Primary School Case Study

Deonarain Brijlall, Aneshkumar Maharaj, Justin Molebale
University of KwaZulu-Natal, Durban, South Africa

The authors explored the teaching and learning of fractions by reflecting on teachers and learners' views on practical work and their classroom practices. The teachers and learners were from two primary schools in a rural area in KwaZulu-Natal (South Africa). Questionnaires, in which teachers and learners expressed their views on practical work and fraction teaching and learning, were administered to teachers and their learners. Lessons on the division of fractions were observed to determine teachers' practices in relation to the researchers' assumptions and claims by literature. Data yielded by these research instruments confirmed assumptions and literature claims. Although it was a small scale qualitative research, interesting observations were made that could have pedagogical implications. The paper is presented in two parts. The first considers the teaching of fractions and the second the learning of fractions.

Keywords: fractions, division, multiplication, teacher-perceptions, learner-perceptions

Part one: The Teaching of Fractions

Introduction

Informal observation of practices by mathematics teachers, coupled with informal interactions at experience-sharing forums, suggested that teachers seldom include practical work when teaching fractions. This led to the formulation of the following research questions: (1) What are the views of teachers on practical work and the teaching of fractions and how do these views relate to their practices?; and (2) What are the factors behind these views? The study was conducted in two South African township schools.

A thorough understanding of the operations' division and multiplication with whole numbers is a pre-requisite for understanding division of fractions (Flores, 2002). Learners' knowledge of working with whole numbers is a valuable reservoir to the learning of multiplication and division of fractions (Murray, Olivier, & Human, 1996). There are different perspectives on fractions. Witherspoon (1993) cited in Kennedy and Tipps viewed fractions as part-wholes, subsets, ratios, quotients and rational numbers. Instruction by most teachers still overemphasizes the part-region perspective of the fraction concept (Sinicrope & Mick, 1992; Witherspoon, 1993). Flores (2002) asserted that children go through several stages to develop the idea of the fraction in the context of subdividing areas. He advised that teachers need to make sure that learners have developed a fairly complete understanding of fractions before discussing division of fractions.

Teachers who understand a topic make connections with other mathematical concepts and procedures (Flores, 2002). Flores suggested that some of the connections needed in the division of fractions are fractions and quotients, fractions and ratios, division as multiplicative comparison, reciprocals (inverse elements) and operators. Therefore, teachers need to understand how the concepts of the fraction $\frac{3}{4}$, a quotient $3 \div 4$ and the ratio 3:4 are related to and different from each other. Limited exposure of learners to a single representation of the fraction concept has been identified to seriously impair learners' full development and understanding of the concepts of the fraction and operations on fractions (Witherspoon, 1993). This includes the division of fractions. Subdivided regions for shading to indicate some fractional part of a real-life pizza or a chocolate bar are among some of the widely used examples for the fraction concept (Moskal & Magone, 2002; Witherspoon, 1993). This singular part-region representation of the fraction concept prevails (Witherspoon, 1993), although there are many other representations and interpretations which could improve the understanding of the fraction concept. To gain a complete understanding of the fraction concept, learners need to be exposed to a variety of concept representations. Witherspoon (1993) suggested the following five representations: (1) symbols; (2) concrete models; (3) real-life situations; (4) pictures; and (5) spoken language.

A conceptual understanding of fractions and operations on them, as clearly distinct from the ability to successfully manipulate algorithms, is a necessary prerequisite if learners are expected to make sense of their learning about fractions. However, Flores (2002) argued that the division of fractions has been traditionally taught by emphasizing the algorithmic procedure "invert the second fraction and multiply", with little effort to provide learners with an understanding of why it results in the correct answer. Witherspoon (1993) warned against assuming an understanding of fractions by learners merely because they are able to carry out an algorithm or recite a definition. In this light, our view is that, to enhance the learners' understanding of fractions, there is a need for practical work which exposes them to different representations and models.

Among key principles that guide the development and implementation of Curriculum 2005 and the follow-up RNCS (Revised National Curriculum), the educational department's policy document listed: (1) participation and ownership; and (2) learner-oriented approach (Department of Education, 1997). The wording of these principles and other related ideals of OBE (outcomes-based education) suggests serious engagement of the learner in the learning process. To this end, Freudental (1991) and Gravemeijer (1994) accentuated the actual activity of doing mathematics; an activity, which they proposed should predominantly consist of organizing or mathematic subject matter taken from reality. Engaging learners with practical activities in learning fraction division provides more than ample opportunity for practical implementation of the ideals of OBE. Practical teaching of fractions by use of concrete models has been observed to be a difficult experience for teachers. Ott, Snook, and Gibson (1991) argued that concrete experiences related to the division of fractions are much more difficult for teachers to devise and for learners to follow.

