Playing with Mathematics: Play in Early Childhood as a Context for Mathematical Learning.

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Play is an essential part of young children's lives. This symposium highlights the integral role of play in young children's mathematics learning and examines the teacher's role in facilitating and extending this. Papers examine key tenets of play, contributing to theoretical understandings and presenting data on teacher's perceptions of play and young children's actions in play. Examination of teacher perceptions and young children's experiences of mathematical play identifies potential for development of mathematical concepts beyond embryonic mathematics inherent in play.

Paper 1: Sue Dockett and Bob Perry; Charles Sturt University. *What makes mathematics play?*

Paper 2: Louise Thomas, *Elizabeth Warren and* Eva deVries; Australian Catholic University. *Teaching mathematics and play based learning in an Indigenous early childhood setting: Early childhood teachers' perspectives.*

Paper 3: Shiree Lee; The University of Auckland. *Mathematical outdoor play: Toddler's experiences.*

Paper 4: Robert P. Hunting; La Trobe University. *Little people, big play, and big mathematical ideas.*

What Makes Mathematics Play?

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This paper considers examples of situations in mathematics learning that are often described as play-based and critiques these in light of conceptualisations of play focusing on children's processes and dispositions. The potential of play in mathematics learning is investigated and the question asked as to whether it matters if children make mathematics play. The role of early childhood educators in using play to build on children's existing mathematical understandings is explored.

Play has long been regarded as a critical element of early childhood curriculum and pedagogy. In addition to being recognised as a vehicle for learning, play is described as a context in which children can demonstrate their own learning and help scaffold the learning of others (Wood, 2008). Despite this, educators often struggle to explain what it is about play than promotes learning and ways in which they can actively facilitate both play and learning (Ranz-Smith, 2007). While this situation applies generally, van Oers (1996) notes that the potential of play to facilitate children's mathematical thinking depends largely on educators' ability to "seize on the teaching opportunities in an adequate way" (p. 71). We argue in this paper that this ability requires: mathematical knowledge; understanding the nature of children's play, particularly the characteristics of play that promote mathematical learning and thinking; and awareness of the role of adults in promoting both play and mathematical understanding. We start this discussion by focusing on the following situations described by educators as involving play and mathematics, asking — *Does this experience involve play*? and then — *Why does it matter?*

Interest Centres

Teachers of the four-year-old group have set up several interest centres around the room as part of their maths program. These include puzzles, boxes of beads and threading patterns, drawing materials, several sets of picture dominoes and Playdough. Children are assigned to an activity and after ten minutes, teachers make a signal and direct them to the next activity.

A Shell from Home

A group of three-year-olds is having a conversation with an adult. One of the children has brought a large shell from home and the children and adult are discussing its features, including shape and colour, where it may have come from, and how it was found. Both the adult and the children have many questions, as well as many possible answers.

Trampoline

A group of children waits patiently for a turn as one girl jumps on the trampoline. She explains that she will finish her turn when she gets to 50. She counts aloud, 33, 34, 25, 26...

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Matching Game

An educator invites three children to sit with her and play a matching game. The educator explains the rules of the game, noting how children locate matching sets of animals. Each child takes their turn and each gathers several sets of cards.

Blocks

One child has seated herself on the floor in the middle of a pile of blocks. There is no room for anyone else in the space. Over a period of forty-five minutes, she proceeds to build up a series of towers, and then knock them down.

Do these Experiences Involve Play?

The answer to this question largely depends on the definition of play adopted. There are many definitions of play, reflecting different theoretical perspectives of learning and development. Drawing on some of the elements of traditional theories of play, recent conceptualisations have adopted critical approaches to assumptions about the universality of play, categorisations of play, the automatic connection between play and learning, and the role of adults in supporting play (Wood, 2007). Current research, while not totally rejecting some of the basic tenets of earlier, traditional approaches to play, focuses on the processes and dispositions of play, the generation of complex and varied forms of play, and recognition of the social and cultural contexts of play (Wood, 2008).

In keeping with the focus on play as a process and disposition, researchers concur that play cannot be defined by its subject matter: “play is a particular *attitude* or *approach* to materials, behaviours, and ideas and *not* the materials or activities or ideas themselves; play is a special mode of thinking and doing” (McLane, 2003, p. 11). In this sense, the process of play is characterised by a non-literal ‘what if’ approach to thinking, where multiple end points or outcomes are possible. In other words, play generates situations where there is no one ‘right’ answer. McLane (2003, p. 11) described this as conferring “a sense of possibility, as well as ownership, control and competence on the player”. Essential characteristics of play then, include the exercise of choice, non-literal approaches, multiple possible outcomes and acknowledgement of the competence of players. These characteristics apply to the processes of play, regardless of the content. In addition, thinking of play as a disposition, or habit of mind (Carr, 2001), helps to link it with other dispositions that are valued in education, including mathematics education, such as creativity, curiosity, problem posing and problem solving (Ginsburg, 2006; NAEYC/NCTM, 2002).

Some of the situations outlined earlier in this paper reflect a context where the children have ownership and control in the initiation, direction and outcome for the activity. For example, the child immersed in block play creates both a physical as well as a conceptual space in which to play and determines the direction and outcomes for the play. By keeping others out, she exerts competence and control. The girl on the trampoline exerts similar qualities. The children talking about the shell also control the experience. It is the one child’s choice to bring the shell to the preschool and all participants – including the children, guide the discussion. In the other situations, control of the experience is much more vested in the teacher who has determined what experiences are on offer, the materials to be used, the ways in which the activity is to be conducted and the desired outcomes for each experience. Each of these experiences can make a valuable contribution to learning and teaching – they are just not play.

