

# *Scratching Below the Surface: Mathematics through an Alternative Digital Lens?*

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A key element in the examination of how students process mathematics through digital technologies is considering the ways that digital pedagogical media might influence the learning process. How might students' understanding emerge through engagement in a digital-learning environment? Interactive software that has cross-curricula implications and facilitates thinking in rich, problem-solving environments is emerging. *Scratch*, a free-to-download graphical programming environment provides opportunities for creative problem solving. This paper is part of an on-going study into the ways mathematical learning evolves through these alternative environments. It reports on a pilot research study involving 10-year-old children using *Scratch* to create mathematical digital learning objects, including games, and examines the ways mathematical thinking was facilitated through this process.

Processing mathematical activity through a digital pedagogical medium frames the nature of the engagement in a distinctive manner, with the understanding that emerges fashioned in alternative ways (Calder, 2008; Keiren & Drijvers, 2006). Building on earlier research involving students processing mathematical tasks with spreadsheets (Calder, 2008), this paper utilises an interpretive lens to examine the manner in which mathematical thinking emerges when children work with *Scratch*, an interactive, programming language. *Scratch* is a media-rich digital environment that utilises a building block command structure to manipulate graphic, audio, and video aspects (Peppler & Kafai, 2006). It incorporates elements of Logo including 'tinkerability' in the programming process (Resnick, 2007). This allows the user to combine the programming building blocks (at times incorporating measurements) and to immediately observe the outcome of that programming. The blocks can be deconstructed and recombined as users logically develop desired movements and effects. *Scratch* facilitates creative problem solving, logical reasoning, and encourages collaboration, and students can use geometric and measurement concepts such as coordinates and the unit circle (Peppler & Kafai, 2006).

When learners engage in mathematical investigation, they interpret the task, their responses to it, and the output of their deliberations through the lens of their pre-conceptions; their emerging mathematical discourse in that perceived area. Social and cultural experiences always condition our situation (Gallagher, 1992), and thus the perspective from which our interpretations are made. Learners enter such engagement with pre-conceptions of both the mathematics, and the pedagogical medium through which it is encountered. Their understandings are influenced by a variety of cultural forms, with particular pedagogical media seen as cultural forms that model different ways of knowing (Povey, 1997). The engagement with the task likewise alters the learner's conceptualisation, which then allows the learner to re-engage with the task from a fresh perspective. This cyclical process of interpretation, engagement, reflection and re-interpretation continues until there is some perceived reconciled interpretation of the situation. Other researchers have likewise perceived learning emerging through digital environments by an iterative process of re-engagements of collectives of learners, media, and other environmental aspects, with the mathematical phenomena (Borba & Villareal,



2005). In essence, the mathematical task, the pedagogical medium, the pre-conceptions of the learners, and the dialogue evoked are inextricably linked. It is from their relationship with the learner that understanding emerges. This understanding is their interpretation of the situation through those various filters (Calder, 2008).

When learners investigate in a digital environment, some input, borne of the students' engagement with, or reflection on the task, is entered. The subsequent output is produced visually, almost instantaneously (Calder, 2009) and can initiate dialogue and reflection. This will lead to a repositioning of their perspective, even if only slight, and they re-engage with the task. They engage in an iterative process, alternatively attending to the task and their emerging understanding. This allows for a type of learning trajectory that can occur in various media (Gallagher, 1992), but is evident in many learning situations that involve a digital pedagogical medium (Borba & Villareal, 2005). There are, however, affordances of the digital medium associated with the process that influence the nature of the engagement (Calder, 2008). These affordances frame the nature of the problem-solving activity. This paper considers two forms of mathematical thinking that emerged when the participants created mathematical games to facilitate understanding of number concepts with their younger, 'buddy' class. One is the evolution of logic and reasoning that developed through the creative problem solving during the programming process, while the other involves the conceptual area of geometry. We examine the first through an iterative, interpretive process, to see how the children's mathematical thinking evolved as the groups created, and then refined their games. We also consider the geometric thinking interspersed through the process as the children transformed their 'sprites' (animated figures), including moving them to specific locations.

