Modelling the Cooling of Coffee: Insights From a Preliminary Study in Indonesia

Wanty Widjaja
Sanata Dharma University
<wanty_widjaja@yahoo.com >

This paper discusses an attempt to examine pre-service teachers' mathematical modelling skills. A modelling project investigating relationships between temperature and time in the process of cooling of coffee was chosen. The analysis was based on group written reports of the cooling of coffee project and observation of classroom discussion. Findings showed that pre-service teachers were able to model the process of cooling of coffee as a decreasing exponential function. Difficulties with interpretation of the constant rate of cooling and re-interpretation of mathematical model were identified.

Mathematical modelling has gained an increased attention in mathematics education community. Almost two decades ago, Blum (1993) reported that modelling was not yet part of the core mathematics teaching component in most countries. Since then, the situation has been improved. Burkhardt (2006) reported that many countries including Germany, the Netherlands, the United States, and Australia, have taken in a significant portion of mathematical modelling in their mathematics curriculum. Publications on mathematical modelling in various journals (see e.g., Barbosa, 2006; Blum, 2002; Blum, Galbraith, Henn, & Niss, 2007; English & Watters, 2006) have documented the global trends of incorporating more modelling in school mathematics.

The growing interest towards modelling is strongly supported by the Organisation for Economic Cooperation and Development's (OECD) Programme for International Student Assessment (PISA), a major international assessment study which promotes mathematical and scientific literacy. PISA (OECD, 2003) contends that mathematical literacy is "the overarching literacy" (de Lange, 2006, 15) that comprises quantitative literacy, numeracy, and spatial literacy. The OECD definition of mathematical literacy depicts a broader spectrum of what constitutes mathematics as it goes beyond school mathematics curriculum:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make a well-founded judgment, and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (OECD, 2003, p.72)

The close relation between mathematical literacy and mathematical modelling is well documented. Recently, Stacey (2009) noted that mathematical modelling plays the most important part of mathematical literacy. This is consistent with an earlier report by Kaiser and Willander (2005) stating that innovative mathematical modelling projects with realistic contexts resulted in a greater comprehension of mathematical concepts. Furthermore, they proposed the inclusion of modelling and applications in learning mathematics starting as early as primary school level.

Despite the growing interest in mathematical modelling at school level, fewer studies at teacher education level have been documented. Concerns about this trend were voiced by Burkhardt (2006) and Doerr (2007). Both highlighted the importance of preparing pre-service teachers with skills of 'doing mathematics' which requires broader teaching strategies. A profound change of view about what constitute mathematics, the role of

L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia.* Fremantle: MERGA.

teachers and students is necessary to support modelling in practice. Hence, teacher education plays a crucial role in training future teachers to acquire modelling skills and its pedagogy.

A reform movement to improve mathematics teaching in Indonesia, called PMRI (Ekholm, 2009; Sembiring, Hoogland, & Dolk, 2010), has been put in practice for a decade. Inspired by Freudenthal's notion of 'mathematics as a human activity' (Freudenthal, 1983, 1991), PMRI puts emphasis on mathematics as a meaningful activity that enables learners to grasp real-world phenomena. Consequently, mathematical modelling is an integral element of learning mathematics. Recent studies in PMRI classrooms (Dolk, Widjaja, Zonneveld, & Fauzan, 2010; Widjaja, Fauzan, & Dolk, 2009) suggest the need to support teachers (both pre- and in-service) with skills of 'doing mathematics' and 'modelling mathematics'. This provides a strong impetus to integrate mathematical modelling as part of a course for mathematics pre-service teachers in Indonesia. This paper will discuss preliminary findings from a modelling project to investigate relationships between temperature and time in the cooling of coffee experiment with samples of Indonesian pre-service teachers.

Method

Setting and Participants

This study was situated in a private teacher training institute in Yogyakarta, Indonesia. A modelling project called "The cooling of coffee" (Keng-Cheng, 2009) was translated into Indonesian and assigned to 20 pre-service teachers (see Figure 1). This study aimed to provide pre-service teachers with experience in mathematical modelling. Pre-service teachers' knowledge and difficulties displayed in completing this project will be examined. The design of project also intended to engage pre-service teachers in a productive classroom discourse based on explanations and justifications of their mathematical models. Having a different pedagogical approach was expected to enable pre-service in carrying out modelling project in the future professional life.

Cooling of Coffee Project

How does a cup of hot coffee cool with time? Is it possible to model the cooling of coffee?

1. List factors or variables in the problem.
2. Collect data to help construct a model. Record your data.
3. What is a suitable model? (Look for an existing model to develop one).
4. What is your method of solution?
5. Carry out your solution method and interpret the solution.
6. Examine assumptions and suggest ways to refine the model

Figure 1. The Cooling of Coffee Project (Keng-Cheng, 2009, 36-37).

