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The Second International Scientific Colloquium

MATHEMATICS AND CHILDREN
(Learning Outcomes)

Drugi međunarodni znanstveni skup

MATEMATIKA I DIJETE
(Ishodi učenja)

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Editor's Note

The main aim of the Organisational Committee of the Second international scientific colloquium *Mathematics and Children* is to encourage additional scientific research in the field of mathematics teaching in Croatia. This represents a contribution to the development of science and education, which is a part of the long-term Education Sector Development Plan 2005–2010. Following the example of European and other countries around the world, special attention in the field of education is given to mathematical literacy of children (PISA* programme) as well as to mathematics teacher training (quality insurance in higher education).

Mathematics teaching in Croatia is facing strategic, organisational, social and technical changes. There is a tendency towards introducing one-shift classes in primary schools, including children with special needs (both gifted ones and those with learning difficulties) in regular classes, organising extended daycare program for all students, two teachers per class, and greater mobility of children and teachers in schools. Special attention is given to defining of the learning outcomes, both within the classroom and within the process of lifelong learning. The presence of economic crisis and recession redirects the attention in mathematics teaching towards reflecting and acting in accord with the demands of the market. New teaching technologies demand changes in the methodology of mathematical education regarding both children and future teachers of mathematics. It is therefore important to develop a lifelong learning programme for teachers of mathematics which also includes doctoral studies.

Unfortunately, in Croatia there is no clearly defined lifelong learning programme for teachers of mathematics which would also include doctoral studies in mathematics teaching methodology. However, there has been some progress, and thus in the academic year 2008/2009 the Mathematics Department of the Faculty of Natural Sciences and Mathematics in Zagreb introduced the Seminar in mathematics teaching as a part of their Ph. D. programme.

Research in the field of mathematics teaching implies multi- and interdisciplinarity. Therefore, cooperation with scientists outside the field of mathematics (psychologists, special education teachers, educators, IT researchers) is an imperative, although we strongly believe that improvements in mathematics teaching should be encouraged within the field of mathematics.

A precondition for developing new approaches and methodologies in mathematics teaching in Croatia is a first-hand experience incorporating the results of international research and standards in mathematics teaching, as well as defining doctoral studies within the same field.

* Programme for International Student Assesment

We believe that the lectures, discussions and experience exchange between Croatian and international participants of the *Mathematics and Children* colloquium will initiate and intensify scientific cooperation in the field of mathematics teaching on the international level. We would also like for this event to initiate the start of doctoral studies in the field of mathematics teaching in Croatia following the examples from Europe and worldwide.

We are very grateful to numerous Croatian and international scientists who have recognised the importance of this event and managed to find the time to attend this gathering. We would also like to thank the heads and entrepreneurs of the local community, as well as the Ministry of science, education and sports, who were kind enough to sponsor this event.

On behalf of the Organizational Committee, I express my deepest gratitude.

Osijek, April 24, 2009

Margita Pavleković

Riječ urednice

Organizacijski odbor Drugoga međunarodnoga znanstvenoga kolokvija *Matematika i dijete* nastavlja s poticanjem znanstvenika na istraživanjima u području metodike nastave matematike. Doprinos je to razvoju znanosti i obrazovanja u okviru prioriteta razvoja Hrvatske (2005.–2010.). Po ugledu na Europu i svijet u okviru obrazovanja posebna se pozornost pridaje kako matematičkom opismenjavanju djece (PISA* program) tako i izobrazbi učitelja matematike (osiguranje kvalitete u visokom obrazovanju).

U nastavi se matematike u Hrvatskoj mijenjaju strateški, organizacijski, socijalni i tehnički uvjeti (Nacionalni kurikul, standardi u nastavi matematike). Teži se uvođenju jednosmjenske nastave u osnovne škole, sveobuhvatnijoj inkluziji djece s posebnim potrebama (darovite i one s teškoćama) u redovite odjele, osiguravanju produženoga boravka za sve učenike, uvođenju dva učitelja u odjel, većoj pokretljivosti djece i nastavnika u školama. Posebna pozornost pridaje se definiranju ishoda učenja kako u nastavi tako i u procesu cjeloživotne izobrazbe. Vrijeme ekonomske krize i recesije usmjerava na promišljanja i djelovanja u nastavi matematike u skladu sa zahtjevima tržišta. Nove tehnologije u nastavi iziskuju promjene u metodologiji matematičkoga obrazovanja, kako djece tako i studenata učiteljskih studija, kao i učinkovitije postupke prepoznavanja matematički darovitih učenika.

Na žalost, u Hrvatskoj još uvijek nije definirana cjeloživotna izobrazba učitelja matematike koja uključuje i doktorske studije iz metodike matematike. No, učinjeni su neki pomaci. Tako je na Matematičkom odjelu Prirodoslovno matematičkoga fakulteta u Zagrebu akademske 2008./09. godine u okviru doktorskih studija pokrenut Seminar iz metodike matematike.

Istraživanja u nastavi matematike pretpostavljaju multidisciplinarnost i interdisciplinarnost. Stoga je pri istraživanjima u nastavi matematike neophodna suradnja sa znanstvenicima izvan područja matematike (psiholozima, defektolozima, pedagozima, istraživačima iz područja informacijskih znanosti), iako i dalje držimo da razvoj metodike nastave matematike treba njegovati u okvirima matematičke struke. Pretpostavka je iznalaženju novih pristupa i metodologija u nastavi matematike u Hrvatskoj upoznavanje *iz prve ruke* rezultata inozemnih istraživanja i strane prakse kako u nastavi matematike tako i definiranju doktorskih studija iz metodike matematike. Vjerujemo da će izlaganja, rasprava i izmjena iskustava domaćih i stranih izlagača na Drugom skupu *Matematika i dijete* potaknuti i ojačati znanstvenu suradnju iz područja metodike nastave matematike na međunarodnoj

* Program međunarodnog ocjenjivanja znanja i vještina učenika.

razini. Također želimo da ovaj skup pridonese bržem zaživljavanju doktorskih studija iz metodike matematike u Hrvatskoj po ugledu na već postojeće u Europi i svijetu.

Važnost ovoga skupa prepoznali su domaći i inozemni znanstvenici, a neki od njih uspjeli su odvojiti dio svojega vremena za zajedničko druženje. Zahvaljujemo njima i vrijednim čelnicima i poduzetnicima lokalne zajednice te Ministarstvu znanosti, obrzovanja i športa koji su sponzorirali održavanje ovoga skupa. U ime organizacijskoga odbora svima od srca zahvaljujem.

U Osijeku, 24. travnja 2009.

Margita Pavleković

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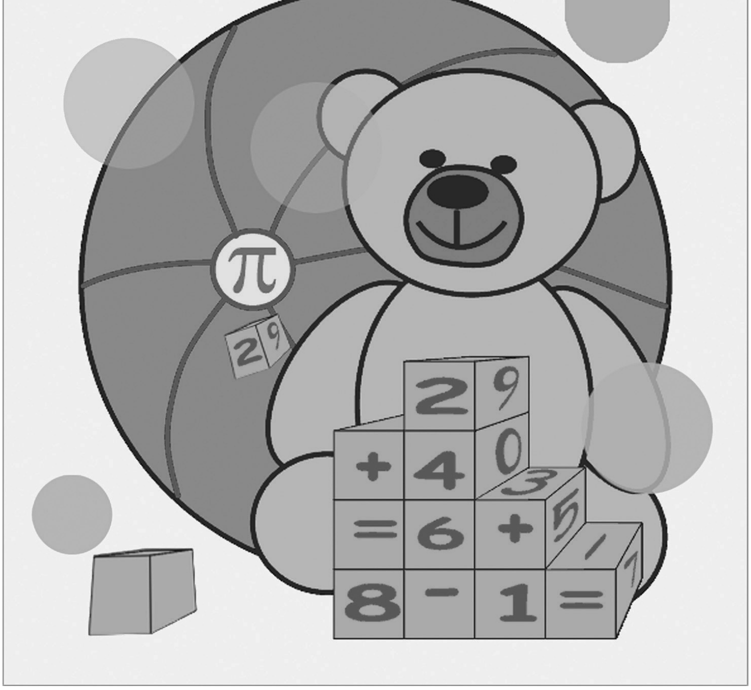
The Second International Scientific Colloquium

MATHEMATICS AND CHILDREN

(Learning Outcomes)

monography



Preface

In this monography, the authors present the results of their researches and provide examples of teaching praxis aimed at raising the quality of mathematics education.

Mathematics is one of the nine academic fields described in the project *Tuning Educational Structures in Europe*. Mathematics education (for students as well as pupils) is organised in accord with the national curriculum framework for mathematics field, which defines objectives, learning outcomes and competences of pupils/students at the end of each educational cycle.

In the scientific and professional circles, the concern for the preparation of pupils/students for everyday life, employment and life-long education is specially emphasized.

Schools (educational centres for pupils) and faculties (educational centres for students) have been connecting with civil society and executives in partnerships aimed at raising the quality of teaching, which is discussed in the first chapter of the book.

When it comes to teaching approaches, there have been two contrasting standpoints throughout history, depending on whether the learning and teaching are teacher-centered or student-centered. Today, we lean more towards the approach that sees teacher as the organiser of activities aimed at pupils/teachers' learning. Therefore, the mathematics education – and not only it – is organised and designed in terms of learning outcomes. (In this text, we will be using the term *teacher* for a person teaching, and it will be obvious from the context whether it refers to a schoolteacher or a university professor).

Learning outcomes are statements that describe what pupils/students needs to *know*, *understand*, and *be able to do* after they have successfully finished an educational cycle. Learning outcomes are coherent with measurable level descriptors in national and European frameworks. Learning and teaching (of mathematics) is more successful if the teacher is familiar with different theories, models and learning styles, and is capable of suiting "their teaching style" to different learning styles of their pupils/students. The most famous learning model based on experience (experiment) is the Kolb's learning model (1984) which emphasizes the four main activities in successful learning: Concrete Experience, Reflective Observation, Abstract Conceptualization, and Active Experimentation, the last one being favoured by the youngest pupils. The papers in the second chapter deal with the learning outcomes and teaching styles.

Today there are a number of learning styles models such as the Sternberg Model (Sternberg 2003), Gardner and Felder-Silverman Model of Multiple Intelligences, and Honey and Mumford Model of Learning. The upsurge of ICT and the development of intelligent educational systems have increased possibilities for monitoring the developmental levels in learning process. One of such generally accepted theories, Van Hiel's Theory (P. Van Hiele i D. Van Hiele-Geldorf 1959), is discussed in the paper in the third chapter of this book. In the third chapter, the results of the research studying students' learning outcomes are presented.

The fourth chapter deals with the influence of learning and teaching strategies on learning outcomes.

The fifth chapter discusses the study of mathematics and mathematics education from the educators' point of view. In the first paper, the author reminds us to mark the occasion of the 100-year birth anniversary of the academic Stanko Bilinski. The paper comments on the article of S. Bilinski, published in *Matematičko-fizički list* (Mathematical-Physical Journal) 50 years ago in order to encourage youth to enrol in mathematics studies. The parallels are drawn to contemporary attitudes towards mathematics. The last article presents the results of the research studying the opinion of mathematics teachers in primary and secondary schools in Croatia of their students' learning outcomes.

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Margita Pavleković

1.
Partnerships of civil society, faculties and schools in raisinig the teaching quality

Civil society organizations as a partner in mathematical education

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Abstract. Civil society organizations, especially vocational organizations, that is, various mathematical societies, are a partner whose presence and work educational institutions have valued for a long time. Current changes in our society and educational system give a new dimension to this partnership. Along with giving comment of the new situation, we will show the example of good practice and, in particular, take a look at the program "Development of scientific and mathematical literacy through active learning".

Keywords: mathematical education, civil society organization

Introduction

Mathematics as a teaching subject is present on all levels of education. It is often a topic of discussions within which numerous problems arise: negative grades within mandatory education, the questions of mathematics teachers' competencies, along with the question: do we need that much mathematics at all? In the end, the central problem is the question of general attitude towards mathematics. So, the main objective in mathematical education becomes the development of positive attitude towards mathematics. In order to solve this problem, it is necessary to include the ones who are formally responsible for the quality of mathematical education, the ones who took this responsibility by choosing their profession, and the ones who possess certain knowledge and competencies.

The social context

We bear witness to the processes in the educational system of the Republic of Croatia whose objectives are the changes in the existing methods of teaching and learning. The professed objective of these changes is the learner's becoming the center of the process in general. In that way, it is expected to harmonize one's ideas and actions with this objective. Today, more than ever, we talk about active learning, learning outcomes and competencies. On the other hand, it is well-known that the educational system is slow and it is not realistically to expect that these changes will come “over night”, no matter how supported they were by the Ministry of Science, Education and Sports. In order to move fast, it is necessary that all the teachers are acquainted with new demands of the educational process. Moreover, they should be in direct contact with the teaching methods which are expected to be implemented in the classroom. There is an attempt to satisfy this demand by organizing various seminars and workshops for the teachers, conducted by the Education and Teacher Training Agency.

Professional associations of mathematicians have always been involved in mathematical education of the students (they organized summer schools for talented students, took part in organizing mathematical contests and quizzes, published various publications for young mathematicians, etc.). It is evident that the interest in mathematics is in decrease. Therefore, this kind of engagement turned out to be insufficient. On the other hand, changes in Croatian educational system and recognition of non-institutional education enabled the intensified involvement of civil society organizations. This possibility has been additionally encouraged in the previous years due to the fact that the Ministry of Science, Education and Sports announced Competition for the Allocation of Financial Support to Projects and Programs of Associations in the Field of Non-institutional Education and Schooling.

The example of good practice

In continuation, we will give the example of the involvement of civil society organization in mathematical education of primary school pupils in Rijeka in cooperation with the City of Rijeka as the founder and with the support of Ministry of Science, Education and Sports. The above-mentioned civil society organization is Golden Section Society, founded in 2004. It is placed in Rijeka, but it operates throughout the entire Republic of Croatia. It is comprised of highly educated experts who are actively involved in education and popularization of sciences and mathematics. Golden Section Society has organized Science Festival Rijeka since its beginning. As the matter of fact, foundation of this organization is a result of the experience gained through the Science Festival. It has been realized that it is necessary to popularize mathematics and science, but at the same time, there is the need for implementing modern educational standards in the Croatian educational system, especially when talking about mathematics and science.

During the previous Science Festivals, which have been organized since 2003, Golden Section Society has organized numerous workshops for pupils, specialized teachers and homeroom teachers. It has been shown that the demand for these contents is larger and it goes beyond the frames of the Science Festival. Supported by Rijeka City Department of Education and Schooling, several programs and projects were applied to Competition for the Allocation of Financial Support to Projects and Programs of Associations in the Field of Non-institutional Education and Schooling, among which some were actualized, and some are in the process of realization.

One of these projects will be mentioned here; it is a three-year program "Development of scientific and mathematical literacy through active learning" which has been conducted in Rijeka elementary schools since school year 2007/2008.

Objectives of "Development of scientific and mathematical literacy through active learning" program are directed towards teachers and pupils' support in active learning and teaching.

Program activities are directed towards:

1) carrying out lessons whose objective is the promotion of active learning that challenges interest, motivation and long-term memory of mathematics and physics contents in full-time education and
2) supporting teachers in active classes, through carryng out teaching classes and making a handbook which will serve to the teachers and students as a support in this working method.

Six different mathematic contents are carried out for the students of fourth to eighth grade within the program framework. Authors of the educational contents which are presented within this program are university professors and their students of teaching course, who are already involved in full-time education. After carrying out each content, a survey is given in order to get an insight in the acceptibility of this method. The results of the questionnaire will be presented in the publication whose printing is planned for the final year of program execution.

"Development of scientific and mathematical literacy through active learning" program is in its central phase. At least one mathematical contents has been presented in each elementary school in Rijeka. More than five hundred pupils from fourth to eighth grade, along with their teachers, have taken part in it. This program is not intended for a certain category of pupils, but for all the pupils of a certain age. Contents presentation tries to promote a deep down access to learning which arises from the need for meaningful engagement in the problem. There is an aspiration to understanding fundamental ideas and principles and developing positive emotions for learning: developing interests, feelings of importance, enthusiasm and satisfaction in learning. Survey results show positive reactions even with the students who don't like mathematics. The majority of the teachers who took part in carrying out mathematical contents express their satisfaction. Many of them accepted it as an example of good practice, which we consider extremely important, especially when talking about homeroom teachers, considering the well-known fact that many of them who choose their profession have a rather negative attitude towards mathematics.

Conclusion

All things considered, it can be concluded that civil society organizations can be a reliable and respectible partner in mathematical education. But, the support of the local community and the Ministry of Science, Education and Sports is of extreme importance. It would have been impossible to carry out this program without the support of the City of Rijeka (founders of elementary schools which carry out the program) and the Ministry of Science, Education and Sports. When talking about mathematical education, there is a need and possibility for including civil society organizations, and it is up to them to offer mathematical education programs of appropriate quality.

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Mathematics teachers as researchers of their teaching and pupils' learning

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Abstract. The teaching of mathematics at any level is a very demanding task for teachers. Firstly, mathematics is one of the most difficult school subject, secondly, pupils do not see any use of mathematics in their every day life, thirdly, quite many of them have difficulties in understanding mathematical concepts. . . There has been a lot of research done in the area of teaching and learning mathematics and it seems that the results (in many cases very encouraging from the researchers' perspective) do not become alive in the mathematics classrooms. What is the reason? We believe that one of the very important reason is that teachers do not very easily change their subjective theories about teaching and learning mathematics and therefore usually do not accept researchers' findings as real, useful. The paper presents the idea of teachers being researchers of their own teaching practice. We invited teachers to join the project Partnership between Faculty of education, University of Ljubljana and some Slovenian schools. The main research area of the project was integrated curriculum and the mathematics teachers involved in the project were asked to develop a teaching approach based on joining different school subjects according to the teaching aims of each subject. The aim of developing such an approach was to deepen pupils' mathematical knowledge and to improve their skills of using and applying mathematics. The methodology teachers mainly used was action research based on three action steps. The paper will present some examples of action research developed and performed by teachers, and analysis of research findings.

Keywords: action research, mathematics, interdisciplinary connections, teaching approach

Introduction

One of the important principles of revitalizing the primary school curriculum is the horizontal integration and intertwinement of knowledge, which can best be achieved by means of interdisciplinary and inter-subject connections. Interdisciplinary connections represent a didactic approach, a teaching approach, where the teacher tries to present or treat a certain content or problem as comprehensively as possible. Such work demands that the teacher clearly defines the objectives and the content of various subjects and combines them. It also requires careful planning and good content-related and organizational implementation based on cognitive constructivist teaching methods. Interdisciplinary connections should be adjusted to the pupils' grade level and their previous knowledge. Sometimes, the cooperation of teachers of various subjects is required as well. Interdisciplinary connections should be implemented when this course of action is logical, when there are reasons for such connections and when this enables us to improve the pupils' creativity and motivation.

There has not been much research done on the interdisciplinary connections in the Slovenian context. The term interdisciplinary connections itself is relatively new. Shoemaker (in Lake, 2002) defines the interdisciplinary connections as education organized in such a way that it combines the common characteristics of various disciplines into a coherent whole. Martin-Kneip, Fiege and Soodak (1995) define the interdisciplinary connections as an example of holistic learning and teaching which reflects the real interactive world and its complexity, bridges the boundaries between the disciplines, and fosters the principle that all knowledge is linked.

Interdisciplinary connections

When we talk about interdisciplinary connections, we do not refer only to conceptual knowledge, but rather emphasize generic skills, for the learning of which content is important but not fundamental. Generic skills could be defined as skills which are independent of content, transferable, and useful in various situations. Let us enumerate a few: critical thinking, problem solving, data processing, use of IKT, execution of project assignments, active learning, reading, writing, listening etc. In theory, interdisciplinary integration is related to the real life, which gives the pupil a reason and a strong motive for learning.

Besides the importance of discipline connections and generic skills development for interdisciplinary connections we have to mention the importance of the interdisciplinary connections learning process. In many practical cases, integration does not prove to be as logical as we would have expected, but the process of integration is much more significant than the content of interdisciplinary integration. With this we wish to emphasize that a child learns from the process of integration itself, recognizing that contents can be integrated, searching through the subjects connected, and, not least, establishing stronger relations between concepts. Inevitably, the logical integrations will be more effective in the adoption of the integration process that the less sensible ones.

Cromwell (in Lake, 2002), for example, explains that the human brain is organized in such a way that it can process many facts at the same time and that information gained by holistic experiences is recalled easily and quickly. We should also mention the Caine and Caine (1997) research findings, which prove that the brain of every person is universal and that every person has their own way of learning. Their research revealed that every person's learning style and use of knowledge depend on the integration of new information with previous knowledge and his/her experience. This leads to the conclusion that we have to approach learning and teaching very carefully and that we must not favour one principle which might seem the most logical and sensible. Naturally, this stands for interdisciplinary integration as well. Sylwester (1995) summarized a number of research studies and, focusing on brain activity, developed a list of teaching advice for the teacher. We will enumerate only a few (Sylwester, 1995):

- Organize cooperative learning (emphasize the social aspect of the experience).
- The emotional element of learning is important.
- Teach what is important to know or understand.
- Employ technology.
- Work in a team.
- When treating problems take into consideration all intelligences. (Gardner (1983) suggests that each person has at least 8 intelligences and claims that schools develop and measure mainly the logical-mathematical and the linguistic intelligence.) When teachers are stimulating all of their pupils' intelligences, the curriculum becomes interdisciplinary, which, among other things, enables the pupils to employ various learning styles (Drake, 1998).

Regarding the curriculum we should emphasise that we can distinguish at least three types of curriculums according to different ways of interdisciplinary integration/connections (Erickson, 1995): the multidisciplinary curriculum, the interdisciplinary curriculum, and the transdisciplinary curriculum. The multidisciplinary curriculum is organized in such a way that a certain theme is treated by various school subjects, which can encourage the pupils to search for connections between the subjects on their own. With the interdisciplinary curriculum the emphasis lies on the subject integration, which is presented to pupils explicitly; certain themes or concepts which are common to all disciplines, are taught together. The transdisciplinary curriculum, as the highest level of integration, is based on the realistic context. The results of the learning process are more often assessed from the perspective of social responsibility development and personal growth than from the perspective of discipline integration.

In the Slovenia context, we practically cannot find a curriculum based entirely on the concept of interdisciplinary integration, even though some possibilities for content integration have been indicated within the curriculums of particular subjects; more in the sense of content correlations. This is mostly the case with lower grades, where all subjects are taught by one teacher and such connections are easier to implement from the organizational perspective. On the other hand, the implementation of interdisciplinary integration in higher grades requires much teamwork and coordination. Some related experiments in school practices will be presented in the empirical part of the paper.

In the paper we will not be discussing the curriculum, but rather the interdisciplinary integration/connections as a teaching approach, as already mentioned in the introduction.

In school practices in Slovenia, the interdisciplinary connections are best achieved in the first trimester, when all subjects are taught by one teacher who knows the curriculum and is quick to find the possibilities for connections. In the second trimester we encounter the interdisciplinary connections mostly in the organizational forms such as summer camps, sports days, project work, sports weekends and months, research assignments, first aid classes, cycling proficiency tests, and competitions.

Kovač, Jurak, Starc (2004) claim that the following criteria have to be met for the success of interdisciplinary connections:

a) the teacher has to know precisely what objectives he/she wishes to achieve in certain subjects by means of interdisciplinary integration, and consequently he/she has to be familiar with the objectives and content of various subjects;
b) joint cooperation of teachers of different subjects is necessary;
c) the choice of contents, the transfer of knowledge, and the organization of lessons have to be adjusted to the children's level of development and education;
d) each interdisciplinary connection has to be planned carefully and has to include individual work of pupils;
e) at the end, the teacher analyzes the realization of the objectives defined.

We wish to emphasize that in Slovenia there are contrasting examples of integration practices and various reasons for integration: transferable knowledge, save time, learning for life, motivation, supplementing the disciplines. As mentioned before, with transferable knowledge we mostly refer to generic skills, which are presently underdeveloped in Slovenia, but some changes in this field have been indicated as well. When supplementing the disciplines in practice, we encounter various forms or ways of implementation. Let us look at some examples of integration between mathematics and other disciplines.

Example 1: *Mathematics: The pupil can arrange objects according to their characteristics.*
Music: The pupil recognizes different instruments and plays them, recognizes different tones (the difference in pitch) and arranges them.
<u>One possible lesson plan:</u> The pupil is arranging tones according to their height. He/She is learning how to arrange objects and how to recognize different tones.
<u>Possible issues, traps:</u> When both themes are new to the pupil, learning is made more difficult and more complex, because he/she has to arrange things that he/she is still learning about.

