Students as Decoders of Graphics in Mathematics

Carmel M. Diezmann
Queensland University of Technology
c.diezmann@qut.edu.au

Tom Lowrie
Charles Sturt University
tlowrie@csu.edu.au

This paper reports on students' ability to decode mathematical graphics. The findings were: (a) some items showed an insignificant improvement over time; (b) success involves identifying critical perceptual elements in the graphic and incorporating these elements into a solution strategy; and (c) the optimal strategy capitalises on how information is encoded in the graphic. Implications include a need for teachers to be proactive in supporting students' to develop their graphical knowledge and an awareness that knowledge varies substantially across students.

Numeracy has been a priority within Australia for the past decade with a national agenda that includes substantial investment in numeracy initiatives and the monitoring of achievement through national testing of Years 3, 5, 7 and 9 students. Underlying these directions is the goal of developing a numerate society in which citizens can cope with the mathematical demands of life at school, at home, at work, and in the community. However, we contend that the achievement of the numeracy goal is dependent (at least in part) on developing students' proficiency in decoding the range of graphics that are used for the communication and organisation of mathematical information. The purpose of this paper is to examine how knowledge of information graphics influences success in mathematics through an investigation of students' performance on items with embedded information graphics. In Hill, Ball, and Schilling's (2008) terms, we are investigating the *knowledge and content of students* (KCS), which is one of three components of pedagogical content knowledge (PCK). As a background, we present an overview of information graphics and how expertise develops. Following the results of the investigation, we consider the findings in relation to the *knowledge of content and teaching* (KCT), another component of PCK. (The final component of PCK relates to *knowledge of curriculum.*)

Background

The Content of Information Graphics

In mathematics, information graphics convey quantitative, ordinal, nominal or spatial information via perceptual elements (Mackinlay, 1999). These elements are position, length, angle, slope, area, volume, density, colour saturation, colour hue, texture, connection, containment and shape (Cleveland & McGill, 1984). Although information graphics is a burgeoning field (Harris, 1996), there are six "graphic languages" that link perceptual elements via particular encoding techniques (Mackinlay, 1999). Examples from the six graphic languages, their perceptual elements and encoding techniques are described in Table 1 and shown in the Appendix. To capitalise on the power of each graphic language, students need to understand the constraints and affordances of each encoding technique. For example, if quantitative information is known about one segment on a pie chart, information can be inferred about another segment on the same pie chart based on its relative size (Affordance) but not about a segment on a different pie chart (Constraint).

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Table 1
Examples of Graphic Languages

Graphic Language	Example Items	Perceptual Elements	Encoding Technique
Axis	Numberline	Points, line, position	Each position encodes information by the placement of marks on axis.
Apposed-position	Line Graph	Points, line, position	Information is encoded by a marked set positioned between two axes.
Map	Street Map	Points, line, position	Information is encoded through the spatial location of marks on a grid.
Retinal-List	Flip Item	Shape, position, connection, containment	Retinal properties of shape and orientation are used to encode information. These marks are not dependent on position.
Connection	Game	Shapes, line, connection	Information is encoded by a set of shapes connected by a set of lines.
Miscellaneous	Pie Chart	Shapes, angles, containment	Information is encoded through angles and containment.

The Development of Expertise with Graphics

Expertise in communication involves proficiency in both the semantic and syntactic dimensions of a language. Like written language, information graphics have a *semantic* dimension (e.g., perceptual elements) and a *syntactic* dimension (e.g., organisation of elements within a graphic). However, unlike (typical) written language, which utilises a sentential mode of communication and requires sequential (cognitive) processing to extract meaning, the syntactic dimension of graphics is underpinned by a visual-spatial mode of communication and requires simultaneous processing to decode information. Thus, expertise with information graphics will be quite distinct from expertise in written language. Within the domain of graphics, expertise should be characterised by three stages of capability according to Alexander‗s (2003) *Model of Domain Learning* (MDL). At the *acclimation stage*, students are orienting to a new domain and knowledge will be limited and fragmented; at the *competence stage*, individuals have foundational and comprehensive knowledge of the domain; at the *proficiency stage*, individuals have breadth and depth of knowledge and may contribute new knowledge to the domain.