The cooling of coffee project was chosen because the starting point of this problem was considered 'experientially real' for pre-service teachers. The whole project took four weeks to complete. Pre-service teachers worked in small groups of four people to carry out this project in the first two weeks. The following two weeks were devoted to poster presentations and whole class discussion. Representatives of each group explained their strategies and answered questions from other groups during the poster presentation.

Framework for Analysis

In analysing group written work and posters, a framework of didactical mathematical modelling process by Kaiser and Blum (in Kaiser & Schwartz, 2006, p.197) will be employed. This paper will examine mathematical knowledge that pre-service teachers display and the difficulties they face in completing the project. Pre-service teachers' knowledge and difficulties will be discussed with respect to the two main didactical modelling processes in Figure 2, i.e., mathematisation and reinterpretation of mathematical solutions to the real-life situations.

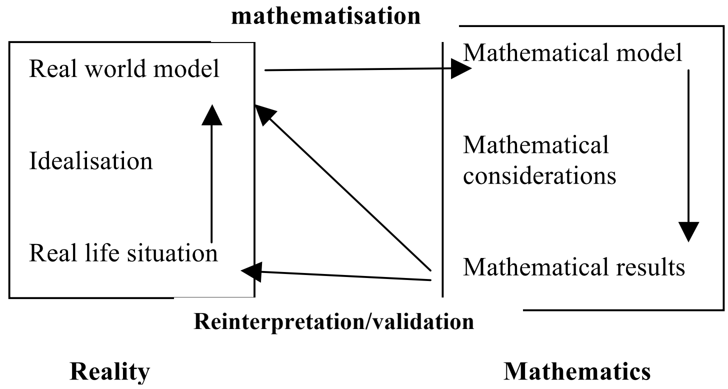

Figure 2. Didactical modelling process (in Kaiser & Schwartz, 2006, p. 197).

Results and Discussion

In this section, data from written project reports of five groups in completing the Cooling of Coffee project will be analysed. The first four steps of the project (see Figure 1) i.e., identification of factors, data collection, mathematical model formulation, and solving the mathematical model will be considered as the mathematisation process. The process of interpreting solutions, examining assumptions, and refining the model will be considered as the re-interpretation or validation process.

Mathematisation from the Cooling of Coffee Data to the Exponential Decay Function

A list of factors affecting the cooling process by various groups was identified. The factors reflected the real life considerations that pre-service teachers brought prior to the experiment. These factors were then tested during the experiment in cooling the coffee. All groups except Group B identified the room temperature as one of the factors affecting the cooling process. Types of cups (a plastic cup, a ceramic cup, a glass cup, etc.), and size of cups (based on the diameter or height of the cups) were also identified by all groups. Table 1 presents factors that influence the cooling process as identified by different groups and the corresponding ways of collecting data.

A variation in combining different factors was observed, suggesting different assumptions about factors that control the cooling process. For instance, Group C collected data on cooling of coffee using the same cups of different sizes. This group also examined whether types of cups (plastic, glass, and aluminium) have different cooling rates. Group D and Group E went further to explore different mixes of substances they put in their coffee.

These groups collected data for coffee without sugar or with sugar using different types of cups (plastic, glass, ceramic). The factors identified by different groups were based on assumptions of daily life experiences. However, it was unclear how the combination of different factors translated to the mathematical model. Figure 3 provides illustrations of some of the data collection process to construct the mathematical model.

Table 1
Factors Affecting Cooling Processes and Data Collection Methods

Group	Factors in the cooling of coffee	Ways of collecting data
A	- room temperature - types of cups (plastic or glass) - size of cups.	Collect data every minute until the coffee reaches around 40^o Celsius.
B	- types of cups (plastic or ceramic) - size of cups	Collect data every minute until the temperature of coffee reaches around 35-39^o Celsius. The data collected twice to find the average of temperature collected of 2 experiments.
C	- types of cups (plastic or glass) - size of cups - room temperature	Collect data every 3 minutes until the coffee reaches around 30^o Celsius (room temperature).
D	- types of cups (plastic, aluminium, glass, ceramic) - size of cups - room temperature - initial temperature	Collect data every 5 minutes until the temperature of coffee reaches around 26-30^o Celsius (room temperature).
E	- types of cups (plastic, ceramic, glass, etc.) - size of cups - ingredients (coffee only; coffee and sugar; coffee, sugar and milk) - room temperature	Collect data every 5 minutes until the temperature reaches around 30^o Celsius (room temperature).

