

# Learning Mathematical Concepts Through Authentic Learning

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This paper explores the infusion of financial literacy into the Mathematics curriculum in a secondary school in Singapore. By infusing financial literacy, a core theme in the 21st century framework, into mathematics education, this study investigated the impact of using financial literacy-rich mathematics lessons by using validated learning environment instruments. This study is part of a larger study to design, monitor, and evaluate an innovative pedagogical approach of using authentic financial literacy examples to reposition mathematics education in schools.

The authentic teaching of mathematics that explicitly connects mathematical concepts, skills, and strategies to purposeful, relevant, and meaningful contexts, therefore promoting a deeper level of understanding in the classroom, is not a new concept. This has been espoused by various national and international organisations and standards, such as the National Council of Teachers of Mathematics (2000) that highlights the aim towards standards that promote understanding and use of mathematics in everyday life and in the workplace. The Programme for International Student Assessment (PISA) mathematics literacy test assessed students' abilities to apply their mathematical content knowledge and skills to a broad range of real-world problems (OECD, 2007). Meanwhile, one of the cognitive domains for the Trends in International Mathematics and Science Study (TIMSS) comprises reasoning skills to solve mathematical problems set in real life contexts (National Center for Educational Statistics, 2009).

## Rationale

The report released by the National Mathematics Advisory Panel (2008) advocated the use of real-world contexts to introduce mathematical ideas. The study discussed preliminary conclusions that for certain populations and for specific domains of mathematics such as fractions, basic equations, and functions, lesson instruction featuring real-world contexts has had positive impact on certain types of problem solving. Another area proposed for authentic mathematics teaching is in financial literacy (FL), which is a core theme of the 21st Century Skills framework (Partnership for 21st Century Skills, 2009). Core concepts of FL such as budgeting, saving, spending, and investing are closely linked with basic mathematical skills as the foundation (Dworsky, 2009; Lipsman, 2004; Lutz, 1999; Roy Morgan Research, 2003; Worthington, 2006). This has, for example, been recognised and developed by the U.S. Department of the Treasury together with the Midwestern University into the "Money Math: Lessons for Life" curriculum. The curriculum, which uses real-world personal financial scenarios to teach mathematical concepts and basic finance to students, is explicitly correlated with the knowledge and skills set forth by the National Council of Teachers of Mathematics (NCTM). (Financial Literacy and Education Commission, 2006)

The importance and relevance of youth FL is becoming ever more critical as both the spending potential and access of young people increases with the increase of affluence in society, such as among Singaporean youth. Fox, Bartholomae, and Lee (2005) noted the importance of one's understanding and knowledge of financial concepts in effective



consumer financial decision making. Most spending and saving habits are developed at an early age and good financial education, beginning even as early as kindergarten, is still seen as the best way to develop this important life skill (Mandell, 2007). According to studies by the National Endowment for Financial Education (NEFE), the most effective financial education comes at critical teachable moments in a person's life (Beck & Neiser, 2009), which occur when a person is motivated by a life circumstance or decision-making event. Understanding this, our study explores the authentic infusion of FL into the Mathematics curriculum of a secondary school in Singapore at the stage of the students' lives when managing personal finance is growing in importance.

## Methodology

Authentic learning of mathematics through real-world concepts related to FL was carried out with a group of students from middle-class families in a secondary school in Singapore over a series of six one-hour lessons. Real-world examples related to taxation, foreign exchange, hire purchase, profit and loss, interest rates, and utility bills were introduced as scenarios for discussion and problem-solving of the mathematics questions. These were infused into the lessons with the teacher still covering the standard syllabus content. The effectiveness of the authentic learning of mathematics in the financial literacy-rich mathematics (FLM) classroom environment was investigated. Quantitative research methods using validated learning environment and FL instruments were used to investigate students' perceptions of the classroom learning environment and FL. Focus group discussions provided the qualitative data to illuminate and triangulate the quantitative findings.