Research Methodology

The nature and quality of data generated by the questionnaires and observation of lessons in response to research questions in the introduction characterised the study as qualitative. Assumptions were made about the practices of teachers when teaching fractions and fraction division, and some of the underlying beliefs that inform these practices. The assumptions on which the study was based were: (1) Minimal use of practical work by teachers is a source of impoverished development of concepts on fractions and operations on them, including division; (2) Limited visual representation of the fraction concept with pictures of

part-regions; and (3) Overemphasis of the algorithm as a goal of instruction. To test assumptions on teachers' practices, lessons on fraction division were observed to ascertain the approach used by teachers. Twelve lessons of each teacher were observed. To find out more about the factors behind teachers' views on practical work in the teaching of fractions and fraction division, a questionnaire was designed for distribution among teachers. Schools that granted access gave three to four weeks to conduct the study. Therefore, this called for a compromise arrangement to generate reasonably credible data on teachers' perceptions of practical work, and the teaching of fractions and fraction division in relation to their practices. It was decided to administer the questionnaire to all four Grade-7 mathematics teachers in the two schools, but observe only the lessons of one Grade-7 group per school.

Observation

Patton (2002) explicitly listed observations among research instruments used in qualitative inquiries. To capture unfolding events in depth, a semi-structured type of observation was deemed as suitable. According to Cohen, Manion, and Morrison (2000), a semi-structured observation has an agenda of issues of interest, but gathers data in a far less pre-determined and systematic manner. This semi-structured character of the observation suited the qualitative nature of this study. The most appropriate role of an observer was observer-as-participant, who was known as a researcher to the group and had less extensive contact with the group (Cohen et al., 2000). Such a role allowed for the capture of events as they unfolded, with a special focus on what teachers did in relation to their assumed practices. The observer tape-recorded each lesson and made notes on teacher-learner interactions. For example, notes was made on: (1) whether practical work was used; (2) the type of representations and models used; (3) whether group work was used; (4) the types of questions posed to learners; and (5) whether sufficient time was allowed for learners' responses.

Questionnaires

Though questionnaires are predominantly associated with quantitative studies (Cohen et al., 2000), if they make provisions for open-ended responses, such questionnaires are capable of generating in-depth data on respondents' feelings, opinions, views, attitudes and perceptions about the phenomenon (the learning of fractions and fraction division by practical means). A questionnaire with all these attributes was designed as a research instrument for a qualitative study. These questionnaires were administered to teachers to find out their views on practical work and fraction learning. The questionnaire mostly consisted of closed items, eight items allowed for open-ended responses for teachers to express their opinions. This questionnaire tried to find a balance between a highly structured questionnaire (with closed items only) and an unstructured questionnaire (open-ended items) to find in-depth information about the role of practical work in learning fractions and subsequent fraction division. Prior to the actual fieldwork, the questionnaire was designed, piloted and refined. Inclusion of open-ended items was the product of these efforts. Teachers were given a week to complete the questionnaire.

Results

Teachers' views. Is there a place for practical work on fractions?

Four respondents answered the questionnaire. Data from questionnaires indicated that these teachers attached a strong value to the role of practical work in teaching fractions and fraction division. All four respondents agreed that fractions offer enough opportunities for the teaching and learning of mathematics

through practical means. Their mostly preferred materials in teaching fractions and operations on them were: (1) groups of objects-sets; (2) pictures/diagrams; and (3) worksheets (with tasks designed to promote practical work). Two respondents preferred each of these materials. Paper-folding and the graded ruler were each preferred by only one respondent.

Table 1

The Role of Practical Work in Teaching Fractions

Statement	Strongly agree	Agree	Disagree	Strongly disagree
Practical work has a place in the teaching of fractions.	4	0	0	0

All four respondents strongly agreed that practical work has a place in the teaching of fractions (see Table 1). Respondents gave different reasons for their preferences of models/aids that they used. The graded ruler, groups of similar objects (sets) and paper-folding were preferred because of their easy accessibility by learners. Sets and pictures/diagrams were chosen for their ease of use by learners. These teachers considered worksheets to be easy for learners to understand and answer. Other approaches were the number line (one respondent) and physical objects that learners could handle (three respondents).

Report on the observation of lessons of the two teachers. In school A, the teacher's approach to the teaching of fraction division embraced the use of visually abstract models. He did most of the work himself and did not allow learners enough opportunities to explore practical means to find solutions to given problems. The lesson was teacher-driven, since it was dominated by the teacher. This teacher used diagrams to demonstrate how the solution to problems, for example, $2 \div \frac{1}{3}$ could be found. Learners were not given sufficient time to use models. The teacher's final solutions contained errors. In some cases, the example used did not relate to division of fractions, which was the intended outcome of the lesson. After giving two definitions of division, i.e., sharing and grouping, the teacher wrote a fraction division problem $2 \div \frac{1}{3}$ on the board and demonstrated the solution. Figure 1 is an illustration of the teacher's solution.

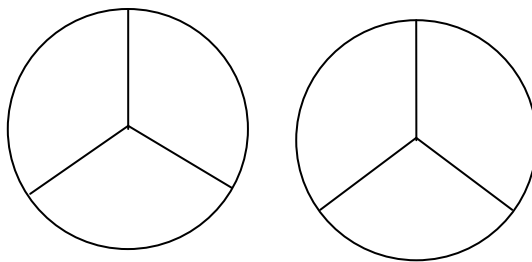


Figure 1. Teacher's circle solution of $2 \div \frac{1}{3}$.

