Mathematical Connections:
A Study of Effective Calculator Use in
Secondary Mathematics Classrooms

Research Paper
by

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Abstract

Mathematics teachers face the challenge of integrating calculator use in their classrooms. Calculators provide advantages for students when performing calculations and they can provide teachers with a versatile instructional tool. Students face high-stakes mathematics tests each year in middle school and must take Regents and college entrance exams during their high school career. It is important to properly integrate calculator use so that students can derive the full benefit of familiarity with the instrument while maintaining a high level of student proficiency with paper and pencil calculations. The goal of my study was to investigate how a student can best learn with the aid of a calculator. I wanted to find out the proper balance of calculator use combined with paper and pencil techniques that work together to give students enduring lessons.

Introduction

Mathematics is a challenging subject for most secondary school students. Students need to pass several high-stakes tests in math during middle school. It is necessary to pass at least one high stake math test in high school in order to graduate. Students are allowed to use calculators on portions of their middle school exam and they are allowed to use graphing calculators on their Regents exams. The big question that faces mathematics teachers is how to best utilize calculator use in the class room to promote learning.

My experience with calculators has made me aware of the issue of student over-reliance on them if they are not monitored for understanding before being allowed to use a calculator. Students need to develop an understanding of the mathematical calculations of a topic before they are allowed to use a calculator. As a college student I learned to rely on my calculator to help me through some pretty challenging math courses. The only way that I learned to use paper and pencil to solve problems was when my professor disallowed calculator use. I learned

Calculus through this monitored approach to calculator use and I still retain the knowledge of how to do most calculations. On the other hand, I took a Linear Algebra course and calculator use was not limited so I only learned how to work with this topic through a graphing calculator. I had to teach myself how to do Linear Algebra with a paper and pencil later because I have to teach it to my students, but it seems that this method would be adequate for a student who do not need to pass the subject on to others. I did get a better grade in Linear Algebra than I did in any of the three Calculus courses that I took. Does a better grade in Linear Algebra mean that unlimited calculator use is more effective? Does better retention of Calculus mean that limited calculator use is more effective?

Mathematics teachers seem to discuss this issue quite frequently. I have spoken with a math teacher who would not allow his students use calculators unless he believed that they had mastered a new idea. His students did not get calculators very often. I found it interesting that his students did not do well on the Algebra Regents; his passing rate was less than fifty percent. Students need to have access to the calculator in order to familiarize themselves with its operation. I believe that he did his student a disservice by limiting their access too severely. I student-taught at a large suburban school and students were encouraged to use calculators for everything, they did not have to understand why it worked, they were just told to push the buttons and read the answer. The students at this school have a passing rate that is much higher than fifty percent.

Somewhere in the middle there is the ideal amount of exposure to calculators for students. I believe it depends on where you are in a series of lessons but the calculator needs to be utilized both for its aid and to give students an opportunity to learn how to use it. I hope to

learn how to maximize calculator integration in my classroom while also ensuring that students acquire enduring paper and pencil computational abilities.

Literature Review

Research indicates that teachers believe that technology, especially graphing calculators, would be helpful in the mathematics classroom. Zembat (2008) found that technology "gave participants a chance to make a conjecture, an opportunity to try that conjecture with the help of dynamic features (GSP, spreadsheets) and to evaluate results." In the same study students were allowed to use calculators only after they had exhausted their pencil and paper techniques. The calculator served as a bridge to higher mathematical ideas. Students would hit a dead end with paper and pencil but the multiple representations afforded by the calculator allowed them to get further in solving the problems.

In a study done in Australia where technology has become a mandatory element of instruction, teachers were surveyed on the topic of technology use during instruction. Nearly 68% of respondents felt that it was difficult to get access to computer laboratories, and over 54% agreed that there were not enough computers available in their schools. (Goos and Bennison, 2008) Calculators can provide an opportunity to integrate technology while also being relatively cost effective. A class set of graphing calculators is about as expensive as a desktop computer but it puts technology into the hands of each student. In the same study done by Goos and Bennison (2008), it was found that a majority of teachers agreed that technology makes calculations quicker, helps students understand concepts, enables real-life applications and allows students to see the link between different representations. They also found that 46.4% of teachers agreed that technology eroded students' basic math skills, 24.9% disagreed and 26.8%

were undecided.(Goos and Bennison, 2008). It would seem then that while technology has many benefits, the surveyed teachers doubt that it is beneficial for the retention of basic math skills; paper and pencil techniques.