Design and Methods

This study employed a longitudinal design to monitor the development of students‗ ability to decode graphics in mathematics items and to solve these items. Students‗ ability was investigated using a multi-method approach. The research questions were:

1. How do students perform on graphic languages over time?
2. How do students solve items from particular graphic languages?

The Graphic Languages in Mathematics (GLIM) Instrument and Participants

The GLIM instrument. This is a 36-item multiple choice test that contains six items of varying difficulty from each of the six graphic languages. The items were sourced from published tests that have been administered to students in the mid-late primary years. Due to the limited Connection items in mathematics tests, some content free items from science tests were also included. A full description of the GLIM instrument is presented elsewhere (Diezmann & Lowrie, 2009a). The GLIM test was administered in mass testing situations over three consecutive years when students were in 9Years 5, 6 and 7. Items were scored 1 and 0 for correct and incorrect responses respectively. Students' performances were calculated on each language subtest (max. score = 6) and the overall test (max. score = 36). Also, over a 3-year period, different cohorts of students were progressively interviewed on sets of twelve GLIM items commencing with the easiest pair of tasks in each graphic language. After completing each pair of items, students then justified their multiple choice response. The interviewer probed their reasoning but no scaffolding was provided. Here we report on five of these items (See Appendix, Items 1, 20, 27, 28 and 36).

Participants. There were three cohorts of participants. Cohort 1 was drawn from two Queensland and five New South Wales schools and completed the mass testing only. This cohort commenced with 371 students in the first year of the study when students were aged approximately 10 years and subsequently reduced to 352 and 325 in successive years. Cohort 2 (N = 67) and Cohort 3 (N = 47) were sourced from two Queensland and three New South Wales schools respectively. In the results, the interview cohorts are indicated by Cohort number (C2 or C3) and year level (Qld) or grade level (NSW). Students were aged 10 years in the first year of their 3-year set of interviews.

Results and Discussion

The results are presented in two parts according to the research questions. Part 1 presents students' performance on GLIM over time. Part 2 reports on students' solutions and their difficulties with GLIM items from five languages excluding Connection languages.

1. How do Students Perform on Graphic Languages over Time?

An Analysis of Variance (ANOVA) (year by correct score) revealed statistically significant differences between the students performances on the GLIM test over a 3 year period [$F(2,1047) = 91.76$, $p<0.001$] with students' performance increasing at a significant rate in each of the three years of the study (Table 2 provides means and standard deviations for total score correctness). Performance increases were statistically significant across all six languages in each of the three years of the study (see Lowrie & Diezmann, in press).

The second level of analysis distinguished participants' performance on mathematics and science items over a three-year period (Years 5 to 7) with means and standard deviations for the two content categories presented in Table 2. (Recall, some of the items in the GLIM test were science items because there were insufficient mathematics tasks in some graphic languages.) An ANOVA (year with content category) revealed a statistically significant difference between the performance of students across mathematics [$F(2,1047) = 86.71$, $p<0.001$] and science [$F(2,1047) = 58.27$, $p<0.001$] content categories. Subsequent post hoc analysis revealed statistically significant differences in the performance of students between Year 5 and Year 6 and Year 6 and Year 7 for both mathematics and science categories [at $p<0.001$for each of the six t-tests].

9 In mass testing results, Year levels are reported as for Queensland students. Year levels for New South Wales students of an equivalent age are one year earlier (i.e., Year 7 in Qld = Grade 6 in NSW). The use of the term ―Grade" signifies a typical NSW age cohort rather than its QLD age equivalent.

Table 2
Means (and SD) for Test Correctness and Mathematics and Science across Three Years

Category	Year			*F* Value
	Year 5 (*N*=371)	Year 6 (*N*=352)	Year 7 (*N*=325)	*F*(2, 1047)
GLIM Total (*N* = 36)	21.74 (5.47)	25.02 (5.3)	27.06 (4.99)	91.76**
Mathematics (*N* = 25)	14.99 (3.72)	17.12 (3.75)	18.64 (3.52)	86.71**
Science (*N* = 11)	6.75 (2.25)	7.90 (2.08)	8.42 (1.97)	58.27**