The problem was not related to any real life situation. Only later did the teacher attempt to contextualize the problem, equating two to two cakes divided by $\frac{1}{3}$, although there was no explanation of what $\frac{1}{3}$ might represent. After depicting his solution, the teacher then asked learners how many pieces of $\frac{1}{3}$ were found in the two circles representing his two cakes. Learners correctly responded with six. Erroneously, the teacher concluded and then wrote $\frac{6}{3} = 2$. This is equivalent to $6 \times \frac{1}{3} = 2$. The correct solution to the given problem

would have been $2 \div \frac{1}{3} = 6$. The following is an illustration of how the same teacher used the number line as an alternative approach to the solution (see Figure 2).

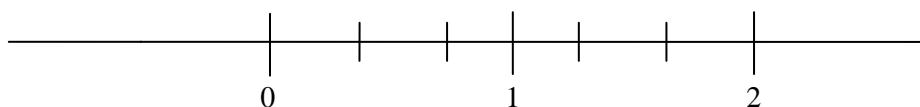


Figure 2. Teacher's number line solution of $2 \div \frac{1}{3}$.

Again, the teacher erroneously concluded that the final solution was $\frac{6}{3} = 2$. In his two attempts at the solution, the teacher never explained how his final solution was related to the original problem. As his last example, the teacher demonstrated the solution to the problem finds $\frac{1}{5}$ of $\frac{1}{2}$. Figure 3 is an illustration of the teacher's solution.

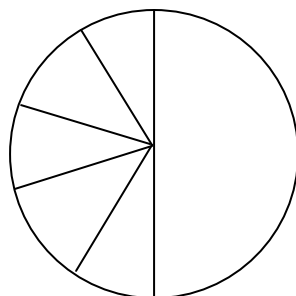


Figure 3. Teacher's circle solution to $\frac{1}{5}$ of $\frac{1}{2}$.

After asking learners a number of leading questions, conclusion was finally reached that there are ten fractions of $\frac{1}{5}$ in the two $\frac{1}{2}$'s, each of which is $\frac{1}{10}$ of the entire circle. Hence, the conclusion is that $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$. This is not an example of a fraction division problem and was, thus, irrelevant to the intended outcome of the lesson. The only visible involvement of learners during the lesson was their responses to teacher's questions which probed desired cues towards final solution.

In school B, the lesson on fraction division focussed on the recalling of terminology and application of the algorithm, the origins of which learners were never assisted to understand, nor did they play any part in developing. The teacher wrote the following fraction division problems, (none of the questions was placed in a context) on the board: (1) $6 \div \frac{1}{2}$; (2) $4 \div \frac{1}{2}$; (3) $\frac{2}{3} \div \frac{1}{6}$; (4) $2\frac{1}{2} \div 5$; and (5) $1\frac{1}{2} \div \frac{1}{4}$. Using $\frac{1}{2}$ as a referent, the teacher revised the definitions of numerator and denominator. To revise reciprocals, the teacher asked learners to give reciprocals of $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{5}{6}$, for which he wrote $\frac{1}{2} = \frac{2}{1}$, $\frac{3}{4} = \frac{4}{3}$ and $\frac{5}{6} = \frac{6}{5}$ on the board. Although this expression of learners' oral responses may be understandable and perhaps acceptable within the context of giving reciprocals, the language of the symbols used suggests a different, incorrect and misleading story. In demonstrating the solution to problem (1), the teacher suggested awareness on his part of learners' prior knowledge of the fraction division algorithm, since he opened with the statement: "We all know that when we divide with a fraction we change the divisor into its reciprocal and multiply the dividend with the reciprocal

instead of dividing with the original fraction”.

Although this was the first lesson on the division of fractions, noting this teacher’s use of the words “We all know that”. It seemed that this was a type of comment the teacher used by force of habit. Through leading questions, the teacher demonstrated the application of the division algorithm to the solution of the problem. When learners demonstrated solutions to subsequent problems, emphasis was also on reciprocals and accuracy in multiplication.

The next lesson dealt mainly with the division of mixed numbers. Here too, focus was mainly on accurate reciprocals, conversion from mixed numbers and correct products. All these distinctive features of rote learning evident in this teacher’s lessons were reminiscent of Siebert’s (2002) parallels between operations involving fractions and seemingly non-sensical algorithms.

Discussion

A major finding was the contradiction between what the teachers said and their actual practice. Further, it appeared that these teachers were not using the problem-centred kind of approach recommended by the new curriculum which requires not just to use practical materials for fractions, but also to choose real-world problems and contexts that can be used as starting points to develop the theoretical constructs and some correct algorithms. Also for a problem-centred approach, learners (not the teacher) ought to first grapple with the problem and attempt to model it. The teacher should mainly be a facilitator to guide their learning and mental constructs in the right direction, and realize that there are several alternative methods and algorithms that learners can come up with.