Graham, Headlam, Sharp, and Watson (2008) did a study on a small group of students to test how well a teacher met her expectations of graphing calculator use in her classroom. The teacher set several goals for calculator use which included raising student confidence and awareness of functionality while working with the calculator, to utilize the calculator as a display and investigative tool and to answer and check examination questions using the calculator. "Overall it can be concluded that with this small group of students the teacher's aims were generally all met to some extent" (Graham, Headlam, Sharp, and Watson, 2008). It is also worthy to note that the students who were least comfortable using the calculator were less likely to use it even when checking their answers.

Calculators are valuable instructional tools and are a necessary element in the modern mathematics classroom. Students need to use calculators frequently in order to develop confidence in the use of the machine. At what point in the learning of mathematical concepts should students be allowed to use calculators? Does calculator use have a negative impact on student acquisition of basic mathematical skills?

Methodology

Population

I conducted this study in order to determine the best use of calculators in a secondary mathematics classroom. I chose to use the students in my regular classes in order to increase the

likelihood of participation and to maintain a continuity of results. I used high school students because I did not wish to allow calculator use in my middle school math classes.

The two classes involved in my study were an Algebra 1A class and a Consumer math class. The Algebra 1A class consists of 23 ninth grade students, one tenth grade student, and one eleventh grade student; these students have shown a history of struggling in math. The Algebra 1A course represents the first half of the Regents Algebra curriculum, the students that successfully complete the course will go on to Algebra 1B and work through the second half of the Algebra curriculum. At the end of the Algebra 1B course, students are expected to take and pass the Algebra Regents exam which is a graduation requirement for high school students in New York State. Algebra is also offered as a one year course where students take the Regents exam after working through the entire curriculum in one school year. The students in the Algebra 1A course are given the opportunity to earn two math credits while working toward passing the Regents exam. The curriculum is offered at a slower pace in the hopes of providing more opportunity for students to master the skills and knowledge necessary to pass the Regents exam.

The Consumer math course is an option for students that have already taken Algebra but do not wish to take more challenging math courses. The class consists of five 10th grade students, fifteen 11th grade students and five 12th grade students; these students need math credits and are taking the course to fulfill their credit obligations. These students have not shown a particular strength in mathematics and can be described as unenthusiastic about learning math. Nine of the Consumer math students have yet to pass the Regents exam.

I chose to use these two classes because they have similar attitudes about mathematics and while the Consumer math class has older students, the ability level of the classes is very

similar when engaged in basic skills. Another factor that I considered was the topic that would be taught and assessed. Solving equations is a topic that is vital in Algebra and can be readily applied to mathematics in the consumer world. I aligned the curricula of these two classes in order to measure their growth in equation solving ability.

Procedure

I received permission from the administration in my building and then I sought volunteers from the two classes. The Algebra 1A class had 16 students who agreed to participate while the Consumer math class had nine students who volunteered to participate in the study.

The execution of my study involved giving a pretest on a topic, teaching that topic for four days and then giving a post-test. The Algebra 1A class was given access to scientific calculators and graphing calculators. During the course of this study students did not select graphing calculators; students chose instead to use scientific calculators that they are more familiar with. Scientific calculators perform mathematical operations but they will not manipulate an equation with a variable. Graphing calculators are much more complex and students were not familiar with the TI-nspire graphing calculators that are available in the classroom. The TI-nspire calculator is a fairly recent development of Texas Instruments and has a great deal of functionality in the hands of a person familiar with manipulating the menus. Graphing calculators are allowed for use during the Regents Algebra exam. Students are much more familiar with the scientific calculators because they are allowed on parts of the seventh and eighth grade New York state mathematics assessments. Students gain familiarity with scientific calculators during their middle school years and can sometimes be reluctant to attempt using a seemingly complicated instrument such as the TI-nspire. The consumer math class was not allowed access to calculators during class.

Instruments

I observed a marked difference in how the students in these two classes approached their work during this study. The consumer math students seemed very diligent and intent on their work compared to the Algebra 1A class. Students asked questions during instruction to clarify their understanding of the material; they seemed to function as a group where every student benefitted from the answer to a question. Classroom discipline is not a large issue in either class but the consumer math students were even more quiet than usual and seemed very keen to complete the class work that they were given. The Algebra 1A students were less prone to pay attention during a lesson and seemed to be unwilling to make note of methods that I was teaching. The Algebra students seemed to wait until I gave them several examples to work on before they got serious about trying to understand the concept being taught. I would have to visit individual students to answer questions or to encourage them to complete the exercises. It seemed like the Algebra students were only interested in learning a quick and easy way to solve examples using their calculators. They knew that the calculator would do most of the work as long as they could figure out how to draw the numbers from the problem in a format that fit the functions of their machine. Meaningful instruction tended to be more one-on-one with the Algebra students, much like the interaction with a calculator is one-on-one.