Example 2: *Music: The pupil understands the concept of pitch, tones of different height (introducing a new lesson)*
Mathematics: The pupil can arrange objects, ideas (to reinforce the knowledge acquired)
<u>One possible lesson plan:</u> The pupil plays different instruments, sings and arranges the tones according to their height. The process of ar-

ranging objects is familiar to him/her, while tones of different heights represent a new concept.

Example 3: *Language: The pupil is familiar with "puzzle" words with "lj", "nj" (spelling and pronunciation differ) and regular words with "lj", "nj" (introducing a new teaching unit).*
Mathematics: The pupil fills in the Carroll diagram of two sets (testing the knowledge gained).
One possible lesson plan: The pupil uses the Carroll diagram to arrange words. The diagram is a tool the pupil is familiar with, used for the practical demonstration of newly acquired knowledge.

By means of interdisciplinary integration we can also increase the pupils' motivation (in this case we refer to correlations and not the actual integration of contents), for example, pairs of socks and the multiplication table for the number 2.

It is clearly evident from the research (Filipič, Hodnik Čadež, 2005) that integration is most commonly achieved by supplementing the disciplines and by correlations, which are in most cases, as already mentioned, motivating examples. Integration of this kind can hardly be called a teaching approach, because these are actually situations where we wish to thematically link two related disciplines, and where the integration of concepts from different disciplines and the development of generic skills and procedural knowledge are not emphasized.

Consequently, we used the MODEL IV project: the Faculties/Schools Partnership (The teacher researcher and interdisciplinary connections), made possible with the co-funding of the European Social Fund of the European Union and the Ministry of Education and Sport of the Republic of Slovenia, (hereinafter called "Model IV") to encourage the teachers to develop a teaching approach which would be based on the interdisciplinary integration and with the help of which they would try to improve the pupils' and students' level of useful knowledge. The teachers acted as researchers of their own teaching practice, which included mastering the skills of interdisciplinary connections, planning, conducting and evaluating a practical case of interdisciplinary connections, and teamwork. The results of the teachers' work are presented in the following chapters.

Teacher researcher. Analysis of examples of good practice of interdisciplinary connections

Background

In Slovenia it is rarely emphasized that research is one of the teacher's roles. Teachers are not trained well enough in the field of practical research and their ongoing professional training is focused on course content and contemporary teaching methods. In Finland, for example, the guiding principle of teacher training is research, the importance of which is based on the following: teachers need to be familiar with the latest research in the field of teaching and learning, they need to know how to implement the research findings in practical situations, and they need to possess the necessary academic and professional competence for research, which enables them to systematically plan the instruction, to help develop social and ethical dimensions of pedagogical work, and to assume a more responsible role in the society (Niemi, Jakku-Sihvonen, 2006).

We are convinced that Slovene teachers should play a more active role in the society. Systematic research would enable teachers to participate actively in the discussions and argumentations about the curriculum, its contents and goals, about the learning process, instruction, pupils' development, and, not least, the ethical questions of their profession.

Training the teachers in the field of research was the main goal of our Model IV project. The guiding principle of the Model IV project was that the teachers must actively participate in the research on the way their pupils are gaining knowledge, how they are learning. When teachers play an active role in the research process and when they enjoy the support of their partners form the Faculty of Education, the probability that they will consider their new role in the classroom and even transform it on the basis of the research findings, is far greater than when they are only informed of the research findings, or when researchers conduct a research study in their groups of pupils. The teachers involved in the project conducted a research on interdisciplinary connections. The teachers played the role of researchers on interdisciplinary integration in classes they teach, and in most cases they worked in teacher teams. We wish to emphasize that we organized a number of training classes in the field of interdisciplinary integration and research for the teachers who wished to participate in the project. The teachers participated in the project for one school year (2007) and they conducted their research during regular classes in close cooperation with their mentors from the Faculty of Education (Krek, Hodnik Čadež, Vogrinc, Sicherl Kafol, Devjak, Štemberger, 2007). The teachers chose various disciplines for interdisciplinary integration. We played the role of mentors to those teachers who chose to integrate mathematics with other disciplines. In total, we cooperated with 33 teachers from 11 schools and kindergartens (2 kindergartens, 1 grammar school, and 8 elementary schools). At the completion of the project we received 9 reports on interdisciplinary connections which included the field of mathematics. One report was not complete, but 8 were satisfactory. A total number of 25 teachers cooperated on the eight reports which are the subject of our analysis.

Teacher researchers' reports analysis

Statement of the problem

At the completion of the year-long research, the teacher researchers who participated in the Model IV project wrote their reports, consisting of the key structural elements of the research report: summary, keywords, theoretical and empirical part, significance of the research for practice and conclusion, and appendixes. All reports have two themes in common: action research and interdisciplinary integration including the field of mathematics. The main objective of this teacher researchers' reports analysis is to combine their findings, present examples of good practice and make recommendations for the implementation of interdisciplinary integration and action research for teachers in school practice.

Research questions

When analyzing the teacher researchers' reports we were trying to answer the following questions: Which research problems the research was focused on?
Which research method was used?
How did they implement the research in practice?
What were their research findings and recommendations for implementation in practice?

Method

We employed the method of descriptive analysis.

Sample

The sample includes 8 teacher researchers' reports (in total, 25 teachers and 420 pupils who were included in the research). The research was conducted during the 2006/2007 school year. The teachers involved work in different Slovenian regions. This is a purposive sample.

Reports analysis

The teachers' research reports are quite extensive, 20 to 40 pages each (without appendixes), or 30 to 60 pages with appendixes. All contributions are structured as reports, consisting of: summary, theoretical part, empirical part (problem definition, methodology, and findings analysis), conclusion and recommendations for practical use, and appendixes (lesson plans, tests, questionnaires).

Coding of reports

When analysing the reports we chose a few key categories which are important for the analysis and interpretation of each teacher's research on interdisciplinary integration. These categories are: the number of teachers per team, grade level and number of pupils included in the research, research problem, research methodology, and research findings.

Reports analysis according to key categories

1. Number of teachers
As already mentioned, 25 teachers handed in their reports (18 class teachers, 2 single subject teachers, and 5 grammar school teachers). They all worked in a team of 2 to 5 teachers, namely 4 pairs of teachers, one team of three teachers, one team of four teachers, and two teams of five teachers. Three pairs, the team of three teachers, and the team of four teachers consisted of class teachers, while one of the five-member teams consisted of class teachers and the other of grammar school teachers. One pair consisted of subject teachers. It turned out that all teachers involved chose to work in teams.

2. Number of pupils
The total number of students involved in the analyzed sample of teacher researchers' reports was 420, namely 30 first-grade pupils, 28 second-grade pupils, 9 third-grade pupils, 151 fourth-grade pupils, 83 fifth-grade pupils, and 34 eight-grade pupils from elementary schools as well as 20 first-year pupils and 25 third-year pupils from grammar schools. The number of fourth-grade elementary school pupils stands out, since the largest number of teachers-researchers teach fourth grade in elementary schools (the total number is 8 class teachers).

3. Research problem, methodology, findings
We established that the research problems of all the reports relate to interdisciplinary connections or mathematics. The reports will be divided into two groups: 1 report dealing with mathematics, action research method (I. group) and 7 reports dealing with action research interdisciplinary connections (II. group). We will present the analyses of these two groups of reports separately.

I. group of reports (the common theme is mathematics, the method is action research)

This group comprises of the report dealing with the mathematical word problem solving in the first five grades of elementary school. This research was conducted by 5 teachers (one teacher of each of the first five elementary school grades) and included 47 pupils (6 first-grade pupils, 8 second-grade pupils, 9 third-grade pupils, 15 fourth-grade pupils, and 9 fifth-grade pupils).

The report presents the research on mathematical word problem solving in the first five grades of elementary school and sets forth the following research problem: "How to employ an effective teaching approach to improve the mathematical word problem solving?" The research team employed the method of action research in 3 steps: assessing the existing knowledge (how successful the pupils are in word problem solving), developing a teaching approach to word problem solving, and assessing the pupils' progress in mathematical word problem solving. The instruments of the first and third steps are tests in mathematical word problem solving. The researchers presented the results of these test by means of tables and diagrams. The teaching approach they developed emphasizes the strategies of mathematical word problem solving (read the text, describe the problem in your own words, underline key information, write down the calculation, check the solving process, check the logic of the result, write down the answer). It turned out that the pupils did follow the process of problem solving, but their results or accurate answers were not much improved from the 1st to the 2nd tests (for example, the fifth-grade pupils had a 53% success rate in the 1st test and a 56% success rate in the 2nd test, third-grade pupils' results were similar: a 65% success rate in the 1st test and a 67% success rate in the 2nd test). The forth-grade pupils' results are somewhat different: their success rate in the 2nd test displayed an average increase of 26%.

It turned out that it is useful to know how to use the mathematic word problem solving process, but that the basic tools remain the knowledge of calculations and the understanding of problematic situations. The latter is undoubtedly complex and not necessarily in direct proportion to the knowledge of the word problem solving processes. The researchers concluded their report by stating that they have observed a visible improvement in the knowledge of the word problem solving processes, but no visible progress in problem solving. This is an interesting observation which leads to the search for new solutions for the successful instruction of mathematical word problem solving, which causes problems for a number of pupils.

II. group of reports (the common subject is interdisciplinary connections, the methodology is action research)

This group comprises of the 7 reports dealing with interdisciplinary integration, namely how the mathematical content can be combined with other contents of the curriculum, and action research. These research studies were conducted by 20 teachers (1 first-grade teacher, 5 fifth-grade teachers, 2 eight-grade teachers and 5 grammar school teachers) and included 373 pupils (Diagram 1).

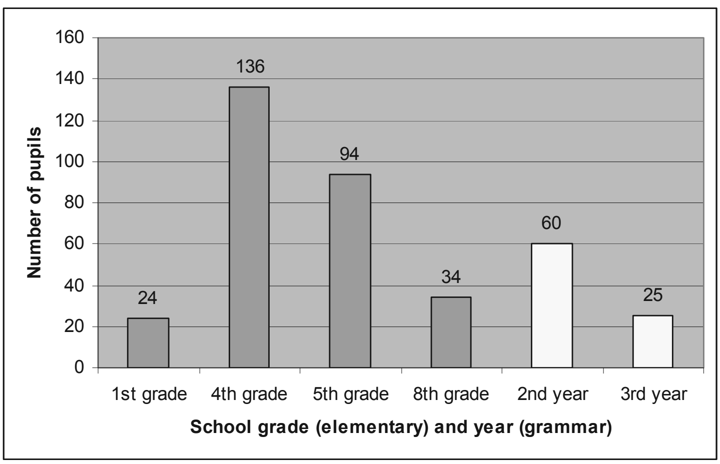

Diagram 1. Number of pupils involved in the research.

All reports have two main research questions in common:

- How to improve the pupils' or students' useful knowledge of data processing?
- How to develop a teaching approach that would enable the teachers to improve the useful knowledge of their pupils or students?

The methodology that was used for all research studies is the method of action research. The three action steps can be summarized in the following way:

1. Assess the level of basic and useful knowledge of data processing (test 1).
2. Based on the results of the test 1, develop a teaching approach based on interdisciplinary connections, namely on combining the data processing contents with various other contents (developing the pupils' useful knowledge).
3. Assess the pupils' progress in useful knowledge and determine the success of the teaching approach developed (use test 2).

In step 1, the teachers/researchers determined that the pupils were familiar with the data processing concepts when these were strictly mathematical, but that they ran into problems when asked to apply or use this knowledge on other contents (especially natural sciences and social sciences). Let us look at some of the results of the first and second tests, which clearly show that the progress in the pupils' knowledge following the teacher's intervention.

a) Two teacher researchers, 4th grade, total number of students: 47, theme: integration of data processing contents with language, natural sciences, physical education, social sciences, and music. The pupils had a 24% success rate in the first test, and a 77% success rate in the second test. The progress in the pupils' useful knowledge is obvious.
b) Two teacher researchers, 8th grade, total number of pupils: 34, theme: integration of mathematics, geography, history, chemistry, and biology. The arithmetic mean of the success rate in the first test was 6.96 in the useful knowledge tasks and 16.84 in the basic knowledge tasks, while the arithmetic mean of the success rate in the second test was 12.02 in the useful knowledge tasks and 18.76 in the basic knowledge tasks. It is obvious that the pupils' knowledge has improved, enabling them to use their knowledge in new situations.
c) Five teacher researchers, 2nd and 3rd year of grammar school, total number of students: 85, theme: integration of mathematics, geography, history, German language, and Slovene language. The increase in the success rate of second-year students was 17% from the first to the second test, while the increase in the success rate of third-year students was 13.3%. The teacher researchers ascribe the success to teamwork, integration of contents, and students' motivation.
d) Three teacher researchers, 5th grade, total number of pupils: 54, theme: integration of language and mathematics, mathematics and physical education, and mathematics and social sciences. The pupils' success rate when combining language and mathematics was 70.2% in the first test and 94.9% in the second test. When combining mathematics and PE, their success rate was 61.4% in the first and 95.0% in the second test. Their success rate when combining mathematics and social sciences was 90.6% in the first test, which is why they were not tested for the second time.

Following the results of the first test (with all teacher researchers this test showed that the pupils' useful knowledge was not as good as their basic knowledge) the teachers developed a teaching approach based on interdisciplinary connections. They combined mathematics, namely the contents of data processing, with other disciplines of the curriculum. The class teacher researchers mostly integrated the fields of natural sciences and social sciences, followed by language, PE, and music. Regarding the subject teachers and grammar school teacher researchers we can set forth the integration of mathematics with geography and history, followed by the integration of mathematics with chemistry, biology, and language.

We can conclude that the teachers planned their teaching approaches according to the objectives and contents of specific disciplines. Schematically, we could present the concept of integration in the following way (Diagram 2).

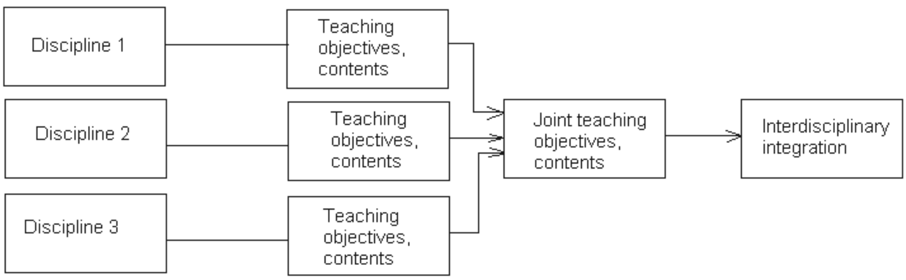

Diagram 2. An example of the interdisciplinary planning (Hodnik Čadež, Filipčič (2005)).

Let us look at some examples of the integration planning process from the reports:

Example 1: *Subject: mathematics, natural sciences, and technical science*
Grade: 4
Theme: The animal kingdom
Contents and objectives: The pupil can distinguish between the living beings according to their physical appearance, habitat, and diet. He/She acquires the new concepts: herbivore, carnivore, omnivore, vertebrates, invertebrates. He/She knows what the animals eat. He/she chooses a demonstration of one of the ways the animals eat. He/She then explains his/her choice of diagram.

Example 2: *Subject: Slovene, Hungarian, German language, mathematics*
Year: 3
Theme: Media
Contents and objectives: To introduce the media (television, radio, internet etc.). Use of media among students. Data processing. Advantages and disadvantages of computers and internet.

Example 3: *Subject: mathematics, social sciences, natural sciences and technical science*
Grade: 4
Theme: Space
Contents and objectives: Creating a mock-up (scale model) of a house. Definition of the living space. Developing measuring and

planning skills, and using them in everyday situations. Creating a floor plan.

In every report the teachers illustrated the approach of work with at least six descriptions of lessons. The teachers analysed each lesson from the perspective of achieving the objectives, the pupils' motivation, and other criteria. Some reports also contain pictorial matter: photographs of pupils working, pupils' products. We have looked in teachers' teaching approaches in depth as well. We were interested in teaching methods and forms teachers have used in their approaches. We have analysed 54 teachers' descriptions of the lessons in terms of teaching methods and forms they have used in order to achieve teaching aims. We came with the following diagram for the teaching methods teachers have attended to in their approaches (Diagram 3).

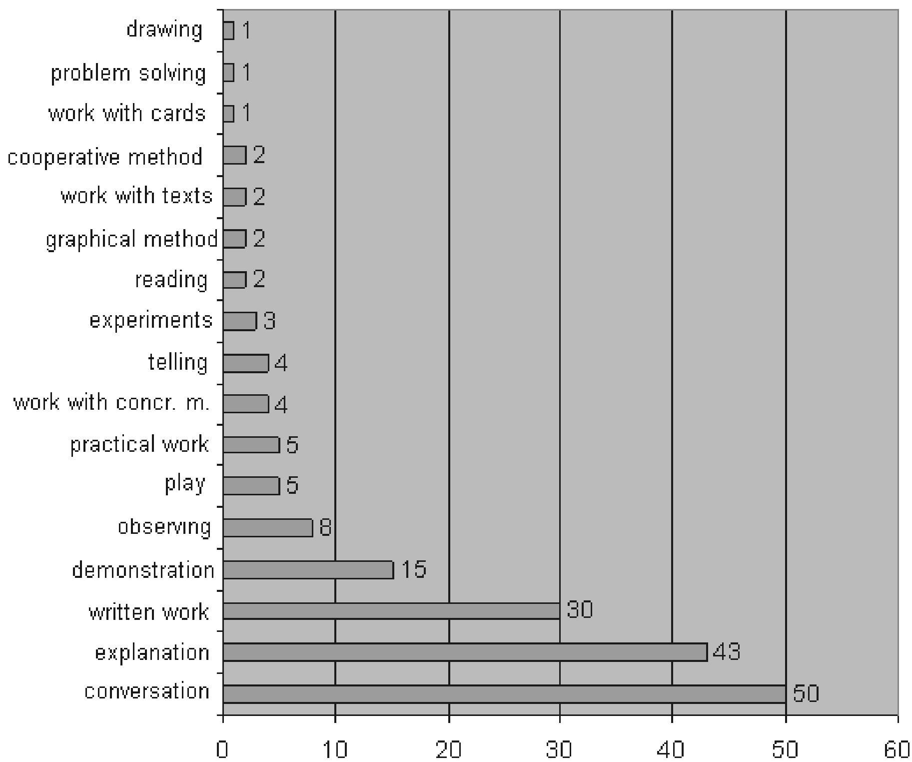

Diagram 3. The teaching methods in the teachers' approaches.

It is clear that the main methods teachers have used in their approaches based on disciplinary connections were conversation (expressed in 92% of all lesson descriptions) and explanation (80%), written work – mainly means solving mathematical tasks on a sheet of paper (56%), and demonstration (28%). Problem solving method was mentioned only once. We can not conclude that the teachers have used only passive methods, or that the pupils were not active during the lessons but we would prefer that the teachers would decide for more active methods (e.g. problem solving) more often. The teachers expressed also some activities which are not the teaching methods (e.g. work with cards, drawing...). This fact can be understood that the teaching material for teachers (in our case a lesson plan framework) is not professionally prepared and that the idea of teaching method could be miss- understood or not properly used by the teachers.

Diagram 4 shows the forms teachers have demonstrated in their new approaches. The analysis is based on 54 teachers' descriptions of the lessons.

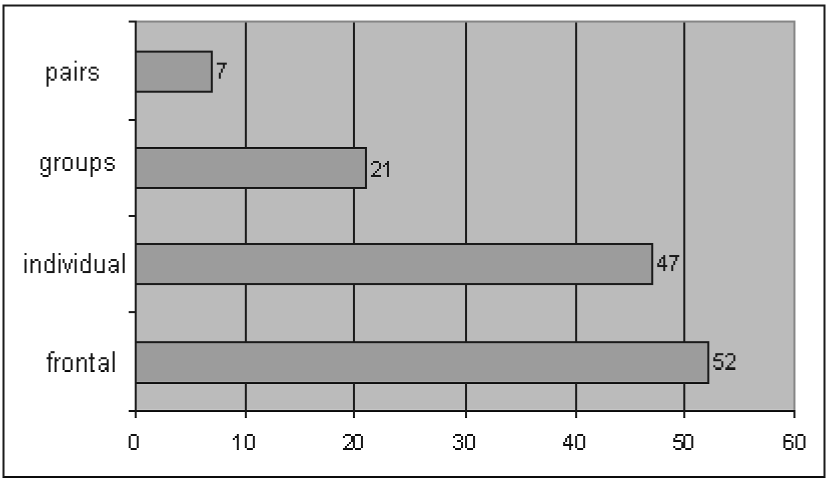

Diagram 4. The teaching forms in the teachers' approaches.

Most frequently used teaching form was frontal work (96%). Frontal work is a teaching form where a teacher performs the teaching with the whole group of pupils in the classroom. Pupils are in a sense "receivers of knowledge", they are inferior to their teachers, their activity is mainly emotional perceiving (Blažič, Grmek, Kramar, Strmčnik, 2003). Individual work (in 87%) was mainly demonstrated in the classrooms by solving mathematical tasks individually, group work was used in 39% of the lessons, and work in pairs in 13% of the lessons.

We believe that the teachers were very much occupied with the new approach on interdisciplinary connections they were developing, with the contents and the processes of the lessons and therefore have used more traditional teaching methods and forms. This could mean a save way of introducing new teaching approach or the standard of their teaching.

Conclusion

We find out that all teachers in their reports described the research they had conducted as a pleasant experience, because the research work had the proper expert support provided by mentors and training, and because the research was though through, systematically organized and executed, and appropriately evaluated.

Based on the reports of teacher researchers involved in the Model IV project (in total, 25 teachers and 420 pupils) we can claim that the research and research findings represented a giant step forward in the pedagogic practice, in each and every case. A step forward in the following areas: The pupils were acquiring knowledge throughout the process (none of the teachers reported a deterioration in their pupils' knowledge).

- This improvement in the pupils' knowledge is the result of the teacher's active intervention in the teaching practice.
- The teachers have gained new dimensions of treating the teaching contents. The disciplines are not separated from one another and they have common objectives and contents.
- The teachers have gained experience in research work and teamwork.
- The teachers were satisfied with the research findings.

The mentors of teacher researchers have observed that teachers need even more training in the field of practical research and that they need to upgrade their professional writing skills. The teacher researchers encountered numerous problems when writing their reports, especially when writing a clear interpretation of their findings. We are convinced that we can influence the implementation of the teaching process and the teachers' research only through the close cooperation between the Faculty and the school teachers. We believe that teachers need the research knowledge and that they need to be informed of the latest research in the field of teaching. To achieve progress in the research we need to integrate the expert and educational contents. Teachers who conduct research gain an analytical insight into their own work, based on which they can draw certain conclusions that help them systematically develop the teaching and learning process, and enable them to play an active part in discussions and decisions related to education.

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Mathematics and informatics in extracurricular activities chosen by pupils and offered by their teachers

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Abstract. After having approved integrated undergraduate and graduate five-year university studies in 2005 and 2006, Croatia expects the first coordination and renewal of class teacher studies curricula. We have carried out research wishing to define learning outcomes for class teacher students in accordance with children's interests in primary education and labour market demands, with special emphasis on extracurricular activities concerning the field of mathematics – informatics.

The article describes results of the research carried out in December 2008 in six primary schools in Osijek, in which 107 four-grade pupils and their 21 teachers were involved. The aim of the research was to determine to what extent the teachers' offer is in accordance with pupils' choice of extracurricular activities in four fields: language and art, physical education, science and ecology, and mathematics and informatics, the first two fields belonging to social sciences and the last two to natural sciences. The article also analyses the number of students in Croatia enrolled in social and natural sciences studies in academic year 2008/09 and relates it to the lack of experts on the labour market having knowledge and skills in mathematics and informatics. The research results have helped the authors become more confident about defining the mathematics and informatics learning outcomes for class teacher students. The authors are likely to encourage school education authorities, school boards, and teachers to direct pupils' interests towards those types of knowledge and skills, which are in demand on the labour market.

Keywords: extracurricular activities, primary school, learning outcomes, labour market

Introduction

Although there has been a four-decade long tradition of extracurricular activities in Croatia, the attention they have received is still insufficient (Jurčić, 2008). The number of families in which both parents work has increased, which leads to a problem of taking care of pupils after lessons. This results in the need for organized activities after regular lessons in order to fulfil pupils' free time until their parents come home from work. However, the main role of extracurricular activities is to enable pupils' normal psychophysical development, their appropriate socialization, to develop the need for meaningful free time spending, to improve pupils' quality of life, and to prepare pupils for involvement in the work society.

