# Utilitarian Model of Measuring Confidence within Knowledge-based Societies

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## **Utilitarian Model of Confidence Testing for Knowledge-based Societies**

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This paper introduces a utilitarian confidence testing statistic called Risk Inclination Model (RIM) which indexes all possible confidence wagering combinations within the confines of a defined symmetrically point-balanced test environment. This paper presents the theoretical underpinnings, a formal derivation, a hypothetical application, and published results of using RIM during mathematics and science testing. This paper advocates the position that such a model provides researchers and policy makers a new statistic to measure an examinee's reflective confidence toward his/her knowledge during testing. Adding RIM to the reflection cluster of a global assessment framework like OCED/PISA could enhance future self-confidence assessments within knowledge-based societies.

Keywords: assessment, confidence testing, knowledge-based economy, reflective knowledge testing

United Nations Educational, Social and Scientific Organization (UNESCO) was established in 1947 to promote growth of international contacts, organizations and achievements (Huxley, 1947). Huxley posed that the task before UNESCO was the emergence of a single world culture with its own philosophy and background of ideas which could meet the minimum physical welfare of humanity. Current issues of global terrorism, dwindling world food supplies, global climate change, discoveries of new infectious diseases, and national economic imbalances threaten such physical wellbeing. To fight against such menaces a single world culture with a knowledge-based economy has been disseminated and actively advocated. The theoretical base of this economy is the New Growth Theory (Johnston, 1996; Taniuchi, 2000). Cortright (2001) states this theory incorporates two components: technology and knowledge. Unlike the old theoretical model which focused upon physical objects that had the potential for "diminishing returns", the New Growth model focuses upon knowledge and technology as infinitely shared and reused with the potential of propelling economic growth. Cortright goes on to say since this theory underscores the importance of investing in new knowledge creation to sustain growth, policy makers need to give careful consideration to all factors that provide incentives for knowledge creation i.e. research and development and the education system (Cortright, 2001; Taniuchi, 2000).

d'Orville (2007) stated the critical catalysts of a country's adaptability and economic and social development was dependent upon the quality of higher education. Such education determined scientific discovery, innovation and future academic exploration. To maintain a competitive edge globally, policymakers, business leaders and universities needed to aggressively seek out ways to create comprehensive learning and discovery environments that were multilateral and international. d'Orville stated these knowledge societies focused upon global knowledge exchange, networking, policy and advocacy. Tapper and Palfreyman (2004) stated the increasingly global character of such higher institutions within an internationally competitive market needed to be redefine so as to serve the interests of society at large and especially its economic interests. The driving factor of future economic success of any nation will be the production and utilization of knowledge and technology in the global competitive market. These products can be infinitely shared and reused and are not dependent upon a country's size, location, and natural resources. Chung (2004) warned with the existence of knowledge societies, there was the potential danger of losing a nations cultural capital i.e. cultural heritage. Culturally-rich communities, if left unprotected, could be pressured to abandon traditional cultural forms. To protest against the loss of cultural capital, d'Orville (2007) suggests the emergence of knowledge societies needed to be monitored and guided by higher levels of education. Such institutions would be responsible for not only to the production, transmission and upgrading of knowledge but also educating its citizens on how to confront differing viewpoints while maintaining its own cultural heritage. Hence, these institutions would not only seek to reduce the "digital divide"; but protect against the loss of cultural capital and seek to reduce the "knowledge divide" among members of knowledge-based societies.

#### Objective of this paper

The objective of this paper is to introduce a utilitarian model for confidence testing that can be used by researchers within knowledge-based societies. This model uses confidence wagering to assess the confidence an examinee has toward reflective knowledge during testing. Such wagering allows the participant in a non-imposed etic structure to make honest evaluations regarding the accuracy of an answer selection. Such self-regulation evaluations are indicated by the desired amount of points the participant feels his/her answer response is worth. This utilitarian model called Risk Inclination Model (RIM) indexes all possible confidence wagering combinations from risk seeking to risk aversion within the confines of a symmetrically point-balanced test environment.