## Approach

This paper continues an ongoing examination of how digital pedagogical media influence the learning process in mathematics. Specifically, it reports on a pilot research project involving a digital-learning class of 26, Year 6 children. Students had access to their own computer and although this was their first experience with *Scratch*, they were confident and experienced with a range of software. Their teacher was the school's ICT coordinator. The students worked in pairs, which were self-selected and single gender. Over the two-week research period, the students wrote daily blogs articulating their progress and reflections, students and the teacher were interviewed, and classroom observations (both written and photographic) were recorded. These data, along with informal observation and discussion formed the data, which were then systematically analysed.

The first week involved the students doing a range of distinct, structured tasks to familiarize them with the *Scratch* environment. All groups were given the same design brief: To design and build a mathematics game suitable for facilitating the number understanding of their Year 1 'buddies'. The students interviewed their Year 1 'buddy' class partners and consulted the Year 1 teacher regarding appropriate mathematics concepts and activities with which the class was familiar. This helped determine the nature of games they would devise. The younger children also gave formative feedback on the games during the development process.

A feature of the approach taken by the teacher was the sharing of the work that had been done each day. Each project was loaded on to a data stick near the end of the session and one student took responsibility to coordinate displaying the work on the data projector. Each group would explain what was being done and any characteristics of their

programming. The other students could ask questions and provide feedback and suggestions. The students' respect for each other and confidence with this process was a feature of the classroom culture and clearly had been engendered before the project took place. The feedback session also gave opportunities for the teacher to formatively assess, to identify aspects that might need individual or whole-class feedback, and for students to identify other class members who could assist them with aspects of their design problems. The projects in these varying evolving stages and the accompanying feedback likewise became part of the data.

## Results and Discussion

### *Problem-solving in Scratch*

While the design brief was set within a mathematics context, a central element of the thinking that took place was in the area of problem solving. The students familiarised themselves with the task and then through iterations of action and reflection modified their game. At each juncture, the feedback to their engagement with the task modified their approach and enabled them to re-engage from a fresh perspective. Thus their thinking evolved and the games became more refined as they reset their investigative sub-goals based on the feedback and subsequent reflection. The feedback was in various forms: immediate visual feedback within the programme as they changed their programming script; fellow student and teacher feedback and suggestions, feedback from the intended users, and feedback involving other groups that unfolded in the public domain. Each of these varying forms of feedback led to reflection, and then re-engagement from a modified perspective.

For instance, the 'Jabadah' group began with a "stage" and explored changing the colour of it, how to move the 'sprites', and some of the pre-programmed effects. They settled on a stage colour and then experimented with moving the 'sprites' that made up the letters of their group name. They wanted to make the J hit the A and set it off spinning, but it moved in a continuous loop. The following observational data, recorded their discussion:

James: We can't get it to go forever-we'll need to explore different loops.

Don: What if we glide until it points to the direction?

James: We can point towards.

Don: What about exploring the use of "sense?"

They tried a few options and considered the visual feedback resulting from each change in the coding. They were developing a sense of the relationship between the programming script they had selected and modified the measurements of, and the associated movement of the 'sprite' on the screen. The next day they continued this relational experimentation by "using existing scripts to see how to manipulate things differently". During this process of experimentation they worked out how to design and operate a spinner. When they reached a point of uncertainty they used a 'predict and check' approach, reflected on the outcome, before refining their evolving script. This involved further relational thinking, as recorded in the written observations, they "looked at how the different scripts affected the action of the sprites" and "experimented with the number scripts in their own project by putting in variables and then running the script to see what would happen." This illustrated a development in their relational thinking as they became more effective at predicting the outcome of their changes to the programming script.