Why Does It Matter?

If a wide range of experiences can support children's learning of mathematics, why does it matter that some of these experiences be deemed to be play and others not? In answering this question we draw on some of the commonalities between play and mathematics. We argue that fluency in the processes underpinning play can, with the skilled guidance of educators, promote a range of mathematical knowledge and understanding.

Children's play can be very complex. Sometimes play develops and evolves over several days, weeks or even longer. There will often be negotiations about roles, rules, materials and scripts. The actual context of the play can also be complex – such as when children play with abstract ideas and possibilities. Mathematics is present in much of children's spontaneous play (Seo & Ginsburg, 2004). Educators who are alert to this, and who themselves feel competent and comfortable playing with mathematics can provoke deep understanding. These educators are also likely to display, and model to the children, the dispositions of playfulness, curiosity, critical and creative thinking (Carr, 2001).

Play is often an inherently social activity. Vygotsky (1967) argued that even solitary play replicates social and cultural contexts, particularly in the rules and roles adopted by players. When play involves others – be they adults or children – opportunities for scaffolding (Bruner, 1986) occur as children interact with more knowledgeable and experienced others. The social interactions within play facilitate joint meaning making, as children test out, explain and enact their perspectives and understandings, at the same time as they encounter those of others. Social interaction in play provides support for the challenges children often construct in play, creating opportunities for innovation, risk taking and problem solving. Such interactions also underpin mathematical thinking.

Play has been described as a context in which children can integrate experiences and understandings, draw on their past experiences, make connections across experiences, represent these in different ways, explore possibilities and create meaning (Bennett, Wood, & Rogers, 1997). If mathematics is as much about understanding connections, processes and possibilities as it is about knowing facts, then play and mathematics have much in common (NAEYC/NCTM, 2002; Perry & Dockett, 2008).

Young children's play often involves mathematical concepts, ideas and explorations (Perry & Dockett, 2008; Seo & Ginsburg, 2004). Ginsburg (2006) described a range of mathematical experiences and concepts embedded in early childhood environments: children's free play; play about mathematics; and children's play with the ideas and approaches that have been introduced by educators. Educators who facilitate children's play and who are aware of the nature and complexity of that play are well positioned to build on children's existing knowledge and understandings – another tenet of early childhood curriculum and pedagogy. It has been noted that "play does not guarantee mathematical development, but if offers rich possibilities. Significant benefits are more likely when teachers follow up by engaging children in reflecting on and representing the mathematical ideas that have emerged in their play" (NAEYC/NCTM, 2002, p. 6). Similar support for play is derived from the AAMT/ECA (2006) position statement in early mathematics, which exhorts educators to: promote play with mathematics as one means of engaging children's natural curiosity; recognise mathematics as a social activity; and promote mathematics that has relevance to children's everyday lives.

Conclusion

In a context of increasing accountability and rising academic expectations, the educational value of play has been questioned (Dockett, Perry, Campbell, Hard, Kearney, & Taffe, 2007; Wood, 2008). When educators evidence a sound knowledge of mathematics, a pedagogical repertoire that includes play, and awareness of the connections between these, there is great potential for early childhood experiences that extend young children's mathematical understandings and dispositions. There is much to be gained from making mathematics play.

References

AAMT/ECA. (2006) Position paper on early mathematics. From http://www.aamt.edu.au/Documentation/Statements/Position-Paper-on-Early-Childhood-Mathematics

Bennett, N., Wood, L., & Rogers, S. (1997) *Teaching through play: Teachers' thinking and classroom practice*, Buckingham, UK: Open University Press.

Bruner, J. (1986). *Actual minds: Possible worlds*. Cambridge, MA: Harvard University Press.

Carr, M. (2001). *Assessment in early childhood: Learning stories*. London: Paul Chapman.

Dockett, S., Perry, B., Campbell, H., Hard, L., Kearney, E., & Taffe, R. (2007). *Reconceptualising Reception: Continuity of learning*. From http://www.earlyyears.sa.edu.au/files/links/final_lit_review.pdf

Ginsburg, H. P. (2006). Mathematical play and playful mathematics: A guide for early education. In R. M. Golinkoff, K Hirsh-Pasek, & D. Singer (Eds.), *Play=learning* (pp. 145-165). New York: Oxford University Press.

McLane, J. B. (2003). "*Does not." "Does too." Thinking about play in the early childhood classroom*. Erikson Institute Occasional Paper Number 4.

NAEYC/NCTM. (2002). *Position statement: Early childhood mathematics: Promoting good beginnings*. From http://www.naeyc.org/about/position/psmath.asp

Perry, B., & Dockett, S. (2008). Young children's access to powerful mathematical ideas. In L. D. English (Ed.). *Handbook of international research in mathematics education* (2nd ed) (pp. 75-108). New York: Routledge.

Ranz-Smith, D. J. (2007). Teacher perception of play: In leaving no child behind are teachers leaving childhood behind? *Early Education and Development*, 18, 2: 271-303.

Seo, K-H., & Ginsburg, H.P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements, J. Sarama, & A-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 91-104). Hillsdale, NJ: Erlbaum.

Van Oers, B. (1996). Are you sure? Stimulating mathematical thinking during young children's play. *European Early Childhood Education Research Journal*, *4*(1), 71-87.

