

Three Primary School Students' Cognition about 3D Rotation in a Virtual Reality Learning Environment

Andy Yeh

Queensland University of Technology

<a.yeh@qut.edu.au>

This paper reports on three primary school students' explorations of 3D rotation in a virtual reality learning environment (VRLE) named VRMath. When asked to investigate if you would face the same direction when you turn right 45 degrees first then roll up 45 degrees, or when you roll up 45 degrees first then turn right 45 degrees, the students found that the different order of the two turns ended up with different directions in the VRLE. This was contrary to the students' prior predictions based on using pen, paper and body movements. The findings of this study showed the difficulty young children have in perceiving and understanding the non-commutative nature of 3D rotation and the power of the computational VRLE in giving students experiences that they rarely have in real life with 3D manipulations and 3D mental movements.

Many existing ICT tools such as Logo and Geometer's Sketchpad utilise 2D computer graphics for geometric visualisation and thus have limited applications for the learning of 3D geometry concepts and processes, especially by primary school students. To address this issue, Yeh (2004) developed a Virtual Reality Learning Environment (VRLE) named VRMath that employs virtual reality (VR) 3D computer graphics to facilitate the learning of 3D geometry concepts and processes. This paper reports on the development of three primary school students' conceptions of 3D rotation within the VRLE.

Background

Explorations within 3D space are concerned with not only the investigation of 3D shapes but also the investigation of moving, positioning, orientating, constructing and building of objects within 3D space. One important element of these explorations is the study of rotations within 3D space (Baturu & Cooper, 1993; Queensland Studies Authority, 2004).

However, 3D rotation activities that can be performed in a real environment with concrete objects are limited by the physical condition of the materials and environment, and also by problems with accuracy when performing 3D rotations with concrete objects. A simple question "Will you face the same direction when you turn right 45 degrees first then roll up 45 degrees, or you roll up 45 degrees first then turn right 45 degrees?" puzzles most students and even adults. Intuitively, most people answer that the two 3D rotations end up with the same direction. Unfortunately, this is wrong because 3D rotations are not commutative in nature.

To gain an understanding of the non-commutative nature of 3D rotation in traditional mathematics classroom activities, one generally must have some prior knowledge of the Cartesian 3D coordinate system, trigonometry, and vector and/or matrix notation for 3D translation, scaling, and rotation. These enable accurate operation of 3D rotation and rigorous proof of the nature of 3D rotations. However, this knowledge is far too complex for most primary school children to comprehend. Therefore, if the investigation of 3D rotations is to be integrated into primary school mathematical programs, then new activities which enable young students without knowledge of the Cartesian 3D coordinate



system, trigonometry, and vector and/or matrix notation to meaningfully experience 3D rotation need to be designed. VRMath, the computational VRLE being presented in this paper, has been designed to provide young students with first- and third-person experiences (Pasqualotti & Freitas, 2002) within 3D space that cannot be provided by explorations with concrete objects in the real world. It is hypothesised that these first- and third-person experiences within the 3D virtual environment provided by VRMath will enable primary school children to develop new ways of experiencing and thinking about 3D rotations.

The VRLE (VRMath)

Informed by the fallibilist philosophy of mathematics (Ernest, 1994), semiotics (Cunningham, 1992; Lemke, 2001), constructivist and constructionist learning theories (Harel, Papert, & Massachusetts Institute of Technology(1991). Epistemology & Learning Research Group., 1991; Kafai, 2006; Kafai & Resnick, 1996), a VRLE named VRMath has been developed by Yeh (2004). VRMath comprises three main interfaces, a virtual reality (VR) interface, a programming interface, and a hypermedia and forum interface (see Figure 1).