Conclusions

Pre-service training. It has been observed that pre-service mathematics teachers regard personal or formal theories of teaching and learning mathematics and classroom practice as separate areas of study (Hobden, 1999, p. 76). In this study, the observed contradiction between teachers’ classroom practices and their self-declared positive attitudes towards practical fraction teaching looked like a continuation of Hobden’s observed pre-service tendencies of trainee teachers to regard theory and practice as two separate entities.

Pre-service teacher training needs to take into account the teachers’ reasons for excluding practical work and implementing teaching strategies that are not centred on practical work. Therefore, teacher training needs to provide programmes that directly address these concerns, especially issues of overcrowded classrooms and perceptions that practical activities take up a lot of time, both during preparation and implementation. The issue of overcrowded classrooms is still a thorn in the side of our public education system. Yet, the approach of our teacher training programmes continues to tailor the training of teachers along methods that are suitable for normal-sized classes. The notion that practical activities are time-consuming suggests a lack of clear understanding, and thus, appreciation of the nature, scope and functional potential of practical work by teachers, the origins of which are summed up by the suggestion that teachers lack proper training in practical work. Therefore, pre-service teacher training on practical fraction teaching needs to be revisited with an eye to addressing these and many other concerns which further research should help bring to the fore.

In-service training. Teachers’ concerns, their observed practices and their acknowledgement that practical fraction division is relevant to OBE requirements for a learner-centred approach, call for a demand to look at how in-service training can assist to address teachers’ needs. Instruction by means of rote application of the algorithm by teachers is a serious impediment to understanding. As practising teachers, in-service training

seems to be the most immediately accessible remedy to their deficiencies. Flores (2002) advised that teachers who understand a topic should be able to make connections with other mathematical concepts and procedures. Recommended and approved in-service training programmes should be informed by teachers' perceptions of their needs directly solicited from them through relevant and appropriate research strategies. Teachers' embracing attitudes towards the relevance of practical fraction teaching to OBE are encouraging points of departure. The ideas of the teachers from school B on aspects of practical fraction teaching that OBE workshops should address and sum up the needs of teachers were in this regard.

Teaching implications. Learners should be assisted with understanding various perspectives of the fraction concept and other meanings of division, e.g., sharing/parting interpretations and using practical representations of fractions. That this is not an easy task is supported by the view that a review of literature indicates that the partitive meaning for division has almost been totally ignored. The partitive meaning of division of fractions has been very resistant to clear concrete explanations (Ott et al., 1991, p. 8). This calls for a commitment from teachers to seek and design effective strategies to help learners with the understanding of partitive and other perspectives of fraction division. For them to be successful, teachers' efforts in this regard need to be the overall outcome of teacher training initiatives both at pre-service and in-service levels.

Part Two: The Learning of Fractions

Introduction

The authors share in this report our exploration of the perceptions of 12-year-old children when engaging with fractions. This work has also been reported in Molebale, Brijlall, and Maharaj (2010a; 2010b) and Maharaj, Brijlall, and Molebale (2007). An experiment was conducted on Grade-7 learners in two township schools, in South Africa, to determine the effects of engaging in practical work during the learning of fractions and fraction division based on worksheets. Selected learners from the experimental groups were interviewed to find out their perceptions about the use of practical work.

In South Africa, learners usually learn operations on fractions through intensive training and drill in the use of appropriate algorithms applicable to specific operations (Maharaj et al., 2007). It has been asserted that procedural knowledge, such as algorithms for operations is often taught without context or conceptual understanding, implying that algorithms are an ungrounded code only mastered through memorization (Sharp, Garofalo, & Adams, 2002, p. 18). This also applies to the division of fractions. Rote learning leaves learners with a shallow understanding of underlying conceptual meanings and processes. The effectiveness of concrete experiences in learning fraction division needed to be examined to find out the benefits and disadvantages, if any, that learners experience as a result of engaging in practical activities when dividing fractions.

Research Methodology

In answering the question "What are the perceptions of learners towards practical work in the division of fractions?", the authors used the experiment and group interviews. The objective of the experiment was to determine what happens when learners engage in practical work to divide fractions, while interviews sought to establish learners' views on practical work in the learning of fractions and fraction division. For the investigation, learners were divided into the control and experimental groups. In school A, 30 learners constituted the experimental group, while 33 learners formed the control group. In school B, the numbers were 38 and 36 for both groups respectively. Learners in each group were evenly spread in relation to levels of

performance (i.e., above average, average and below average). Learners' performance abilities were determined according to previous assessment records provided by the subject teachers.

Worksheets with an exercise on the division of whole numbers were used to introduce learners to the measurement and partitive interpretations of division. Solutions to these exercises included the use of a drawing ruler and sets of bottle-tops. The control group was exposed to a training and drill approach which excluded practical work. Both groups wrote a pre-test and post-test to enable comparison of performance levels. Afterwards, some learners were interviewed to find out learners' preferences between the two modes of fraction division that they had been exposed to i.e., the ruler (part whole) and bottle-tops (subsets). General comments by learners were also catered for.