Vygotsky, L. (1967). Play and its role in the mental development of the child. *Soviet Psychology*, *5*(3), 6-18.

Wood, E. (2007). New directions in play: Consensus or collision? *Education 3-13*, *35* (4), 309-320.

Wood, E. (2008) Conceptualising a pedagogy of play: International perspectives from theory, policy and practice, in D. Kurschner (Ed.) *From children to red hatters: Diverse images and issues of play*, Play and Culture Studies, 8, 166-190.

Teaching Mathematics and Play-based Learning in an Indigenous Early Childhood Setting: Early Childhood Teachers' Perspectives.

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In recent years there has been a surge in attention concerning early childhood settings and mathematics learning. The literature provides several reasons for this. In brief these are (a) a recognition that children enter school with a great deal of intuitive knowledge about mathematics and that this knowledge can serve as a base for developing formal mathematics in a school setting, young children do not need to be made ready to learn mathematics (Carpenter, 1996), (b) there is a relationship between early mathematical knowledge and later achievement (Aubrey, Dahl & Godfrey, 2006), (c) the main determinants of later achievement is quality early mathematical experiences (Young-Loveridge, Peters & Carr, 1997), and (d) young children are capable of engaging in mathematically challenging concepts (Balfanz, Ginsburg & Greenes, 2003). Yet many early childhood teachers are reluctant to embrace an active role in the teaching of mathematical concepts (Grieshaber, 2008a) and continue to perceive teaching and play-based pedagogies as incompatible (Ryan & Goffin, 2008). More specifically, early childhood practitioners are often fearful of mathematics and see a mathematics curriculum as having the potential to restrict children's choices and thus "inhibit their ability to be self regulatory and autonomous" (Macmillan, 2009, p. 110).

A suggestion presented in this paper is that the core business of young children is *playing* and the core business of teachers (including early childhood teachers) is *teaching.* To give consideration to these statements we first need to ask: What does teaching look like? *and* Would one know 'teaching' if one saw it in an early childhood setting? The work that early childhood teachers do with young children is referred to by an array of terms and it is rare to find that the term 'teaching' is used to describe this work. Children learn through play and a dominant theory in early childhood education suggests that they need adult guidance to assist them to reach their full learning potential (Balfanz, Ginsburg & Greenes, 2003; Vygotsky, 1962). This paper argues that play is a pedagogical tool that can enable learning and this learning can be maximised with appropriate, timely and effective adult input.

The research project reported here (Warren, deVries, & Thomas, 2009), looked at the mathematical experiences of pre-prep children in a Brisbane Indigenous kindergarten (average age 3 years and 6 months). The sample comprised two teachers and twenty-eight children. Given that Tayler, Thorpe and Bridgstock (as cited in Fleer & Rabin, 2007) report that, as compared with other cohorts of early years children, Indigenous children gain even less from attending play-based programs, this project endeavoured to investigate ways in which pre-prep children, engaged in play-based programs, could be supported in their mathematical learning. The overarching objective was to develop culturally appropriate best practice/research grounded teacher and parent materials to support the transition of Indigenous children from home to school with regard to their numeracy learning.

The focus of this paper is to examine the teachers' perspectives on play in early childhood education and their reflections as they incorporated mathematical experiences

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into their play-based program. Of particular interest, in the reporting of this aspect of the project was ways in which the teachers spoke of what they did when engaging with young children in a mathematical context.

Method

The research methodology involved initial interviews with the two early childhood teachers that took the form of focused conversations. These interviews were followed by professional development sessions based on numeracy for young children and the supported implementation of various numeracy based experiences into the pre-prep program. The model used for the professional learning was the Transformative Teaching in Early Years Mathematics [*The TTEYM Model* (Warren, 2009)], a model underpinned by the theories of Vygotsky which state "children's [and teachers'] development is best guided by people who are experienced in using these tools (i.e., language, mathematical systems, and technologies)" (Hill, Stremmel & Fu, 2005, p. 15).

After several weeks when the teachers and children had engaged with these experiences as part of the pre-prep program, video data was collected of children and teachers at work (and play) in the classroom. The teachers then individually viewed these videoed sessions with the researcher. Audio recordings were taken of each teacher as they viewed and discussed their own facilitation of these numeracy experiences as recorded on the video. The transcriptions of these recordings were analysed to identify ways in which teachers spoke about children's engagement with mathematical concepts and their use of a play-based pedagogy.

Results

Interview 1: – In their initial interview (prior to TTEYM) the teachers spoke of play as an important element of learning processes and something over which children have control:

> Play to me is an important part of learning. … Most importantly they (the children) have control over their play. … I allow the children to help plan their play. [Interview 1 (b)]

Play was seen as part of learning. This implied that there was something more to learning than just 'play'. Children were identified as active participants with control over their play.

However, this was not all that these teachers spoke of when identifying play as a feature of their curriculum and pedagogy. Of significance for these teachers was their role in the children's play:

> To listen (…) to help them achieve (…) to extend and enhance their play… [Interview 1 (b)] (…) as the teacher you try and investigate what they're doing first (…) and then you guide that conversation and do not do the play for them [Interview 1 (a)]. You guide that play (…) and once they get that concept and are able to communicate it, well their play will be better [Interview 1 (a)]

Interview 2: - Following the professional learning (TTEYM) and the introduction of mathematics experiences into the classroom, the teachers spent time viewing their own practice (videoed by the researcher). The following points reflect how the teachers spoke of the interplay between their role as *supporters* of play and *teachers* of mathematics.