## **Theoretical framework**

The theoretical position of this paper proposes the balancing of wager points during testing can reveal an examinee's willingness to differentiate between what he knows and what he does not. Brewer and Sampaio (2006) stated as the time between a memory recognition task and the time of confidence judgment regarding that task increased, the accuracy of memory was negatively affected. This paper promotes the position that the confidence a participant has toward an answer selection and the wagering of points determined by the participant for that selection be treated as the same variable.

#### Theoretical underpinnings of RIM

Risk Inclination Model (RIM) uses the Principle of Moments or Varignon's Theorem to calculate the nth factorial moment of probability among the distribution of risk wagers within a test set. Varignon's Theorem states, "The moment caused by the resulting force about some arbitrary point is equal to the sum of the moments of the system." In physics, the first factorial moment (labeled as M) is known as Center of Mass. M represents the balance point among the distribution of examinee wagers. A flat or un-weighted distribution which represents a zero risk or risk aversion gives a 1st moment (labeled M<sub>0</sub>) as schematically plotted in Figure 1 (A).

A deviation made to one item must be counter balanced by another item in the test set with a given total wager, as plotted in Figure 1 (A) through (F). Figure 1 shows the possible combinations of risk a student could make within a 100-point parameter using three risk values: 5, 10, or 15. Using RIM, a Risk Inclination Index (RII) can be constructed reflecting the possible combinations of risk inclination an examinee could exhibit at the moment of answer selection.

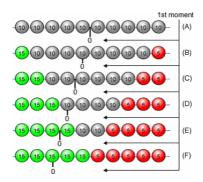


FIGURE 1. Position of 1<sup>st</sup> moments

## Formal Derivation of RIM

The following formal derivation of RIM is divided into three sections of calculation: 1) 1<sup>st</sup> moment, 2) inclination and 3) risk level.

## Calculation of 1<sup>st</sup> moment

Consider a n-item test with the wager (W) made by the student for the  $i^{th}$  item in the test set. The value of the wager (W) belongs to the set {  $W_{min}$ ,  $W_{min} + 1$ , ...,  $W_{max}$ } with the middle value of

$$W_0 = (W_{max} + W_{min}) / 2. (1)$$

The middle value has to be an integer. The 1st moment of this example has the form

$$M = \sum_{i=1}^{n} W_i *_i / \sum_{i=1}^{n} W_i$$
 (2)

The denominator is a constant for a given Total Wager (TW) in the test set. The TW of Figure 1 is 100. The 1<sup>st</sup> moment of Figure 1(A) i.e. zero risk or risk aversion is calculated as

$$M_0 = \sum_{i=1}^{n} W_0 * i / TW.$$
 (3)

Therefore, the  $1^{st}$  moment of risk aversion is equal to  $M_0$ .

## Calculation of inclination

Any  $1^{st}$  moment (M) deviation away from risk aversion (M<sub>0</sub>) is termed as inclination (I) and is calculated as

$$I = \sum_{i=1}^{n} (W_i - W_0) *_i / TW.$$
 (4)

The maximum inclination of wagers are calculated as

$$I_{\text{max}} = \left\{ \sum_{i=1}^{n/2} (W_0 - W_{\text{max}}) *i + \sum_{n/2+1}^{n} (W_0 - W_{\text{min}}) *i \right\} / TW. \tag{5}$$

#### Calculation of risk level

The student's level of risk is calculated as

$$R = (I/I_{max})*100\% . (6)$$

## Hypothetical application of RIM

The following wagers (5, 15, 5, 15, 15, 15, 15, 5, 5) represent a hypothetical multiple-choice test set. These wagers ( $W_i$ ) are redistributed in descending order: 15, 15, 15, 15, 15, 5, 5, 5, 5, 5, 5, 5. W<sub>max</sub> (i.e. student's maximum wager) is assigned 15, and the  $W_{min}$  (i.e. student's minimum wager) is assigned 5. Equation (1) is used to calculate the middle point ( $W_0$ ) of wagers. The  $W_0$  is calculated to have a value of 10. Equation (2) is used to estimate the sum of 1<sup>st</sup> moment values of these wagers. The Total Wagers (TW) for this hypothetical test is set to 100. Therefore, the 1<sup>st</sup> value for the redistributed wagers is M = 425/100 or M = 4.25. This value ( $M_0$ ) indicates the degree of inclination the student has in wagering that his answer is correct. This degree of inclination is in reference to  $M_0$  (i.e. risk aversion). The risk aversion ( $M_0$ ) value for the redistributed wagers is  $M_0 = 5.5$ . Therefore, the  $M_0$  (i.e. greatest point of risk aversion) is when each wager value on a 10-item test are all ten. The balls shown in Figure 1( $M_0$ ) represent such a situation. If these balls were physically connected on a rod, the Center of Mass (CM)