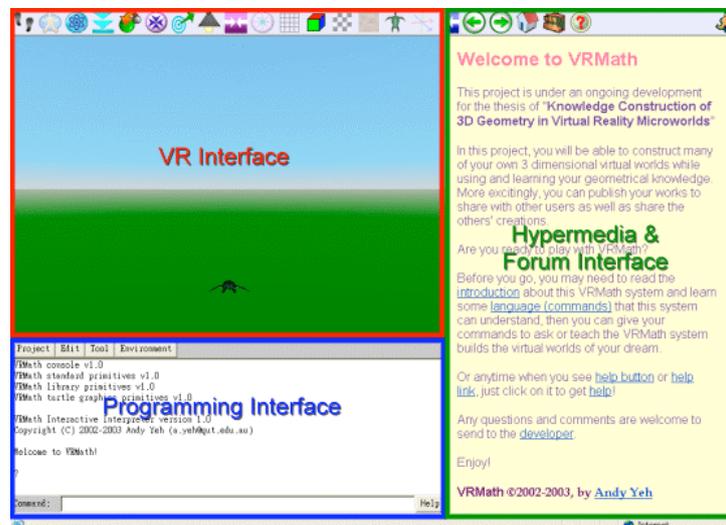


Figure 1. VRMath

VR interface: This is the interactive 3D computer graphics that allows real time visualisation of a 3D virtual space. Users can use mouse and/or keyboard to navigate within the 3D virtual space and view the geometrical objects within the 3D virtual space from different and continuous viewpoints. This kind of 3D navigation is a first-person experience (Pasqualotti & Freitas, 2002) in which the users constantly feel that they are moving. The VR interface also provides the visualisation of the manipulations (e.g., changing location and orientation) of geometrical objects created through the use of programming interface. The manipulation of objects is a third-person experience (Pasqualotti & Freitas, 2002) in which users stay stationarily and the objects are moving. Moving oneself or objects represents two distinguishable human spatial abilities termed spatial orientation and spatial visualisation (McGee, 1979), which can be mapped to first- and third-person imagery respectively. This interface thus enables the cultivation of both spatial orientation and visualisation abilities (Yeh & Nason, 2004a). Amorim, Trumbore, and Chogyen (2000) suggested that giving opportunities to switch between first- and third-person imagery might be of great benefit for the virtual traveller to anticipate new vantage

points and appropriate actions. Therefore, VRMath also implements an Avatar View function in which users can view from the turtle's (see programming interface) viewpoint. In Avatar View mode, the navigation within VR space can only be controlled by the programming commands such as FORWARD, BACK or turning commands. Thus, when manipulating the turtle through programming interface, the Avatar View enables users to switch between first- and third-person experiences. Moreover, when in Avatar View mode and the turtle's orientation is manipulated by a mathematical program through programming interface, users can also perceive what has been termed by Elliott and Bruckman (2002) as "mathematical movement" (e.g., the movement of sine wave in parametric equations).

Programming interface: This interface implements a Logo style language with an extended set of 3D related commands. Because of the nature of the VR interface, many geometric concepts in the VRLE environment differ from the traditional 2D Logo environment. For example, VRMath has a 3D turtle in VR space. VRMath uses metre and centimetre as the distance unit while traditional Logo uses pixels on the screen. To enable 3D rotation and movement, VRMath implements another four rotational or turning commands: ROLLUP (RU), ROLLDOWN (RD), TILTLEFT (TL), TILTRIGHT (TR) in addition to the traditional LEFT (LT) and RIGHT (RT). VRMath also has many built-in 3D shape commands such as CUBE, SPHERE, CYLINDER and CONE for easy creation of 3D models in the VR space. Figure 2 presents visual images of the effect of the 3D turning commands.

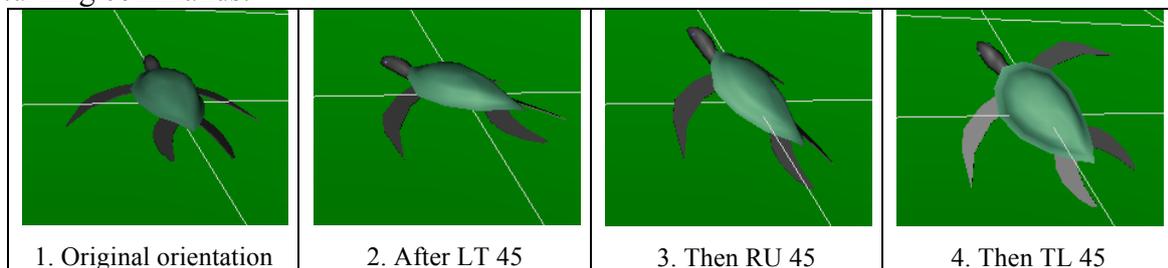


Figure 2. 3D Rotation in VRMath

Hypermedia and forum interface: This is the frame on the right side of VRMath containing hypermedia documentations and an online discussion forum. This is designed to provide non-linear and rich information and a channel for users to express and communicate ideas. With proper scaffoldings, this interface can be a pertinent vehicle for collaborative learning (Yeh & Nason, 2004b).