The mathematical experiences presented in TTEYM were seen as interesting enough to entice the children and capture their interest. This enabled the teachers to operate within their pedagogical philosophy that play is a key component of children's learning and that the children should direct it:

> they are confident and they want to have a go, they want to play all the games before they know they can because they have prior knowledge about number which you build up as a teacher. And then when they go on to the next level like pre-prep before they go to formal schooling that they can build upon that knowledge again, again and again. [Interview 2 (a): 259]

There was also support for the teaching of skills as a means of enhancing children's confidence in themselves as learners and teachers.

> And once the child gets very good at it and understands all the concepts of learning it, they (the child) will begin to teach (i.e. they will play the game with other children) it. Because they know they have become very good at it [Interview 2 (a): 271]. When some of the more … confident ones came along, and start the game themselves, they sort of encourage some of the not so confident ones to come over. [Interview 2 (b): 125-127]

The teachers were able to identify that these children were beginning to transfer their mathematical knowledge across tasks, something they had not previously evidenced.

> ... I find that because we did the fly game and then I put out the fishing game another time, that they were more interested in the fishing game also, but in a different way because when they did the fly game (and then) went over to the fishing game, then they were taking the fishes off the line and lining them all up. So you know, normally they would sort of just catch the fish and put it in the bucket and that's it they wouldn't do anything more with it [Interview 2 (b): 23]

The teachers could also identify which children were able to incorporate the knowledge and skills experienced through the direct teaching element of the introduction of the games into their play:

> …it (the introduction of activities) sort of helps them in their other play to extend it a bit more … in what they want to get out of it - not what we want them to get out of it [Interview 2(b):31-35].

One teacher also indicated how the project had changed the ways in which she engages in teaching practices:

> ..if they don't pick it up first time around .. I ask them a different way, then they pick it up that way. … they are really thinking about … what we are saying, what they need to listen to, It's good … it's made me as a teacher more aware of … how I speak to the children. [Interview 2 (b):106-118]

The shift in teachers' talk between Interview 1 and Interview 2 raises for consideration the positioning of both children and teachers as either active or passive participants in the dance between *teaching* mathematics and *play* based *learning*. Specifically one teacher was able to articulate the different focus for children and teachers as active participants in learning contexts that draw on play as a tool to enable active engagement in learning.

> …it has made me think a lot more…, what to plan for the children …, for them also to help me plan …. If they suggest something then I try to work it out, … they can get this out of that activity … what can I throw in to help them, with their numbers, extend it a little bit more I've added something else in there to extend … to … try … their numbers and recognise their numbers and language … Not overwhelm them, but…, try to help them get the best out of the activity, that they enjoy and stay at it longer [Interview 2 (b):213-223]

The children were focused on play and the teacher's role was to focus on how and when she could 'step-in' and engage in teaching. In this way the teacher was able to talk of her responsibility to influence children's learning and her belief in children's autonomy in planning their play. It was evident in the shift in the teachers' talk that they had added to their focus on children as active participants in play (and therefore learning) to include a view of themselves also as active participants in children's play (and therefore learning).

Analysis of the teachers' talk enables the holding together of play and learning as teachers actively engage in appropriate processes of teaching.

The literature informing this analysis raises for deliberation the question – Is there a silencing of the term 'teaching' in play based contexts and to what extent are early childhood teachers complicit in this silencing? Teaching as a missing term in early childhood contexts has been recently addressed (Grieshaber, 2008b). When early childhood teachers speak of 'sustained shared thinking', 'the teachable moment' 'facilitation of learning', 'guided participation' 'scaffolding', 'co-constructing' are these responses to an early childhood discourse that excludes use of the term teaching or is it a way of disguising their talk of teaching – given that teaching is a discourse not generally considered acceptable in an early childhood context (McArdle & McWilliam, 2001).

The results of this project begin to evidence that at the commencement of TTEYM the teachers perceived play based learning and teaching to be at odds. That is, that play is child directed and a teacher's role is to extend and enhance the play in a passive manner. Two key components of TTEYM appeared to challenge these perceptions. These were (a) the types of mathematical activities presented in the professional development session (engaging and hands on), and (b) the maths expert's active implementation of these activities with 'real children' in a play-based context.

Ryan and Goffin (2008) called for a shift in research targets in the area of early childhood education that allows for a greater focus on teachers' perspectives and thinking about their work with young children. This paper calls for further work by both researchers and practitioners in the fields of early childhood education and mathematics with a focus on how early childhood teachers construct themselves as teachers engaged in both a play-based pedagogy and mathematics as a curriculum discipline.

References

Aubrey, C., Dahl, S., & Godfrey, R. (2006). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal 18(1),* 21-46.

Balfanz, R., Ginsburg, H., & Greenes, C. (2003). The big maths for little kids early childhood mathematics program. *Teaching Children Mathematics,* 264-268.

Carpenter, T. P. (1996). Cognitively guided instruction: A knowledge base for reform in primary Mathematics Instruction. *Elementary School Journal, 97 (1),* 3-20.

Fleer, M., & Raban B. (2007). Constructing cultural-historical tools for supporting young children's concept formation in early literacy and numeracy. *Early Years, 27(2),* 103-118.

Grieshaber, S. (2008a). Marginalization, making meaning, and mazes. In G. Genishi, & A. L. Goodwin (Eds.). *Diversities in early childhood education: Rethinking and doing*. New York: Routledge.