At the moment a new National Curriculum is being created in the Republic of Croatia, therefore it is necessary to pay attention to this type of educational activity in primary school. Analysing the curriculum for the pupils from grade one to four, one can notice that one session of advanced individual counselling and tutoring, as suggested in the curriculum, is insufficient to fulfil pupils' free time after regular lessons and does not meet pupils' needs for extracurricular activities ([9]). The benefits and advantages of extracurricular activities are numerous, therefore the issue of their complete inclusion in educational system has become even more important and has had wider implications. Based on the report by the US Ministry of Justice and their Ministry of Education in June 1998, the following can be concluded about the advantages and benefits of time spent purposefully after the regular lessons. They include:

- improving pupils' school success
- attending lessons more regularly and decreasing the number of drop-out pupils
- doing homework more successfully and regularly
- improving pupils' behaviour at school
- increasing the number of students with a plan for their future
- decreasing violence and youth crime, school vandalism, drugs and alcohol abuse (Šiljković et al., 2007).

For better organization of extracurricular activities, it is necessary to provide essential education for teachers, even at universities, so that they could recognize pupils' potentials and interests on time and develop them appropriately. Of course, teacher's satisfaction and motivation for organising extracurricular activities must not be neglected.

Characteristics and categorization of extracurricular activities

Extracurricular activities encompass different contents apart from regular lessons, and a school organises them on its premises. They are supposed to fulfil pupils' individual and actual needs, interests and abilities, expand and broaden their motor knowledge, develop appropriate interrelations, and bring pupils and their teachers closer together.

Extracurricular activities can be categorised, among other, according to subject fields, their purpose, where they take place, etc.

According to subject fields, they can be classified into the following categories: physical education, language and arts, mathematics and informatics, and science and ecology. The first two belong to social sciences and the last two to natural sciences. Furthermore, they can be classified according to place where they are organised into those which take place on school premises and those which are organised outside school. Some authors (Jurčić, 2008) also suggest classification of extracurricular activities into artistic, educational and informational – instructional.

Extracurricular activities are organised at the beginning of the school year for pupils from grade one to eight. It is up to each pupil to choose activities according to their interest, needs and abilities, but the choice is partially limited by teachers' offers. It is necessary to familiarise pupils' parents with the purpose, time, place and the way of teaching extracurricular activities. The aim of extracurricular activities is to include as many pupils as possible and direct them to appropriate activities according to their wishes and abilities. Extracurricular activities provide and create conditions for pupils to take part in different organisational types of work that are meaningful and satisfactory.

The pupils are expected to participate actively in the organising of extracurricular activities, more precisely, they should initiate the offer of activities, participate in curriculum development, decide on the activities' content, as well as their course and importance. Additionally, it is important to enable a pupil to attend the activities regularly and to try to fulfil pupil's expectations.

Offer and choice of extracurricular activities concerning the teacher – pupil relations

Teachers are the organisers of extracurricular activities and their decision on the choice of activity directly influences the offer. They usually choose only one activity, rarely two or more. Their decision on offer partially limits pupils' choice. Motivation and adequate working conditions are important factors which also influence teacher's choice of activities. Teacher's task is to recognise pupil's abilities and predispositions, which are going to influence teacher's decision on what activities to offer. Of course, teachers should be adequately and sufficiently educated so that their offer does not include their personal views. This primarily refers to teachers and pupils' prejudice against certain extracurricular activities. The most common prejudice refers to extracurricular activities concerning natural sciences, especially mathematics and its familiar subjects. The prejudice affects all population from pre-school children to college students. One can often notice gender dominated extracurricular activities, it is believed that girls are usually better at activities connected to social sciences and boys at natural sciences.

From 1998 to 2008 first-year students of the Faculty for Teacher Education have had a choice of three modules in their further course of studies: developmental module A, informatics module B and foreign languages module C. The offer of modules is directed to labour market demands, especially the one regarding mathematics and informatics fields. During this course of time, the least percentage of students chooses the informatics module, i.e. the one belonging to mathematics and informatics fields. Hence, the students are still not aware of the labour market needs despite the existing initiatives on the faculty's behalf.

Pupil's choice should be independent from teacher's offer because the chosen extracurricular activity should create a feeling of satisfaction and desire to explore and create. Pupil's choice is also influenced by their general school success, especially by the grades that they have in the subject related to a certain extracurricular activity. Our research has shown that pupils usually choose extracurricular activities according to subjects in which they have excellent grades.

There should be an endeavour for an extracurricular activity to be a result of pupils' wishes, teachers' abilities and the labour market demands. It is necessary to continuously educate teachers and provide them with professional development in order to achieve the satisfactory balance among teacher's offer, pupils' choice of the extracurricular activity and market demand. School is obliged to ensure the working conditions for teachers (temporal, material and spatial conditions) to organise an extracurricular activity, as well as support in life-long learning.

Research methodology and results

In December 2008 a questionnaire was conducted about the participation of fourth-grade pupils in extracurricular activities, and about the teacher's choice of extracurricular activities. It was conducted in 6 primary schools in Osijek, on the sample of 107 pupils and 21 teachers.

The research aim was to establish whether there were differences between the pupils' choice and the teachers' offer of extracurricular activities, as well as to find out the possible reasons for pupils' choices and the consequences of these choices. We have grouped the activities into four fields: language and arts, physical education, science and ecology, and mathematics and informatics. In the analysis of the research results we have included data about the number of students enrolled in natural and social science studies. We have also used the data about the current needs on the labour market for experts with mathematical and informatics knowledge and skills. Thereby we have tried to find out whether there is a connection between pupils' choice of an extracurricular activity from a certain field and the future choice of university and occupation. We have also looked at the differences in the choice of extracurricular activities between students of different gender.

Data analysis of extracurricular activities offered by teachers (Figure 1) shows that the offered activities are most commonly the ones from the following fields:

- language and arts (activities: reciting, drama class, art class, scenery class, tamburica* class), 46,15%
- mathematics and informatics (activities related to mathematics and informatics technology), 26,92%

* i.e. a very popular Croatian national string instrument (*translator's remark*)

- science and ecology (activities related to science and ecology), 15,38%
- physical education (activities: rhythmic, dodge ball, small field football), 11,54%

Figure 1 shows that majority of teachers offer activities from the language and arts fields, whereas the activities from the mathematic-informatics field and science and ecology fields are much less represented. The lowest percentage of teachers offers activities from physical education field. To conclude, most teachers offer extracurricular activities from the field of social sciences.

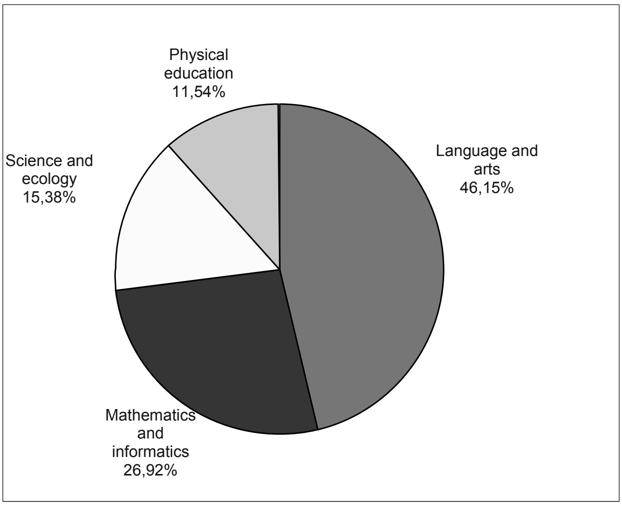

Figure 1. Teachers' offer of extracurricular activities.

Data analysis of pupils' choice of extracurricular activities has shown the following percentage of representation:

- language and arts, 38,04%
- physical education, 36,26%
- mathematics and informatics, 19,94%
- science and ecology, 5,76%

It is obvious (Figure 2) that most students choose activities from the language and arts fields, as well as the physical education field, whereas fewer activities are chosen from mathematics and informatics fields, and the fewest from the fields of science and ecology. Therefore, a higher percentage of students choose extracurricular activities from the field of social sciences, as well.

In relation to examinees' gender, we have observed certain differences in the choice of extracurricular activities according to fields. Male pupils choose extracurricular activities from the field of physical education, and mathematics and informatics, whereas female students are more prone to choose activities from the language and arts fields. If we take a look at their choices and compare them to the science field, we can see that the percentage of male pupils choosing extracurricular activities which belong to natural sciences is 6,42 higher than the percentage of female pupils choosing them.

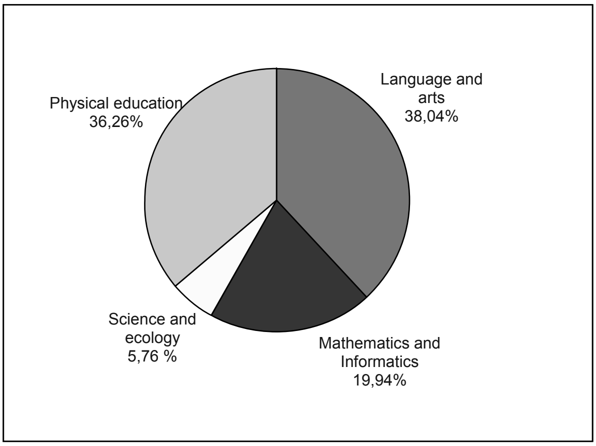

Figure 2. Students' choice of extracurricular activities by field.

The choice of extracurricular activities from certain fields through different grades shows that pupils' interests for these fields oscillate. Throughout the grades, what varies most is the interest for extracurricular activities in the field of science and ecology, i.e. there is a negative correlation.

Moving to the upper grades, pupils considerably lose interest for extracurricular activities belonging to the field of science and ecology (Figure 3).

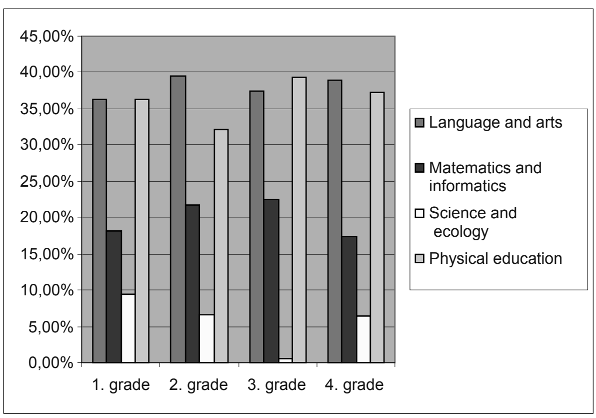

Figure 3. Students' choice of extracurricular activities throughout grades.

Considering the pupils' gender, we have done the same analysis of pupils' choice throughout grades. The female pupils' results reveal a positive correlation between interest and choice of extracurricular activities from the language and arts fields, and a negative correlation between interest and choice of extracurricular activities from the physical education field, and science and ecology fields. As female pupils move to the upper grades their interest for the above stated fields rises or falls, in the order mentioned.

The male pupils' results show that there is a positive correlation between interest and choice of extracurricular activities from the field of physical education, and negative correlation between interest and choice of extracurricular activities from all other fields. Moving to the upper grades, the interest of pupils does not grow for any other field but physical education.

If we compare the relation between the offer and the demand for extracurricular activities, we can see that there is an imbalance between the teachers' offer of extracurricular activities and the pupils' choice by fields (Figure 4). Teachers' offer of extracurricular activities from the field of science and ecology is 9, 62 % higher than pupils' demand in that field. On the other hand, pupils' demand for the activities from the field of physical education is 24, 72 % higher than teachers' offer.

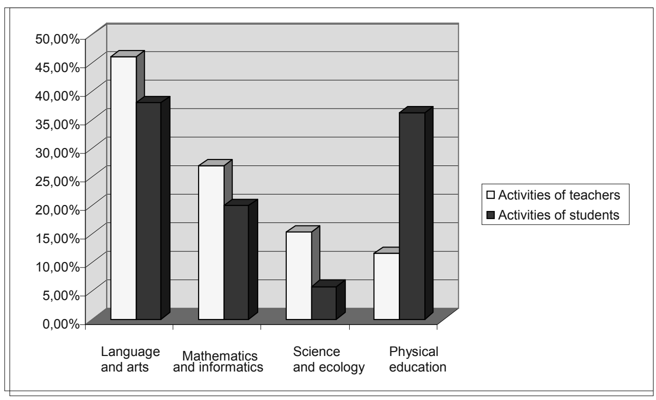

Figure 4. Relation of offer and choice of activities (Teachers/pupils).

We can conclude that pupils' choice is still partially conditioned by teachers' offer. So, the pupil does not have a big influence on the offer of extracurricular activities, and cannot completely pursue his/her free choice of preferred extracurricular activities.

Furthermore, pupils' low tendency to choose extracurricular activities in the field of mathematics and informatics, and negative attitudes towards personal computers and subjects related to mathematics could generally be brought into relation with the choice of universities related to natural or social sciences, as well as the choice of occupations which do not require the knowledge of complex and higher mathematics. The overall process is repeated cyclically because the former teachers' choice of extracurricular activities is created in the same way students choose extracurricular activities today, and later faculties and future occupations. Figure 5 shows the percentage of pupils enrolled in natural and social sciences universities in the academic year 2008/2009 in Croatia.

We can conclude that there is a significant difference in the choice of natural or sciences universities in favour of social science universities (92,92% of all students enrolled) ([6]).

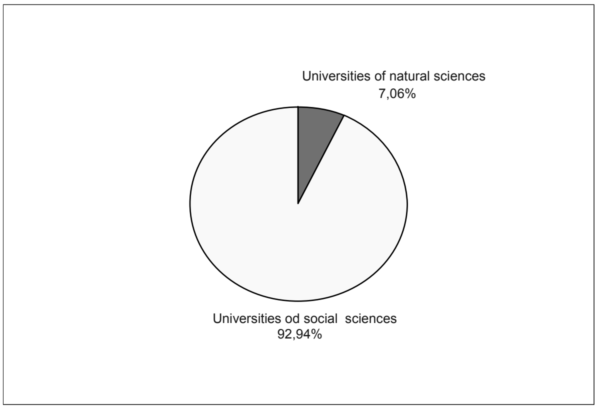

Figure 5. The percentage of students enrolled in natural and social sciences universities in 2008/2009.

The situation is reflected on the labour market, where consequentially there is a lack of experts in natural sciences, especially the ones with knowledge and skills in mathematics and informatics. According to the data from the Croatian Institute For Employment, the number of experts needed in mathematical and informatics field in 2008 was 830, which is 47,6 percent of the overall working positions offered ([7]). This means that almost half of the offered working positions in education are related to the experts in mathematical and informatics field.

Conclusion

The research results have confirmed our hypothesis that the teachers' offer of extracurricular activities is not completely in coordination with pupils' wishes. To be more concrete, looking at the choice of activities by teachers and pupils in the field of physical education and science and ecology, the results of the chi-square test have shown that we can claim with the 95% certainty that there is a statistically important difference in the choice of activity. Also, teachers' offer and pupils' choice of extracurricular activities are not in coordination with labour market demand.

The results of chi-square test show that there is a statistically important difference in the pupils' choice of activities in natural and social science at all levels of reliability. It is clear that pupils prefer activities from the field of social sciences, whereas the activities from natural science are still unfairly neglected. This leads to the conclusion that those prejudices that exist towards natural sciences and mathematics in our society are passed down from generation to generation and sometimes from teachers to pupils, so they remain present from the pre-school age to the university enrolment and later occupation choice. The gained data in the year 2008/2009 are a clear signpost that a significantly bigger number of pupils choose social science studies rather than the natural science studies, in spite of the fact that the labour market requires education experts in the field of mathematics and informatics. The research results indicate the need for forming teams of experts who shall motivate pupils and students to direct their interests towards the activities needed on the labour market. The results will be helpful in the process of coordination and renewal of the curriculum of teacher education studies.

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2.
About learning outcomes in teaching mathematics to pupils

Mathematics learning built on pictures

Katalin Munkácsy

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Abstract. We studied learning of mathematics in small schools of remote areas of Hungary. Disadvantage pupils speak other social dialect than the teachers, that because even the gifted students can not reach enough good results. We used pictures, ppt presentation to help for the students to understand the context of the mathematics tasks and to help for the teachers to use int he classroom relatively new didactics tools of education, like group work, ICT, narrative elements of learning process.

(Our teachers knew very well these methodical tools, but they thought that one can use them only in top10 schools.)

We taught some elements of history of mathematics, too. Egyptian number writing was one of the best tool to motivate pupils.

We could see that every student reach better results and the gifted pupils could solve inverse tasks, some of them were hard also for my students.

Keywords: new methods of mathematics education, visual approach, “hands-on” activity, historical thinking, gifted pupils, disadvantage pupils

Introduction

We studied learning of mathematics in small schools of remote areas of Hungary. Disadvantage pupils speak other social dialect than the teachers, that because even the gifted students can not reach enough good results. We used pictures, ppt presentation to help for the students to understand the context of the mathematics tasks and to help for the teachers to use in the classroom relatively new didactics tools of education, like group work, ICT, narrative elements of learning process. (Our teachers knew very well these methodical tools, but they thought that one can use them only in top10 schools.)

We taught some elements of history of mathematics, too. Egyptian number writing was one of the best tool to motivate pupils.

We could see that every student reach better results and the gifted pupils could solve inverse tasks, some of them were hard also for my students.

First I would like show for you our educational program, which we elaborated in the frame of international NEMED project. In the second part of my talk I would like work together with the audians as a computer aided workshop.

The public opinion and part of the literature regards low motivation as the cause of poorer learning results among underprivileged children. Sociological research provide alternative answers for the problem: the primary causes for school failure are communication disorders and low motivation is already a consequence (Tuveng, E. – Wold, A. H., 2005).

There may be significant differences in language use even among those people who speak the same language. Linguists describe these deviations as diverse social dialects. Language differences may be significant not only among people from various geographical regions but from different social classes as well. Earlier Bernstein marked a qualitative difference between two ways of language use – elaborated and restricted codes – but present day sociolinguistics considers different social dialects equivalent, hence they equally ensure communication between people. In spite of this children speaking dialects other than the standard used at school face serious problems. At conventional schools only students are expected to adapt without being given any help. Children, partly because of their disadvantages, cannot ask for help; thus the teaching-learning process is constantly disturbed by the mostly unrecognized communication barriers. We would like help talented pupils to get possibility to reach specialised education projects (Balogh, 2004, 2007).

Aim

In our research we study how these disorders occur and by what means is it possible to reduce them.

Metod

Our research method is participant observation. The conventional observation-based diagostic survey in this field is inappropriate because it is communication that is mostly disturbed during classroom observation. It is difficult to notice disorders only by observation; and the solution of occurring disorders are usually postponed after the observers are gone. That's why we collaborate with educators on the development of the teaching-learning process and we design each step as a reflection to the observations. With this method we recognize the problems and the possible solutions at the same time.

To collect information about motivation of the pupils we used OMT test (Kuhl, 1999).

W adapted new results of Hungarian mathematics didactics (Ambrus, 2004, Czeglédy 1992, Szendrei 2005).

Learning theory background

In the case of underprivileged children it is a common method to reduce the curriculum content and the requirements. In contrast to this we wanted to offer the children an elaborated, high standard curriculum that meets the optimal requirements of the syllabus. We ensure the children's understanding by providing a substantial amount of help. Our primary task is to clarify the context of the exercise without solving the problem on the behalf of the students. In our research we apply Bruner's theory of learning (Bruner, 1965 and 1991); or to be more precise we build upon the idea itself and other achievements resulting from the development of the same learning theory (Tall, 2005). The principle of our work is that it is required to bring the whole process of learning into the classroom; not only its third, symbolic level, but the children's everyday experiences and the complicated middle class language use, as well. It is the active pedagogical intervention that bridges the gap between experience and language. The familiar everyday activities are performed in the classroom in a way that enables us to easily gain mathematically relevant experiences and reach conclusions.

Population

Our survey is done in multigrade (mixed age pupils in the same classroom) schools. Here the research is up-to-date from a sociological perspective, since the need for the economy of education and the preservation of the population of the villages would turn the hard situation of the small rural schools into the opposite direction. From a pedagogical viewpoint the fact that most of the children are underprivileged makes the situation clearer because there is no stigmatizing effect which would place certain students in the role of the 'bad' student which actually happens in the normal majority schools. Therefore emotional conflicts and self-esteem disorders occur less frequently; there is more opportunity to observe and shape cognitive processes.

Sample

First part: In the frame of international research project, NEMED we worked together with 16 schools.

Second part: the work continued with 4 schools.

Learning and development program in mathematics

Our project contains an integrated art program and different afterschool activities.

Our aims in mathematics education:

- Justifying the existence of communication barriers
- Elaborating and testing the development program

Principles

- On the course of learning children face problems in a way that they are expected to use their problem-solving skills (Pólya) and they are led to the recognition of higher level mathematic correlations (Varga Tamás).
- Aspects from the history of mathematics is taken into consideration.
- A wide range of mathematic topics are chosen. The topics are connected to the curriculum but they are set up to allow an insight into deeper mathematic correlations. Due to the lower level of basic skills the arithmetic, logical and number theory tasks are moved to a later stage; the children are directed to the understanding of mathematical correlations by space geometry and function-related data management problems.
- Image driven learning: during the class the children watch a slideshow; the methodological instruction focuses on the images, reflects to the activity, and points forward to the symbolic level.
- Computer use: communication device and information source.
- Group work.
- Profound effects on communication.

Results

- We had to distinguish preparedness for school from preparedness for learning. Preparedness for school expects a fairly rich vocabulary, basic counting skills knowledge of the function of reading-writing (experiences on the basis of which the children know that to write and to read is good and useful), monotony-endurance – as a basic condition of working together; and which is characteristic of most of the Hungarian students. Moreover in the case of middle class circumstances handicap in this field usually cooccurs with mental injury to a certain degree.
- Preparedness for learning embraces curiosity, the ability to pay attention, the sufficient state of mental development in order to learn how to write, the ability to express own experiences (Bruner).

According to the OMT test our students also possess the inner conditions for learning (in general we knew this, but it can be seen in this case as well), but in comparison a large percent of them show poor results (e.g. on the basis of their style of writing)

- often, the students do not understand the words the teacher says but they still not ask questions
- as the Norwegian research also points out the phrasing of the problem and its expression would require much better communication skills than we can expect in this age; in this case lower communication skills are proven
- the word and concept inventory of the children is much smaller than it is presumed in the curriculum; but if there is sufficient support available they are curious and have much pleasure in learning new words

- the children were willing to do the activity-based problem solving tasks in the playful, fairy tale-like context; the main aids for this were the powerpoint slides
- their teachers saw through the mathematic content of the teaching process and applied their experiences in other fields in a reflective way
- every observed student was very uncertain in the meaning of the concepts on the borderline of mathematics and the everyday life (e.g. edge, face, pick). In our case study, eventually, as a result of the problem solving embedded in activities all of the students reached the correct calculation algorithm, while the most gifted children could even solve difficult inverse problems on their own, correctly.
- In the school we should stick to the obligatory content of the syllabus; and during the teaching-learning process find occasions to discuss the occurring problems in a more profound way, develop the need for these discussions and find opportunities to take part in extracurricular activities outside the school.
- the ICT, the group work and the narrative were proven to work
- a community of teachers through direct and virtual relationships was established and worked well.

Conclusions

- The children welcomed the program. Their motivation became more concrete. As it was observed previously their personality has a rich motivational system and a desire to achieve good results. Now this has expanded to classroom learning, and it was a joyful experience for most of them and they can't wait for the program to continue.
- The educators took part readily in the development process, they engaged in the program in a creative way; they can apply various aspects of their experience in other fields of their work. According to the teachers some methods – such as ICT, group work – previously supposed to be as extraordinary events of teaching are indeed applicable in the given difficult circumstances and they successfully employ them.
- The long-lasting effects require that the well-chosen parts of the curriculum should be processed by outsider experts and the educators should be given the opportunity to consult continually with them to discuss the problems that occur during their work.

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Through games into the world of probability

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Abstract. Probability is a mathematical content which school children of Slovenia started to learn rather late in comparison to other countries (in secondary schools), which was in addition only on a formal level in most secondary schools. The new curriculum of mathematics for the nine year primary schools introduces this mathematical content already in the primary schools.

Today mankind lives in a rapidly changing world and has to face new and uncertain situations more and more often. We have to make sure that children, the future adults, get ready for this world in the way that they shall be able to interpret it critically and act within it. We therefore need an alphabet of probability which demands a special way of thinking alien to deterministic way of thinking which is prevailing in our schools. Also those adults who have gone through that kind of schooling quite often hit at the difficulties in understanding the basic concepts of probability, since the two-valiant logic (right/wrong, true/not true) most often fails. With our research we have come to a conclusion that school children already at the lower primary school class level accept and understand very well the most principal concepts of probability through play at the intuitive and experience level.