Note: ** $p < 0.001$

A third level of analysis was restricted to the 25 mathematics items. This analysis was used to identify if there were any mathematics items on which student performance *did not* increase over time. Consequently, multiple ANOVAs (year with item) revealed statistically significant differences between performance on 22 of the 25 items [with a Bonferroni correction method set at p = 0.002, i.e., 0.05/25]. Thus, for 22 of the mathematics items, performance increased across the three years. The means and standard deviations for the three insignificant items (Appendix, Items 1, 27, 35) are presented in Table 3. Item 1 was of limited concern because the ceiling effect precluded statistical improvement over time. However, there was substantial scope for improvement on Items 27 and 35 (in relation to mean scores) over the 3-year period. Items 1 and 27 are discussed further shortly.

Table 3
Means (and SD) for the Items, Which Did Not Result in Performance Increases Over Time

Item	Graphic language	Year 5	Year 6	Year 7	% increase	*F*(2,1051)
1	Axis	.88 (.33)	.89 (.31)	.93 (.26)	6	2.5
27	Retinal	.65 (.47)	.68 (.46)	.71 (.45)	9	1.3
35	Connection	.21 (.40)	.21 (.40)	.27 (.44)	3	2.1
Total	25 Maths Items	14.99 (3.72)	17.13 (3.75)	18.64 (3.52)	24	86.71**

Note: ** $p < 0.001$

2. How Do Students Solve Items from Particular Graphic Languages?

Apposed Position Item. Whilst in Year 6, Cohort 1 found the column graph item (Appendix, Item 20) to be relatively difficult (= 0.66). In interviews, 60% of Grade 5 students (C3) were successful in considering the information on the x and y axes simultaneously. Jane, for example, indicated that ―Most are 9 or some are 10 or older. They are the two highest (9 + 10). I looked at this (points to x axis) and I knew that (the y axis) wasn‘t the ages of children". Unsuccessful students focused on the data as positioned on *either* the x or y axis rather than considering the relationship between these data and the axes (Lowrie & Diezmann, 2007). A high proportion of students (70% of the inappropriate solution strategies) focused on the y axis and selected a solution that corresponded with the highest point on the graph: ―I put A because most children are 9 because it is the highest bar" (Terry). Elsewhere (Lowrie & Diezmann, 2007), we have argued that students‘ incorrect responses can be derived from prior, and prototypical, tasks that routinely required them to select the highest point on the graph without an additional requirement to interpret other data. As Roth (2002) suggested, when students are not required to read beyond simple components of the data, they tend to concentrate on perceptual features rather than attending to all elements in a holistic manner.

Map Item. By contrast to the other map items, Cohort 1‘s performance on Item 28 (Appendix) plateaued from Year 6 (= 0.71) to Year 7 (= 0.75). In interviews (C3, Grade 5), approximately 70% of the successful approaches involved students understanding the relationship between location and direction: ―First I had a look at where the pool was [location] ... He drives North and takes the first right, which is Wattle Road. Then he takes the 2nd left, that‘s first, that‘s second, that‘s School Road [direction] (Larry)". Unsuccessful responses involved students overlooking the need to simultaneously attend to ordinal and positional information. On this item, unsuccessful students tended to either focus on the ordinal information (second left) <u>or</u> the directional information (left rather than right turn). Ellen‘s incorrect solution (Post Rd) was reached by ―start[ing] at the pool, then (they) took (a) right turn (Wattle Rd) then (a) left turn and it‘s Post Rd". Ellen was able to carry out directional instructions but overlooked the requirement to take the *second* road on the left. Thus, she was not able to *interpret information* and distinguish between what information should and should not be included (Wiegand, 2006).

Axis Item. With the exception of a number line item (Appendix, Item 1), there was a significant improvement in Cohort 1‘s performance over time on Axis items (Table 3). Although the lack of improvement on Item 1 was due to a ceiling effect, it was evident that some students lacked knowledge of how information is encoded on a number line. In interviews, 91% of students (C2, Year 5) were successful on the number line item. The most common explanation (67.2%) for the selection of the correct response of D related to it being closest to the identified number: ―I chose D because it‘s closest to 20 and C is too far away (Jo)." The six students who selected an incorrect response all used the same strategy of counting back: ―I think it should go there (D) because it‘s next to 20 and it goes 19, 18 then 17" (Tracy). Thus, unsuccessful students treated the number line as a *counting model* in which all marks irrespective of spacing were one number apart. In contrast, successful students‘ explanations focused on distance and number implying that they conceptualised the number line as a *proportional* or *measurement* model.