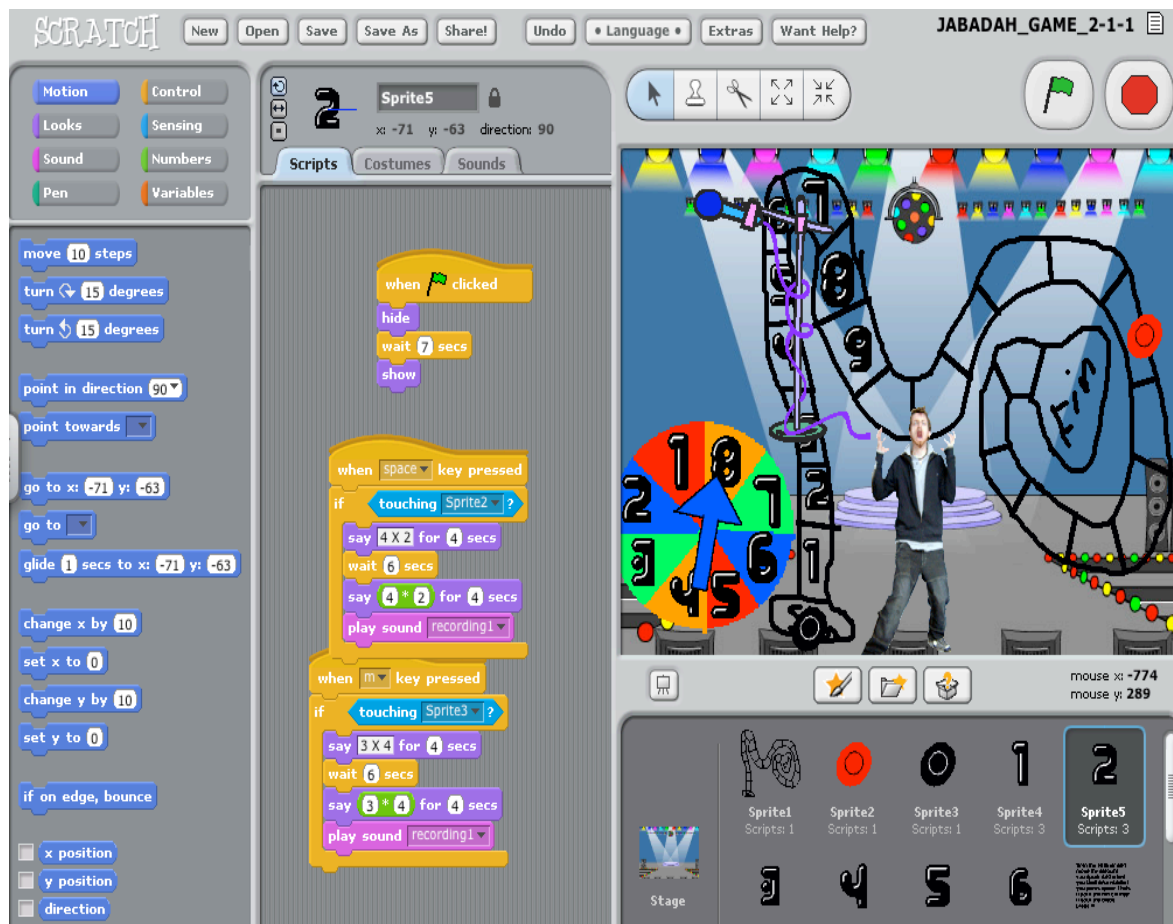


Figure 1: The group ‘Jabadah’s workspace as their game is in development.

While the spinner was now operating successfully, they had encountered another problem. Although they were able to move the blue and red counters on their board game whilst *Scratch* was in design mode, they had not been able to move the counters in full screen mode. They experimented with other scripts line by line. Eventually, through evaluation of the feedback to their input, they were able to achieve this aspect. With each engagement they reflected on the digital feedback, modified their interpretation of the situation, and re-engaged with the task from this modified perspective. Their thinking evolved through the problem-solving process. As well as the relational thinking, they also used logic and reasoning to evaluate and interpret the situation, before resetting their sub-goal in the investigative process. They generalised from a range of actions and after reflection, determined the type of command that produced the desired effect. They also responded to other feedback:

MF: How does the code work, tell me what that code means?