Learning of probability in primary schools is not explicit and formal, but it is rather systematic gathering of experiences, on the basis of which we can later on much easier teach probability (at secondary schools). We therefore do not speak of formal definition of probability within the curriculum of mathematics for primary schools, but rather prepare pupils for later mathematical analyses of chance events through didactic games and logically graded other activities. Pupils describe what they consider possible or impossible; they establish differences between certain, chance and impossible event; they compare probabilities of chance events among themselves; in simple games of chance they establish logic hypotheses and try to back them up with experiences. . . Pupils are supposed to acquire experiences through chance events and develop capabilities of anticipating them through play. From the aspect of accepting uncertainty pupils are expected to move from the level of anticipation of an event to the level of comparing the probability, before they comprehend the statistical and classical definition of probability later on (in secondary schools).

Keywords: nine year primary school, didactic games, activities, learning of probability, certain, impossible and chance event

Introduction

We do not teach probability in primary schools in an explicit formal way, but gradually by learning of probability through the acquisition of experiences which are useful for children later in secondary schools when they learn about formal contents of probability. From the teaching aspect the area of probability is very demanding. Despite non-disputable lessons secondary school children and students have twisted ideas of probability. In the primary school mathematics curriculum we do not speak of formal definitions of probability during lessons, we do not either mention its classical or statistical definition. We do not even calculate probability but we rather prepare school children by using intuition and ludism to help them know later on in their further education how to mathematically analyse chance events. As in case of combinations we also here in these contents do not surpass the level of experience. Pupils are expected to acquire experiences with chance events by gradually increased activities and this way obtain concepts, principles and capacities of anticipation in such chance events which is in the today's world full of uncertainty and unpredictability a very important objective. People have to know how "to face" them, how to anticipate them and to know how to make decisions among various alternatives and eventually know how to solve problems which cannot be solved by two-valiant logic and as Fischbein says: "to develop thinking which is different from deterministic thinking" (Fischbein, 1984, p. 35).

Learning objectives for probability in the first triad

At the beginning of their schooling (in the first triad) pupils are expected to gain through games and various other activities the following objectives:

- to describe, what is for them possible or impossible ;
- to differentiate between certain, chance and impossible event;
- to use by way of practical activities (throw of dice, lot, toss of coin) the following terms in a logic and consistent way: possible, impossible, don't know, maybe, it is possible, it is not possible, accidentally, less probable, more probable;
- to compare the probabilities of different events among them;
- to set logic hypotheses in games of chance and to try to support them with experiences;
- to put down the results of chance events (at the throw of the dice, and toss of the coin) into a table with a histogram (Cotič, 1998).

Levels which lead to classical definition of probability

It is not enough to explain only why "probability" needs place in the new curricula of mathematics, but we also have to show in what way it has to be introduced. The scheme below shows what levels school children should go through in order to understand the classical and statistical definition of probability in secondary schools.

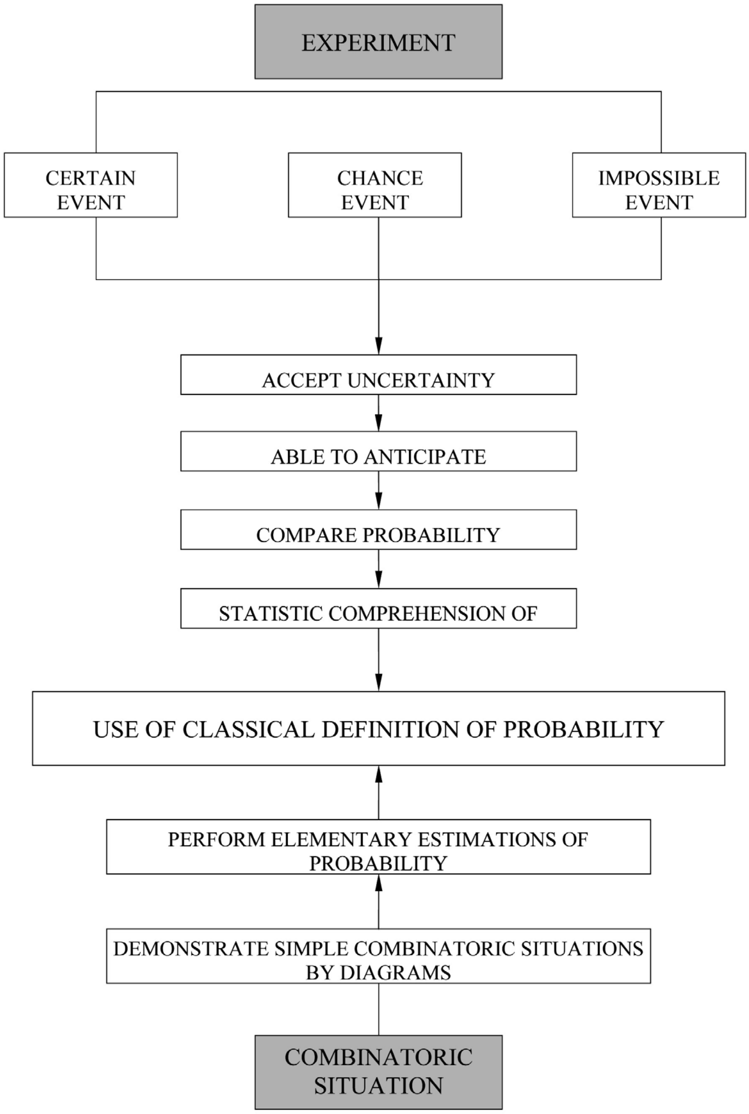

Figure 1. Levels of probability.

Accept uncertainty

As a first step into the world of probability children should be led towards the acceptance of uncertainty without being disturbed: thus towards the concept of chance event. According to the Piaget's development theory 6 to 7 years old children do not only lack a clear idea about the probability of a certain event but they also do not differentiate between chance and non-chance events, although they have experienced them by themselves. Children attribute purposefulness also to objects and phenomena not only to humans, for instance: "The sun is in the sky to warm me up; the sea has waves to move ships. . . " Nothing happens by chance, everything is intended, planned and determined. The acceptance of a chance event is not only a cognitive process, but for children also an affectionate problem, as they accept every uncertainty with anxiety. Children should gradually become aware that an event is not only certain or impossible but also accidental. The easiest way to achieve graduality is through various games where an important part is "luck" (Ludo, Tombola. . .) and the capability of choosing among different possibilities the one that gives the biggest probability to win (Spinning Top, Playing Cards. . .).

Ability to anticipate

The concepts like "certainly", "accidentally" and "impossible" should be built up for school children first of all by way of different activities in which they are involved themselves. Many experts of mathematical didactics have been describing this as a subjective probability which is the origin of understanding the empirical and mathematical probability later on. In their every day lives children correctly use the words like: for sure, not possible, possible, by chance: "In the afternoon I *may* go to the cinema. The capital of Slovenia is *most certainly* Ljubljana. *It is impossible* to be on the planet Jupiter in an hour."

Figure 2. Possible, impossible.

Those concepts are so simple that the adults can be sure that children understand them as well, however, they forget that children most often equalise *impossible* with *wrong*, and *maybe* with *certainly* or *right* (Fischbain, 1975).

It is not enough to offer pupils only situations which happened independently from them (e.g., observing the weather, counting certain types of cars. . .), but above all the situations which schoolchildren can master themselves and which offer opportunities to be repeated under the same conditions (Throw of Dice, Lot, Toss of Coin. . .) in order to be able to make conclusions after several repetitions (Fischbain, 1985):

- If I throw the dice, I will most certainly "hit" a number lower than seven.
- It is impossible for me to throw ten.
- I may possibly throw one.

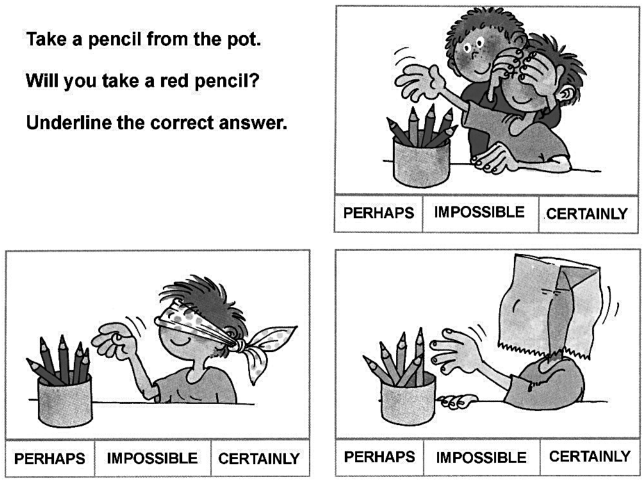

Figure 3. Perhaps, impossible, certainly.

To accept uncertainty means also to accept the fact that the anticipated event does not happen. It is therefore necessary for children to perform such activities which give them a chance to foretell the probability of events in uncertain situations. They afterwards check their forecasts and find out that it is not necessary that their forecast comes true. As we have already mentioned before children should accept a result without being excited regardless, whether what he anticipated, happened or not. Here are two examples:

Example 1: *You have in your plastic bag* 15 *black and* 6 *yellow balls. What colour may be the ball pulled out of the bag? What colour ball have you pulled out?*

Example 2: *In the bag there is* 1 *yellow,* 1 *red and* 1 *blue dice. Forecast what colour may be the dice when you pull out for the first time; what colour dice is impossible when you pull out for the second time? And what colour will the dice most certainly be when you pull out for the third time? When doing that do not put the dice back into the bag. In what order have you pulled out the dices?*

According to Piaget and Inhelder (1951) children, who are in their operational and concrete period, are neither capable of separating events between certain and accidental nor formulating predictions respecting experiences of previous analogous situations. Their criteria are usually based on the criteria of repetition (if the last pulled out dice is red, also the next one will be red) and on the criteria of compensation (the colour of the pulled out dice should be of the colour which has not yet been pulled out). That type of children's behaviour is determined by affectionate motivation, by their trust in the correctness of their choice, by their need of correctness as well as their need for order and by attributing purposefulness to the elements (in our case the dices), (Piaget, Inhelder, 1951). It is therefore very important that school children repeat their attempts several times under the same conditions because this is the only way they become aware that it is possible to formulate forecasts which are not subjective (Cotič, Hodnik, 1995).

Compare probabilities

At the beginning the pupils' concept of probability develops as the capability of mastering chance events. Of course this development is based on experience since children differentiate between certain, chance and impossible event exactly on the basis of experience. Only after that they will become aware that among chance events some are more probable and others less probable, or equally probable. This way pupils are introduced into the qualitative assessment of the probability of a chance event.

In the case of plastic bag which contains 20 red, 5 white and one black small balls, there are events which are less probable, however probable (pulling out black ball) and very probable and yet not certain (pulling out red ball). It is therefore urgent to propose to children games which offer a chance to compare chance events having different probabilities to take place. Those are, as we already know, games with dices, cards, coins... Pupils, of course, build their criteria of forming their hypotheses slowly and gradually by repeating their attempts for several times under the same conditions. We have to point out that the emphasis in those activities (attempts) is on team work (consultations, distribution of work, coordination in the group, communication in the group).

This way they are slowly introduced into the statistical comprehension of probability.

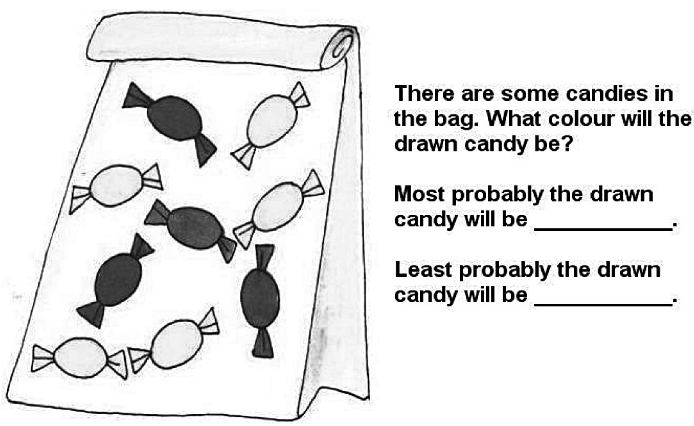

Figure 4. Pupils compare probabilities.

Statistical comprehension of probability

Teachers should for example bring to the classroom a vessel with sweets (or tokens, balls or. . .) of different colours. Pupils are expected to count the sweets of particular colour before they put them into the vessel. On the basis of their knowledge on how many sweets of a certain colour are there in the vessel, children should put hypotheses on the probability of different events. Teachers afterwards check together with pupils the answers in an empirical way. Not looking, each pupil should pull a sweet out of the vessel and write down the result into a table. Pupils then return sweets into the vessel. Pupils should repeat the experiment several times under the same conditions (for example 100 times). With teachers' help pupils shall find out that after more and more attempts the relative frequency of certain event is approaching a certain number. We measure the probability of the event by that number. Of course, pupils shall formulate their findings in their own words.

Conclusion

In the area of probability we therefore derive from the subjective probability in the first triad. It is important to talk to children about the subjective experiencing of probability and harmonise the descriptive ways of expressing it. During the lessons of mathematics of the first triad pupils have to reach their cognition through reflective experiences, which means that for certain accidental phenomena certain regulations are true. Hence we transfer to the empirical comprehension of probability. In later years of schooling pupils should slowly and gradually meet with slightly more demanding assessments of probability, where first of all a combinatoric problem should be systematically solved in order to forecast the result on the basis of the analysis of combinatoric display and carry out the elementary assessment of probability already leading towards the classical definition of probability. Pupils this way slowly and progressively learn about simple rules for the world of probability which in the beginning seemed to them entirely unpredictable or even uncertain. While teaching probability, we have to bear in mind that the concept of probability has in mathematics always occupied a special position, since it is difficult to determine it by rigour which is required almost by all other mathematical disciplines. Even the great mathematician Laplace wrote about probability the following: "It is, indeed, unbelievable that the discipline whose roots originate from the teaching of hazard and games of chance has become one of the most important mathematical disciplines."

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Producing plane figures and selecting plane figures in the fourth class of lower primary school

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(Poster)

Abstract. In Hungary during the first four years of geometry teaching the basis is laid down and the aim in the initial stage of primary school is to develop the abilities of learners for being able to gain knowledge on their own. The basis of learning geometry lies in inductive cognition gained from experience. Starting out from the concrete, gathering experience from various activities can lead to the formulation of general relationships.

The evolvement of some geometrical concepts, such as squares, rectangles, parallelism, perpendicularity, symmetry have been examined in an educational development experiment conducted with fourth class pupils, the aim was to put the van Hiele model of geometry teaching into practice. In the lesson plans what we aimed at was that children could be able to discover the geometrical concepts first on the basis of concrete experience in real games and activities then visually (drawing) and finally at an abstract level.

The presentation aims at presenting exercises connected to producing plane figures and to selecting plane figures. We are going to outline their working up, as well as some individual solutions and thoughts. We also intend to draw the attention to several typical problems and errors that occured in the process of thinking during the educational development experiment.

Keywords: mathematics teaching, producing plane figures, selecting plane figures

In Hungary during the first four years of geometry teaching the basis is laid down and the aim in the initial stage of primary school is to develop the abilities of learners for being able to gain knowledge on their own. The basis of learning geometry lies in inductive cognition gained from experience. Starting out from the concrete, gathering experience from various activities can lead to the formulation of general relationships. The third educational principle laid down by Farkas Bolyai also emphasizes the importance of starting with the concrete: "(The teacher)... should always start with what learners can see and touch, and not with general definitions (it is not grammar that the first utterance is based on) and he should not torture prematurely with longwinded reasoning.... We should start with geometric shapes and reading... and we also should get out of the sheet..."

Among others producing plane figures has been examined in an educational development experiment conducted with fourth class pupils, the aim was to put the van Hiele model of geometry teaching into practice.

According to *P. H. van Hiele*, the process of acquiring knowledge in geometry can be divided into five stages.

(*Level 1*) At the level of global recognition of shapes children perceive geometrical shapes as integral whole. Children easily recognize various shapes according to their forms, they also learn the names of shapes however, and they do not see the connection between the shapes and their parts. They do not recognize the rectangular prism in a cube, the rectangle in the square, because these seem to be totally different for them.

(*Level 2*) At the level of analyzing shapes, children break down the shapes into parts, and then they put them together. They also recognize the planes, edges and the vertexes of geometrical forms, and the plane figures of geometric shapes which are delineated by curves, sections, and dots. At this level a great importance is attached to observation, measuring, folding, sticking, modeling, laying parquet, using mirrors etc. By means of these activities children can recognize and list the properties of the shapes (parallel planes or sides, perpendicular, the properties of symmetry, right angles, etc), but they do not come up with definition and they do not see the logical relationship between the properties. Even if children perceive what squares and rectangles have in common, it cannot be expected that they could draw the conclusion that squares are actually rectangles. At this level children are not able to notice the relationships between shapes.

(*Level 3*) At the level of local logical arrangement learners can see the relationships between the properties of a given shape or between various shapes. They can also come to a conclusion from one property of shapes to another. They also realize the importance of definition. The course of logical conclusions is however determined by the textbook and the teacher actually. The demand for verification is being started, although it applies only for shapes. At this level the squares are considered as rectangles.

Level 4 (aiming at a complete logical setup) and *level 5* (axiomatic setup) are to be reached in secondary and tertiary education.

In the lesson plans what we aimed at was that children could be able to discover the geometrical concepts first on the basis of concrete experience in real games and activities then visually (drawing) and finally at an abstract level.

Activities with concrete objects

1. Cutting the rectangle into two along the diagonal. Producing other plane figures by fitting the triangles gained in this way, and naming them. Gathering experience on plane figures and describing them.
2. Producing plane figures cut out from paper without restriction, and describing their characteristics.
3. Producing planes figures from the 2, 3, 4 and 6 regular triangles from the set of logics, which consists of 48 various plane figures, which can be red, yellow, blue or green. Their sizes are, small or large, their shape can be circle, square or triangle, their surface can be smooth or there is a hole in them.
4. Producing various plane figures from paper strips by one cut. Naming them and describing their characteristics and shared characteristics.
5. Cutting general rhombus from rectangle, its characteristics.
6. Cutting general deltoid from rectangle, and its characteristics.
7. Making rectangles and then the "frame" of a general parallelogram from six match sticks. Comparing the characteristics of rectangles and parallelograms and highlighting their differences.
8. Making squares then general rhombus from four match sticks. Comparing the characteristics of squares and rhombuses.
9. Making 2 rectangles, a pentagon and a triangle, a triangle and a quadrangle, 2 quadrangles and 2 triangles from a rectangle by one cut.
10. Selecting plane figures according to given characteristics.

Tasks at visual level

1. Drawing squares on square grid.
2. Drawing various quadrangles on square grid.
3. Drawing various triangles on square grid.
4. Drawing quadrangles of given characteristics.
5. Drawing plane figures which have no symmetry axis, and which have exactly 1, 2, 3 and 4 symmetry axes.
6. Drawing plane figures according to given requirements.

In a task related to polygons Kornél raised the following question:

> *'Are polygons which have two angles? '*
> The teacher asked.
> *'Can you draw anything like that?'*
> *'Yes, I can'*, said Kornél and he drew a semi-circle.
> Looking at the drawing the teacher asked:
> *'Are polygons delineated by curves?'*
> Kornél thought about it for a while and responded:
> *'Not really, only by straight lines. Then this is not a polygon.'*

Abstract level

Twenty questions is one of the favourite games among children, which is also suitable for racticing the characteristics of solids and plane figures. During a game what the children had to guess was the symmetrical trapezoid. These are the questions and answers of a game:

— *Is it a quadrangle?*
— *Yes, it is.*
— *Are the opposite sides parallel?*
— *No, they aren't.*
At this point the teacher realized that child need some help.
— *We can as the question in another way: Are all the opposite pairs of sides parallel? And I said no, they aren't.*
— *Does it have parallel sides?*
— *Yes, it does.*
— *Does it have a right angle?*
— *No, it doesn't*
— *Are its sides of the same form?*
— *We can also put the question like this,* said the teacher. *Are all the sides equal? No, they aren't.*
— *Does it have sides of equal length?*
— *Yes, it does. Anyone, who already knows it, can draw it. Those who don't, keep on asking.*
— *Is it symmetrical?*
— *Yes, it is.*
— *Does it have one reflection axis?*
— *Yes, it does.*

Then we drew the plane figure on the board.

In lower primary (grades 1–4) geometry teaching learners can reach the first two stages of geometric thinking according to the van Hiele levels. It is not feasible to reach level 3 by the completion of lower primary. Although sets of concepts are established, but there is no relationship whatsoever between them. Actually children do not recognize the logical relationships between the characteristics of a shape and they are not able to draw conclusions from one characteristics of a shape to another.

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The concept of zero among 7 – 12-years-old children

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(Poster)

Abstract. During the past few years I invegestigated for several times the concept of zero among the teacher trainees of Baja, and typical errors occurred. This academic year I have made investigations among the pupils of a first, second, third, fourth, fifth and sixth form class in the Training School of Eötvös József College. I have been interested in the way the concept of zero developed within the teaching process, and I intend to compare these results with the data gained earlier.

A written survey has been done among the pupils of a first, second, third, fourth, fifth and sixth form class in The Training School of Eötvös József College. The test sheets dealt with basic operations in which one of the terms or factors was zero, pupils were aked to write the sign of the operation between two numbers that were the same in such a way as the result should be zero, with true/false statements connected to zero, and students were asked to write an essay about zero. The aim of this presentation is to report about the results of the above mentioned survey.

Keywords: mathematics teaching, zero

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Mathematical requirements in PISA assessment

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Abstract. This work presents the results of mathematical-didactical analysis of PISA questions in the field of mathematical literacy. The analysis is based on 26 publicly available PISA questions that represent a profile of PISA mathematical questions. The results of the analysis are furthermore compared to the results of PISA testing in Croatia in 2006.

The intention of the research is to determine mathematical requirements in PISA assessment, as well as to establish the extent in which these requirements match the requirements of traditional mathematical education in Croatia. In the light of these findings, an additional goal is to explain the results of the conducted PISA testing in the field of mathematical literacy.

The analysis shows that PISA requirements are in many ways different from the requirements in Croatian mathematical education, with respect to the content, competences, complexity and in the forms of questions. Content-wise, PISA questions emphasize fields of mathematics such as statistics and probability, but Croatian pupils who took part in PISA testing in 2006 have not encountered those subjects in any part of their curriculum. Competence-wise, PISA emphasizes interpretation and argumentation, while Croatian mathematical education is dominated by operationalization and automatization. We argue that the differences between requirements in PISA assessment and the Croatian mathematical praxis have lead to poor results of Croatian pupils on PISA testing in the field of mathematical literacy. In the conclusion, we suggest the guidelines on what components of PISA tests could lead to improvement in the field of mathematical education in Croatia.

Keywords: analysis, PISA questions, PISA testing, mathematical literacy, mathematical education, mathematical requirements

Introduction

In the past few years we could hear mentioning PISA assessment in the public and educational circles, mostly in the context of national ranking on the international scale. PISA is a Programme for International Student Assessment developed by the OECD participating countries (Organization for Economic Co-operation and Development). The first PISA assessment took place in 2000, after that it takes place every third year and examines reading literacy, mathematical literacy and scientific literacy among 15-year-old students (OECD, 2003). Each cycle has a "major" domain, to which two-thirds of the testing time is devoted. The major domain in 2000 was reading literacy, in 2003 it was mathematical literacy, and in 2006 it was scientific literacy. In 2009, the major domain will be reading literacy again and so on. Croatia participated for the first time in the PISA assessment in 2006 and was ranked as 36th on the scale of all 57 participating countries in the field of mathematical literacy (Braš Roth et al, 2008). With respect to the OECD average (500 points), this result is considered significantly below the OECD average. The reader can find more information about PISA in Croatia in the book by Braš Roth et al (2008).

Mathematical literacy is described as *"individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen"* (OECD, 2003). In order to properly measure mathematical literacy, in the domain of mathematical literacy three components are distinguished: mathematical contents required for solving problems successfully, mathematical competencies, and mathematical context in which the problems are located. The reader can get more information about PISA mathematical literacy components in OECD (2003), Braš Roth et al (2008) and Glasnović Gracin (2007a, 2007b).

The reason for the worldwide media interest in PISA assessment and PISA results lies mostly in international ranking and competition among nations. On the other hand, experts on mathematics education (De Lange, 2005, Peschek, 2006) emphasize the importance of deeper research of PISA requirements as well as research of national curricular tasks with the purpose of improvement of mathematical education.