Findings

On the identification and representation of fractions, the following findings were made:

(1) Learners' competence in representing fractions improved significantly after exposure to practical work based on ruler and bottle-tops which includes fractions that initially proved to be very difficult for learners;

(2) Test performances showed that learners found it difficult to work with equivalents, whether part-regions or subsets. Learners often did very well when working with non-equivalents;

(3) Learners from the experimental groups in both schools, showed remarkable improvement in post-test items on representation of equivalent subsets after exposure to practical work;

(4) In most post-test items, even those in which both groups (experimental and control) performed poorly, experimental groups from both schools often did significantly better than their counterparts in the control groups;

(5) Basing on learners' performance in the identification and representation of fractions (worksheets and tests), it can be concluded that practical work contributes to improve learners' understanding of the fraction concept by learners.

The use of the ruler and bottle-tops as concrete embodiments of fractions in fraction division represented two perspectives of the fraction concept: (1) the ruler part-region perspective; and (2) bottle-tops subset perspective. Comparison of learners' performance using either of the practical aids became necessary to determine which of the two instruments was more efficacious and expedient in fraction division. Table 1 compares results with the ruler against results using bottle-tops. It shows the performance of learners in both schools for selected division problems from some worksheets. W is the abbreviation for worksheet and accompanied by a number to indicate the worksheet the problem is taken from e.g., W3 for worksheet 3. Table 2 indicates that the highest scores were obtained when learners used bottle-tops, except in (2) where learners from school A scored higher with a ruler. Therefore, it can be concluded that generally learners were more successful with bottle-tops than they were with the ruler when dividing fractions. It suggested that learners do better when working with fractions if they can actually cut up the whole.

The authors explain the probable reasoning behind $\frac{2}{3} \div \frac{1}{2}$ by the use of bottle-tops. In worksheet 3, successful learners realised that division by $\frac{1}{2}$ made the result larger. So, $1 \div \frac{1}{2}$ gave an answer of two by using a ruler. This meant replication of the one when dividing by $\frac{1}{2}$. For $\frac{2}{3} \div \frac{1}{2}$, the successful learners knew that two out of the three bottle-tops needed to be replicated to yield a result of four out of three. During

interviews, most learners confirmed their preference of the concrete approach to fraction division over all others they had been exposed to. Although the majority of learners favoured bottle-tops as preferred tools of practical fraction division, two learners favoured diagrams. When asked for reasons for her preference of diagrams, Phumla responded: "... when I'm using diagrams it's easy for me to see my problem". Phumla had successfully demonstrated her faith in diagrams when she used them in solutions to a number of fraction division problems in the pre-test and post-test. Phungula, a co-admirer of diagrams justified his affinity towards diagrams: "It's because drawings can help you to represent fractions. This helps you to see the fractions as you divide them".

Table 2

Comparison of Learners' Performance: Ruler Versus Bottle-Tops

Division problem	Correct responses (%): Ruler			Correct responses (%): Bottle-tops		
	Worksheet	School A	School B	Worksheet	School A	School B
(1) $1 \div \frac{1}{2}$	W3	63	95	W7	86	100
(2) $\frac{1}{2} \div \frac{1}{4}$	W4	97	100	W8	62	100
(3) $\frac{2}{3} \div \frac{1}{2}$	W6	27	51	W11	30	94
(4) $\frac{3}{4} \div \frac{1}{3}$	W6	00	17	W12	83	97

The problem of familiar and unfamiliar fractions. Learners' constant reference to $\frac{1}{2}$ as an example of a fraction led us to assume that $\frac{1}{2}$ was a familiar fraction to them, hence our inquiry if fractions $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ were familiar fractions to learners and to which the response was positive. Learners' concurrence with the researcher's suggestion that $\frac{1}{2}$ was easy "Because it's a fraction that you work with most of the time" confirmed the familiarity of half and other fractions listed thereafter. Working with the same familiar fractions all the time proved to be a handicap to learners when they worked with less familiar fractions. When asked why it was difficult to show the fractions $\frac{3}{4}$ and $\frac{2}{3}$ (using the ruler or bottle-tops) during the experiment, Sihle's response was "Because we were not familiar with $\frac{3}{4}$ and $\frac{2}{3}$ " and Xaba concurred. When asked why these fractions were not familiar, Xaba gave the following response (and Thobile agreed):

Xaba: Besingawafundi, Sir! (They were not taught to us, Sir!)

Interviewer: Beningawafundi? (You never learned them?)

Xaba: Yes sir!

Interviewer: So, whenever your teachers teach you fractions, they don't usually use these fractions $\frac{3}{4}$ and $\frac{2}{3}$? Is that correct? (Thobile nods approvingly and the interviewer offers her a chance to respond) Thobile!

Thobile: Yes sir!