James: It just spins randomly and lands on a random place.

Although the question hasn’t been answered in the detail intended, the student nevertheless has reflected on the question and articulated their response in terms of both language of movement (spins, lands) and chance (random). The children also articulated the movement of the spin in mathematical language, which they understood, even though the script was modified rather than created:

Don: All we did was go: “When sprite 15 clicked repeat random 3 to 100 and turn 45 degrees.”

This indicated development in the children’s understanding of rotation and the link between the numerical size and the movement of the turn. In the interviews, they articulated the value of the ‘buddy’ feedback and how they responded to it by adapting the game.

James: They said it was fun. They thought the spinner was cool.

Don: We changed the questions from multiplication to addition because it was too hard for them.

Eventually, they were satisfied with the game and the way it operated. Data from other groups also highlighted the way *Scratch* facilitated problem solving. For instance, when Geoff had run into a problem with the logic of the scoreboard of their game:

Geoff: I’ll need to problem solve that.

He then investigated spacing, proportion, colour, and size aspects of the scoreboard. The challenge of the problem-solving process was evident in the blog from another group:

We are trying to figure out how to use a gravity effect and how to use the variables. We are finding it challenging to make our character Jetman jump in the air without spinning 15 degrees.

The teacher also discussed problem solving in her final interview. She talked about some of the benefits:

The communication and competencies coming through with the use of it. That whole problem solving and questioning (aspects). So the whole thing of exploratory learning was where it was a very valuable bit of software.

She further stated that two of the benefits for the children using the programme were in problem solving and mathematics.

### *Further Thinking in Geometry and Measurement*

Instances of thinking in geometry and measurement emerged during the iterative, hermeneutic process through which ‘Jabadah’s’ game evolved. The trialling of variations of movement, angle size and coordinates, and linking these to the instantaneous effects would have enriched their understanding of these aspects. There was also evidence of geometric thinking from other groups. These are recorded as different snippets rather than being situated within each group’s overall process.

The ‘Jigsaw’ group explored changing the length of time for repeat movements and varying angle sizes. They later articulated their attempt to make the letters glide into place, eventually figuring out how to use coordinates to specify where the ‘sprite’ was to glide to and how to keep the characters in place. The ‘Mats’ group likewise aimed to explore animation and movement. They worked out how to use the glide command and x- and y-coordinates to move to different positions on their stage. From the interview data:

Stan: We have to remember where the numbers (their ‘sprites’ for the game they were devising) go, so they all move to the middle and then they mix around to different places.

Matiu: We want to put the numbers in position.

Later, they applied this learnt skill in their game to moving asteroids through space. They were observed manoeuvring a spaceship and dodging spinning asteroids. Interestingly, at another point, when writing the script for their ‘sprite’ to move they initially recorded: “turn 90 degrees, wait one second” 10 times, rather than using a more efficient: “repeat 10 times command.”

After choosing a stage and 'sprite', another group, XE2, were observed immediately engaging with movement and the positioning of their 'sprite'. This involved the use of coordinates to indicate the position they wanted the 'sprite' to glide to. They were not concerned with the exact position of the coordinates, but more the general position associated with them. They spent time exploring different coordinates and how this affected the position of the 'sprite', gaining a sense of the relationship between the values of the coordinate and the position on the screen. They also programmed 'wait time' of 5 seconds and 'hide time' of 6 seconds. 'PC' also experimented with time, and what the interval signified, when creating their game. They formulated a programme that offered simple addition equations such as '7 + 4 = ' and the 'buddy' children needed to match the solution to the appropriate number of aliens.

Peter: If you get this right, it tells you, and then it changes to the next question in fifteen seconds.

In the 'Hemzie' group blog, data indicates that they had marked plots on a pencil-and-paper map they had made to help them work out how to move the 'sprite' from one place to another. Their aim was to explore 'sprites' and how to change from one 'sprite' to the next. They thought this aspect was challenging. Later they worked out the movement effects they required and were incorporating sounds. They recorded and linked an appropriated sound for each movement. They eventually enabled a car 'sprite' to be moved by inputted commands.