In general two ‘benchmarks’ were observed in ways of collecting data, namely temperature and time for cooling a cup of boiling coffee. Four groups considered temperature as a criterion for cooling of coffee. As seen in Table 1, three groups (C, D, and E) used room temperature (i.e., 30^o Celsius) as a benchmark temperature for cooling off coffee whereas Group A selected 40^o Celsius as the ‘cool’ temperature for coffee. It was unclear whether 40^o Celsius was the actual room temperature when Group A collected their data. In contrast, Group B used a length of time (i.e., 60 minutes) to derive a mathematical model and to determine the rates of cooling under different circumstances. This choice led ultimately to different temperatures of coffee for different types of cups. Interestingly, in comparing different rates of cooling for different cups, Group B calculated differences between the initial temperature and the temperature after 60 minutes of the cooling process began.

*Figure 3.*Data collection processes in Cooling of Coffee project

All five groups were able to relate the mathematical model of cooling of a cup of hot coffee to an exponential decay function. They derived their mathematical model first by plotting their data into a Microsoft *Excel* spreadsheet. By observing the plots of the data, all groups noticed that a decreasing exponential function was the best mathematical model to fit this set of data. Hence, all groups found a model of exponential decay that could explain the cooling of coffee process. Samples of group posters and the interaction during poster presentation are presented in Figure 4.

Figure 4. Samples of group poster and interaction in The Cooling of Coffee project.

Explanations in deriving mathematical models showed difficulties in understanding and interpreting the rate of cooling. In this case, the rate of cooling was represented by the value of k in general function of $T(t) = A.e^{-k.t}$. Of the five groups, only three groups provided reasonable explanations for deriving the mathematical model and interpreting the value of *k.* Two groups (Group D and Group E) utilized the trend line tool to approximate the exponential functions that best represented their data. However, in deriving the value of

k, these groups started with the general function $y=A.e^{-k.t}$ and calculated an average of initial temperature for different types of cups and an average of temperature at a particular time. This approach contradicted earlier efforts to find different mathematical models in different experiments. Calculating averages for various cups under different circumstances indicated lack of meaningful interpretation of k. In contrast, the other three groups worked out the mathematical model by relating the temperature changes over time to the 'rate of change' notion. Group A, and B expressed this as $\frac{\Delta y}{\Delta t} = \frac{y(t+\Delta y)-y(t)}{\Delta t} = k.y$. However they did not take into account the decreased temperature in their equation, expressing $\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt} = k.y$ or $\frac{dy}{dt} = k.y$ instead of $\frac{dy}{dt} = -k.y$. Using integration, the function of $y(t) = y_0.e^{k.t}$ was obtained. The negative value of k was determined later after working with the data. Similarly group C also did not express decreased temperature change over time in the mathematical model. This group was the only group that showed not only knowledge about the rate of change notion but also knowledge of physics by referring to Newton's Law of Cooling. Using this law, the rate of change of the temperature of an object is directly proportional to the difference between the object's temperature and the room temperature. Hence this group represented the change of temperature over time as $dT/dt = k(T - 30)$ which led to the mathematical function $T_{(t)} = (T_0 - 30)e^{k.t} + 30$. In this function, k represents the rate of cooling process, T_0 represents the temperature of boiling coffee, t represents the time and $T_{(t)}$ represents the temperature of coffee after the cooling process going on for t.

Re-interpreting the Exponential Decay Function Back to the Data

In examining their assumptions, all groups reviewed the role of factors, which were identified at the beginning of the project. By observing the graphs of experiments carried out in 26^o Celsius and 29^o Celsius at the same time, a conclusion about the impact of room temperature was drawn. It was noted that when room temperature is higher, the coffee takes a longer time to cool. The influence of types of cups (plastic, glass, ceramic) in the cooling process of was also recorded. It was found that a cup of coffee made of glass took longer time to cool of compared to the same cup made of plastic. Size of cups also affected the rate of cooling. A cup with larger base area took shorter time to cool as compared to a cup with a smaller base area for the same height. Group D and Group E experimented with different mix of substances. Group D found no difference in trends of cooling of coffee using four different cups with or without sugar. They concluded that a cup of coffee made of aluminium cools the fastest in both conditions. However, this conclusion should be considered cautiously. It turned out that in carrying out the experiment, containing sugar was not the only different factors in the experiment. The Group also used different types of cups with varying sizes so it was difficult to pinpoint which factors were the 'control variables'.

A reasonable interpretation for the value of k was articulated by Group B. This group observed that the value of k was a constant. Note that Group B formulated the mathematical model as $T_{(t)} = (T_0 - 30)e^{k.t} + 30$, hence they found negative value for k. Furthermore "as k gets smaller, the coffee cools faster", indicates a sensible interpretation of k as the constant rate of cooling. In addition, Group B noticed that the temperature of coffee was close to the room temperature at the end of the cooling process. The fact that only one out of five groups was able to articulate a sensible interpretation of k suggested that re-interpretation of a mathematical model is a challenging part of the modelling process.