value zero would be located between balls 5 and 6 (see Figure 1). When statistically calculated, this value is 5.5 i.e.  $M_0$ . As the student's inclination toward risk moves away from aversion, the position of the 1<sup>st</sup> moment moves to the left, toward a higher risk value. Figure 1(F) represents the redistributed wagers in a maximum state of inclination. The zero value in this figure represents the physical CM position. The calculated M value is the statistical 1<sup>st</sup> moment position. A decreasing value results as M moves away from  $M_0$ . Calculating the movement of M for the redistributed wagers uses equation (4). The inclination (I) of the redistributed wagers is 1.25. The I value calculated for the redistributed wager example is the same at the maximum inclination ( $I_{max}$ ) value calculated for the entire test. This similarity indicates the wagers used in the redistribution example constitute the highest level of inclination toward risk-seeking behavior a student could indicate. The risk potential of this level of inclination is calculated from equation (6) as  $R = I/I_{max} *100\%$  or R = (1.25/1.25) \*100% or R = 1.

#### Conclusion

The RIM statistic is flexible and can be used with any set of risk values and Total Point value (TW). Within the confines of any symmetrically point-balanced test environment, inclination toward risk seeking or risk aversion can be mathematically calculated. Jack et al. (submitted) conducted an empirical study using the RIM statistic and found significantly evidence that showed the measurement of risk inclination during between-subject analysis can predict the accuracy of female students' retrospective confidence of their knowledge during testing. A multiple-regression analysis was conducted to determine the best linear combination of quiz score and risk inclination for predicting semester grade among female students. Correlational significance was found among three variables: 1) quiz score: scores from three quizzes using confidence wagering, 2) S. grade: the student's semester grade, and 3) RII

TABLE 1. Mean, Standard Deviations, and Intercorrelations for Risk inclination index (RII) and Predictor Variables among female students from mathematics and science classes (n=45)

Variable	M	SD	1	2
S. grade	58.60	17.98	.870***	.272*
Predictor variable				
1. quiz score	.59	.20		.330*
2. Risk inclination (RII)	.68	.28		

<sup>\*</sup>p<.05; \*\*p<.01, \*\*\*p<.001

TABLE 2. Simultaneous Multiple Regression Analysis Summary for Risk inclination, Quiz score and Semester grade (n=45)

Variable	В	SEB	β
Quiz score	75.76	6.97	.736***
Risk inclination	-1.17	5.43	203

Note.  $R^2 = .75$ ;  $F_{(2, 42)} = 64.48$ , p<.000

ratings from the three quizzes using confidence wagering (see Table 1). This combination of variables significantly predicted semester grade, F  $_{(2,42)}$  = 64.48, p<.000, with only quiz score contributing to the prediction. The beta weights, presented in Table 2, suggest that quiz scores using risk wagering contribute to predicting the student's final semester grade. The adjusted R squared value was .75. This indicated that 75% of the variance in the semester grade was explained by the model.

#### Educational importance

As seen above, the RIM statistic is a viable means of using the utility of wager to mathematically calculate the confidence and accuracy of middle school students during multiple-choice science and mathematics tests. Jack et al. (submitted) proposed adding the RIM statistic to the reflection cluster component of the OCED/PISA assessment framework could enhance future PISA assessments by evaluating if students have confidence to use their knowledge to meet real-life challenges and if such confidence is accurate. If knowledge within knowledge societies is to be valued, the participants of these societies must be confident in its

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#### **NOTES**

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