### *Quantitative Data-Collection Instrument*

School students' perceptions of their classroom environments were assessed using the Constructivist Learning Environment Survey (CLES) (Taylor, Dawson, & Fraser, 1995; Taylor, Fraser, & Fisher, 1997), which assesses Personal Relevance, Uncertainty, Critical Voice, Shared Control, and Student Negotiation. The CLES was selected because of its ability to characterise specific dimensions of the constructivist learning environment and thus measure students' perceptions of the perceived (actual) and preferred forms of the learning environment and the extent to which these constructivist approaches are present in classrooms, as created by the teachers. The modified version of the CLES has five scales and for the current study, such scales are reflective of the interest in a pedagogy that makes use of students' everyday experiences as a meaningful context for the development of their mathematical content knowledge, skills, and values. The CLES incorporates a critical theory perspective on the socio-cultural framework of the classroom learning environment (Grundy, 1987; Habermas, 1972, 1984) as the students were engaged in mathematical reasoning of the FL issues when applying what they have learnt.

The Cronbach's alpha coefficients for the CLES from past research in various countries are tabulated in Table 1.

Table 1  
*Internal Consistency (Cronbach's Alpha Coefficient) for CLES in Past Research*

Scale	Alpha Reliability Coefficient				
	Taiwan <sup>a</sup>	Australia <sup>a</sup>	Korea <sup>b</sup>	South Africa <sup>c</sup>	USA <sup>d</sup>
Personal Relevance	0.87	0.88	0.78	0.61	0.75
Uncertainty	0.83	0.76	0.79	0.60	0.74
Critical Voice	0.73	0.85	0.80	–	0.77
Shared Control	0.92	0.91	0.87	0.63	0.84
Student Negotiation	0.85	0.89	0.84	0.63	0.85

*Note.* <sup>a</sup> Aldridge, Fraser, Taylor, & Chen (2000); <sup>b</sup> Lee & Fraser (2001); <sup>c</sup> Aldridge, Fraser, & Sebela (2004) (this study used 4 CLES scales only); <sup>d</sup> Nix, Fraser, & Ledbetter (2005).

### *Sample*

The study involved a purposeful sampling (Merriam, 1998) comprising willing and chosen participants from selected classes of secondary four level students.

- 57 students from 2 classes make up the experiment group, where the classes were taught using FLM lessons.
- A control group of 39 students from 2 classes in the same school and level were taught in the same period using the conventional, whole class instruction for mathematics lessons.

### *Research Questions*

The research questions for this study are as follows:

1. Are students receiving FL-rich mathematics lessons showing a more positive attitude and perception towards their mathematics classroom learning environment as compared to students receiving traditional mathematics only instructions?
2. Are students receiving FL-rich mathematics lessons outperforming students receiving traditional mathematics only instructions in FL scores?

## Results

### *Quantitative results*

All students responded to the CLES questionnaire post-intervention. Students also answered a FL questionnaire designed to measure their attitude toward FL and toward the FL Programme. Quantitative analyses of the results are shown in Table 2 and Table 3.

Table 2

*CLES questionnaire: Cronbach's Alpha, Mean, Standard Deviation, and T Values of Students' Perception toward Their Actual Course Learning Environment (N=96)*

Scale	Number of Items	Cronbach's Alpha		Group 1 (N=57)		Group 2 (N=39)		T value
		Actual	Preferred	Mean	SD	Mean	SD	
Personal Relevance	4	0.73	0.71	3.71	0.67	3.73	0.64	-0.18
Uncertainty	4	0.71	0.73	3.88	0.83	3.87	0.69	0.02
Critical Voice	4	0.83	0.77	3.45	1.05	3.80	0.86	-1.78
Shared Control	4	0.83	0.89	3.14	1.13	3.66	0.79	-2.48*
Student Negotiation	4	0.85	0.84	3.30	0.96	3.60	0.73	-1.76

*Note.* Group 1 = Experiment group. Group 2 = Control group. From Table 2, the CLES questionnaire is a robust instrument as it has reliability Cronbach's alpha ranking from 0.71 to 0.89. \*Significant at  $p=0.05$  level.

Table 3

*Attitudes towards Maths, FL and FL Programme: Cronbach's Alpha, Mean, Standard Deviation, and T Values of Students' Perception (N=96)*

Scale	Number of Items	Cronbach's Alpha	Group 1 (N=57)		Group 2 (N=39)		T value
			Mean	SD	Mean	SD	
Attitude Towards Maths	8	0.92	3.88	0.74	4.00	0.75	-0.79
Attitude Towards FL	12	0.64	4.15	0.32	3.96	0.53	0.66
Attitude Towards FL Programme	12	0.86	3.55	0.66	3.54	0.66	2.02*

*Note.* Group 1 = Experiment group. Group 2 = Control group. The Attitude towards FL and FL programme questionnaires and Math test scores show adequate reliability (Cronbach's alpha ranked from 0.64 to 0.92). \*Significant at  $p=0.05$  level.