Method

There were three participants involved in this research study, Rosco, Bonbon, and Grae (their pseudonyms), who were aged 9 or 10 years old. They came from an inner city primary schools in eastern Australia. The three students were introduced to VRMath through 6 hours of instruction which covered the six rotational or turning commands and 3D navigation within the VR space. The question posed to them was: "Will you face the same direction when you turn right 45 degrees first then roll up 45 degrees, or when you roll up 45 degrees first then turn right 45 degrees?"

The students were videotaped as they experimented with the VRMath environment as they attempted to solve the problem. They spent about one hour each on the problem. The author, the researcher, sat with the students during this time, asking questions to draw out the reasons for any interesting activity. Field notes also were made by the researcher.

The videotapes were transcribed and the students' posts on the VRMath forum were also collected. The transcriptions and the posts were analysed to provide rich descriptions of the thinking of each student, which in turn was analysed for evidence that the students' experiences on the VRMath environment were assisting them to understand about 3D rotation.

Results

The initial thinking of all participants was that the two 3D rotations (RU 45 RT 45 and RT 45 RU 45) would end up in same direction regardless of the performance sequence. This thinking was challenged when the students interacted with VRMath. The processes by which students changed their conceptual understanding of 3D rotation will be presented in turn.

Rosco's experiment: Avatar View

When Rosco was asked to justify his thoughts about the 3D rotation problem, he immediately came up with the idea of using the "Avatar View" in VRMath. Avatar View is a function by which the user temporarily becomes the turtle and views actions within the 3D virtual space from the turtle's perspective. In this mode, the 3D navigation by mouse and keyboard in VR space are disabled to prevent changing the viewpoint by mouse dragging. The programming commands become the only way to manipulate the turtle's position and orientation as well as to change the viewpoint. Bonbon suggested that Rosco switched on the Compass in VR space in order to see the degrees. Rosco thus began his experiment as illustrated in Figure 3.

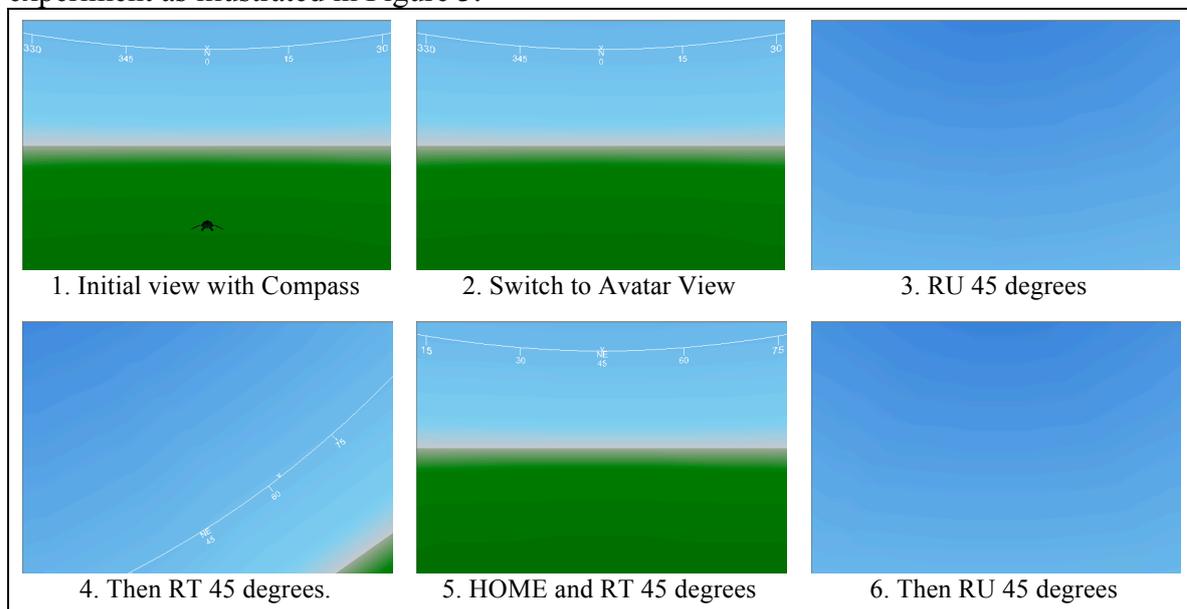


Figure 3. Avatar View experiment about 3D rotation

To his surprise, Rosco found that the views of Picture 4 (RU 45 RT 45) and Picture 6 (RT 45 RU 45) in Figure 3, which he originally thought to be the same, looked different. Because of the different part of the sky he (or the turtle) saw, he then started to think that different order of two 3D rotations may end up with different directions. He also contributed his idea of using Avatar View in the forum in the following posting titled "How to determine if":