The first topic of instruction was solving one-step algebraic equations. An example of this type of problem is: $[x + 9 = 21]$. The student had to determine the inverse operation and apply it to both sides of the equation to solve for the variable. This simple of a problem is also solvable using the guess and check method. This type of problem is addressed in middle school math so students were mostly familiar with the methods necessary to solve for the variable. The

second topic was solving two-step equations: this involves performing two inverse operations to solve for the given variable. An example of this type of problem is: [2x + 9 = 21]. Solving this type of equation requires more care and a solid understanding of the operations necessary to correctly solve for the variable. If a student could not perform the algebra, he or she could also use a guess and check approach to solve this type of equation. I extended this topic into solving multi-step equations that involved variables on both sides of the equation, my goal was to reach the topic of solving systems of equations algebraically. A multi-step equation has this form: [2x + 9 = 21 – 4x].   Both classes struggled with solving multi-step equations. We did reach the topic of solving linear systems but the assessment from solving multi-step equations and the lack of necessary skills and understanding of linear systems caused me to go back and revisit solving multi-step equations.

Solving One-Step Equations

Solving one-step equations is a fundamental skill involved in algebra. I started my study with this topic because in order to solve a one-step equation, students have to be able to perform basic operations like addition, subtraction, division and multiplication. Students seemed to have the most problem when dealing with an equation that had a negative quantity that had to be moved to the other side of the equal sign. It is possible that a student can understand how to solve this type of problem but they would incorrectly carry out the operation and end up with an answer that had the wrong sign. Students who had access to calculators only had to correctly determine the inverse operation and the sign of the quantity to be moved and then they would plug it into the calculator to arrive at the answer. A student that did not have a calculator had to be aware of the rules that apply to operations with negative numbers.

Solving Two-Step Equations

These problems require a student to be aware of the proper order of operations involved in arithmetic. To properly "undo" the operations that are being performed on the variable, a student has to be careful to reverse the order of operations in order to get the variable by itself. The biggest cause of error with this type of problem was the same issue that student had with one-step equations, working with negatives. Even students with calculators would make errors with assigning a negative or positive value to a quantity. The second largest contributor to student error was that students would perform multiplication when they should have divided or divide when they should have multiplied.

Multi-Step Equations

These problems can be challenging to students if they have not acquired the necessary proficiency with solving two-step equations. These equations have the added complexity of moving a variable from one side of the equal sign to the other in order to combine like terms, allowing a student to then get the variable alone on one side of the equal sign after carrying out the necessary inverse operations. Mistakes were compounded as students tried to move quantities across the equal sign. Students with calculators would tend to not write work down, instead they would try to keep track of values using their calculators. This may have contributed to their error when trying to solve these equations. It seemed that the students who showed work and used a step by step approach were more successful than those who tried to simply provide an answer.

Assessments

Assessments consisted of sixteen multiple choice questions. I chose multiple choices because they are more easily measured and they also reflect the majority of questions that students face when taking the Algebra Regents exam. This did not allow me to break down the types of errors that caused students to arrive at incorrect solutions but I felt that I had a sense of the types of mistakes being made through class work and homework examples. I gave the pre-tests on Monday. My instructional strategies varied, I used guided notes that I went through on my laptop computer as it was projected on the screen for students. I assigned homework and then worked through various examples and answered student questions. I printed former Regents questions and worked through them on the overhead and then gave several examples to students to work on in class. I allowed students to work in small groups when I felt that they would benefit from working together. I regularly toured my classroom offering help and clarification to students as they worked. I was available for students to receive help after school on Mondays and Tuesdays and before school on Thursdays. I gave the post-tests on Friday.

Results

Table 1

	Algebra 1A	Consumer Math
One-step Pre-test	69.8%	92.8%
One-step Post-test	80.2%	88.9%
Gain	10.4%	-3.9%

Students seemed to benefit from having the use of a calculator (see Table 1). The Algebra 1A class had a class average increase of over ten percent. An 80% success rate is only 5% below the mastery level of 85%. The consumer math students showed a small decrease in their class average but they are still above the mastery level. Problem solving techniques were not measured. Since the questions were multiple choices, it is possible that students could have used test taking strategies such as checking each available choice to arrive at the correct answer. This first result would seem to argue for the use of calculators.