Retinal List Item. Cohort 1 found the Retinal-List items to be the most difficult of the graphic language items to solve in Year 5. This is a concern because some of these items, such as the Flip (Appendix, Item 27), are typical tasks in national numeracy tests for Years 3 and 5 (e.g., Curriculum Corporation, 2009a, 2009b). In interviews, 56.7 % of students (C2, Year 5) successfully completed the Flip item. The majority of successful students (63.2%) used a *Symmetry* strategy in which they sought a pair of symmetrical shapes: ―A flip is when you flip it over (moves one hand over) so it‘s kind of like symmetrical if you join them together (joins hands together) and if you join B together (pair of shapes) it would look symmetrical (Donna)". Over 20% of students used the *Deduce and Check* strategy (21.1%). These students reached a solution by visualising changes to the perceptual elements of each shape in turn and deducing the correct response by eliminating incorrect responses. These students exhibited functional rather than proficient knowledge of graphics because they employed a laborious strategy in lieu of the optimal strategy of identifying a symmetrical pair of shapes. The perceptual elements of the graphic and their relationship were not evident to all students with some unsuccessful students demonstrating a lack of knowledge about the spatial relationships between shapes (Del

Grande, 1990). Hannah, for example, eliminated the correct answer of B because the noses on the shapes pointed in opposite directions. Thus, on the Flip item, there was substantial variance in knowledge of graphics between the most and least capable students.

Miscellaneous Item. The pie chart is a Miscellaneous item of particular interest because only 71% of Year 8 students in the 2003 Trends in International Mathematics and Science Study (TIMSS) were successful on a pie chart showing crop distribution (National Center for Education Statistics, n.d.). Some insight into students‗ performance on pie charts is evident from interviews on Item 36 from GLIM (Appendix). On this item, 93% of students were successful (C2, [10]n = 15, Year 7). However, as with the Flip item, there was a distinct difference between students who were proficient with graphics and those who were functional. Successful students predominantly used two strategies. The *Fraction* strategy (53.3%) focused on calculating the total amount of Jemma‗s budget from the known value of the clothes segment of $30 and the area of this segment being a quarter of the circle. For example, Amy said ―Clothes for a quarter of the circle then if you times that ($30) by four because there are four quarters in the circle … she would have had around ($)120." Thus, the Fraction strategy capitalised on the affordance of the encoding technique of a pie chart through the utilisation of angles and containment in solution. By contrast, the *Estimate Quantity and Add* strategy (20%) overlooked the proportional nature of the pie chart and involved an estimation of the cost of objects and a sum of their total cost. For example, Holly selected $120 as her response: ―I reckon she spent about ($)10 or no … $15 for that (food) ... Which is ($)35 if you add them together and then books would probably be about ($)50 (that‗s) ($)85 and um they (banking and games) would probably be about $40". Although this strategy can lead to success, it is laborious and errors can occur at multiple points. The only unsuccessful student guessed the answer. An elaboration of the cohort‗s performance on this item is presented elsewhere (Diezmann & Lowrie, 2009b).

In sum, with consideration for students‗ performance and Alexander‗s (2003) MDL, there are three types of decoders based on their ability (a) to identify and utilise critical perceptual elements and (b) to exploit the encoding technique in a solution strategy. *Emergent decoders* are acclimating to the graphical domain and have limited knowledge. Thus, these students try to employ a solution that incorporates some critical perceptual elements. *Functional decoders* display some competence in the graphical domain in that their ability to interpret perceptual elements enables them to create a workable strategy. By contrast, *proficient decoders* have developed the expertise to exploit the encoding technique in the graphic to employ an optimal strategy.