Methodical analysis of PISA problems

This work deals with methodical analysis of PISA mathematical tasks and it avoids questions related to competition among nations at PISA assessment. The goal of this research was to analyze mathematical requirements in the publicly available PISA items, and to compare them with the requirements of elementary school mathematical education in Croatia. The analysis examined 26 publicly available PISA test items (OECD, 2007), which altogether consist of 42 mathematical problems. They include all released PISA examples from 2000, 2003 and 2006 and give examples of unreleased PISA mathematical problems.

The systematization basis is taken from "Bildungsstandards", i.e. the Austrian educational standards for mathematics (Institut für Didaktik der Mathematik, 2007). Every test item is classified according to required mathematical content, mathematical activities and its complexity. Also, the analysis included questions about whether a particular task is common for mathematical education in Croatia, and whether it is common in everyday life, i.e. in natural, social and cultural setting in which the individual lives.

The *content* field of this analysis is divided into numbers and measures, variables and functional dependences, geometric shapes and solids, and to statistic representations and parameters. The *mathematical activities* field is divided into representations and modeling, calculation and operation, interpretation and of argumentation and reasoning. The *complexity* field is divided into applying basic knowledge and skills, finding connections, and applying reflection knowledge.

Mathematical requirements in PISA assessment

The results show that the analyzed PISA problems greatly differ from the problems common in mathematics education in Croatia. We find that 22 out of 42 test items are uncommon in Croatian mathematics textbooks. It means that more than half of the analyzed PISA problems are quite different from the school mathematics problems. Furthermore, additional 26% of problems are estimated as partly common for mathematics education (i.e. only according to some components). It means that only one-fifth of given PISA test items are common in Croatian mathematics education.

Deeper analysis showed that some of the PISA problems, which are indicated as not typical for mathematics classes, are more common for physics classes, geography classes, technical classes, puzzle-mathematics and so on. This finding indicates that PISA mathematical literacy exceeds the boundaries of pure mathematics education and that it involves other subjects and fields as well. On the other hand, a high percentage of 95% of analyzed PISA test items are estimated as common in everyday life. This observation is in accordance with the description of mathematical literacy as *"the ability of students to analyze, reason and communicate ideas effectively as they pose, formulate, solve and interpret solutions to mathematical problems in a variety of situations"* (OECD, 2003). This description evidently puts emphasis on using mathematics in many different situations in life. Here we find the discrepancy between the main goal of PISA mathematical literacy and the main goal of "classical" mathematics education. The goal of mathematics education is not only to apply mathematics in everyday life, but also acquiring the abstract mathematical knowledge and competencies, while PISA mathematical literacy assesses only the appliance of mathematics in different life situations. For example, in PISA assessment in 2003, when the main domain was mathematical literacy, there were only 3 problems out of 84 from the field of algebra (Schneider and Peschek, 2006).

As already said, in this paper the PISA mathematical test items are analyzed according to the content, competencies and complexity. With respect to the content, it is interesting to remark that 21% of published mathematical PISA tasks represented the field of statistics and probability. Moreover, PISA assessment in 2006 gave as much as 17 out of 48 problems that represented the field of uncertainty, which fully covers statistics and probability (Braš Roth et al, 2008, p. 161). This means that in more than one-third of given test items in PISA assessment in 2006, a student needed to apply the knowledge of statistics and probability. PISA puts strong emphasis on this field. *"This type of statistical thinking should be used by every individual"* (Braš Roth et al, 2008, p. 133). But statistics and probability were introduced to Croatian curriculum for the first time in fall 2006 (MZOS, 2006) and to a very limited extent. This means that Croatian students who took part in PISA assessment in the spring of 2006 had not studied those subjects in any part of their schooling.

The second big mathematical field, to which PISA gives a lot of attention, is the field of *functions*. Croatian students learn in mathematics classes about this field to a very limited extent and using a fully different approach than it is required in PISA problems. According to Croatian mathematical program, linear functions are taught in the 7th grade, in a very strict environment of analytical geometry. In the 8th grade, the quadratic function and possibly the square root function are presented. This way of approaching and teaching functions in Croatian elementary schools should be reconsidered because the initial tests in the secondary mathematics education show the worse results exactly in the field of understanding functions (Dakić, 2000, Rac Marinić Kragić 2007). On the other hand, PISA offers a completely different approach to functions. The analyzed PISA tasks show that the function graphs come from the real situations, and this means that they do not represent only linear relations. Related to this, PISA puts emphasis on the skills of interpreting graphical representations. Regarding to the school praxis, those PISA problems are more common to physics than to mathematics classes. *The concept of function is mentioned in our schools only in examples of analytical geometry and in no other context, but PISA test items offer a different context and concept"*, wrote in their report the Croatian PISA 2006 working group for mathematics (Braš Roth et al, 2008, p. 161).

We notice a big difference between PISA requirements and the requirements of Croatian school mathematics in the field of expected students' mathematical activities as well. In more than one-third of analyzed tasks students needed the interpreting abilities, i.e. interpreting given mathematics relationships in particular context. We can compare this skill with the interpretation of numerous non-linear functional representations, with pictures of statistical data etc. One half of the analyzed questions required the competencies of elementary computing and operating, while the other half required modeling, argumentation, representation and interpretation. This percentage of required mathematical activities differs a lot from the Croatian mathematical school praxis, which is mostly dominated by operating and automatization, and where one can barely find any argumentation and interpretation. These differences can help in understanding why Croatian students had problems in PISA assessment in 2006.

The required mathematical content and mathematical activities influenced the complexity of particular problems. 27% of PISA problems in 2006 required reflection skills, which consist of posing complex problems, reasoning, original mathematical approach, generalizations etc. Such approach is very rare in regular mathematics classes and is neglected because of the domination of operationalization and solving procedural problems. Thus, it is not surprising that Croatian students have showed remarkably poor results in the field of reflection problems, some of them even significantly below the world's average. Also, the open-answer problems made approximately one-fourth of all analyzed problems. This percentage matches the percentage of open-answer problems in regular PISA testing in 2003 (Schneider, Peschek, 2006). Our students found open-answer problems very difficult because those types of problems require very good understanding of mathematical content as well as very developed competencies like interpretation, argumentation, proving and reflection.

It is also important to mention that PISA mathematical test items are often textually very rich, so they require students' skills of fully understanding the text, distinguishing important and non-important parts in text, reading concentration and verbal competencies.

Conclusion

The results of this analysis and the report of the Croatian working group for mathematics of PISA 2006 reveal major differences between PISA problems and the mathematical school exercises. Those differences surely affect our students' results on the international PISA scale. The analysis of the published PISA tasks shows that OECD emphasizes the rich textual mathematical questions which are situated in everyday situations, and which often require interpretations of graphical representations, mostly of functional dependences or statistical representations. Also, it is required to have skills of modeling problems, interpretation of different mathematical contents, argumentation, and explaining the open-answer problems. The given characteristics of PISA problems don't dominate in the primary mathematics education in Croatia. In the report by the Croatian PISA 2006 working group for mathematics it is suggested that we should take from the PISA assessment those elements which can bring an improvement to Croatian mathematics education (Braš Roth et al, 2008).

According to the content, PISA program suggests a new approach to functions that could be embedded in our mathematics classes, but also in the physics education, and any other subject where students use the interpretation of graphical representations. Also, the probability and statistics field should be more extensively embedded in the whole mathematics education, so that students would have more time for data representation and interpretation, starting already in lower school grades.

According to required mathematical activities and problem complexity, PISA puts emphasis on problems that require skills of modeling situations or interpreting problems that are often connected with reflexive thinking. Such mathematical test items are often combined with open-answer problems because they require argumentation, reasoning etc. This finding can be compared with the estimate of Croatian working group for mathematics of PISA 2006: *"PISA results from the domain of mathematical literacy point that we should put more emphasis on interpretations, reflective competencies, argumentations and verbal expressions of mathematical contents in our school praxis"* (Braš Roth et al, 2008, p. 163).

This analysis and described differences between PISA test items and the Croatian mathematical school praxis can hopefully help in concrete discussions and conclusions about the improvement of quality in mathematical education in Croatia.

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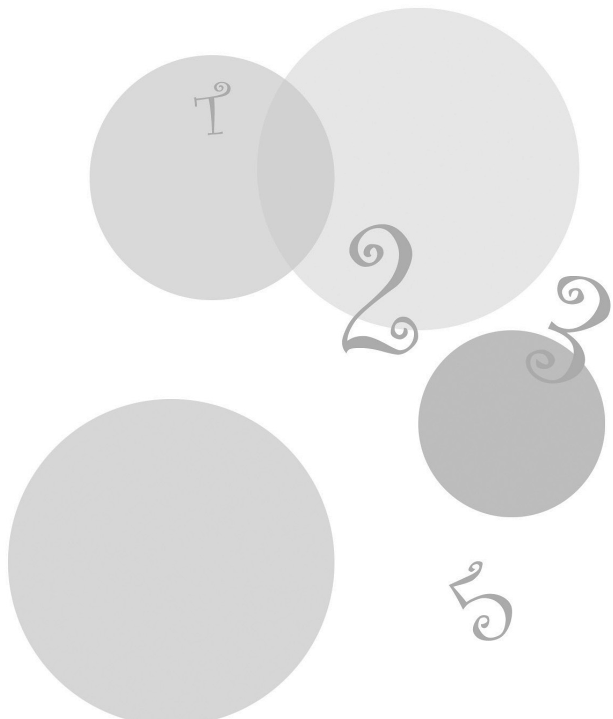
About learning outcomes in teaching mathematics to students



Learning outcomes in mathematics: Case study of their implementation and evaluation by using e-learning

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Abstract. Learning outcomes are considered to be a key tool for student-centered teaching and learning. In implementation of learning outcomes both the top-down approach and the bottom-up approach need to be combined. Whereas the former takes into account the overall study program and the level of study, the latter departs from the level of a particular unit and course. In devising the instruction of mathematics for non-mathematics majors it is essential to recognize the role that mathematical tools and models play in such a study program. In doing so, students' pre-knowledge of mathematics should by no means be disregarded.

In our paper we aim to present a case study of implementation of learning outcomes in several mathematical subjects within the Information and Business Systems study program at the Faculty of Organization and Informatics of the University of Zagreb. In the first phase, after the learning outcomes have been recognized, they are harmonized with students' pre-competences, teaching methods, student workload (ECTS), continuous monitoring of students' achievements and their assessment, while taking into account different learning and motivation styles. During the second phase the learning styles evaluation model is elaborated and the relation and interactions between different elements of the learning and teaching process are verified.

The entire process is heavily supported by ICT and executed through blended e-learning and the use of social software such as wiki, e-portfolio, etc. Such a delivery mode does not only enhance student motivation for learning mathematics and the availability of teaching and learning materials but also improves communication between the student and the teacher, as well as that among the students themselves. In addition, it enables the teacher to store a lot of students' artifacts, which opens many possibilities for the evaluation of learning outcomes.

Keywords: learning outcomes, mathematics, ICT, e-learning, taxonomy, e-portfolio

Learning outcomes and other elements of the curriculum

Institutional level

The prerequisite for the systematic and the consistent introduction of learning outcomes into the study programme is the project at the institution and the support of the management and the faculty board of the institution. In this matter, the project can be internal or it can have an external sponsor (grant). For example, at the Faculty of Organization and Informatics University of Zagreb the foundations for the implementation of learning outcomes have been set within the structure of the project entitled Learning outcomes in interdisciplinary study programmes INTER-OUTCOMES which were executed at the Faculty of Organization and Informatics (FOI) of the University of Zagreb in the period from February 2008 to February 2009, and which were financed by The National Foundation for Science, Higher Education and Technological Development of the Republic of Croatia. Partner institutions on the project were the Faculty of Science – Mathematical department of the University of Zagreb and the Faculty of Electrical Engineering and Computing of the University of Zagreb. The leader of the project was Blaženka Divjak from the Faculty of Organization and Informatics in Varaždin. The objective of the project was to develop the methodology of learning outcomes and their dissemination within the framework of the system for quality insurance in higher education and their implementation with the emphasis on interdisciplinary area of informatics. The three mentioned partner institutions were associated and had the aim to define, develop and compare the learning outcomes for the study programme of informatics, which necessarily includes computing science and mathematics. In this project, teachers were educated in the learning outcomes and this was the prerequisite for the agreement about methodology and for the implementation of learning outcomes. First we will continue with the basic theoretical precepts for the introduction of learning outcomes.

Learning is a complex process which enables the perception and understanding of the world and as such, it encompasses a high spectrum of the activities that include the mastering of reading and understanding of what has been read, as well as the understanding of abstract principles and mathematical evidence and the development of appropriate behaviour for specific situations (Fry and el, 2003).

Modern literature gives us different theories about how one learns. Today a constructivist theory of learning prevails, which postulates that it is the experience which leads us to formulate general concepts (constructs) that serve as the models of our reality. According to constructivism people participate actively in the development of their knowledge. The most significant representatives of constructivism of the twentieth century are Swiss psychologist Jean Piaget and American psychologist Jerome Bruner. In professional literature there is a fair number of the critics of constructivism and parallely some other theories are developing such as rationalism, behaviorism, cognitive science, etc. (Fry and el, 2003).

Implementation of learning outcomes

In professional public the topic of recognizing the key principles of teaching in higher education is widely discussed. Being inspired by (Ramsden, 2003) we give some more important principles in Table 1.

Principles	Instruments
Clear goals and intellectual challenge	Learning outcomes and goals
Interest and understanding	Good teaching and appropriate literature
Concern and respect for students and student learning	Appropriate student's workload (ECTS)
Appropriate assessment and feedback	Implementation of taxonomy
Development of generic skills	Learning outcomes
Learning from students	Quality assurance and enhancement of teaching

Table 1. Teaching principles in higher education.

In theory, as well as in practice, we distinguish between three basic approaches to teaching. The first one, often called traditional, is the one in which the teacher is at the centre of the teaching process. Moreover, the teacher can also appear as the one who organizes the activities directed to learning. The third approach puts the student at the centre of the teaching and learning process. In Table 2. we give basic characteristics of these three approaches. The table has been taken from (Ramsten, 2003, p. 115).

	Teaching as telling	Teaching as organising	Teaching as making learning possible
Focus	Teacher and content	Teaching techniques that will result in learning	Relation between student and subject matter
Strategy	Transmit information	Manage teaching process; transmit concepts	Engage; challenge; imagine oneself as the student
Actions	Chiefly presentation	"Active learning"; organising activity	Systematically adapted to suit student understanding
Reflection	Unreflective; taken for granted	Apply skills to improve teaching	Teaching as a research-like, scholarly process

Table 2. Theories of university teaching.

Teaching planning should in fact deal with the organization of the teaching process. Thus, we should bear in mind the students' pre-knowledge, the goals of the study programme and the role of a single subject in the programme. Furthermore, we should be aware of different learning, organizational–technical possibilities which we have at our disposal and also of available teacher resources. In the end, it should be clear how this affects student workload, i.e. students' activities should be expressed in ECTS points. This helps us to formulate learning outcomes for the study programme or the subject. The constructing of objectives and learning outcomes calls for conscious decisions about a great number of challenges and problems in the teaching and learning process on the part of the teacher and the institution.

Let us emphasize that learning outcomes are statements about what is expected of the student to know, to understand, to do and to evaluate as a result of the learning process. They are connected with measurable level descriptors in national and European qualifications framework. In the literature there are many discussions about differences between objectives, outcomes and competences. Learning objectives determine what the teacher wants the student to learn and to understand, so those who support the studentàcentered learning prefer using learning outcomes in the organization of the teaching process. Lately, in the professional literature there is more discussion about learning outcomes than teaching objectives, although the objectives can be formulated in a way that they reflect the modern approach to teaching. Moreover, by achieving learning outcomes through the process of studying, the student acquires competences necessary for finding employment and self-employment.

Consequently, after determining the levels of the study programme and agreeing about professional competences, learning outcomes of the study programme are developed. Learning outcomes at the subject level take into consideration learning outcomes of the programme obtained in such a way, and their expression is based on a chosen taxonomy. With this, one should be aware of the specific quality of the observed subject and initial students' competences. Afterwards, learning outcomes at the level of teaching units are detailed and appropriate methods of teaching and assessment are chosen. Further, we should bear in mind that all the activities in the subject can be recognized and measured in the student workload expressed in ECTS points. Finally, in order to reach the system improvement, evaluations of all parts of the curriculum should be done regularly, as well as those relating of learning outcomes at all levels, and information obtained in this way should be integrated in the system (Picture 3).

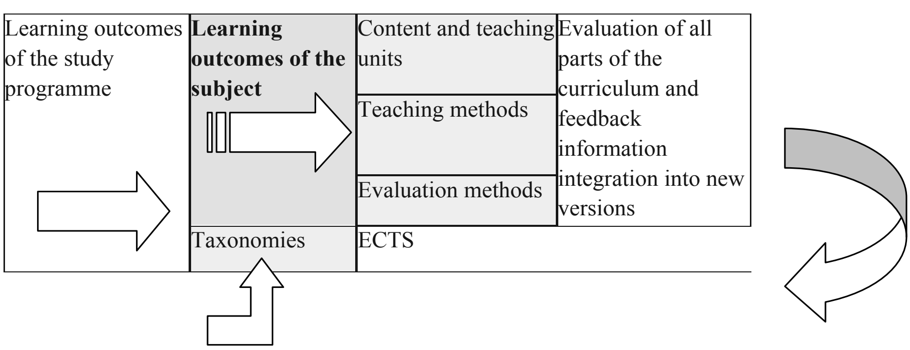

Figure 1. Learning outcomes context.

Learning outcomes evaluation

Having developed learning outcomes at a certain levels, the regular periodical evaluation of their achievement and relevance has to be done. This procedure should be a part of internal assurance the quality of teaching. On the other side, it shall be evaluated by external professionals in the framework of external evaluation process.

The validation of learning outcomes should comprise that kind of the evaluation process and it should also consist of regular students' feedback information about whether specific outcomes are achieved and whether all the outcomes are covered. Furthermore, study verification based on learning outcomes is needed. In this context, the connections between learning outcomes, teaching methods and knowledge testing should be checked, and one should also assess how set outcomes influence the student workload. Finally, there is also a question of teaching literature and e-learning material which will enable students to learn in order to gain set learning outcomes.

As a result of such evaluations, learning outcomes should be revised at the end of each year or semester at all necessary levels. The easiest is to introduce changes at micro levels (teaching units and partially subjects). Unfortunately, innovations in the curriculum demand certain verification on the part of the faculty and university councils, the senate, and often of the National Council for Higher Education. Periodical repetition of this step leads towards the improvement of students' knowledge and employability. We shall continue with presenting a few examples of learning outcomes implementation and evaluation by using e-learning. First, we explain how to test efficiently students' knowledge and pre-knowledge by using taxonomy. Furthermore, we give some examples of the social software use, specifically e-portfolio, then how to test students' understanding and how to evaluate learning outcomes of the subject. In the end, we shall show the use of survey and the work diary on the subject in order to get feedback information about fulfilling learning outcomes and student workload.

Case study of implementation and evaluation of learning outcomes

Blended learning of mathematics and implementation of learning outcomes

At the FOI, for some years, we have been considering e-learning as an unavoidable and a very important element of the teaching process at our institution and which essentially contributes to the quality of the teaching process and especially to the accessibility of the teaching materials. The result of such approach is the acquired E-learning strategy of the Faculty of Organization and Informatics (the E-learning Strategy of the FOI), which relies on the E-learning Strategy of the University of Zagreb (the E-learning Strategy of the UniZg). The fundamental guidelines of the strategy are:

- E-learning is a legal and a desirable way of learning and teaching at the University of Zagreb, and also at our faculty

- The level of e-learning introduction into the teaching process at our faculty is guided by pedagogical needs, and not exclusively by the imperative of modern technology application
- Different aspects of e-learning represent the area of scientific research at the faculty, since they are directly connected with information science.

By introducing and actively using e-learning, FOI intends to improve the quality of the teaching process and learning outcomes, render students (future citizens of the society of knowledge) capable for a lifelong learning, enable a widening participation to higher education and ensure visibility of the faculty on the international educational market.

In the framework of the strategy the blended learning has been chosen as the most appropriate one for the needs of teaching at our faculty, and conforming to this, three levels of blended learning have been determined.

Students have also recognized the possibilities and advantages of blended learning in relation to classical learning. In the survey, which was done in the academic year 2007/2008 and in which 240 students of the first year participated, we asked: "Do you prefer when teaching is done: a) mostly with the support of a computer b) in a classical way with oral teacher's lecture c) with a combination of the first two ways." 69% of the questioned students prefer blended learning, 24% classical way, and 7% computer-supported teaching.

Taxonomies in mathematics

In order to construct more successfully the learning outcomes according to "depth of knowledge", we observed several taxonomies created for mathematics. All observed taxonomies define the "depth" of the mathematical content, that is, they do not dwell only on the content defining. Bloom's taxonomy (Bloom, 1956) is the most frequently used taxonomy in creating the learning outcomes. It consists of 6 categories (knowledge, comprehension/understanding, application, analysis, synthesis and evaluation). The categories are also arranged according to weight. According to Bloom, the highest level of taxonomy includes a very complicated level of cognitive thinking. However, Bloom's taxonomy is not suitable for creating learning outcomes in mathematics because it is too complicated for everyday use, especially if the teacher wants to use it to test the students' knowledge. Moreover, we studied the following taxonomies: the MATH taxonomy (Smith and others, 1996), the TIMSS (Chrostowski and O'Connor, 2001) and the MATH-KIT (Cox, 2003) and we finally decided for the MATH-KIT. We have to mention that good results in implementation in mathematics (Chick, 1998) were given by the SOLO taxonomy (Structure of the Observed Learning Outcomes) which was developed by Biggs and Collis in 1982, where the evaluation of the students' progress was shown in five levels, so it correlates with the grade scale that we use in Croatia (from 1 to 5). However, for our needs of preparing the data base of questions and problems, five levels are too much for effective work.

Cox's taxonomy ensures the creation of the teaching process following the learning outcomes, it is simple for the classification of the depth knowledge, suitable for assessment purpuses, and especially assessing homework (tests) via web. Taxonomy defines classification in three categories:

- K (Knowledge) – basic knowledge. It implies concept defining and understanding, knowing examples, use of concepts and facts, use of theorems and formulas in tasks which demand a simpler application, a practical use of calculation techniques.
- I (Interpretation) – interpretation: comprehension, understanding, analysis and synthesis. It implies the fact that the student can reproduce the learned theorem, understand it and know some consequences and limitations, deduce heuristic evidence, adjust set problem so that the theorem can be applied.
- T (Transfer) – translating knowledge into a new context, application, creation, synthesis. It implies the fact that the student can apply and observe the theorem in the new and unfamiliar context, correlate the set material with other aspects of mathematics, develop new and improve the existing models, formulate hypothesis.

Task base according to taxonomy

The first example of the learning outcomes implementation by using of e-learning is given in subjects Mathematics 1 and Mathematics 2 in the first year of undergraduate study Information and business systems. More about the methodology of teaching in these subjects is described in (Divjak & Erjavec, 2006). Besides classical ways of knowledge assessment through preliminary exams, we also use on-line knowledge self-testing in the e-learning system Moodle.

All forms of testing and knowledge assessment in Moodle and in classical tests are prepared according to Cox's taxonomy. For example, self-testing in Moodle is a test in electronic way that a student does individually and the moment he hands the test he gets the information about his success. Having finished the test each student not only gets an insight into the correct answers but also feedback information about the solution, especially if the *self-test* consists of the tasks of the highest level in terms of taxonomy (type T).

In the subjects Mathematics 1 and Mathematics 2 we have created a number of self-tests and homework tasks in Moodle that have been done according to Cox's taxonomy. For each homework or self-test we have created the data base of questions that has three groups (K, I and T). For example, in homework associated with determinants in each of the three groups there are 30 tasks, totally 90 tasks for each homework. To each student Moodle generates his set of questions/tasks by taking out predetermined number of questions/*tasks* from each group. In this way we have achieved that each student gets a test of a defined knowledge width and depth in advance. For Mathematics 1 we have a base of totally 300 questions and tasks, and for Mathematics 2 a total of 532 questions and tasks. The reason for greater number of tasks in Mathematics 2 is that its material deals with calculus (functions, limits, derivations, integrals) so it is easier to do more calculation tasks than in Mathematics 1, which mainly includes linear algebra (matrices, determinants, linear equations systems). However, Cox's taxonomy is not the only key according to which categories of questions within the task base for these two subjects have been developed. Single self-tests have been created so that they are mainly graphic or geometrical. The objective of this kind of testing is to strengthen the sense for geometrical cognition of a problem, to encourage interest for mathematics and to popularize topics which are not popular among our students (relations and sets in Mathematics 1, or derivations and integrals in Mathematics 2), but which are important for their professional competences.