Writing the remainder as a fraction. When nine is divided by seven, the division algorithm yields

$9 = 1.7 + 2$. This means that seven divides into nine once with remainder two. In the division throughout by seven yields $\frac{9}{7} = 1 + \frac{2}{7}$, the remainder two is expressed as a fraction of seven. Evidence that writing the remainder as a fraction was a problem for learners in division problems that involved the remainder first emerged in learners' performance in worksheets. During interviews several learners confirmed their difficulties and gave reasons thereof. When asked why they experienced difficulties with problems to which the answer was a mixed number, Sabelo gave the following response:

Sabelo: (Interrupting the interviewer) Because sir, when you said $\frac{2}{3} \div \frac{1}{2}$, there is a remainder.

Interviewer: What was difficult? (Pause) Why did the remainder make it difficult for you to give the answer? (Silence) Yes? Sabelo has said because there was a remainder, then that was the problem. What was the problem with the remainder? (Silence and then he directed the question to a specific learner) Mvubu! (Silence again) because you had to write the remainder... as a fraction?

Sabelo: (Emphatically) Yes! Yes sir! We can't write the remainder as a fraction.

The ruler's problem of repeated counting. Evidence that the ruler posed difficulties for learners as an instrument for practical fraction division can be found in Table 2. Learners clearly articulated the difficulties they experienced with using the ruler for fraction division. One learner detailed the problem with the ruler in the following exchange:

Interviewer: (Confirming his understanding of the meaning behind Sihle's gestures) You can count better with bottle-tops?

Sihle: (Together with Sabelo) Yes! sir! (Pause) Sir! When you count with a ruler, you can get disturbed and you forget where you were.

Interviewer: (Nodding in understanding) Okay, okay! Because with a ruler there's a lot of counting, it's easy for you to get disturbed...

Sihle: Yes sir!

Interviewer: And you can't remember where you stopped?

Sihle: And you start afresh.

Overemphasis of the algorithm. Using the algorithm, learners from school B were, therefore, more successful in fraction division problems than their counterparts in school A. In the following discussion, Sihle's preference of the algorithm and the reasons he advanced for his position were further proof that learners can be comfortable with application of the algorithm (referred to as the rule), provided that they have been adequately trained and drilled:

Interviewer: Now if you say the rule, I'm going to ask you... did you understand (With emphasis) what was happening when you...when you used the rule?

Sihle: Yes sir! Because we used... we used to use the rule when we divide (interrupted)

Interviewer: (Interrupting) Sihle! Am I correct to say... you think you understood because you got the answers correct?

Sihle: Yes sir!

However, when asked if he understood the rationale behind the algorithms requirement that we inverted and multiply, Sihle was silent, from which we concluded that he could not explain it.

Overemphasis of the part-region fraction perspective. Overemphasis of the part-region perspective of the fraction concept was seen to cause problems for learners on a number of occasions. Figure 4 shows learners' erroneous partition of two rectangular shapes in an effort to show $\frac{1}{3}$ as they tried to find a solution to $2 \div \frac{2}{3}$. The exercise had been given earlier by the teacher before learners were engaged with practical fraction division. It is evident that learners did not understand that all the three parts of the partitioned figure representing $\frac{1}{3}$ must be equal.

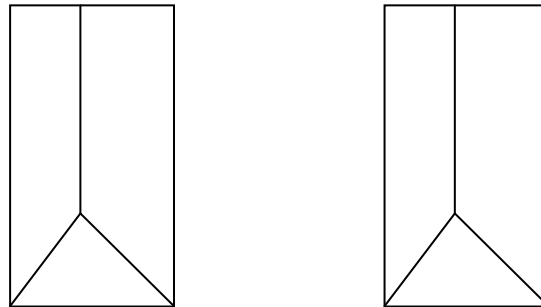


Figure 4. A group's solution of $2 \div \frac{2}{3}$.

Figure 5 shows an incorrect representation of the subset perspective of $\frac{3}{4}$ by some learners in the post-test.

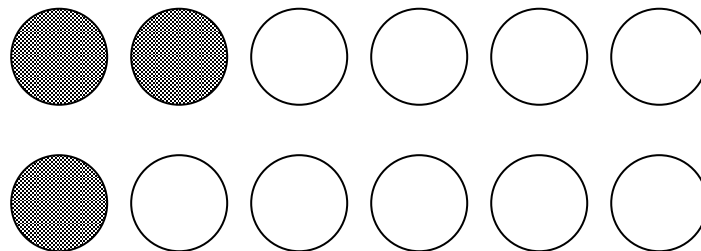


Figure 5. Learner's representation of $\frac{3}{4}$ of 12 in post-test.

Here, the learner did not understand that the "3" in the fraction $\frac{3}{4}$ represents sets of items, and not three items. Figure 6 illustrates another learner's response to the same problem. We noted that the representation is incorrect because the learner shaded $\frac{3}{4}$ of nine and not 12.