Grieshaber, S. (2008b). Interrupting stereotypes: Teaching and the education of young children. *Early Education and Development. 19 (3), 505-518*.

Hill, L., Stremmel, A., & Fu, V. (2005). *Teaching as inquiry: Rethinking curriculum in early childhood education.* Boston: Pearson.

Macmillan, A. (2009). *Numeracy in early childhood: Shared contexts for teaching and learning*. South Melbourne: Oxford University Press.

McArdle, F., & McWilliam, E. (2005). From balance to blasphemy: shifting metaphors for researching early childhood education. *International Journal of Qualitative Studies in Education, 18(3), 323-336.*

Ryan, S. & Goffin, S. (2008). Missing in action: Teaching in early care and education. *Early Education and Development, 19 (3), 385-395.*

Vygotsky, L. (1962). *Thought and language* (E. Hanfmann & G. Vakar, Trans.). Cambridge, MA: Massachusetts Institute of Technology. (Original work published in 1934).

Warren, E. (2009). Early childhood teachers' professional learning in early algebraic thinking a model that supports new knowledge and pedagogy. *Mathematics Teacher Education and Development, 10, 30-45.*

Young-Loveridge, J., Peters, S., & Carr, M. (1997). Enhancing the mathematics of four year-olds. An overview of the EMI-4S Study. Jo*urnal for Australian Research in Early Childhood Education, 2, 82–93.*

Waren, E., deVries, E., & Thomas, L. (2009). *Early numeracy in pre-prep settings: Final report*. ACU

Mathematical Outdoor Play: Toddler's Experiences

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Mathematics is a subject area that is generally understood and accepted as an important part of academic learning and therefore has an important part to play in the formal education of our children. However, in New Zealand there is no such formal requirement in early childhood education and therefore mathematics is an area often overlooked by early childhood teachers and parents. This paper reports a summary of the findings of a case study where observations of toddler's outdoor play episodes showed evidence of mathematical knowledge and skill in unstructured play activity.

Within NZ early childhood settings it is generally accepted that the very highest quality learning and teaching occurs through play. However the term 'play' can have a wide variety of definitions. Within this study the incidents of toddlers exploring and engaging in what has been termed 'mathematical play' were situated within a child-centred and integrated curriculum. The toddlers directed their own experiences within the outdoor area and engaged in a wide variety of play activity that has the potential to be analysed from a variety of perspectives. Learning within early childhood, and the experiences that can constitute learning, occur in the socio-cultural, holistic environment of a learning community (Rogoff, 1998; Burton, 2002), the play curriculum.

Another key tenet of mathematical play that was evident within this study is the notion of enjoyment. For children to be involved in play it must be fun. "We can influence young children's keenness to learn mathematics by making the tasks we do of interest to them … by showing that we really think maths is important and fun", (Clemson & Clemson, 1994, p. 19).

The notion of an integrated curriculum in early childhood, underpinned by socio-cultural theory, includes all actions, interactions, experiences and routines that children are involved in - that is, all subject domain areas as well as routines such as meals and hygiene practices. This curriculum, and therefore the environment of the setting, was facilitated, supervised and set up by the teachers, but the children were free to explore the environment in an unstructured manner, and to add additional resources from the indoors as they wished.

Mathematical Play

Each of the following mathematical categories arose from the analysis of the observations of children's play. These are presented in order of frequency of occurrence and give examples related to that particular category. The mathematical process of problem solving has not been specifically reported in this paper but is inherent within each of the other categories.

Space

Space (and spatial concepts) arose as the most common area of mathematical understanding that the toddlers displayed in this study. This is in direct contrast to previous research describing young children's apparent lack of skill as being due to the inability to make abstractions, apply logic or understand representation (Piaget, 1952). For example,

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Sarah aged two years, two months [2:2] was able to figure out how to ride a bike and wear a circular skirt at the same time without it getting caught in the wheel. This showed that she was able to apply logic to the situation in order to move within the space. She may have been thinking 'if I lift the skirt it will not be in the way anymore, so therefore I need to hold the skirt up as I ride'. Sarah's previous experience of manipulating objects and herself in space enabled her to solve her problem. In other words, she was applying logic to her situation.

Toddlers' knowledge of ways to manipulate objects to create space was evident when Lyle [1:10] and Ricky [2:7] moved large cardboard cartons around another child in order to fit themselves into them. The children's experience with both the movement of the boxes and their bodies enabled them to complete their self-chosen task successfully. These incidents show the children's understanding of underlying concepts of space, and that in order to create space, move within space, and manipulate space, they must think and act logically, abstractly, and at times draw from previous experience.

If toddlers did not possess skills of abstraction, logic or representation they would be unable to explore space in the ways observed in this study. They would not have the ability to solve problems, move objects in a logical way or negotiate with others.

Number

Number is the most commonly discussed, debated and reported upon mathematics within the readily available literature. However, no examples could be sourced that discussed or described the foundational concepts of number being developed in children younger than three years of age. Yet this was the second most common category of mathematics observed within the toddler's play experiences.

Number skills, including counting sequences both forward and backward, using number to name and classify objects, counting as timing and quantification, were all evident in the toddlers' outdoor play experiences. This skill in number concepts occurred at a much younger age than has previously been recognised. For example; Fraser [2:10] counted out loud (rote counting) from one to six; Trent, [2:0] stated that he was holding two sand scoops; and Anne, [2:7] counted from one to ten in both forwards and backwards sequences.