*How to determine if ru 45 lt 45
and lt 45 ru 45*

have da same veiwpoint

1. go to avatar view
2. copy this text ru 45 lt 45
3. copy this text lt 45 ru 45
4. SEE FOR YOURSELF



Hi peoples im rosco!!! 😊😊 hello

Bonbon's exploration: Look at the turtle

Bonbon used her hands to simulate the two 3D rotations, and was pretty sure that the two 3D rotations were the same. She did a straight forward experiment by watching the turtle turns, but she decided to try on RU and LT (left) instead of RU and RT (right). The processes of her experiment are illustrated in Figure 4.

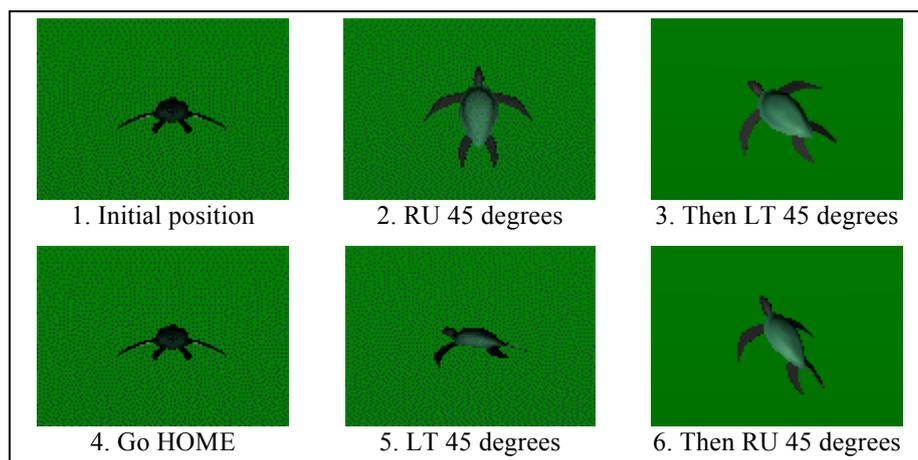


Figure 4. Turtle experiment about 3D rotation

Bonbon carefully compared the two views of Picture 3 (RU 45 LT 45) and 6 (LT 45 RU 45) in Figure 4 and noticed that they were different. However, before she made a conclusion, she also tried tilting rotations (TL and TR) with RU and smaller degrees, and together with Rosco's Avatar View experiment, she convinced herself that the two 3D rotations ended up with different results.

Grae's experiment: Create 3D objects

After seeing Rosco's and Bonbon's experiment, Grae could not think of any idea to show the difference between the two 3D rotations. The researcher encouraged him to try to create a 3D object after each 3D rotation. Grae then decided to create a sphere after each 3D rotation. He used commands "RU 45 RT 45 BALL" to create the first sphere, and then "HOME RT 45 RU 45 BALL" for the second sphere. The processes are illustrated as in Figure 5.

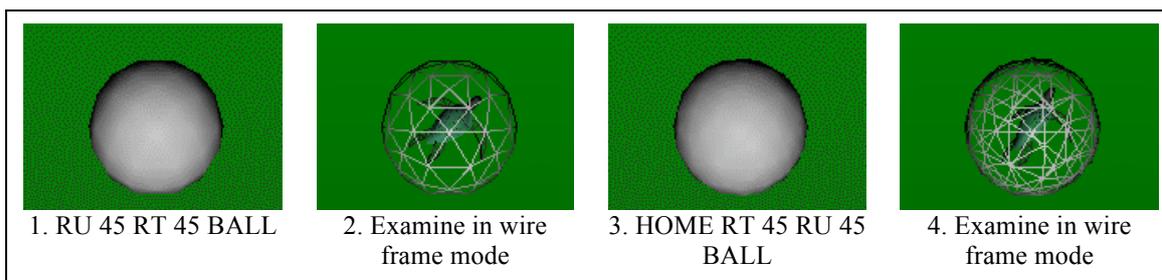


Figure 5. Create 3D Objects experiment about 3D rotation (1)

Grae originally thought that the two spheres should be somewhat overlapped but located at different place. However, he was confused when he navigated to see the two balls from different viewpoints; they seemed to be one ball. The researcher then suggested him to try on CUBE instead of BALL and with different colours. Figure 6 shows the processes of creating cubes after each rotation.