Table 2

	Algebra 1A	Consumer Math
Multi-Step Pre-test (two–step equations)	77.8%	77.1%
Multi-Step Post-test	52.1%	81.3%
Gain	-25.7%	4.2%

The Algebra 1A class did acceptably well on the pre-assessment but showed a big loss when measured for understanding with the post-test. The Consumer math class showed a modest gain in their class average. The Algebra students tried to rely on their calculators to help them solve these problems. The scientific calculator is not equipped to help students manipulate variables. These students had not taken the time to properly learn a methodical approach to solving these problems. The added complexity of combining like terms made the calculations too difficult to keep track of while trying to solve for the value of the variable. Calculators can give students an incorrect answer if the student does not properly input the problem. Calculators

carry out the order of operations and if students do not properly input parentheses, they can be given an incorrect solution. The Consumer math students were more used to showing work and these students seemed to take a more methodical approach to solving these problems.

Discussion and Interpretation

Summary

The purpose of this study was to determine the impact of calculator use on how well students learn basic math skills. My research experiment had clear results that show the impact of uninhibited calculator use while introducing new material. Results from the study were gathered from pre-test and post-test scores. Students were given a pre-test to determine a baseline score. The post-tests were given after students were given several lessons and afforded ample practice opportunities. The results show that calculator use can inhibit basic skill acquisition for students when learning new material.

Discussion

Learning new mathematical ideas requires a measured problem solving approach where new ideas are understood and often based on previously learned ideas. Solving simple one-step algebraic equations is an idea that students encounter when they are attempting to learn Algebra. An understanding of the principles of the operations that allow for solving these types of equations is a fundamental building block for further exploration and learning. During the experiment, one group of students was allowed unlimited calculator access. These students performed well while learning how to solve one-step equations. They increased their average from a 69.8% on the pretest to an 80.2% on the post-test. The questions were multiple-choice so it is possible that students approached solving the problems a number of different ways. Class

instruction focused on "undoing" the operation to the variable by performing the inverse operation to both sides of the equation. This is a basic idea and is necessary in order to move forward in the subject. Students also learn that it is fairly simple to plug the available answers into an equation to solve multiple choice questions. This is a test taking strategy that is fairly common. Since the test was multiple-choice, problem solving techniques were not measured. It became apparent that students in the calculator group did not acquire the basic skills for solving these equations when the topic involved solving multi-step equations. Their average on the multi-step equations pretest was 77.8% and decreased to 52.1% on the post-test. It became apparent that they were relying on mental math or guess and check to solve the one-step equations, the added complexity of the multiple steps disallowed their techniques. Their choice in calculators, scientific, is not supportive while solving for a variable. They became too reliant on using the calculators to help them solve the problems, the added steps and the required skill in algebra proved to be out of their reach.

The second group of students were not allowed access to calculators during instruction or while performing an assessment. This group scored 92.8% correct on the pretest and fell to 88.9% on the post-test of solving one-step algebraic equations. Class instruction focused on a step by step measured approach to isolating the variable using algebraic properties, as it did with the former group. The group without calculators, paper and pencil group, seemed to focus more on showing work and getting the steps correct. They did not seem to be looking for a shortcut but instead they seemed to strive for an understanding of the process. On the multistep equation pretest they averaged 77.1% correct and achieved 81.3% correct on the post-test. The lack of a calculator forced the paper and pencil group to focus on learning the steps necessary to solve

these types of questions. When they moved forward into solving more complex equations they had the necessary skills to perform the operations.

Limitations

The study was conducted over a four week period. The time limit did not allow for measuring how well students retained their learning over an extended period. The assessments were multiple-choice which did not allow for measuring problem solving strategies. The two classes that participated in the study were not exactly alike in their make-up. The calculator class consisted of ninth grade students while the pencil and paper class consisted of mostly eleventh grade students. The students were very similar in their attitudes towards math and their basic skills.

Implications

This study indicates that it is vital that students gain proficiency in basic skills before being allowed access to technology which can short-cut their understanding. Mathematics instructors need to be aware of the harm that can come from over use of technology. Calculators need to be used as tools that can help make large calculations easy or as a means of checking work for correctness. It seems that calculators have become too readily available because of the pressure to move forward in the yearly curriculum. They provide a quick means to provide each student access to a lesson but they can also hinder students' ability to focus on problem solving methods.