It should be emphasized that on-line tests do not have a big influence on forming a pass grade (D), since students do them mainly in uncontrolled conditions but they have a motivational role in the teaching process.

Students' pre-knowledge

It is important to evaluate initial students' competences (pre-knowledge) in every subject and to compare them with output competences, in order to evaluate the students' progress in a specific subject. It is clear that this evaluation is not an easy or unambiguous procedure. Initial competences are described through a prerequisite in the form of whole subjects, but also as a set of necessary pre-knowledge that should be acquired through the previous formal, non-formal and informal learning. In the first year of the undergraduate study initial competences for some subjects are tested by an entrance exam, but it is also necessary to conduct a test of pre-knowledge for individual study groups in order to prepare the teaching process more effectively, but also to give students usable feedback information about possible deficiency in their competences. For this purpose, in the subjects *Mathematics 1 and 2* we do the pre-knowledge testing and we use taxonomy for classifying tasks and students' success.

The test at the beginning of the second semester consists of 12 questions divided in three groups (according to K, P and T taxonomy) in a way that each of the four units (quadratic equation, Pythagoras' theorem, logarithmic function, trigonometric functions) expands through these three levels. The objective of this testing is to determine the pre-knowledge which is necessary in order to follow the lectures in Mathematics 2, not only in width but also in depth.

Since we have been following students' pre-knowledge for the last three years, it has been determined that all three generations lack in knowledge and that some tasks are rarely or never solved. These are mainly graphical or geometrical tasks which relate the function to the practical problem, or the graph of a function with properties of the function. In order to lessen these lacks in the pre-knowledge, tutorial classes have been organized where older students help those attending specific subject classes, then frequent teacher consultations (6 hours per an assistant and 4 per a professor) and extra material for revision in the e-learning system. To stimulate students to do graphic tasks and to achieve better results we have created a great number of self-testing tasks in Mathematics 1 and 2, with graphical tasks and problems, for example the self-test 3 in Mathematics 2 (Table 3).

Table 3 shows the tasks solubility for groups (K, I, T) in Moodle for two self-tests and for the test which is done at the beginning of the second semester in the last three academic years. In the academic year 2008/09, 240 students did the test at the beginning of the semester, and the analysis of solubility was based on two seminar groups, i.e. on totally 62 students. The test analysis in 2006/2007 was also based on two seminar groups (i.e. on 80 students out of total of 324 who did the test that year).

In the academic year 2008/2009, 234 students did the self-test 3 in Mathematics 1, and 146 students in the year 2006/2007 (Table 3). We should note that one of the reasons for such a low response in self-testing in 2006/2007 was the fact that, in that year, e-learning was introduced for the first time in the subjects Mathematics 1 and 2 and all self-tests and tasks show a low students' response than in later years. This self-test includes linear algebra material (determinants and systems of linear equations) and these are the types of questions: true-false, multiple answer questions, linking, short answer (the student needs to write the correct answer). All questions are put in groups K, I and T (each group has 17 questions). The distribution of tasks solubility in these self-tests confirms the good distribution of tasks according to depth (K, I, T) and the justification of introducing taxonomy in subjects. One of the reasons why similar distribution of solubility does not occur in the test done at the beginning of the semester lies in the fact that it examines pre-knowledge from secondary school, and self-testing examines acquired knowledge during the semester and this is also an indicator that students who come to our faculty do not have some basic knowledge which we consider that they should have upon finishing secondary school.

The self-test 3 in Mathematics 2 (Table 3) is a test that has graphical and geometrical tasks from the area of function derivations and derivation application. In the year 2006/2207, 244 students solved the test and this academic year the test will be done only in the second part of the semester. The results from the year 2006/2007 show that students solve geometrical tasks better than those algebraic (self-test 3 in Mathematics 1), which we did not expect, if we consider the results from the beginning of the semester in which graphical and geometrical tasks were poorly done.

		2006/07	**2008/09**
Mathematics 1, Self-test 3 in Moodle	**K**	64.8%	82.73%
	I	52.7%	67.03%
	T	40.6%	43.82%
Mathematics 2, Test at the beginning of the 2nd semester	**K**	79.95%	48.79%
	I	23.1%	16.33%
	T	28.3%	18.68%
Mathematics 2, Self-test 3 in Moodle	**K**	73.9%	
	I	64.3%	
	T	44.3%	

Table 3. Results of self-testing in Moodle and the test from the beginning of the semester.

Learning outcomes evaluation by using e-portfolio

We introduced e-portfolio in the subject Selected chapters of mathematics (4th semester of the undergraduate study) in order to monitor students and evaluate the learning outcomes. The portfolio represents systematic, multidimensional and organized collection of evidence about students' knowledge, skills and attitudes. With e-portfolio this collection is available in electronic way or on-line. "In a nutshell, portfolio assessment is considered an effective means of measuring the change sin students' cognition and learning process, involvement and interaction, and assessing higher-order cognition abilities and attributes." (Frankland (ed), 2007). We use the open source e-portfolio system Mahara.

For each chapter (out of 6) the students have to make one artefact (homework, test, presentation exercise, model, to describe a possible application, systematized lecture notes) and reflection on learned material, acquired skills and their entire progress.

Therefore, the e-portfolio role in the subject Chosen mathematical chapters is dual.

- Reflection on the subject, the students' activities in the subject and their execution. The subject is described by its role in the programme, application, learning outcomes and etc. Moreover, there is a discussion about difficulties and success, explanation of the subject concepts and their correlation with other subjects as well as opinion on mathematical modelling and its role in the ICT profession. On one side, the activity related to e-portfolio represents a contribution to the use of technology in teaching, and on the other side, it serves to raise students' awareness of their own work and progress on the subject. This progress is monitored by the means of emphasizing the personal choice of the best artefacts on the subject and reflecting about the material and progress that is written in a free form.
- Evaluation of learning outcomes of the subject. This activity will mainly be done by the teachers on the basis of students' e-portfolio. The results of such evaluation will give valuable information for the analysis of learning outcomes achievement, but also about the subject role in the study programme. Since there are more than 300 students on the subject, and more than 200 of them have chosen e-portfolio activity as a part of constant monitoring, we believe that the sample will be statistically significant.

Learning outcomes evaluation by using the survey and the diary

The last example we give is related to the subject Project cycles in research and development which is taken in the postgraduate doctoral study program of Information sciences. The subject has been delivered for three years consecutively and the total of 45 students took it. We have to mention that these were very serious and mature students, and therefore the methodology of work and monitoring of students' progress had to be done very cautiously and in an elaborated way. You can find more about the learning outcomes and teaching methodology of the subject in (Divjak & Kukec, 2007).

With the purpose of learning outcomes evaluation two methods were used. The first one is the survey method in which we ask students, among other things, to what extent set learning outcomes have been achieved. The answers showed that 90% of students assessed that the learning outcomes were completely achieved and the remaining 10% answered that the learning outcomes were achieved rather well. Since these are very responsible students, their quantitative and qualitative answers can be considered relevant.

The second method of evaluation of learning outcomes, which is at the same time estimation of student workload during subject execution and exam preparation, is through a learning diary that is written by students in the e-learning system. The qualitative analysis of these artefacts shows to what extent a single student, with given initial competences, has managed to achieve required learning outcomes and how much effort he/she needed to fulfil specified learning goals. Average diary has a length between one and two A4 pages and contains very useful information for improvement of quality of teaching and learning as well as data on students' pre-knowledge and motivation for the study.

Conclusion

Information and communication technologies (ICT) and the need for the lifelong learning represent an unavoidable reality of the present age. Our task is to use the technology with the purpose of improving the teaching process quality and insuring the accomplishment of the learning outcomes, bearing exclusively in mind the pedagogical needs and not the imperative of the application of modern technologies. In this article we have presented some new examples of the e-learning use with the purpose of better implementation and evaluation of the learning outcomes. First, we described the use of the data base for testing created in the taxonomy in order to serve the needs of self-testing, and afterwards, the use of taxonomy to determine the students' initial competences. The example of the e-portfolio use follows, and on one hand, it serves for students' reflection on their own progress in the subject, and on the other hand, it enables the teacher to evaluate the learning outcomes achievement. In the end, we showed the example of learning outcomes evaluation and student workload by keeping the learning work in the subject. We conclude that it is very important to adjust methods of evaluation of learning outcomes to the study level, to the specific subject as well as to the students' characteristics and combine them with the evaluation of the other elements of the teaching and learning process.

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Mathematic competencies of students interested in teaching studies
(an analysis of an entrance exam in mathematics)

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Abstract. Unsatisfactory results in mathematics at all levels of education suggest a need to review all the factors that can influence such results. Elementary school teachers are the first experts who are entrusted with the development of children's basic systems of mathematic concepts, processes and terminology as well as with creating the very attitude towards mathematics.

Competencies we expect the teachers to have can be defined as an in-depth conceptual knowledge and understanding of contents of mathematics, understanding of pedagogic concepts and psychological characteristics of children along with a positive attitude towards mathematics. Some of the above mentioned competencies can be and are developed and improved at teaching colleges, but many are expected to be inherent in the personality and the attitudes of the teachers to be. This idea prompted a study of mathematic competencies of students interested in the teaching career. To that purpose an analysis of the results of a very simple mathematics entrance exam was carried out. The exam was taken by 123 students in the year 2008. The results of the analysis show a thorough lack of knowledge and undersatnding of the basic concepts of mathematics, many of which are encountered already at the very early stages of teaching mathematics.

Keywords: mathematics, elementary school teachers, mathematic competencies, teachers colleqes

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Investigation of spatial ability in the population of students of mathematics teacher education programmes at the Department of Mathematics, University of Zagreb

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Abstract. Development of spatial ability is one of the key goals of mathematics education in primary and secondary school. Being an important aspect of geometric thinking, it can be defined as an intuition about shapes and the relationships among shapes, that is, as the ability to generate, retain, retrieve, and transform well-structured visual (mental) images. Individuals with spatial ability have a feel for the geometric aspects of their surroundings and the shapes formed by objects in the environment. The most widely recognized model for the development of geometric thinking is described by the Van Hiele theory (P. Van Hiele and D. Van Hiele-Geldorf, 1959). The theory identifies five hierarchical levels of individuals' understanding of spatial ideas, visualization, analysis, informal deduction, deduction and rigor, respectively. A pilot study was carried out to identify the Van Hiele levels among students of mathematics teacher education programmes at the Department of Mathematics, University of Zagreb. The analysis of the study will be presented and discussed in relation to the expected learning outcomes in geometry taught in primary and secondary school as well as in preservice mathematics teacher education. Gender analysis of the results will also be presented.

Keywords: geometric thinking, spatial ability, Van Hiele theory, pilot study, students – prospective mathematics teachers

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Application of basic mathematical concepts and skills in physics

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Abstract. Educational research worldwide has identified several fundamental mathematical concepts and skills that pose particular difficulties to students and that are significant both in mathematics and physics. Some of these are graph interpretation, vectors, proportionality and interpretation of mathematical symbols and expressions.

In this talk we will discuss the expected learning outcomes related to the mentioned concepts and skills in the context of mathematics and physics education. Furthermore, we will present the analysis of a pilot study of student understanding of graphs in mathematics and physics that has been carried out on a sample of students in Croatian gymnasiums.

Keywords: mathematics education, physics education, graph interpretation

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4.
Impact of learning and teaching strategies on learning outcomes

A number theoretical game with chess figures

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(To the memory of my father Ernő Molnár,
my first impressive teacher in mathematics)

Abstract. As it is well-known, in the usual chess game we have $N = 32$ figures. (The following game can be carried out not only with chess figures, but with stones and other tools, moreover with paper and pen, etc.). Two players, the beginner **B** and the second player **S**, consecutively take off the figures, each at least $a = 1$ piece, at most $A = 4$ pieces.

The winner is the one who takes off the last figure.

We can ask several questions: E.g. is it true that **B** always wins? If yes, what is his strategy? How to modify the game rules, e.g. **A** or **N**, so that **S** has the winning strategy?

Similar questions probably make clear some important concepts in game theory, and also in elementary number theory, hopefully in an amusing form.

Keywords: game with numbers, subtraction with a given rule, division with remainder (rest), primes

Reformulation of the game

Just at the beginning we reformulate our game for a paper-pen version for two children, or even more for a blackboard-chalk version between the lecturer and a member of the audience, as here at a conference. Or, between the teacher and a pupil of the class in a primary school, as this happened to me when I was a teacher-candidate (more than 40 years ago).

We start with the number $(N =)32$, written in the blackboard. The beginner pupil **B** has to write the next number, smaller than 32 at least by $1(= a)$ but at most by $4(= A)$. Then comes the teacher as second **S** with the last number by the same rules, and so on. *The winner is the one who writes* 0 (zero) *at the end.*

Let us say that the pupil writes first the number 29, and the teacher **S** follows with the number 28 , which is, again, less than 29 by a number between 1 and 4. And so on, **B**: 24,

S: 20, **B**: 16, **S**: 15, **B**: 12, **S**: 10, **B**: ?

1. Soon comes the endgame, and the pupil notices that he will lose. That is, he can write any number out of 9, 8, 7, 6, **S** will write 5, and next time **S** can write 0 for any choice of **B**: from among 4, 3, 2, 1. Now comes the critical questions to the game documentation in the blackboard. Where did **B** lose, or **S** won the game?

2.That means that *the game analysis is carried out in the opposite direction from the end to the beginning.* It turns out that **S** won the game when he wrote down 20, since then he could write down 15, 10, 5, 0 in a consequent way. Of course, **B** could have won if he had written 25 and earlier 30, just at the beginning.

3. *Then the beginner pupil* **B** *can formulate his winning strategy.* He starts with writing down 30, then 25, 20, 15, 10, 5, 0, and he can proceed like this, because of the game rules

$a = 1, A = 4, A + a = 5$ and $32 = 6 \times 5 + 2$ by division with remainder. The rest is 2, as a result of successive subtractions of $5 = A + \quad a$ from $N = 32$.

4. Of course, the second player **S** *has a winning strategy, if* $N = 30$ was the starting number. Or with $N = 32$, if the maximal subtrahend A was 7 (or 3, 15, 31), since $32 = 4 \times 8$

$(8 \times 4, 2 \times 16, 1 \times 32)$.

5. We can summarize the essence of the game by the numbers N, A, aEg (natural numbers) $N > A > a > 0$ from the rules, and assume that

$N = k \times (A + a) + r \ (r, k \quad Eg, 0 \leq r < A + a)$

by division with remainder (i.e. with successive subtraction).

i) **B** *wins,* if $a \leq r \leq A$. In this case, he can write down first $N - r = k \times (A + a)$, and so on $\ldots, A + a$ and 0, as a winning strategy.

ii) **S** *wins* (i.e. **B** loses), if $r = 0$.

iii) If $0 < r < a$ or $A < r < A + a$, then neither **B** nor **S** has a winning strategy, but both can reach draw (remis, in French) for the best play of the partner. Namely, **B** decreases with a , if $0 < r < a$ (and $A + a < N$); and decreases with A if $A < r < A + a$. Similarly does **S** when it is his turn. If somebody fails, the other may win, although there are more draw positions, if $1 < a$ is large enough.

Some remarks regarding elementary number theory

Of course, the game rules can be changed, by the teacher **S** or by the pupil **B**, as we indicated above.

If the beginner **B** fixes the values of $N > A > a > 0$, then he wins. If first **B** says N, **S** fixes A and a, then **S** wins.

To exclude draw, we fix $a = 1$. To exclude trivial play, we assume $N >> A$, i.e. N is much bigger than A. *Then prime numbers play a special role*. If **B** fixes a prime for N, e.g. 31, then **S** can not say a convenient A for winning the game, since $A = 30$ obviously *is not a fair rule*.

And so on, *this game is clear now*.

Other games, chess

We can conclude some general consequences which can be applied to *games between two players*, e.g. to chess; but also to some single-player games, e.g. playing with a Rubik-cube which is not our topic now.

In *chess*, the well-known game rules are more than a thousand years old. They need some abstractions with the chess-board and with the *moving rules of the chess-figures*, just as in our starting game with the numbers N, A, a and an operation (subtraction) with them. These rules are the same for the beginner **B** (with white figures in chess) and for the second player **S** (with black figures).

1. *The endgame is the most important part of the game*, in chess as well.
2. *We have to understand the essence of the game from the end in the opposite direction*. In chess, *the chess-mate procedures* are fundamental (e.g. with King and Queen against the other King, etc.). In general, *the known winning positions* in the endgame determine the former strategies.
3. *A winning strategy from the beginning in chessz/*, if it exists at all, has not been known and not realistic. It is the same with draw strategies. These facts provide the wonder, beauty, art, science, . . . of the chess game.
4. *But endgames with 5 figures have already been solved* by computers (it is my last information). That means, for any chess position with 5 figures the computer decides, whether the beginner wins, loses, or the party finishes with draw. Of course, only if both players give their best, that means, two perfect computer programs play against each other.

The axiomatic method as a game, concluding remarks

Some parts of modern mathematics which are *axiomatized enough* can be considered as *individual games*. Mathematician(s) as player(s) represent human society. The basic concepts like numbers, geometric objects (points, straight lines, planes, space itself), etc., with their basic relations, operations, logical thinking rules, etc., have been created as game rules, based on experiences, discoveries, agreements of human society by its excellent persons, scientists, etc. These game rules are collected in axioms for some restricted fields of mathematics, but also for physics and other sciences, perhaps even for social sciences as well.

A mathematical result with its proof can be considered as a product of a game procedure on the base of game rules. The steps are similar as illustrated above. We can start with an analysis of the endgame in the opposite direction to find a winning strategy if it exists at all. We have already learned that such a strategy does not exist in general. Activities of *David Hilbert*, *Kurt Gödel*, the Hungarian *John von Neumann*, as instances, are related with our topic. Human history, culture, science and art show our conclusion: "Life is a game".

We have reached far from our above mentioned game now. As far as I remember, I learned playing chess from my mother, then my father, and I play chess also today. Father, Ernő Molnár (1912-1994) taught me the game that we analysed in the lecture, just with chess figures. Before that time he had just solved this game as a problem appeared in the journal for teachers *Teaching Mathematics* of the János Bolyai Mathematical Society and of the Hungarian Ministry of Education and Culture. This journal still exists now. Enthusiastic teachers can solve mathematical problems for training their ability and for entertainment.

I thank my colleagues Gábor Molnár Sáska and Zsolt Lángi for their kind suggestions.

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Amusing mathematics in the teaching of mathematics

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Abstract. In all the reforms in the teaching of mathematics so far, there was a search for a more contemporary way of knowledge acquisition, a better way of learning transmitting certain new ideas from science to the subject. A more modern approach to the teaching of mathematics, decreasing the burden on the students, can be achieved in different ways. The article discusses one of the new ways of teaching leading in this direction – amusing mathematics.

Keywords: mathematics, amusement, amusing mathematics, relief

The first impression created by the students and teachers of mathematics by the above title is: mathematics and amusement! The common idea of mathematics is that it is a difficult subject, demanding a lot of time, effort and hard work. This is true. No doubt, mathematics belongs to more difficult subject in the curriculum, and the students feel a certain psychological pressure when acquiring new material. Traditional forms still prevail in the teaching of mathematics, and its essential aim is the mere acquisition of knowledge prescribed by the curriculum, and the acquisition of knowledge based on a number of rules, formulas and the skill of solving standard exercises. A successful mastering of the curriculum often means only the acquisition of new information and can result in further burden on the students. This is not good. Instead of burdening the students to memorize a great number of facts, we should stimulate and motivate their thinking and try to make them acquire a considerable part of new knowledge by their own efforts and abilities. A constant modernization in the teaching of mathematics is necessary to achieve this.

All the reforms in the education of mathematics so far required the students to acquire a better way of knowledge acquisition, a better way of learning and transmission of the knowledge into the school subject. Modernization, especially in the teaching of mathematics in primary school, can be primarily achieved by a more frequent variation of familiar teaching methods and their improvement. But this is not enough. It is necessary to introduce **new forms of work**, to change the conventional ideas about mathematics and demonstrate that mathematics can be easy and amusing. Naturally, the introduction of new ways of work requires serious preparation and additional effort on the part of the teacher of mathematics. All this, however, should not compare to the satisfaction of the mathematics teacher when seeing the interest of the students and their acquisition of new knowledge without psychological pressure and coercion.

The students have already been faced with unusual mathematical exercises in mathematical magazines and other periodicals, and from time to time teachers of mathematics in some schools work in a different way, introducing new forms of work, with interesting and amusing mathematical contents which relieve the burden of the subject. In this way we realized that mathematics can really be easier and amusing in itself. The students with the inclination to mathematics already find it interesting and amusing in itself. Mathematics can become more interesting to other students if the teaching of mathematics is imbued with different contents.

Here are some interesting modern possibilities:

Production of panels with interesting mathematical contents, production of geometrical solids, mathematical quizzes, mathematical games, mathematical crosswords, mathematical projects, computer aided mathematical discoveries, mathematics in the nature, school mathematical magazine, amusing mathematical exercises, entertaining lessons, etc.

Each of these forms of work contains an element of amusement. Thus, the production of panes is a creative act which reveals a satisfaction of creative work to the students. Beside a definite educational effect, this activity has a certain entertaining character: getting knowledge through fun. Mathematical crosswords are an interesting form of work which enables students to relax, rest and prepare to continue new hard work. Like all crosswords, mathematical crosswords are fun, but they have an educational character, especially as a means of checking the acquired knowledge. Computer aided mathematical discoveries can inspire enthusiasm in students, which is the highest level of interest. An enthusiastic student is amused rather than burdened by mathematics! Both the role and the education affect of other new forms of work could be mentioned in a similar way.

All these entertainment element can be integrated into a unique whole known as **amusing mathematics**. The objective of this paper is precisely this mathematical discipline. Amusing mathematics can best be recognised by amusing exercises. The choice of amusing exercises is great. A series of exercises dealing with the subject in an amusing way can be found for any topic, any teaching unit, for any mathematical idea. An amusing exercise can nice introductory motivating example before dealing with a certain idea or as a refreshment of a sometimes rigid teaching situation.

What are amusing exercises in fact? Here is a list of main features distinguishing these exercises:

1) Amusing exercises are mathematical miniatures requiring minimum knowledge in arithmetic, algebra and geometry.
2) Formulation of exercises is simple and clear to everybody.
3) The texts are written in the form of shot, amusing stories from everyday life.

4) Greater previous knowledge of mathematics does not always guarantee quick solution.
5) The problems are not always easy, many of them require considerable mental effort, logical reasoning and especially ingenuity in finding the way to solve them,
6) Amusing illustrations often play an important role in the beauty of such exercises.
7) Basic values: development of logical reasoning and ingenuity, stimulating an interest in mathematics, making mathematics popular.

Areas of amusing mathematics: *unusual properties of numbers, number and letters in figures, games with numbers, geometrical solids, same digits, one draw of a pencil, combinatory problems, figures, logical miniatures, magic squares, mathematical games, figure covering, cryptograms, cutting up figures, composing figures, matches, sticks, tests, etc.*

A few words on immediate purpose of amusing exercises. Amusing mathematics is intended for EVERYONE: students, teachers of mathematics, parents and all other enthusiasts! For there is something interesting and amusing for everyone.

The problems in amusing mathematics are firstly designed for primary school pupils with specialized teachers (age 11-15), as well as students in the lower forms of secondary schools (age 15-17) because of different levels of complexity.

Amusing exercises are not always amusement. The can be a supplement to the mathematics curriculum because it introduces elements in the learning and teaching of mathematics, for which there is not enough time in regular teaching: an amusing short story, humour, ingenuity, mental relief.

The mathematics teacher should pay special attention to the difficulty of exercises because it is not irrelevant, for instance, whether we speak of prime numbers in primary school or in secondary school. Even not in an amusing way. An amusing exercise does not always mean an easy exercise. The solution of need not always be complete. They should be sufficient for understanding, but they should provide enough space for thinking. The objective of amusing mathematics is not painstaking computation, but rather the idea and satisfaction of discovering something interesting and unusual. Sometimes, even a look at the solution of an unusual problem refresh and amuse.

Suitable moments for application:

Beginning of school year (teaching of mathematics is still not ùseriousò; amusing exercises are a very suitable means to give an impetus and stimulate the interest of the students for learning mathematics; exercises from the area pf MEASUREMENT, WEIGHING, POURING, are very suitable for the purpose).

Regular teaching (amusing exercises can be used as introductory motivating examples, refreshment, games).

Homework (selection of examples for homework should be periodically enriched by some example of extraordinary contents and amusing character; such homework could also be an occasion for the whole family to come together and participate).

Dealing with less advanced students (a tiring and exhausting atmosphere; more advanced students are neglected).

Amusing lessons.