Measurement

The third most common category of mathematics evident within the children's play involved toddlers exploring measurement concepts. However, some of the toddlers' 'measures' were not accurate but naïve (Wellman & Gelman, 1992). With children in the toddler stage this is to be expected and the refining of these skills will continue to occur with further experiences. Interestingly, measurement is the category that showed evidence of mathematical exploration by the youngest child in the findings. Ryder, [1:3], was observed exploring volume by placing handfuls of sand onto his feet. The change of measuring tool from his hand to a sand scoop showed an understanding that the tool increased the volume of sand moved. This showed his developing knowledge of a way to move more sand and, by implication, to measure more efficiently.

Pattern

Recent mathematics education research has been focused upon the development and knowledge of patterning, which has been found to influence children's reasoning and the ability to generalise patterns (e.g. Mulligan, Mitchelmore & Prescott, 2006; English, 2004). However, this research was conducted with four to six year old children and was focused upon graphical representation (picture patterns) rather than play behaviour.

Toddlers were observed repeating the refrain of a popular television tune, expressing knowledge that mealtime was approaching, and repeating of behaviours in play episodes. Similar to the previous categories of mathematical play, toddlers' prior experiences of these patterning concepts had laid the foundations for their knowledge and skill, and showed their conceptual knowledge of the routines and patterns of the day.

Shape

Knowledge of the geometrical properties and names of shapes is a common feature of academic programmes and this is evident in the available research concerning children four years of age and older (e.g. Geist, 2001; Pound, 2006; Willis, 2001). Shape concepts, as well as most mathematical ideas, are not commonly described within the literature as appropriate for infants or toddlers.

In this study, only four observations were relevant to the category of shape, and only one included a toddler applying a name to a shape verbally. However, three other observations showed evidence of the toddlers' understanding of similarity between shapes (even when unable to name them). Awareness of shape can be clearly seen in the incident with Gene [2:8] in the sandpit where he was carefully creating 'sandcastles' with a castle-shaped bucket. Each time the 'castle' was not perfect he placed the bucket back over it in an attempt to 'correct' the shape. When the 'castle' did turn out correct, he smiled and then demolished it. He understood the shape he wanted and was persisting in his effort to achieve it.

Classification

In order to apply abstraction of concepts, logical solutions to problems, and gain understanding of how objects, people and places can be grouped, skills in classification must be gained (Geist, 2001; Babbington, 2003). In this study toddlers classified a range of objects for their own purposes. For example, Bree [2:5] had clear understanding of which napkins were hers and became quite distressed when she had to wear a different type.

Conclusion and the Way Forward.

This paper reports on a larger study that provided convincing evidence that toddlers engage in outdoor play experiences that contain mathematics and highlights toddlers as competent and confident learners of mathematics. While maintaining a play-based and integrated curriculum, the mathematical knowledge of children, particularly toddlers, requires more explicit attention from teachers.

Purposeful teaching and learning in mathematics is enabled when teachers provide resources and environments that encourage exploration in the outdoors. They must also hold adequate subject content knowledge and ideally an interest in mathematical ideas. Finally, we must all strive to ensure that mathematical learning is meaningful and enjoyable for children. The curriculum needs to retain a sense of playfulness and fun to ensure that children develop positive dispositions for learning through their play experiences.

References

Babbington, S. (2003). *The magic of mathematics*. Paper presented at the 8th Early Childhood Convention. Palmerston North, September 4-8.

Burton, L. (1994). *Children learning mathematics: Patterns and relationships*. Hemel Hempstead: Simon and Schuster Education.

English, L. D. (2004). Mathematical and analogical reasoning in early childhood. In L. D. English, (Ed.), *Mathematical and Analogical Reasoning of Young Learners* (pp. 1-22). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Geist, E. (2001). Children are born mathematicians: Promoting the construction of early mathematical concepts in children under five. *Young Children, 56*(4), 12-19

Mulligan, J. T., Mitchelmore, M. C., & Prescott, A. (2006). Integrating concepts and processes in early mathematics: The Australian Pattern and Structure Mathematics Awareness Project (PASMAP). In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 209-216). Prague: Program Committee.

Piaget, J. (1952). *The child's conception of number*. New York: Norton. (Original work published 1941).

Pound, L. (2006). *Supporting mathematical development in the early years* (2nd ed.). Berkshire, United Kingdom: Open University Press.

Rogoff, B. (1998). Cognition as a collaborative process. In W. Damon (Ed.), H*andbook of child psychology (*5th ed., vol. Two: Cognition, perception and language), pp. 679-744. New York: John Wiley and Sons.

Wellman. H. M., & Gelman, S. (1992). Cognitive development: Foundational theories of core domains. *Annual Review of Psychology*, *43*, 337-375

Willis, S., (2001). *Becoming Numerate: Who's at risk and why?* Paper presented at Early Years Numeracy Conference, 31 May 2001, Melbourne, Vic: Monash University.

Little People, Big Play, and Big Mathematical Ideas

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The term *big play* is proposed to draw attention to the need for teachers and carers to become aware of embryonic mathematics inherent in activities of young children. Candidates for big ideas of early years mathematics are outlined, following brief discussion of what the expression *big mathematical idea* might mean.