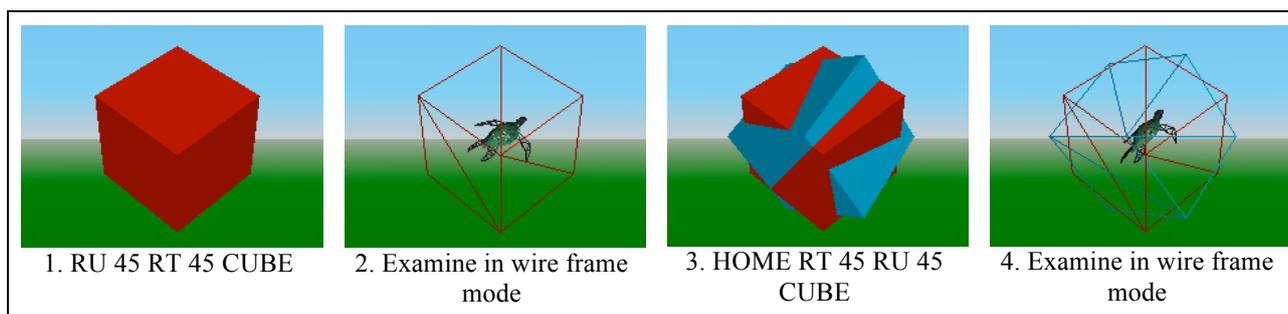


Figure 6. Create 3D Objects experiment about 3D rotation (2)

Grae was then satisfied with this result, and with the help of this researcher, Grae posted a message titled “two turns must take turns” in the forum:

Hi,
 if you lt 45 ru 45, or if you ru 45 lt 45 Will these be the same?
 you can check the answer by doing:
 1. home ru 45 lt 45 cube so you have a cube...
 2. you pick another color from the material editor.
 3. home lt 45 ru 45 cube
 so you have another cube but this time the turtle go lt 45 first then ru 45
 do you think that the two cubes are in the same place???

--
 grae 😊😊😊😊😊😊😊😊

Discussion and Conclusion

From “the two 3D rotations are the same” at the beginning to “the sequence of performing 3D rotations does matter” at the end, the three young participants experienced a conceptual shift after their interactions with VRMath.

The non-commutative nature of 3D rotation may be easily understood by one who can perform trigonometry in 3D coordinate system, but it would be very difficult for most people if they can only use their body movements, senses or feelings, mental reasoning, and other concrete objects. It is evident that although students live in a 3D space, they have limitations on manipulating or thinking three dimensionally.

The VR interface of VRMath which enabled the students to switch between first- and third-person experiences facilitated dynamic visualisations of the 3D rotations. Rosco, for example, utilised the Avatar View to simulate the body movement, which was a typical example of using a computer to address a limitation with real world experiences within 3D space. In the Avatar View, Rosco temporarily became the turtle and viewed the rotations from the turtle's perspective. At the same time, he also manipulated the turtle's orientation by using 3D rotation commands. This operation of switching from third-person experience (watching the turtle turning) to first-person experience (turning himself) allowed Rosco to see different portion of the sky, and as a result, to realise the non-commutative nature of 3D rotations and thus correctly solve the 3D rotation problem posed by the researcher. Rosco's experiences confirmed the benefit of switching between first- and third- person imagery (Amorim et al., 2000).

Bonbon and Grae used the Logo-like programming language to manipulate the turtle and build 3D objects in VR space to solve this 3D rotation problem. Bonbon's experiment demonstrated again that the computational environment VRMath easily and accurately showed the two 3D rotations were different, which was in contrast to the use of her hands to simulate the 3D rotation. Grae's experiment of creating objects was another approach to successfully solve this 3D rotation problem. Nevertheless, he also found that creating a sphere after each set of 3D rotation would not show any difference of the two 3D rotations because as long as the turtle doesn't move, the centre for spheres remains the same.