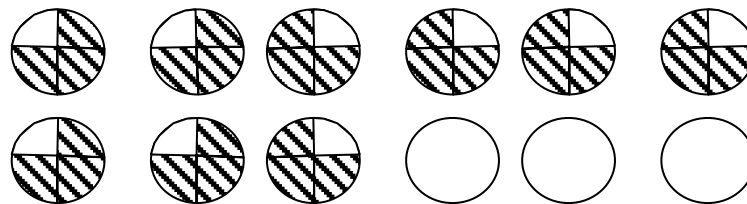


Figure 6. Learner's representation of $\frac{3}{4}$ of 12 in post-test.

Figure 7 shows a learner's successful representation of $\frac{2}{3}$ of six. We noted that this approach does not guarantee a successful solution to "How many marbles make $\frac{2}{3}$ of six marbles?".

Discussion

The preparatory exercise which served as an introduction to worksheets on fraction division supported the use of whole numbers in aiding learners' understanding of fraction division. The exercise contributed immensely to learners' better understanding of fraction division. It also helped to prepare them for related worksheet tasks. This study confirmed the advice of Murray et al. (1996) and Flores (2002) that an understanding of whole number division is a pre-requisite for understanding fraction division when the sharing view of division is emphasised.

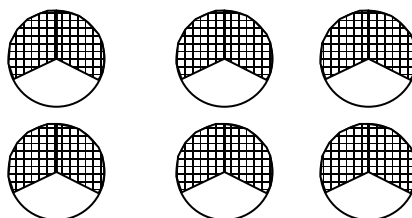


Figure 7. A learner's representation of $\frac{2}{3}$ of six in post-test.

Learners' successes in fraction division in worksheets and post-test, coupled with learners' pronouncements in favor of practical work confirmed the claim that "with the help of concrete models of fractions, learners can see that $\frac{1}{4}$ fits two times into $\frac{1}{2}$, therefore $\frac{1}{2} \div \frac{1}{4} = 2$ " (Flores, 2002, p. 238). Views by Phumla and Phungula (on diagrams) confirmed the claim that "many students who make drawings understand mathematical operations better than those who use only symbols or observe the drawings made by someone else" (Dirkes, 1991, p. 28).

The direct link between learners' failure to write the remainder and their inability to interpret the remainder as a part of another fraction (the divisor) supported Hart's (1981) observed tendency among learners to ignore the question "a fraction of what?".

The use of bottle-tops supported an interpretation of the division concept as repeated subtraction/removal of an equal quantity from the original group, and then counting the number of subsets formed. This is the essence of the measurement interpretation of the division concept. Measurement situations involve finding how many groups can be made when the total amount and amount per group are known. Therefore, the subset interpretation is a powerful alternative to the part-region perspective of the fraction concept. It is a useful option that teachers should consider seriously to enhance and broaden learners' understanding of the concepts of fractions and fraction division.

By restricting their examples of fractions to unit fractions (with numerator 1), teachers deny learners a broader view of what the fraction symbol actually entails. This, together with the practice of confining the idea of the fraction to the part-region perspective result in a very narrow and limited understanding (in learners) of what the overall concept of the fraction is all about.

The ruler as a tool has a limited capacity for practical fraction division. It compounds the problem of dividing fractions, by adding the extra dimension of the need for accuracy. Besides having to represent fractions and divide them correctly, learners have to deal with the issue of accuracy to determine fractions correctly, which becomes an obstacle in learners' progress towards acquisition of and competence relating to the concepts fractions and fraction division.

Overemphasis of the algorithm was one of the assumptions behind the motive for the study. The inclination of learners in school B turns to the algorithm when confronted with fraction-division situations and the remarkable successes that they scored (in post-test), contradicted Hart's (1981) claims of the algorithm being difficult for learners to apply. These learners, well trained and drilled in the application of the fraction division algorithm, were able to effectively and successfully use it in the solution of fraction division problems. However, school B learners' poor performance in pre- and post- test on some fraction identification and representation items compared to much better performance in fraction division items justified warnings that we should be careful not to assume that students understand fractions merely because they are able to carry out an algorithm or recite a definition (Witherspoon, 1993, p. 84).

Conclusions

Instruction needs to capitalize on the subset perspective, i.e., bottle-tops, to extend learners' understanding of fractions. It could be useful to overcome difficulties learners experienced in the identification and representation of the subset perspective of fractions, especially the equivalent type. Use of the subset perspective should not be limited to understanding fractions, but be extended to help learners understand fraction division situations. Learners should be assisted with understanding the sharing/partitive and other meanings of division using practical representations of fractions. It is not an easy task since it is supported by the view that "... a review of literature indicates that the partitive meaning for division has almost been totally ignored... the partitive meaning of division of fractions has been very resistant to clear concrete explanations" (Ott et al., 1991, p. 8). Commitment from teachers to seek and design effective strategies to help learners with the understanding of partitive and other meanings of fraction division is required. For the teacher training initiatives both at pre-service and in-service levels need to focus on the partitive perspective and other meanings of fraction division.

The design of teaching programmes and sessions should anticipate problems that learners are likely to encounter in their divisions of fractions by practical means. Learners' problems that should be accommodated and addressed by instruction are: (1) the remainder problem; (2) overemphasis of the part-region perspective; (3) familiar and unfamiliar fractions; (4) the problem with Graded instruments; and (5) the algorithm problem.