Young children's play has many facets: free or self-directed (Smith, 1994), structured or teacher-directed (Bruner, 1986; Vygotsky, 1978), symbolic (Edo, 2009; Van Oers, 1994), constructive (Smilansky, 1968), and imaginative (Fein, 1981), to name a few (Keizer, 1983). Play may incorporate the invention and extension of action schema, and it may incorporate rehearsal and practise of those schema (Piaget, 1962). It is well accepted that play has an important role in early years mathematics learning (Griffith, 1994; Perry & Dockett, 2007). So the challenge for harnessing play for advancing young children's mathematical thinking raises questions such as: What kinds of mathematical thinking are young children capable of? What mathematics should young children be learning? Is some mathematics more appropriate or desirable than other mathematics? Can young children learn mathematics through their play experiences? If so, how can carers and teachers nurture and establish mathematical thinking through children's play?

Children's play, from the perspective of the mathematics educator, may not be uniformly efficacious for mathematical development, even though Piaget (1962) believed that "everything during the first few months of life, except feeding and emotions like fear and anger, is play" (p. 90). So it is suggested that *big* mathematical ideas potentially accessible to preschoolers be identified, so that carers and teachers might be alert to possible play contexts in which these ideas might become manifest, albeit in embryonic form. Once understood, appropriate interventions or activities may be planned so as to assist development of children's mathematical thinking. Hence the term 'big' play. It is first necessary to briefly discuss what the expression *big mathematical idea* could mean.

Differing Views on What Makes a Mathematical Idea 'Big'

There is no consensus at the levels of policy, curriculum development, or tertiary mathematics education as to what the *big ideas* of school mathematics might be (Hunting & Davis, 2010). To simplify, allow me to identify two poles or extreme positions representing this matter – what might be called the *soft* big idea and the *hard* big idea. The soft big idea is essentially to accept the status quo of school mathematics curriculum as we have experienced it for the past 100 years or so, and identify major curriculum topics that warrant attention. Examples might be: fractions, place value, long division, ratio and proportion, and so on. We call this meaning soft because of acceptance of the general belief that the selection and sequencing of school mathematics topics is the way we have always done it, based primarily on a logical analysis of elementary mathematics from an adult point of view, in the face of demonstrable overall failure to achieve success in teaching these. The hard big idea is to first ask what conceptual tools professional mathematicians have found to be fundamental and potent in the history of mathematics, and in their own mathematical education. Once established, attempt to develop ways and

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means to establish preparatory foundations at school level, mindful that children's mathematics and mathematical thinking is not the same as that of the adult (NAEYC & NCTM, 2002). Examples of hard big ideas include variability and randomness in chance processes, the notion of unit system, scale and similarity, boundary and limit, function, equivalence, infinity, recursion, and so on. The intersection between soft and hard big ideas is by no means the null set.

Other definitions of bigness might include those topics, or concepts or ideas in school mathematics that cause the most misunderstanding – those highly robust misconceptions. Or those topics, concepts or ideas that stimulate the greatest interest, or have the most interconnectedness across major strands, or capacity to unify specific conceptual clusters of ideas, or represent transition points at which major conceptual reorganisations or accommodations (Piaget, 1974) are necessary in order for deeper understanding to occur.

Candidates for Big Ideas of Early Childhood Mathematics

Table 1 provides a provisional list of candidates for big ideas in early childhood mathematics, together with brief comments.

Table 1
Candidates for Big Ideas of Early Years Mathematics

Candidate	**Comment**
The pigeonhole principle[34]	Dirichlet first formalised this principle (Elstrodt, 2007). One-to-one correspondence is at its root, and rational counting depends on it.
Negation	Some actions can be undone, neutralised or annulled, leading to reversibility (operations) (Piaget, 1954).
Class inclusion	Seriation, order, and asymmetric relationships including nested number relationships (Inhelder & Piaget, 1964)
Symmetry	Mirror images through paper folding (Schuler, 2001)
Partitioning into equal subsets; halving first	Sharing items using systematic dealing procedure (Davis & Hunting, 1990; Davis & Pitkethly, 1990).
Ratios	Two for me, one for you
Composition and decomposition of continuous and discontinuous material	Part-whole relationships (Bjorklund, 2008). Comparisons between set and subset foreshadow subtraction. Supersets may also be created through addition (Hunting, 2003).
Congruence	An exact copy, based on analyses of similarities and differences of item attributes: Game of Snap.
Similarity, scale, and proportion	Enlargement and contraction: Using an overhead projector or Smartboard.
Randomness	Events that cannot be predicted; effects without causes
Variability	Some days are warmer than others; some children are taller than others.
Approximation	Find some items about as long as your shoe

[34] If you have fewer pigeon-holes than pigeons and you put every pigeon in a pigeon-hole, then there must result at least one pigeon-hole with more than one pigeon.

Algorithm	Repetition of action sequences, such as climbing up and over a low 2-step ladder, over and over.
Conservation of quantity and number	Investigations of classic Piagetian tasks (see Piaget & Szeminska, 1960)
Infinity	Possibility of endless subdivision of a line segment (see Piaget & Inhelder, 1956; Fischbein, Tirosh, & Hess, 1979)
The counting complex	Subitising, counting-all, counting-on, skip counting (Fuson, 1988; Gelman & Gallistel, 1978; Sophian, 1987; Steffe et al., 1983)
Combinatorial thinking	If Mary has 2 different shirts and 3 different skirts, how many different outfits could she make (English, 2006)?
Trial and Error	Solving spatial puzzles and jigsaws.
Visualising	Mental re-presentation of spatial patterns, action sequences.
Imagining	Anticipation of event outcomes based on prior experiences.
Representing	Recording events graphically, tallying.
Naming	Development of a mathematics lexicon.