From Table 2, there was no significant difference between both groups' perception of their classroom learning environment as indicated by the T values for all scales, except on Shared Control. Students in the experiment group have less favourable view on Shared control as compared to their counterparts in the control group. From Table 3, comparison between the two groups' perception indicated that the experiment group students had a lower score on their attitudes towards mathematics. However, this difference was not statistically significant. The difference between the experiment and control groups could be attributed to the fact that the experiment group comprised an academically lower ability group based on streaming of the previous years' school examination results.

On the other hand, the students in the experiment group have more positive views towards FL and FL Programme as compared to the control group students who are of the higher-ability group. More importantly, the difference is statistically significant on the

Attitude toward FL Programme scale. Hence, we can assert that the FL programme may have positive impact on students' attitude.

### *Qualitative results*

Pre- and post- intervention focus group discussions were conducted with 10 selected students from the experiment group in 2 separate groups of 5 students each.

At the stage of the pre-intervention focus group discussion, the students had already been informed by the teacher that FL concepts were to be introduced into their lessons, and they were looking forward to them. Generally, the students already showed a good interest in and enjoyed learning Mathematics. The students also demonstrated awareness of their classroom learning environment by being able to provide practical suggestions of how the lessons could be made more exciting and relevant, such as through the use of technology, group work, games, or project work. A greater interest in the subject was equated with better grades. Acknowledging therefore the importance of application to reinforce knowledge and understanding of the concepts, the students were supportive of the idea for introduction of real-world examples in their classroom as they believed that this would also make the lessons more interesting.

Before the FLM intervention, the students only had a vague understanding of FL as “something about finance” or “something related to the economy”. FL was seen as important for “some people” or for the “needy people”. Many of the students believed that FL should be a separate topic to be learnt and not to be infused into the existing mathematics lessons as this might make the learning and especially preparation for the upcoming national level examinations more stressful with the additional content. The relationship between mathematics and FL did not seem clear to some students. While some could not as yet see opportunities for application of the concepts in their student life, they nevertheless agreed that these were useful.

After the FLM intervention, the students demonstrated an increased awareness of financial literacy, being able to give more descriptive explanations of their understanding.

S1: Initially, financial literacy to me is just about saving money, but I think now, financial literacy is about planning your budget and knowing the difference between a need and a want.

S2: Financial literacy is about how you save your money and how you use and manage your money. It is also about how to look at profits, losses, and cost prices.

In general, they did not face much difficulty in understanding the content of the FLM lessons. Interestingly, the infusion of FL concepts was done so subtly that some students did not even perceive any difference in the mathematics content being taught as it was still according to the textbook syllabus, albeit using real-world examples related to finances as the scenarios for problem-solving. Both a present and future awareness of the utilisation of the mathematics and FL concepts learnt were evidenced during the discussion.

S3: I have learnt how to calculate bills and how to reduce bills so that we can save money.

S4: I know money will grow when I put it in the bank due to compound interest.

S5: I learnt how to see your profits and losses like when you make an investment, you must know when to withdraw your money.

The students demonstrated their FL budgeting skills when given two separate planning scenarios during the focus group discussion. Some even made reference back to their classroom learning, for example, applying what they had learnt in their lessons about

utility bills in planning their household budget, and about comparison of prices and discounts in their use of the class budget to buy gift items.

## Summary

This study is part of a larger study to design, monitor, and evaluate an innovative pedagogical approach of using authentic examples to reposition mathematics education in schools. From this study, it is clearly evident that students receiving FLM lessons outperformed participants receiving “traditional mathematics only” instructions in their attitude towards FL, showing both a higher FL score and also demonstrating knowledge and application of FL concepts in their daily-life. Therefore, teachers may want to rethink the way mathematics lessons are taught and replicate this study to evaluate the success of using day-to-day real-life FLM intervention in Mathematics lessons.

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