Brian: The reward is that you get to steer the car around for 20 seconds.

The 'Lissa' group also had initial difficulty with the movement, but learnt from the feedback sessions. They were eventually able to have a beach ball move around the screen through a maze, controlled by the keyboard arrows.

The 'Pig' group explored similar areas but with an additional transformation. They wrote in their blog:

We have learnt how to move letters and characters by programming a key on the keyboard to move an object. We learnt that if you use a text box you can't make an animation with effects, it will just enlarge your 'sprite'.

They articulated that their initial aim was to find out about position and effects. They were exploring movement and angles.

While engaged in the programming experiences, *Scratch* appeared to facilitate the children's understanding of angles and measurement, with experimentation enabling them to find what was appropriate to use in their particular context. Errors with programming appeared to have a positive effect in that they prompted the children to willingly experiment with commands to achieve the desired appearance and effects. The 'tinkerability' of *Scratch* facilitated exploration with angles and the measurement of time and length. Students could actively experiment with angle size, for example, in ways that would not be possible without the digital medium. Likewise, the understanding that emerged regarding coordinates was inherent in the process of exploring the movement and position of the 'sprites'. Clements, Sarama, Yelland, and Glass (2008) discussed how game contexts and practice can significantly improve spatial performance. The study not only involved participants with spatial movement and location while designing the games, the trialling and modification process would also have influenced the children's spatial awareness.

## Conclusion

*Scratch* software proved to be an engaging and relatively easy to use space for problem solving. Additionally, it proved to be an effective medium for encouraging communication and collaboration (Otrell-Cass, Forret, & Taylor, 2009). Each of the above episodes illustrated how *Scratch* provided a worthwhile and motivating programming environment to explore some mathematical ideas. The challenge of creating a mathematical activity or game for younger students overtly positioned the programme in mathematics, while implicitly, it simultaneously demanded that mathematical ideas be utilised to develop their game. What is not quite so certain is the extent to which new mathematical learning occurred during this process. The students in this digital class were quickly able to access and understand the programming capabilities and used mathematical thinking in their approach to problem solving. In the classroom, where electronic media and an environment where discussion and sharing were the norm, the students were able to transform their ideas into workable programmes. It proved to be a medium whereby programmes were easily composed and decomposed, thus encouraging the use of critical, meta-cognitive and reflective skills. *Scratch* was also intrinsically motivating. The sharing sessions were pivotal in that they provided a forum for displaying work, and as a way of collectively helping each other to solve programming problems. *Scratch* therefore, provided an opportunity for students to develop their thinking, a key competency that is integral to the *New Zealand Curriculum* (Ministry of Education, 2007). The facilitation of logical thinking from initial empirical concepts, as students test ideas in response to feedback, and the influence of programme feedback in the evolution of students' geometric ideas has been reported elsewhere (e.g., Clements et al., 2008).

While not specifically designed to facilitate conceptual thinking in a particular mathematical area, there were clear indications of the children engaging with mathematical ideas and to some extent enhancing aspects of their mathematical thinking through the use of *Scratch* in the development of the digital learning objects. Their spatial awareness, understanding of angles, and positioning sense through the use of coordinates, were all engaged to varying degrees. There was also evidence of relational thinking as the children made links between their input, the actions that occurred on screen, and the effect of specific variations of size and occurrence of single or iterative procedures. However, the process the participants undertook more directly facilitated mathematical thinking through the creative problem-solving process it evoked, and the development of logic and reasoning as they responded to the various forms of feedback. These mathematical conclusions can nevertheless only be tentative. While consideration of mathematical thinking was one intention of the research study, it was predominantly set up as an open investigation into the potential of the software across a range of learning areas. A more focussed study on the mathematical learning implications may have been more productive in the revealing of mathematical thinking, and may have reached less tentative conclusions.

## Acknowledgement

We wish to acknowledge Dr Mike Forret, Dr Kathrin Otrell-Cass, and Sheena Saunders, who were part of the *Scratch* research team.

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