End of school year (last days of the school year should be a natural end to an educational period without coercion, tension, major testing; the end should be amusing, but still educational; MATHEMATICAL CROSSWORDS are especially suitable for the purpose.

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Metacognition and self-regulation in learning and teaching mathematics

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Abstract. Mathematics is considered to be one of the most important subjects learned in school. Yet, many students face considerable difficulties in studying mathematics (e.g., TIMSS, PISA). Such evidence points to the need of developing effective instructional methods that have the potential to promote mathematics knowledge and reasoning. The NCTM (National Council of Teachers of Mathematics) emphasizes that meaningful mathematical teaching has to enhance knowledge construction via problem solving, building connections, developing mathematical communication, and using various kinds of mathematical representations. In particular, the NCTM (2000) enhances the importance of developing students' **metacognition** as a means for improving students' mathematical problem solving and reasoning. The present study is rooted in this approach, aiming to elaborate the importance of metacognitive instructional method, as well as selfregulation on students' mathematical reasoning and on their general and domain specific meta-cognitive knowledge.

Keywords: metacognition, self-regulation, mathematics, learning and teaching

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5.
Discussion about studies of mathematics and teaching mathematics

"Shall we study mathematics?" — fifty years after (About the 1959 article of academic S. Bilinski)

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Abstract. Fifty years ago Stanko Bilinski (1909.-1998.) wrote his article "Shall we study mathematics?" The aim was to encourage future math students. Here we discuss and analyse the article, cited in its entirety.

Keywords: mathematical education, motivation

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Math teachers' perceptions of mathematics education in elementary and secondary schools in Croatia — results of an empirical research

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Abstract. The presentation presents the results of an empirical research which investigates math teachers' perceptions of mathematics education in elementary and secondary schools in Croatia. The sample included 292 participants of The Third Congress of Mathematics Teachers' held in Zagreb in 2008. The applied questionnaire consisted of mainly closed questions which measured teachers' opinions on a 4-point Likert scale, and a few open-ended items.

Research results show that the traditional approach and traditional teaching methods prevail in Croatian elementary and secondary schools. For instance, when defining mathematical competence, the teachers emphasize solving mathematical tasks, learning basic mathematical facts and applying mathematical knowledge. In teaching mathematics, they put the biggest emphasis on solving mathematical tasks and the development of logical thinking, while the application of mathematics in everyday life and critical thinking about mathematical concepts and procedures they consider less important. Teachers' responses indicate that they mostly develop students' skill of asking questions and seeking information, concept understanding, and students' interest in mathematics, while not spending a lot of effort on developing students' skill in written communication of ideas and results, and skills in learning mathematics. If we exclude the usual assigning and examining homework, the most common teaching methods are dialogue (94.8%), demonstration of procedures for solving mathematical problems (93.6%), and lecture when introducing a new mathematical concept (86.2%). According to teachers' estimates, during mathematics lessons students most often listen and copy from the board (87.8%), solve mathematical tasks on their own (87.8%), and comment on a previous or upcoming test (87.8%). Activities that require autonomous students work, like writing of their own ideas about mathematical concepts (4.1%), autonomous research and experiments (17.0%), and autonomous learning from the textbooks, are the least often used students' activities. Teachers say that, on average, they spend 34.8% of time on teaching mathematical concepts, and most of the time they teach solving mathematical tasks.

The results reveal that critical reflection on mathematical concepts, autonomous thinking and concluding about mathematical concepts and procedures aren't developed enough in Croatian mathematics education, and that inquiry based method and problem solving are rarely used. However, these methods are essential for the development of mathematical competence.

Keywords: mathematics education, math teachers, mathematical competence, math teaching methods and procedures, students activities in class

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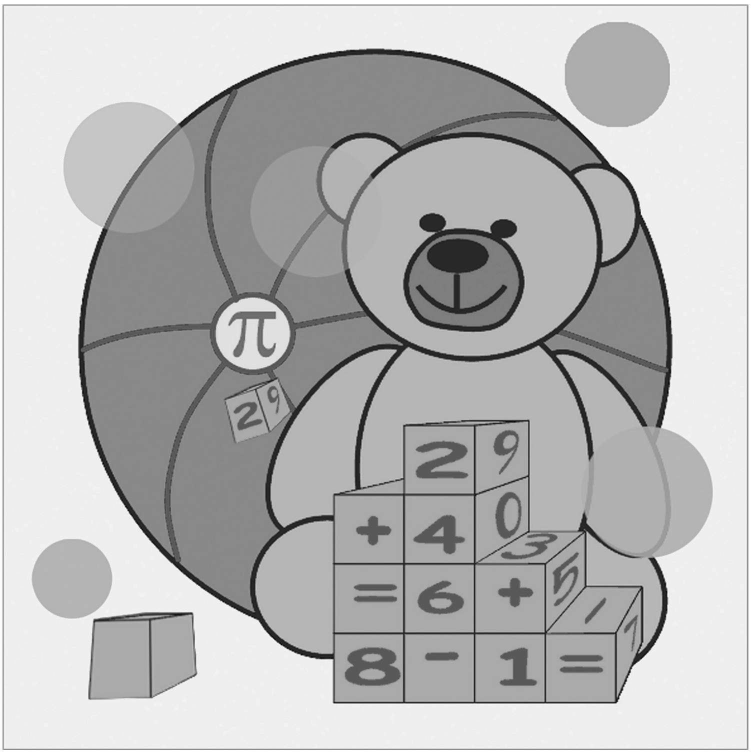
Drugi međunarodni znanstveni skup

MATEMATIKA I DIJETE

(Ishodi učenja)

monografija



Predgovor

U monografiji *Ishodi učenja* autori predstavljaju rezultate svojih istraživanja i navode primjere iz nastavne prakse usmjerene podizanju kvalitete nastave matematike.

Matematika je jedno od devet akademskih područja koja su opisana u projektu *Usklađivanje obrazovnih struktura u Europi* (*Tuning educational structures in Europe*). Nastava matematike (kako za učenika tako i za studenta) organizira se u skladu s nacionalnim kurikulom matematičkoga područja. U nacionalnome kurikulu matematičkoga područja definirani su ciljevi (engl. objectives), ishodi učenja (engl. learning outcomes) i kompetencije (engl. competences) učenika/studenta na kraju svakoga obrazovnoga ciklusa.

U znanstvenim i stručnim krugovima posebno se naglašava briga o pripremi učenika/studenta za svakodnevni život, zapošljivost i cjeloživotnu izobrazbu.

Škole (središta za izobrazbu učenika) i fakulteti (središta za izobrazbu studenata) povezuju se s civilnim društvom i gospodarstvenicima u partnerski odnos s ciljem podizanja kvalitete nastave, o čemu se govori u prvome poglavlju knjige.

U pristupima poučavanju tijekom povijesti nalazimo dva suprostavljena stajališta, ovisno o tome je li učitelj ili učenik u središtu pozornosti procesa učenja i poučavanja. Danas smo bliži stavu da je učitelj organizator aktivnosti koje su usmjerene učenju učenika/učitelja. Stoga se organizacija nastave matematike (i ne samo nje) promišlja u terminima ishoda učenja. (U ovom tekstu rabit ćemo termin *učitelj* za osobu koja poučava, a iz konteksta će se razumjeti govori li se o učitelju u školi ili sveučilišnome nastavniku).

Ishodi učenja tvrdnje su koje opisuju što učenik/student treba – *znati*, *razumjeti* i *moći učiniti* nakon što je uspješno završio pojedini obrazovni ciklus. Ishodi učenja povezani su s mjerljivim pokazateljima razina (engl. level descriptors) u nacionalnim i europskim okvirima. Učenje i poučavanje (matematike) uspješnije je ako učitelj poznaje različite teorije, modele i stilove učenja pa je u stanju promišljati o "svojem stilu poučavanja" kako bi ga prilagodio različitim stilovima učenja svojih učenika/studenata. Najpoznatiji model učenja temeljen na iskustvu (pokusu) jest Kolbov model učenja (1984) koji ističe četiri glavne aktivnosti uspješnoga učenja: konkretno iskustvo, promatranje i promišljanje, stvaranje apstraktnih koncepata i aktivno eksperimentiranje, što je blisko najmlađim učenicima. O ishodima učenja i stilovima poučavanja učenika izlaže se u radovima iz drugoga poglavlja.

Danas nailazimo na veliki broj modela o stilovima učenja kao što su Sternbergov (Sternberg, 2003), Gardnerov i Felder-Silvermanov model višestrukih inteligencija te Honeyjev i Mumfordov model učenja. Napretkom ICT-a i razvojem inteligentnih sustava u obrazovanju povećavaju se mogućnosti praćenja razvojnih razina u procesu učenja. Jedna takva općeprihvaćena teorija, Van Hieleova teorija (P. Van Hiele i D. Van Hiele-Geldorf, 1959.), spominje se u radu iz trećega poglavlja ove knjige. U trećem poglavlju predočeni su rezultati istraživanja o ishodima učenja studenata.

U četvrtome poglavlju govori se o utjecaju strategija učenja i poučavanja na ishode učenja.

U petome poglavlju raspravlja se o studiju i nastavi matematike s gledišta onih koji poučavaju. Prvim radom autor nas prigodno podsjeća na obilježavanje stogodišnjice rođenja akademika Stanka Bilinskog. U radu se komentira članak S. Bilinskog, objavljen u *Matematičko-fizičkome listu* prije pedeset godina sa svrhom poticanja upisa na studij matematike. Tadašnji se stavovi uspoređuju s današnjim stavovima prema matematici. U posljednjem članku navode se rezulatati istraživanja o mišljenju učitelja matematike u osnovnim i srednjim školama u Hrvatskoj o ishodima učenja svojih učenika.

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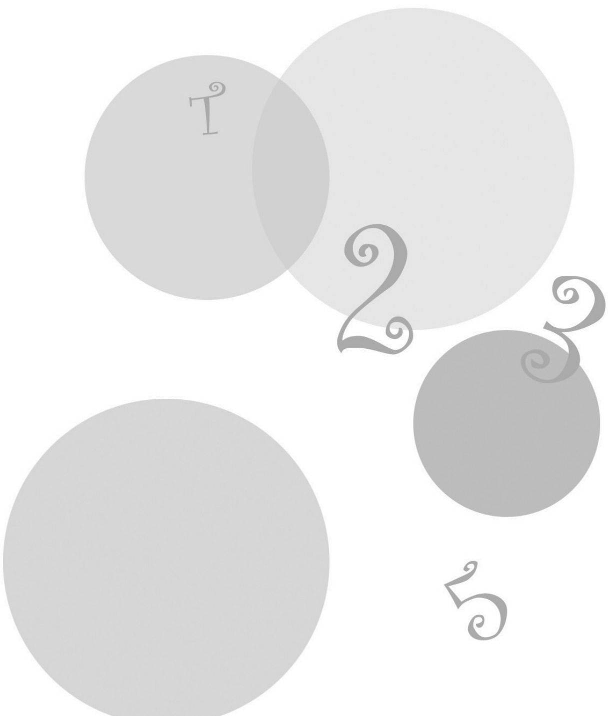
Margita Pavleković

1.
Partnerski odnosi civilnoga društva, fakulteta i škola u podizanju kvalitete nastave



Organizacije civilnoga društva kao partner u matematičkom obrazovanju

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Sažetak. Organizacije civilnoga društva, posebice strukovne organizacije, odnosno različita udruzzenja matematičara, partner su na čiju su se prisutnost i djelovanje obrazovne institucije odavno navikle. Aktualne promjene u društvu i obrazovnom sustavu daju novu dimenziju tom partnerstvu. Uz komentar novonastale situacije dat ćemo i primjer dobre prakse te se posebice osvrnuti na program "Razvoj prirodoznanstvene i matematičke pismenosti aktivnim učenjem".

Ključne riječi: matematičko obrazovanje, organizacija civilnoga društva

Uvod

Matematika je, kao nastavni predmet prisutan na svim razinama obrazovanja, često predmetom diskusija u okviru kojih se postavljaju brojna pitanja; od pitanja zašto je u sustavu obveznoga obrazovanja toliko negativnih ocjena iz matematike, preko pitanja kompetencija nastavnika matematike, pa sve do pitanja treba li nam uopće toliko matematike. U konačnici se vrlo često kao središnji problem pokaže pitanje općega stava prema matematici, pa se briga o razvoju pozitivnoga stava prema matematici pokazuje ključnim zadatkom u matematičkom obrazovanju. Uključe li se u rješavanje toga problema, uz one koji su za kvalitetu matematičkoga obrazovanja formalno nadležni i one koji su izborom svojega zanimanja preuzeli na sebe odgovornost za kvalitetno matematičko obrazovanje učenika, i svi ostali koji posjeduju odgovarajuća znanja i kompetencije, moguće je postići određene pomake.

Društveni kontekst

Svjedoci smo da se na svim razina obrazovnoga sustava u Republici Hrvatskoj događaju procesi koji za cilj imaju promijeniti postojeći način učenja i poučavanja. Deklarirani cilj svih tih promjena jest da onaj koji uči treba postati središtem cjelokupnoga procesa, pa se od svih ostalih očekuje da svoje razmišljanje i djelovanje usklade s tim ciljem. Više no prije govori se o aktivnom učenju, ishodima učenja, kompetencijama... S druge strane, poznato je da je obrazovni sustav po svojoj prirodi trom sustav i nije realno očekivati da se značajne promjene unutar sustava obrazovanja dogode "preko noći", ma koliko bile podržane i forsirane od strane nadležnoga ministarstva. Kako bi se pomak ipak što brže dogodio, potrebno je da što veći broj nastavnika bude ne samo upoznat s novim zahtjevima procesa obrazovanja, već da, koliko god je to moguće, bude u neposrednom doticaju s metodama poučavanja za koje se očekuje da ih primjenjuje u svom radu. Ovaj se zahtjev nastoji zadovoljiti organiziranjem različitih seminara i radionica za nastavnike, što je zadatak kojim se pretežito bavi Agencija za odgoj i obrazovanje.

U matematičko su obrazovanje učenika uvijek u određenoj mjeri bila uključena i strukovna udruženja matematičara koja su se prvenstveno posvećivala radu s nadarenim učenicima (organizacijom ljetnih škola za nadarene, sudjelovanjem u organizaciji matematičkih natjecanja i kvizova, izdavanjem različitih publikacija za mlade matematičare i sl.). Kako smo svjedoci da je interes za bavljenje matematikom prisutan kod sve manjega broja učenika, takav se angažman pokazao nedostatnim. S druge strane, promjene u hrvatskom obrazovnom sustavu i prepoznavanje potencijala izvaninstitucionalnoga odgoja i obrazovanja su omogućili da se u obrazovanje općenito, pa i u matematičko obrazovanje, intenzivnije uključe organizacije civilnoga društva. Takva mogućnost posljednjih je godina dodatno potaknuta i činjenicom da Ministarstvo znanosti, obrazovanja i športa (MZOŠ) jednom godišnje raspisuje natječaj za raspodjelu financijskih potpora projektima i programima udruga u području izvaninstitucionalnoga odgoja i obrazovanja.

Primjer dobre prakse

U nastavku ćemo navesti primjer uključivanja organizacije civilnoga društva u matematičko obrazovanje učenika osnovnih škola u Rijeci u suradnji s Gradom Rijeka kao osnivačem škola te uz potporu MZOŠ-a.

Organizacija civilnoga društva o kojoj se u konkretnom slučaju radi jest udruga *Zlatni rez*, osnovana 2004. godine sa sjedištem u Rijeci te područjem djelovanja u cijeloj Republici Hrvatskoj. Udrugu čine visokoobrazovani stručnjaci koji su svojim stručnim i znanstvenim radom aktivno uključeni u problematiku odgoja i obrazovanja te u aktivnosti popularizacije prirodnih znanosti i matematike. Od svoga je osnivanja udruga *Zlatni rez* organizator Festivala znanosti u Rijeci. Zapravo se može reći da je i samo osnivanje Udruge proizašlo iz iskustva Festivala znanosti i saznanja da uz potrebu za brojnim popularizacijskim aktivnostima kada su u pitanju matematika i prirodoslovlje postoji i potreba za implementacijom modernih obrazovnih standarda u hrvatski obrazovni sustav, posebice u području prirodnih znanosti i matematike.

Tijekom proteklih je Festivala znanosti u Rijeci, koji se organiziraju od 2003. godine, udruga *Zlatni rez* organizirala brojne radionice za učenike, nastavnike i učitelje razredne nastave. Pokazalo se da je potražnja za tim sadržajima mnogo veća i da nadilazi okvire Festivala znanosti te je, uz podršku Odjela za odgoj i školstvo Grada Rijeka, na natječaj za raspodjelu financijskih potpora projektima i programima udruga u području izvaninstitucionalnoga odgoja i obrazovanja, što ga raspisuje MZOŠ, prijavljeno nekoliko programa i projekata, od kojih su neki realizirani, a neki su u realizaciji. Među njima istaknut ćemo trogodišnji program "Razvoj prirodoznanstvene i matematičke pismenosti aktivnim učenjem", koji se u riječkim osnovnim školama provodi od školske godine 2007./2008.

Ciljevi su programa "Razvoj prirodoznanstvene i matematičke pismenosti aktivnim učenjem" usmjereni na podršku učenicima i nastavnicima u aktivnom učenju i poučavanju.

Aktivnosti su programa usmjerene:

1) na izvođenje nastavnih sati čiji je cilj u redovnoj nastavi poticati aktivno učenje koje izaziva interes, motivaciju i dugotrajno pamćenje sadržaja iz matematike i fizike, te
2) na davanje potpore nastavnicima u primjeni aktivne nastave kroz izvođenje nastavnih sati i izradu priručnika koji će služiti nastavnicima i učenicima kao potpora pri ovakvom načinu rada.

U okviru programa izvodi se šest različitih sadržaja iz matematike za uzrast učenika od četvrtoga do osmoga razreda. Autori su edukacijskih sadržaja koji se prezentiraju u okviru ovoga programa sveučilišni nastavnici te njihovi studenti nastavničkoga smjera od kojih su mnogi već uključeni u redovni sustav obrazovanja. Nakon svakoga se izvedenoga sadržaja provodi anketa kako bi se po završetku programa dobio daljnji uvid u prihvaćenost ovakvoga načina rada, a objedinjeni će rezultati ankete biti prikazani u publikaciji čiji se tisak planira u posljednjoj godini izvođenja programa.

Program "Razvoj prirodoznanstvene i matematičke pismenosti aktivnim učenjem" nalazi se u svom središnjem dijelu i do sada je u svakoj osnovnoj školi Grada Rijeka u okviru toga programa predstavljen barem jedan matematički sadržaj. U njemu je do sada sudjelovalo više od 500 učenika od četvrtoga do osmoga razreda te njihovi nastavnici. Značajka je ovog programa da nije namijenjen samo određenoj kategoriji učenika, već svim učenicima određenoga uzrasta. Načinom se prezentacije sadržaja želi potaknuti dubinski pristup učenju koji proizlazi iz potrebe za smislenim angažiranjem na zadatku, pri čemu se teži razumijevanju temeljnih ideja i principa te razvijanju pozitivnih osjećaja prema nastavi i učenju: razvijanju interesa, osjećaja za važnost, oduševljenja i zadovoljstva u učenju. Do sada provedene ankete pokazuju pozitivne reakcije čak i kod onih učenika koji ne vole matematiku. Zadovoljstvo sudjelovanjem u programu iskazuje i većina nastavnika koja je prisustvovala izvođenju matematičkih sadržaja. Mnogi su od njih prihvatili to kao primjer dobre prakse, što smatramo izuzetno važnim, pogotovo u slučaju kada se radi o nastavnicima razredne nastave, s obzirom na poznatu činjenicu da taj poziv odabiru mnogi koji prema matematici nemaju pozitivan stav.

Zaključak

Iz svega rečenoga možemo zaključiti da organizacije civilnoga društva mogu biti pouzdan i respektabilan partner u matematičkom obrazovanju učenika. No, za uspjeh je njihova djelovanja svakako izuzetno bitna podrška kako lokalne zajednice, tako i nadležnoga ministarstva. Prikazani program sasvim sigurno ne bi bilo moguće realizirati bez podrške Grada Rijeka (osnivača osnovnih škola u kojima se provodi program) i potpore MZOŠ-a. Kada je matematičko obrazovanje u pitanju, postoji potreba i mogućnost za uključivanjem organizacija civilnoga društva, a na organizacijama civilnoga društva je da ponude programe matematičkoga obrazovanja odgovarajuće kvalitete.

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Učitelj matematike v vlogi raziskovalca poučevanja in učenja

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Povzetek. Poučevanje matematike je zelo zahtevna naloga. Matematika je za učence eden težjih predmetov v šoli, učenci ne prepoznajo uporabne vrednosti matematike v vsakdanjem življenju, abstraktni matematični pojmi so jim večkrat nerazumljivi oz. imajo z njimi težave...Po drugi strani pa poznamo veliko raziskav s področja poučevanja in učenja matematike in zdi se, da rezultati (ki so največkrat zelo spodbudni z vidika raziskovalcev) nimajo nobenega vpliva na delo v razredu. Zakaj je temu tako? Prepričani smo, da učitelji težko spremenijo svoje subjektivne teorije o poučevanju in učenju matematike in največkrat so rezultati raziskav zanje nerealni, nekoristni... Prispevek predstavlja pomen raziskovanja za učitelja, če je sam v vlogi raziskovalca. Povabili smo učitelje v projekt Partnerstvo, v katerem so sodelovale Pedagoška fakulteta Univerze v Ljubljani in šole v Sloveniji, ki so se odločile za sodelovanje. Temeljna ideja projekta je bila medpredmetno povezovanje matematike z drugimi predmeti, raziskovalna metoda, ki smo jo želeli podrobneje predstaviti učiteljem, pa akcijsko raziskovanje. Učitelji so imeli nalogo razviti učni pristop, ki je temeljil na medpredmetnem povezovanju in ga evalvirati v praksi z vidika doseganja ciljev izbranih vsebin in učenčevega napredovanja v uporabnem znanju. Učitelji so uporabili akcijsko raziskovanje, ki je potekalo v treh korakih: ugotavljanje predznanja učencev, izvajanje novega učnega pristopa ter ugotavljanje napredovanja učencev v znanju. Prispevek prikazuje poglobljeno analizo raziskovanja učiteljev in nekaj primerov dobre prakse.

Ključne besede: akcijsko raziskovanje, matematika, medpredmetno povezovanje, učni pristop

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Matematika i informatika izvan obvezne nastave u odabiru učenika i ponudi njihovih učitelja

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Sažetak. U Hrvatskoj predstoji prvo usklađivanje i prinavljanje kurikula učiteljskih studija nakon odobravanja integriranih preddiplomskih i diplomskih petogodišnjih sveučilišnih studija 2005. odnosno 2006. godine. U želji da se ishodi učenja studenata učiteljskih studija, posebice kada se radi o izvannastavnim aktivnostima unutar skupine predmeta iz područja matematika – informatika, definiraju u skladu s interesom djece u primarnom obrazovanju, ali i potražnji na tržištu rada, proveli smo istraživanje.

U članku se raspravlja o rezultatima istraživanja provedenog u prosincu 2008. godine u šest osnovnih škola u Osijeku kojim je obuhvaćeno 107 učenika/ca četvrtoga razreda i 21 njihov učitelj/ica. Cilj istraživanja bio je uvidjeti koliko je učiteljeva ponuda u našem okruženju usklađena s učenikovim izborom aktivnosti izvan nastave u četiri područja, i to: jezično-umjetničkom, području tjelesne i zdravstvene kulture, prirodoslovlju i ekologiji te matematičko-informatičkom području, od kojih prva dva pripadaju društvenim, a druga dva prirodnim znanostima. U članku se također analizira omjer broja studenata u Hrvatskoj upisanih na studij društvenih odnosno prirodnih znanosti u akademskoj 2008./09. godini te ga se dovodi u vezu s manjkom stručnjaka sa znanjima i vještinama iz područja matematike i informatike na tržištu rada. Nalazi istraživanja povećali su sigurnost i samopouzdanje autora članka pri definiranju ishoda učenja studenata učiteljskih studija za skupinu predmeta iz područja matematika – informatika. Kreatore školskoga odgoja i obrazovanja, uprave škola i učitelje iz svoga okruženja autori članka potaknut će na usmjeravanje interesa učenika onim zanimanjima i vještinama koja potražuje tržište rada.