One important misconception about 3D rotation found in this study was thinking that a turning could be eliminated by its opposite turning performed later in a series of 3D rotations. For example, in the four rotations RU 45 RT 45 RD 45 LT 45, students with this misconception believe that RU can be eliminated by RD and RT by LT. However, as VRMath showed, a rotation of another dimension in between the two opposite turns means that the two rotations of the same dimension still cannot eliminate each other.

To conclude, VRMath with its computational power provided the young children with new ways of thinking about and doing 3D geometry. The small number of cases reported in this study makes conclusions from this study tentative. Further studies, which are currently in progress, will provide further support for the educational efficacy of VRMath. However, this study does provide initial indications that VRMath, with its VR visualisation interface, fully implemented and extended Logo-like 3D programming language (e.g., mathematical functions and recursive procedures), and online forum for collaborative learning, could be a most powerful environment for young children to experience 3D mathematical modelling, simulation and problem solving.

Acknowledgements: The author would like to thank Professor Tom Cooper and Adjunct Professor Rod Nason for their valuable comments and input to this paper.

References

- Amorim, M.-A., Trumbore, B., & Chogyen, P. L. (2000). Cognitive repositioning inside a desktop VE: The constraints introduced by first- versus third-person imagery and mental representation richness. *Presence: Teleoperators and Virtual Environments*, 9(2), 165-186.
- Baturo, A. R., & Cooper, T. J. (1993). Four perspectives for teaching spatial concepts and processes in the primary school. In A. R. Baturo & L. Harris (Eds.), *New directions in geometry education -- Conference Proceedings of The Third Annual Conference on Mathematics Teaching and Learning* (pp. 1-6). Brisbane, Australia: Queensland University of Technology.
- Cunningham, D. J. (1992). Beyond educational psychology: Steps toward an educational semiotic. *Educational Psychology Review*, 4(2), 165-194.

- Elliott, J., & Bruckman, A. (2002, June). *Design of a 3D interactive math learning environment*. Paper presented at the DIS 2002 (ACM conference on Designing Interactive Systems), London, UK.
- Ernest, P. (1994). *Constructing mathematical knowledge: Epistemology and mathematical education*. London Washington, D.C: Falmer Press.
- Harel, I., Papert, S., & Massachusetts Institute of Technology. Epistemology & Learning Research Group. (Eds.). (1991). *Constructionism: Research reports and essays, 1985-1990*. Norwood, N.J.: Ablex Pub. Corp.
- Kafai, Y. B. (2006). Constructionism. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 35-46). Cambridge New York: Cambridge University Press.
- Kafai, Y. B., & Resnick, M. (Eds.). (1996). *Constructionism in practice: Designing, thinking, and learning in a digital world*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Lemke, J. L. (2001). *Mathematics in the middle: Measure, picture, gesture, sign, and word*. Retrieved June 20, 2001, from <http://academic.brooklyn.cuny.edu/education/jlemke/papers/myrdene.htm>
- McGee, M. G. (1979). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin*, 86(5), 889-918.
- Pasqualotti, A., & Freitas, C. M. D. S. (2002). MAT3D: A virtual reality modeling language environment for the teaching and learning of mathematics. *CyberPsychology & Behavior*, 5(5), 409-422.
- Queensland Studies Authority. (2004). *Years 1 to 10 mathematics syllabus*. Retrieved July 30, 2004, from <http://www.qsa.qld.edu.au/yrs1to10/kla/mathematics/docs/syllabus/preprint.pdf>
- Yeh, A. (2004, April). *VRMath: Knowledge construction of 3D geometry in virtual reality microworlds*. Paper presented at the Doctoral Consortium, CHI2004, Vienna, Austria.
- Yeh, A., & Nason, R. (2004a, June). *Knowledge building of 3D geometry concepts and processes within a virtual reality learning environment*. Paper presented at the World Conference on Educational Multimedia, Hypermedia & Telecommunications, Lugano, Switzerland.
- Yeh, A., & Nason, R. (2004b, June). *VRMath: A 3D microworld for learning 3D geometry*. Paper presented at the World Conference on Educational Multimedia, Hypermedia & Telecommunications, Lugano, Switzerland.