<u>Recommendations for Further Research</u>

Future research should focus on teaching methods that maximize calculator use while maintaining basic skill acquisition. Professional development that focuses on proper calculator integration needs to be provided for middle school mathematics instructors. Middle school mathematics determines student success at the high school level. Students need to acquire problem solving methods and critical skills that will help them learn the higher level math that is required for High School graduation. More information needs to be provided to teachers about when and how to properly introduce calculator use in mathematics. There must be a proper balance for different kinds of assessment questions. The major focus on New York State tests is multiple-choice with some extended response questions. The implications for instruction and how it is geared towards getting students to pass these tests could be studied to better understand why graphing calculators are allowed while taking these tests. Multiple-choice questions seem to be more accessible to the majority of students but it would be interesting to see how accurate these tests are when compared to extended response answers.

<u>References</u>

Borte, C.M. (1998). *Teachers' perceptions of calculator use in the classroom.* Oswego, NY: SUNY Oswego.

Burke. G. (2001). Computers and calculators in the school: a status report. *National Council of Teachers of Mathematics.*

Cavanagh, M., & Michael, M. (2003). Graphics calculators in the learning of mathematics teachers. *Mathematics Teacher Education and Development, 5*. Retrieved from ERIC.

Fey, J.T., & Hirsch, C.R. (1992). *Calculators in mathematics education.* Reston, Virginia: The National Council of Teachers of Mathematics.

Goos, Merrilyn, and Anne Bennison. (2008) Surveying the Technology Landscape: Teachers' Use of Technology in Secondary Mathematics Classrooms. *Mathematics Education Research Journal* 20(3), 102-28. Retrieved September 29, 2009, from ERIC.

Graham, E., Headlam, C., Sharp, J., & Watson, B. (2008). An investigation into whether student use of graphics calculators matches their teacher's expectations. *International Journal of Mathematics Education in Science and Technology*, *39*(2), 179-196. Retrieved September 29, 2009, from ERIC.

Zembat, I. O. (2008). Pre-service teachers' use of different types of mathematical reasoning in paper-and-pencil versus technology-supported environments. *International Journal of Mathematical Education in science and Technology*, *39*(2), 143-160. Retrieved September 29, 2009, from ERIC.

Appendices

Appendix A	Letter of Consent to Administration
Appendix B	Letter of Consent to Administration
Appendix C	Sample Assessment

Appendix A   **Consent Letter to School Administrator**

Dear Sir,

  This Spring I would like to conduct a study involving student volunteers at our school. I am a graduate student at SUNY Oswego and I am working on my thesis. The question that I seek to answer is; at what point in the learning of new mathematical ideas is it appropriate to allow students to use calculators?

  I would like to ask for student volunteers to participate in our regular class setting. During our meetings students will be given short lessons and then assessed on how well they learn with the aid of a calculator versus how well they learn without it use. I can only see a benefit for the students who participate as they will be exposed to mathematical ideas that are extensions of their current topics. I believe that I will also benefit as this study will give me an insight into how to better use calculators during instruction.

  To protect student privacy all participants will remain anonymous and I will ask for permission from parents before allowing students to participate in my study. Thank you for your time and consideration.

Sincerely,

Jeff Clark

Appendix B   **Consent Letter to Parents**

Dear Sir or Madam,

       I am the seventh grade, Algebra A and Consumer Mathematics teacher. I am also a graduate student at SUNY Oswego. As part of my graduate program I have to complete a thesis. My thesis involves studying how well students learn with the use of calculators versus how well students learn without them.

       With the consent of the administration I am conducting a study involving volunteers who are in my regular math classes. Students will be assessed on how well a calculator helps them learn basic skills and new ideas versus the traditional method of using paper and pencil.

       All results will be kept confidential, identities will be protected and participation is strictly voluntary. Students will not suffer any consequences if you do not wish them to participate. This is not a part of our regular class and has no effect on a student's grade. Please sign and return the attached permission form if you consent to let your child participate. Thank you.

Sincerely,

Jeff Clark
Mathematics Teacher

_____

I have read the above statements and I allow _____
to participate in you study.

Parent Signature_____ Date_____

Name_____   Date_____

Without Calculator   With Calculator

1.) Add $\dfrac{x}{x-1} + \dfrac{2x}{x-2}$ Add $\dfrac{y}{y-3} + \dfrac{4y}{y-1}$

2.) Simplify $\sqrt{300}$ Simplify $\sqrt{500}$

3.) Solve for x: $\dfrac{6}{x} = \dfrac{x+2}{5}$ Solve for x: $\dfrac{x}{7} = \dfrac{7}{x+2}$

Note: Questions do not reflect a single topic. Samples are given to show that there is no bias associated with calculator use allowed.