What Then is Big Play?

Big play is a child's self-motivated and self-directed activity that – alone or with other children – features embryonic mathematical thinking, which, in the estimation of the astute teacher or carer, may present an opportunity for conversation, discussion, a question, or just observation and recording for later investigation (Perry & Dockett, 2007). As Van Oers (1996) observed,

> I draw the cautious conclusion, that play activity can be a teaching/learning situation for the enhancement of mathematical thinking in children, provided that the teacher is able to seize on the teaching opportunities in an adequate way. To what extent this approach also leads to lasting learning results in all pupils is an issue for further study (p. 71).

Final Comment

The view of some, that young children's natural propensity to play and mathematics learning are irreconcilable, is surely invalid. Young children, as do carers, live in a world of relationships. Mathematics is the science of relationships, so is inherent in play. Our challenge is to identify those play occasions where overt intervention may stimulate and extend children's mathematical thinking and problem solving skills, and to consider the nature of such interventions.

References

Björklund, C. (2008). Toddlers' opportunities to learn mathematics. *International Journal of Early Childhood, 40*(1), 81-95.

Bruner, J. (1986). *Actual minds: Possible worlds*. Cambridge, MA: Harvard University Press.

Davis, G., & Hunting, R. P. (1990). Spontaneous partitioning: Pre-schoolers and discrete items. *Educational Studies in Mathematics, 21*, 367-374.

Davis, G. E., & Pitkethly, A. (1990). Cognitive aspects of sharing. *Journal for Research in Mathematics Education, 21*(2), 145-153.

Edo, M., Planas, N., & Badillo, E. (2009). Mathematical learning in a context of play. *European Early Childhood Education Research Journal, 17(3), 325-341* (retrieved 31-1-2010 from http://dx.doi.org/10.1080/13502930903101537).

Elstrodt, J. (2007). The Life and Work of Gustav Lejeune Dirichlet (1805–1859). *Clay Mathematics Proceedings, 7* (retrieved 31-1-2010 from www.uni-math.gwdg.de/tschinkel/gauss-dirichlet/elstrodt-new.pdf).

English, L. (2005). Combinatorics and the development of children's combinatorial reasoning. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 121-141). New York: Springer.

Fein, G. G. (1981). Pretend play in childhood: An integrative review. *Child Development, 52*, 1095-1118.

Fischbein, E., Tirosh, D., & Hess, P. (1979). The intuition of infinity. *Educational Studies in Mathematics, 10*(1), 3-40.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Gelman, R., & Gallistel, C.R. (1978). *The child's understanding of number*. Cambridge: Harvard University Press.

Griffith, R. (1994). Mathematics and play. In J. R. Moyles (Ed.), *The excellence of play* (pp. 143-157). Buckingham: Open University Press.

Hunting, R. P. (2003). Part-whole number knowledge in preschool children. *Journal of Mathematical Behavior, 22,* 217-235.

Hunting, R. P., & Davis, G. (2010). *What are the big ideas of school mathematics?* In preparation.

Inhelder, B., & Piaget, J. (1964). *The early growth of logic in the child: Classification and seriation*. New York: Humanities Press.

Keizer, B. A. (1983). *The role of play and direct instruction in problem solving in 3 to 5 year olds*. Masters thesis, Simon Fraser University (retrieved 4-2-2010 from http://*ir.lib.sfu.ca/bitstream/1892/7807/1/ b16673219.pdf).*

National Association for the Education of Young Children (NAEYC) and the National Council for Teachers of Mathematics (NCTM). (2002*). Early childhood mathematics: Promoting good beginnings: A joint position statement.* Reston, VA, and Washington, D.C.: Authors.

Perry, B., & Dockett, S. (2007). *Play and mathematics.* Adelaide: Australian Association of Mathematics Teachers (retrieved 1-2-2010 from http://www.aamt.edu.au/Documentation/Statements/Early-Childhood-Mathematics-support-paper-Play).

Piaget, J. (1954). *The construction of reality in the child*. New York: Basic Books.

Piaget, J. (1962). *Play, dreams and imitation in childhood* (trans. C. Gattegno & F. M. Hodgson). London: Routledge & Kegan Paul.

Piaget, J. (1974). *Understanding causality.* New York: W**.** W**.** Norton.

Piaget, J., & Inhelder, B. (1956). *The child's conception of space* (trans. F. J. Langdon & J. L. Lunzer). London: Routledge and Kegan Paul.

Piaget, J., & Szeminska, A. (1960). *The child's conception of geometry* (trans. E. A. Lunzer). New York: Basic Books.

Schuler, J. (2001). Symmetry and young children. *Montessori Life, 13*(2), 42-48.

Smilansky, S. (1968). *The effects of socio-dramatic play on disadvantaged preschool children*. New York: Wiley.

Smith, P. K. (1994). Play and the uses of play. In J. R. Moyles (Ed.), *The excellence of play* (pp. 15-26). Buckingham: Open University Press.

Sophian, C. (1987). Early developments in children's use of counting to solve quantitative problems. *Cognition and Instruction, 4,* 61-90.

Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Theory, philosophy, and application.* New York: Praegar Scientific.

Van Oers, B. (1994). Semiotic activity of young children in play: The construction and use of schematic representations. *European Early Childhood Education Research Journal, 21*, 19–33.

Van Oers, B. (1996). Are you sure? Stimulating mathematical thinking during young children's play. *European Early Childhood Education Research Journal, 41*, 71–87.

Vygotsky, L. S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.