‗The Report Card‗ on Graphical Expertise

The investigation of students‗ decoding of graphics (i.e., KCS) informs *Knowledge of Content and Teaching*. Over time there was a significant improvement in students‗ performance on most graphic items but there were some items on which teachers needed to provide explicit support to students. Additionally, teachers needed to be able to identify students as *emergent, functional* or *proficient* decoders of graphics and to tailor support to students‗ expertise in decoding. Emergent decoders need opportunities to learn about the critical (and non critical) perceptual elements in various graphics. Functional decoders need opportunities to compare solution strategies and identify the optimal strategy

10 Participants were from one intact class in one of the two C2 schools.

according to the encoding technique of a particular graphic. Proficient decoders need opportunities to apply their graphic knowledge and create new strategies for solving tasks.

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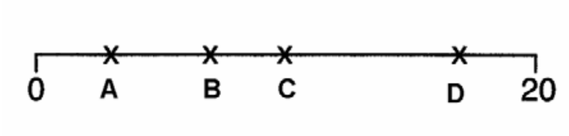
Appendix

Estimate where you think 17 should go on this number line.

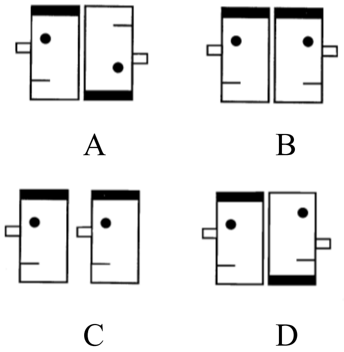


Answer

☐ **A** ☐ **B** ☐ **C** ☐ **D**

Item 1: Axis-*Number Line* (QSCC, 2000, p. 11)

Which two faces show a flip?

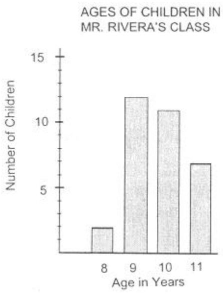


Answer

☐ **A**
☐ **B**
☐ **C**
☐ **D**

Item 27: Retinal List – *Flip* (QSCC, 2001, p. 13)

The graph above shows how many of the 32 children in Mr Rivera's class are 8, 9, 10 and 11 years old. Which of the following is true?

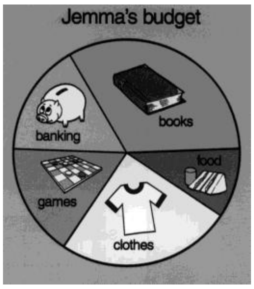


Answer

☐ **Most are younger than 9**
☐ **Most are younger than 10**
☐ **Most are 9 or older**
☐ **None of the above is true**

Item 20: Apposed Position-*Column Graph* (NCES, 1992, Q 229)

In 2004, Jemma budgeted $30 on clothes. Approximately how much money did she get that year?

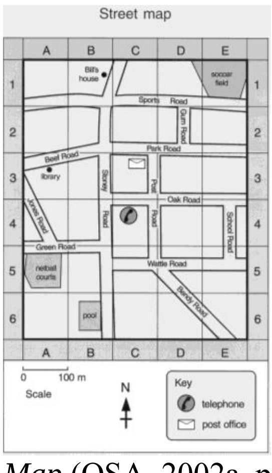


Answer

☐ **$90**
☐ **$120**
☐ **$150**
☐ **$180**

Item 36: Miscellaneous-*Pie Chart* (QSA, 2002b, p. 6 & p. 1 of insert)

Bill leaves the pool. He drives north and takes the first road on the right, then the second road on the left. Which road is he in?

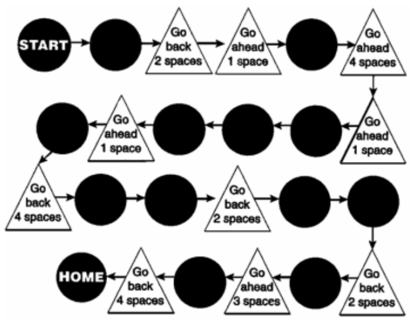


Answer

☐ **School**
☐ **Post**
☐ **Beef**
☐ **Jones**

Item 28:Map-*Street Map* (QSA, 2002a, p. 7 & p. 3 of insert)

Two children are playing a board Game. They toss a standard dice and move forward the number of spaces to match the number on the dice. What is the least number of tosses of the dice needed to reach ?



Answer

☐ **5**
☐ **4**
☐ **3**
☐ **2**

Item 35: Connection: *Game* (ETC, 2002, p. 9)