References

- Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education*. London & New York: Routledge Falmer.
- Cramer, K., & Bezuk, N. (1991). Multiplication of fractions: Teaching for understanding. *Arithmetic Teacher*, 39(3), 34-37.
- Department of Education. (1997). *Senior phase policy document (Grades 7-9)*. Pretoria: Department of Education.
- Dirkes, M. A. (1991). Draw to understand. *Arithmetic Teacher*, 39(4), 27-29.
- Empson, S. B. (2002). Organizing diversity in early fraction thinking. In B. Litwiller, & G. Bright, (Eds.), *Making sense of fractions, ratios and proportions* (pp. 28-39). Virginia: National Council of Teachers of Mathematics.
- Flores, A. (2002). Profound understanding of division of fractions. In B. Litwiller, & G. Bright, (Eds.), *Making sense of fractions, ratios and proportions* (pp. 237-246). Virginia: National Council of Teachers of Mathematics.
- Freudental, H. (1991). *Revisiting mathematics education*. Dordrecht: Kluwer Academic.

- Fraser, C., Murray, H., Hayward, B., & Erwin, P. (2004). The development of the common fraction concept in Grade three learners. *Pythagoras*, 59, 26-33.
- Gravemeijer, K. P. E. (2001). Developmental research: Fostering a dialect relation between theory and practice. In J. Anghileri, (Ed.), *Principles and practice in arithmetic teaching*. London: Open University Press.
- Hart, K. (1981). Fractions. In K. M. Hart (Ed.), *Children's understanding of Mathematics: 11-16* (pp. 66-81). Oxford, London & Northampton: Alden Press.
- Hobden, S. (1999). The beliefs of pre-service teachers about mathematics teaching and learning (Master's Thesis, University of Natal).
- Langa, R. (2003). Teachers' understanding of and use of assessment in the context of outcomes-based education: A case study of a Hammarsdale farm school (Master's Thesis, University of Natal).
- Lubienski, S. T. (1999). Problem-centred mathematics teaching. *Mathematics Teaching in the Middle School*, 5(4), 250-255.
- Moskal, M. B., & Magone, M. E. (2002). Making explicit what students know about fractions. In B. Litwiller, & G. Bright (Eds.), *Making sense of fractions, ratios and proportions* (pp. 128-129). Virginia: National Council of Teachers of Mathematics.
- Maharaj, A., Brijlall, D., & Molebale, J. (2007). Teachers' views of practical work in the teaching of fractions: A case study. *South African Journal of Education*, 27(4), 597-612.
- Molebale, J., Brijlall, D., & Maharaj, A. (2010a). Exploring the learning of fractions at grade seven. *Proceedings of the 18th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education* (Vol. 2, pp. 215-220). University of KwaZulu-Natal, South Africa.
- Molebale, J., Brijlall, D., & Maharaj, A. (2010b). Exploring the teaching of fractions at grade seven. *Proceedings of the 18th Annual Meeting of the Southern African Association for Research in Mathematics, Science and Technology Education* (Vol. 2, pp. 221-236). University of KwaZulu-Natal, South Africa.
- Murray, H., Olivier, A., & Human, P. (1996). Young students' informal knowledge of fractions. In K. Morrison (Ed.), *Second National Congress of the Association of Mathematics Education in South Africa: Conference Proceedings* (pp. 41-48). Natal: Interpak.
- Ott, J. M., Snook, D. L., & Gibson, D. L. (1991). Understanding partitive division of fractions. *Arithmetic Teacher*, 39(2), 7-10.
- Patton, M. (2002). *Qualitative research and evaluation methods*. Thousand Oaks, London & New Delhi: Sage.
- Sharp, J. M., Garofalo, J., & Adams, B. (2002). Children's development of meaningful fraction algorithms: A kid's cookies and a puppy's pills. In B. Litwiller, & G. Bright (Eds.), *Making sense of fractions, ratios and proportions* (pp. 18-28). Virginia: National Council of Teachers of Mathematics.
- Siebert, D. (2002). Connecting informal thinking and fractions: The case of division of fractions. In B. Litwiller, & G. Bright (Eds.), *Making sense of fractions, ratios and proportions* (pp. 247-256). Virginia: National Council of Teachers of Mathematics.
- Sinicrope, R., & Mick, H. W. (1992). Multiplication of fractions through paper folding. *Arithmetic Teacher*, 40(2), 116-121.
- Sinicrope, R., Mick, H. W., & Kolb, J. R. (2002). Interpretations of fraction division. In B. Litwiller, & G. Bright (Eds.), *Making sense of fractions, ratios and proportions* (pp. 153-161). Virginia: National Council of Teachers of Mathematics.
- Whitworth, D., & Edwards, M. (1969). *Teaching mathematics to the ordinary pupil*. London: University of London Press Ltd..
- Witherspoon, M. L. (1993). Fractions: In search of meaning. *Arithmetic Teacher*, 40(8), 482-485.