Similarly, refining the mathematical model was found to be difficult for the students in this study. Ideas for refinement were limited to general limitations of the current mathematical model related to technical aspects of data collection. Lack of accuracy in reading the temperature and calculation error caused by rounding were offered as explanations for discrepancies of temperature based on mathematical model and the real data. All groups only proposed increasing the number of experiments to refine their mathematical model.

Concluding Remarks

This study sought to enhance pre-service teachers' modelling skills in a project to investigate the cooling of coffee. Analysis of written reports of the project indicated that pre-service teachers in this study were able to obtain a decay exponential function as a model for the cooling of coffee. Relevant factors affecting the process of cooling including room temperature, types and size of cups were identified. However, pre-service teachers did not attend to the complexities of combining different factors (e.g., sizes of cups, types of cups) in mathematical models.

Evidence in this study showed that re-interpreting and linking variables in the mathematical model back to the real-world data was problematic. Difficulties with interpreting the meaning of the constant rate of cooling (k) and incorporating room temperature into the mathematical equation were observed. Sound mathematical knowledge of calculus and functions is essential to support rich mathematical discussion in modelling cooling of coffee. Findings from this preliminary study indicated that a modelling project is a rich learning tool for pre-service teachers to 'do mathematics' and to 'see mathematics' in daily life.

References

Barbosa, J. C. (2006). Mathematical modelling in classroom: A socio-critical and discursive perspective *Zentralblatt für Didaktik der Mathematik*, *38*(3), 293-301.

Blum, W. (1993). Mathematical modelling in mathematics education and instruction In T. Breiteig, I. Huntley & G. Daiser-Messmer (Eds.), *Teaching and Learning Mathematics in Context* (pp. 3-14). Chichester: Ellis Horwood Limited.

Blum, W. (2002). Applications and modelling in mathematics education: Discussion document. *Educational Studies in Mathematics, 51*(1/2), 149-171.

Blum, W., Galbraith, P. L., Henn, H.-W., & Niss, M. (2007). *Modelling and applications in mathematics education*. New York Springer

Burkhardt, H. (2006). Modelling in mathematics classrooms: Reflections on past development and the future. *Zentralblatt für Didaktik der Mathematik, 38*(2), 178-195.

de Lange, J. (2006). Mathematical literacy for living from OECD-PISA perspective *Tsukuba Journal of Educational Study in Mathematics, 25*(13-35).

Dolk, M., Widjaja, W., Zonneveld, E., & Fauzan, A. (2010). Examining teacher's role in relation to their beliefs and expectations about students' thinking in design research In R. K. Sembiring, K. Hoogland & M. Dolk (Eds.), *A decade of PMRI in Indonesia* (pp. 175-187). Bandung, Utrecht: APS International.

Ekholm, M., & van den Hoven, G. H. . (2009). *PMRI - Majulah!* Naarden: Zet&Print.

English, L., & Watters, J. (2006). Mathematical modelling in the early school years *Mathematics Education Research Journal, 16*(3), 59-80.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht: D. Reidel Publishing Company.

Freudenthal, H. (1991). *Revisiting mathematics education, China lectures*. Dordrecht: Kluwer Academic Publishers.

Kaiser, G., & Schwartz, B. (2006). Mathematical modelling as a bridge between school and university. *Zentralblatt für Didaktik der Mathematik, 38*(2), 196-208.

Kaiser, G., & Willander, T. (2005). Development of mathematical literacy: Results of an empirical study. *Teaching Mathematics and Its Applications, 24*(2-3), 48-60.

Keng-Cheng, A. (2009). *Mathematical modelling in the secondary and junior college classroom*. Singapore: Prentice Hall.

Organisation for Economic Cooperation and Development. (2003). *The PISA 2003 assessment framework – mathematics, reading, science and problem solving knowledge and skills.* Paris: OECD.

Sembiring, R. K., Hoogland, K., & Dolk, M. (2010). *A decade of PMRI in Indonesia*. Utrecht: Ten Brink.

Stacey, K. (2009). Mathematical and scientific literacy around the world. In U. H. Cheah, Wahyudi, R. B. Devadson, K. H. Ng, W. Preechaporn & J. C. Aligaen (Eds.), *Proceedings of the 3rd International Conference on Science and Mathematics*. (pp. 1-7). Penang: SEAMEO RECSAM.

Widjaja, W., Fauzan, A., & Dolk, M. (2009). The role of contexts and teachers' questioning to enhance students' thinking. In U.H. Cheah, Wahyudi, R.B. Devadson, K.H. Ng, W. Preechaporn & J. C. Aligaen (Eds.), *Proceedings of the 3rd International Conference on Science and Mathematics.* (pp. 466-474). Penang: SEAMEO RECSAM.