Ključne riječi: aktivnosti izvan obvezne nastave, primarno obrazovanje, ishodi učenja studenata, tržište rada

Uvod

S obzirom na četiri desetljeća dugu tradiciju izvannastavnih aktivnosti u osnovnoj školi u Hrvatskoj, pozornost koja im se pridaje još uvijek nije dostatna

(Jurčić, 2008.). Povećanjem zaposlenosti obaju roditelja dolazi do problema zbrinjavanja učenika nakon nastave. Time se stvara potreba za organiziranjem aktivnosti nakon redovne nastave kako bi se ispunilo slobodno vrijeme koje učenici imaju do povratka roditelja s posla. Međutim, glavna uloga izvannastavnih aktivnosti jest osiguravanje pravilnoga psihofizičkog razvoja učenika, odgovarajuće socijalizacije, razvijanje potrebe sadržajnog iskorištavanja slobodnog vremena, poboljšanje kvalitete života učenika te njegova priprema za uključivanje u društvo rada.

U Republici Hrvatskoj trenutno se oblikuje novi nacionalni kurikul pa je potrebno obratiti pozornost i na ovaj oblik odgojno-obrazovne djelatnosti u osnovnoj školi. Gledajući nastavni plan i program za učenike od prvoga do četvrtoga razreda osnovne škole, može se uočiti da jedan sat dopunske i dodatne nastave, koliko je predviđeno važećim planom, ne ispunjava u potpunosti slobodno vrijeme učenika nakon nastave te ne zadovoljava njihove potrebe za dodatnim aktivnostima ([9]). Višestruke su prednosti i korist izvannastavnih aktivnosti pa je problem njihove potpune ugradnje u obrazovni sustav još važniji i poprima šire razmjere. Na osnovi izvještaja Ministarstva pravosuđa i ministarstva obrazovanja SAD-a iz lipnja 1998. može se donijeti zaključak o prednostima i koristima kvalitetnoga organiziranog vremena nakon nastave. One se odnose na sljedeće:

- poboljšanje učeničkog uspjeha u školi
- redovitije pohađanje nastave i smanjen broj djece koja prijevremeno odustaju od škole
- uspješnije i učestalije pisanje domaće zadaće
- poboljšanje učeničkog vladanja u školi
- značajno povećanje broja učenika s planovima za budućnost
- smanjenje nasilja i maloljetničkog kriminala te školskog vandalizma, zlouporabe droge i alkohola. (Šiljković i dr., 2007.)

Radi optimalne organiziranosti izvannastavnih aktivnosti potrebna je odgovarajuća edukacija učitelja već na fakultetima kako bi mogli pravovremeno prepoznati učeničke sposobnosti i sklonosti te ih razvijati na odgovarajući način. Dakako, ne smije se zanemariti ni učiteljevo zadovoljstvo i motivacija u vođenju aktivnosti.

Obilježja i podjela izvannastavnih aktivnosti

Izvannastavne aktivnosti obuhvaćaju različite programske sadržaje kojima se učenici bave izvan nastavnih obveza u organizaciji škole i u njezinim prostorijama. Njima želimo zadovoljiti individualne i stvarne potrebe učenika, interese i sposobnosti učenika, proširiti i produbiti njihova motorička znanja, razvijati uljudne odnose među učenicima te zbližavati učenike i učitelje.

Izvannastavne aktivnosti možemo podijeliti, primjerice, prema nastavnim područjima, po namjeni, po mjestu održavanja itd.

Prema nastavnim područjima izvannastavne aktivnosti možemo podijeliti na sljedeća područja: tjelesna i zdravstvena kultura, jezično-umjetničko područje, matematičko-informatičko područje, prirodoslovlje i ekologija. Prva dva područja pripadaju društvenim znanostima, a druga dva prirodnim znanostima. Nadalje, možemo ih podijeliti prema mjestu održavanja na one koje se održavaju u školskim prostorima i one koje se održavaju izvan njih.

Neki autori (Jurčić, 2008.) predlažu još i podjelu izvannastavnih aktivnosti na umjetničke, obrazovne i informativnopoučne.

Izvannastavne aktivnosti organiziraju se za učenike od prvoga do osmoga razreda na početku školske godine. Izbor aktivnosti prepušten je svakom učeniku prema vlastitom interesu, potrebi i mogućnostima, ali je djelomično ograničen učiteljevom ponudom izvannastavnih aktivnosti. Roditelje učenika potrebno je obavijestiti o svrsi, vremenu, mjestu i načinu rada aktivnosti u kojima njihova djeca sudjeluju. Cilj izvannastavnih aktivnosti jest obuhvatiti što veći broj učenika i usmjeriti učenike na odgovarajuću aktivnost u skladu s njegovim željama i sposobnostima. Izvannastavnim aktivnostima želimo omogućiti i stvoriti uvjete učenicima da se uključe u razne organizacijske oblike u kojima će naći smisao i zadovoljstvo.

Od učenika se očekuje da bude aktivni sudionik izvannastavnih aktivnosti, točnije, trebao bi sudjelovati u poticanju ponude aktivnosti, oblikovanju programa i sadržaja samih aktivnosti te njihovog tijeka i značaja. Osim toga, važno je omogućiti učeniku redovno pohađanje izvannastavnih aktivnosti te nastojati ispuniti njegova očekivanja.

Ponuda i odabir izvannastavnih aktivnosti izmedu učitelja i učenika

Učitelji su organizatori izvannastavnih aktivnosti te njihova odluka o odabiru aktivnosti izravno utječe na samu ponudu. Oni najčešće odabiru samo jednu izvannastavnu aktivnost, a rijetko dvije ili više. Njihova odluka o ponudi djelomično ograničava učenikov izbor. Motivacija i prikladni radni uvjeti važni su čimbenici koji utječu na učiteljev odabir izvannastavne aktivnosti. Učiteljev je zadatak prepoznati učeničke sposobnosti i sklonosti na temelju kojih će, također, donijeti svoju odluku o tome koju će izvannastavnu aktivnost ponuditi. Dakako, učitelj treba biti odgovarajuće i dovoljno educiran kako njegova ponuda izvannastavnih aktivnosti ne bi uključivala njegove subjektivne aspekte. Posebice se to odnosi na učiteljeve predrasude i učeničke predrasude prema određenim izvannastavnim aktivnostima. Najčešće predrasude vezane su upravo uz izvannastavne aktivnosti iz prirodnih znanosti, posebice iz matematike i njoj srodnih predmeta. Predrasude se protežu od predškolske do studentske dobi. Često se uočava podjela izvannastavnih aktivnosti prema spolu pa je uvriježeno mišljenje kako su djevojčice uspješnije u izvannastavnim aktivnostima iz društvenih znanosti, a dječaci aktivnostima iz prirodnih znanosti. Tako, od 1998. do 2008. godine na Učiteljskom fakultetu u Osijeku studenti prve godine imaju mogućnost odabira među trima usmjerenjima daljnjeg školovanja oblikovanih po modulima: modul A-razvojni smjer, modul B-smjer informatika i modul C-smjer strani jezik. Ponuda modula usmjerena je potražnji na tržištu rada, posebno modula vezanog uz matematičko-informatičko područje. Tijekom navedenoga razdoblja uočava se pojava da studenti u najmanjem postotku odabiru modul vezan uz matematičko-informatičko područje. Još uvijek dakle nije razvijena svijest studenata o potrebama na tržištu rada unatoč postojećim poticajima fakulteta.

Učenički odabir trebao bi biti nezavisan od učiteljeve ponude jer bi mu odabrana izvannastavna aktivnost morala pružati osjećaj zadovoljstva i želju za istraživanjem i stvaranjem. Također, na učenički odabir značajno utječe opći uspjeh, a posebno ocjena iz predmeta vezanog uz izvannastavnu aktivnost koju će odabrati. Iz našeg istraživanja vidljivo je da učenici pretežno odabiru izvannastavne aktivnosti prema predmetima iz kojih imaju odličnu ocjenu.

Treba težiti tomu da izabrana izvannastavna aktivnost bude rezultanta učeničkih želja, učiteljevih sposobnosti te potreba na tržištu rada. Nužno je kontinuirano obrazovati i usavršavati učitelje kako bi se ostvario zadovoljavajući sklad između učiteljeve ponude, učeničkog odabira izvannastavne aktivnosti i tržišne potražnje. Obveza škole jest osigurati učiteljima radne uvjete za ostvarenje izvannastavne aktivnosti (vremenske, materijalne, prostorne) te potporu u cjeloživotnom usavršavanju.

Metodologija istraživanja i rezultati

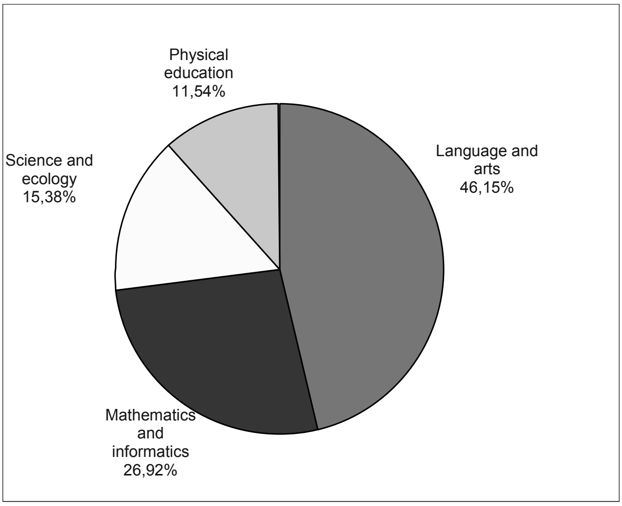
U prosincu 2008. provedena je anketa o sudjelovanju učenika i učenica četvrtih razreda u izvannastavnim aktivnostima te o učiteljevom odabiru izvannastavnih aktivnosti. Anketa je provedena u šest osnovnih škola u Osijeku, na uzorku od 107 učenika i 21 učitelja.

Cilj istraživanja bio je utvrditi postoje li razlike između izvannastavnih aktivnosti koje izabiru učenici i izvannastavnih aktivnosti koje nude učitelji te moguće uzroke i posljedice njihovih izbora. Aktivnosti smo grupirali u četiri područja: jezično-umjetničko, područje tjelesne i zdravstvene kulture, prirodoslovlje i ekologija te matematičko-informatičko područje. U analizu rezultata istraživanja uključeni su i podatci o upisanim studentima na fakultete prirodnih i društvenih znanosti te podatci o stanju potražnje stručnjaka za znanjima i vještinama iz područja matematike i informatike na tržištu rada. Time smo htjele uvidjeti postoji li povezanost između učeničkog odabira izvannastavnih aktivnosti iz određenih područja te kasnijeg izbora fakulteta i zanimanja. Također osvrnule smo se i na razliku u odabiru izvannastavnih aktivnosti prema spolu učenika.

Analizom podataka vezanih uz učiteljevu ponudu izvannastavnih aktivnosti (Slika 1) došli smo do zaključka da su najčešće ponuđene izvannastavne aktivnosti iz područja:

- jezično-umjetničkog (aktivnosti: recitatorska, dramska, likovna, scenska, tamburaška), 46,15%
- matematičko-informatičkog (aktivnosti vezane uz matematiku i informatiku), 26,92%
- prirodoslovlja i ekologije (aktivnosti vezane uz ekologiju te prirodu i društvo), 15,38%
- tjelesne i zdravstvene kulture (aktivnosti: ritmika, graničar, mali nogomet), 11,54%

Iz slike 1 vidljivo je da učitelji najviše vode aktivnosti iz jezično-umjetničkog područja, značajno manje iz matematičko-informatičkog te prirodoslovlja i ekologije, a u najmanjom postotku iz područja tjelesne i zdravstvene kulture. Učitelji dakle većinom nude izvannastavne aktivnosti koje se mogu svesti u područje društvenih znanosti.



Slika 1. Učiteljeva ponuda izvannastavnih aktivnosti po područjima.

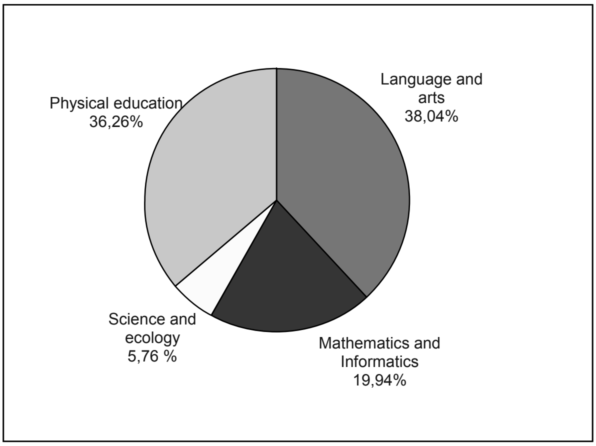
Analizom podataka vezanih uz učenički odabir izvannastavnih aktivnosti došle smo do sljedećih podataka:

- jezično-umjetničko, 38,04%
- tjelesna i zdravstvena kultura, 36,26%
- matematičko-informatičko, 19,94%
- područje prirodoslovlja i ekologije, 5,76%

Možemo zaključiti (Slika 2) da učenici najviše odabiru aktivnosti iz jezično-umjetničkog područja te područja tjelesne i zdravstvene kulture, značajno manje iz matematičko-informatičkog područja, a najmanje iz područja prirodoslovlja i ekologije. Učenici dakle također u većem postotku odabiru izvannastavne aktivnosti iz društvenih znanosti.

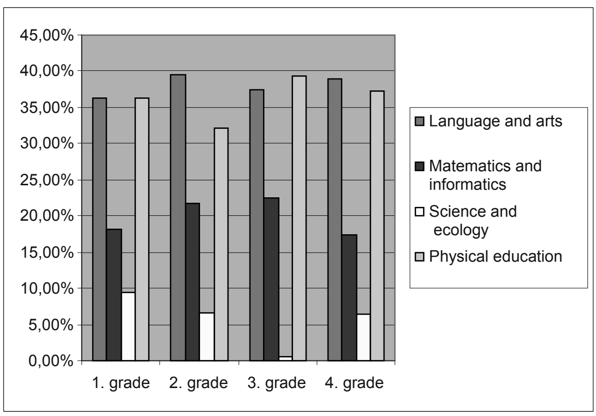
U odnosu na spol također smo uočile neke razlike u odabiru izvannastavnih aktivnosti prema područjima. Učenici češće biraju izvannastavne aktivnosti iz područja tjelesne i zdravstvene kulture te matematičko-informatičkog područja dok učenice češće biraju izvannastavne aktivnosti iz jezično-umjetničkog područja. Osvrtom na odabir područja po znanostima učenici odabiru izvannastavne aktivnosti iz prirodnih znanosti za 6,42% više u odnosu na učenice.

Promatrajući učenički odabir izvannastavnih aktivnosti po područjima i razredima primjetna je oscilacija u njihovom zanimanju za određena područja. Po razredima, najviše se razlikuje učeničko zanimanje za izvannastavne aktivnosti iz područja prirodoslovlja i ekologije, tj. postoji negativna korelacija.



Slika 2. Učenikov odabir izvannastavnih aktivnosti po područjima.

Prelaskom u više razrede učeničko zanimanje za izvannastavne aktivnosti iz područja prirodoslovlja i ekologije značajno opada (Slika 3).

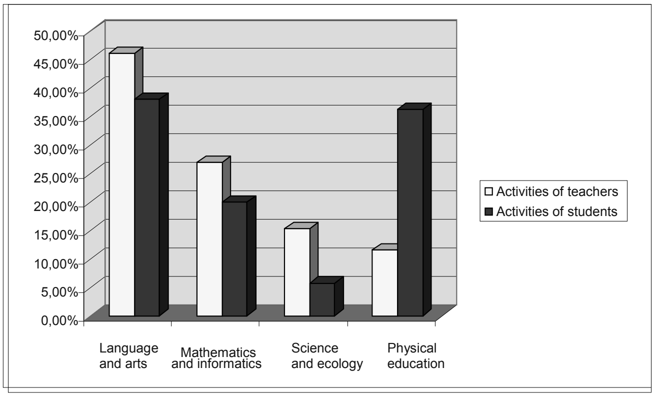


Slika 3. Učenički odabir izvannastavnih aktivnosti po razredima.

S obzirom na spol provele smo istu analizu učeničkog odabira izvannastavnih aktivnosti po razredima. Kod učenica rezultati pokazuju pozitivnu korelaciju u odabiru izvannastavnih aktivnosti iz jezično-umjetničkog područja, a negativnu korelaciju u odabiru izvannastavnih aktivnosti iz područja tjelesne i zdravstvene kulture te prirodoslovlja i ekologije. Prelaskom u više razrede interes učenica po navedenim područjima raste, tj. opada, redom.

Kod učenika rezultati pokazuju pozitivnu korelaciju u odabiru izvannastavnih aktivnosti iz područja tjelesne i zdravstvene kulture, a negativnu korelaciju u odabiru izvannastavnih aktivnosti iz svih ostalih područja. Prelaskom u više razrede interes učenika raste jedino za područje tjelesne i zdravstvene kulture.

Usporedbom odnosa ponude i potražnje izvannastavnih aktivnosti može se uočiti nesklad između učiteljeve ponude izvannastavnih aktivnosti po područjima i učeničkog odabira izvannastavnih aktivnosti po područjima (Slika 4). Učiteljeva ponuda izvannastavnih aktivnosti iz područja prirodoslovlja i ekologije veća je za 9,62% u odnosu na učeničku potražnju iz tog područja. S druge strane, učenička potražnja izvannastavnih aktivnosti iz područja tjelesne i zdravstvene kulture veća je za 24,72% u odnosu na učiteljevu ponudu iz tog područja.



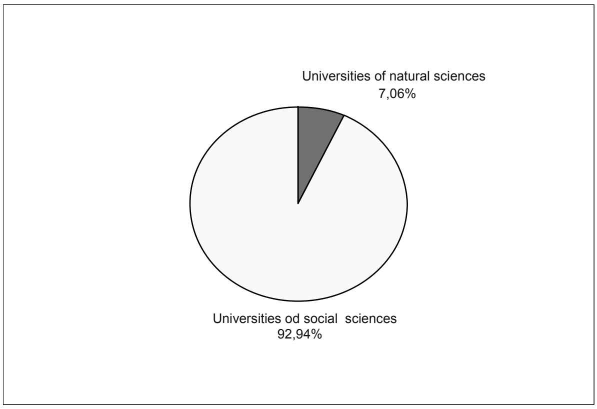
Slika 4. Odnos ponude i odabira aktivnosti (učitelji/učenici).

Možemo zaključiti da je još uvijek učenikov odabir izvannastavnih aktivnosti djelomično uvjetovan učiteljevom ponudom. Učenik dakle nema dovoljan utjecaj na ponudu izvannastavnih aktivnosti i ne može u potpunosti slobodno odabrati željene izvannastavne aktivnosti.

Nadalje, općenito možemo dovesti u vezu slab učenički odabir izvannastavnih aktivnosti iz matematičko-informatičkog područja,, negativne stavove prema osobnim računalima i matematici srodnim predmetima s odabirom željenog fakulteta prirodnih odnosno društvenih znanosti, ali i odabirom onih zanimanja koja ne zahtijevaju znanja složene i više matematike. Ukupni proces kružno se ponavlja jer je nekadašnji učiteljev odabir izvannastavnih aktivnosti također kreiran analogno načinom kako danas učenici odabiru izvannastavne aktivnosti pa kasnije fakultete i buduća zanimanja. Slika 5 prikazuje postotak upisanih studenata na fakultete prirodnih i društvenih znanosti u akademskoj 2008./09. godini u Hrvatskoj.

Možemo zaključiti da je značajna razlika u odabiru fakulteta prirodnih i društvenih znanosti u korist fakultetima društvenih znanosti (92,94% od ukupno upisanih studenata) ([6]).

Situacija se odražava i na tržište rada gdje je posljedično tomu nedostatak stručnjaka iz prirodnih znanosti, a posebno onih sa znanjima i vještinama iz matematičko-informatičkog područja. Prema podatcima dobivenim od Hrvatskog zavoda za zapošljavanje, broj traženih stručnjaka iz matematičko-informatičkog područja 2008. godine iznosio je 830 što čini 47,6% od ukupnog broja ponuđenih radnih mjesta iz obrazovanja ([7]). Gotovo polovica dakle ponuđenih radnih mjesta iz obrazovanja odnosi se na stručnjake iz matematičko-informatičkog područja.



Slika 5. Postotak upisanih studenata na fakultete prirodnih i društvenih znanosti 2007./08.

Zaključak

Rezultati istraživanja potvrdili su našu pretpostavku da učiteljeva ponuda izvannastavnih aktivnosti nije u potpunosti usklađena s učeničkim željama. Određenije, promatrajući odabir aktivnosti učitelja i učenika iz područja tjelesne i zdravstvene kulture te područja prirodoslovlja i ekologije rezultati provedenog hi kvadrat testa pokazali su da s 95%-tnom sigurnošću možemo tvrditi da postoji statistički značajna razlika u odabiru aktivnosti. Također, učiteljeva ponuda i učenički odabir izvannastavnih aktivnosti nije usklađen s potražnjom na tržištu rada

Rezultati hi kvadrat testa pokazuju da postoji statistički značajna razlika u odabiru aktivnosti iz prirodnih i društvenih znanosti kod učenika na svim razinama pouzdanosti. Vidljivo je da učenici daju prednost aktivnostima iz društvenih znanosti dok su aktivnosti iz prirodnih znanosti još uvijek nepravedno zanemarene. To upućuje na zaključak da se predrasude koje vladaju prema prirodnim znanostima i matematici u našem društvu prenose svakodnevno s roditelja na dijete te ponekad i s učitelja na učenika pa time ostaju neprestano prisutne od predškolske dobi do upisa na fakultete i kasnijeg izbora zanimanja. Dobiveni podatci iz akademske 2008./09. godine jasan su pokazatelj da studenti u značajno većem postotku odabiru studije društvenih znanosti nego studije prirodnih znanosti unatoč činjenici što na tržištu rada postoji najveća potreba za stručnjacima iz obrazovanja na području matematike i informatike. Rezultati istraživanja ukazuju na potrebu oblikovanja timova stručnjaka koji će potaknuti interese učenika i studenata prema onim aktivnostima koje potražuje tržište rada. Dobiveni rezultati pomoći će u procesu usklađivanja i prinavljanja kurikula učiteljskih studija.

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2.
O ishodima učenja iz nastave matematike za učenike

Képekre épülő matematikatanulás

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Eötvös university (ELTE), Budapest, Hungary

Összefoglaló. Vizsgálatainkat nehezen megközelíthető magyar kisiskolákban végeztük. Azt tapasztaltuk, hogy a hátrányos helyzetű tanulók más társadalmi nyelvváltozatot beszélnek, mint tanáraik, ezért még a tehetséges tanulók sem tudnak elég jó eredményeket elérni. Képeket, vagyis PowerPoint prezentációkat használtunk, amelyek egyrészt segítettek megérteni a tanulóknak a feladatokat, másrészt segítettek a pedagógusoknak, hogy viszonylagosan új pedagógiai módszereket alkalmazzanak. Ezeket a módszereket már korábban is jól ismerték, de úgy gondolták, hogy azok csak a legkiválóbb, jól felszerelt iskolákban használhatóak, mint például a csoportmunka, az informatikai eszközök és a tanulási folyamat elbeszélő elemei, párhuzamosan a logikailag szervezett ismeretekkel.

Tanítottunk néhány érdekességet a matematikatörténetből is. Az egyiptomi számírás volt az egyik legjobb motiváló eszköz.

Azt tapasztaltuk, hogy minden tanuló a korábbinál jobb eredményeket ért el, a legtehetségesebbek pedig olyan inverz feladatokat is meg tudtak oldani, amelyek közül néhány az egyetemi hallgatóimnak is gondot okozott.

Kulcsszavak: A matematikatanítás új módszerei, szemléltetés, tárgyi tevékenység, történeti gondolkodásmód, tehetséges tanulók, hátrányos helyzetű tanulók

A nyelvi hátrány és a matematikatanulás

Gyakran előfordul, hogy gyerekeknek nem az anyanyelvükön kell elkezdeniük iskolai tanulmányaikat. Ennek nehézsége nyilvánvaló. Azonban az azonos nyelvet beszélők között is jelentős nyelvhasználati különbségek lehetnek. Ezt az eltérést a nyelvészek eltérő regionális és társadalmi dialektusként írják le. Tapasztalataink és a nemzetközi szakirodalom szerint is az eltérő nyelv és az eltérő nyelvváltozat is hasonló tanulási akadályokat okozhat.

Sokan úgy vélik, hogy a matematikatanulást befolyásolja legkevésbé a nyelvi különbség.

Ezt a nézetet sok tapasztalat erősíti meg, pl. ha egy tanuló, néhány év otthoni tanulás után külföldre, idegen nyelvi környezetbe kerül, akkor gyakran előfordul, hogy nyelvi hátrányai ellenére ő lesz a legjobb matematikus az osztályában.

Érdemes azonban felfigyelni arra, hogy ebben az esetben a tanuló biztos anyanyelvi alapokkal és jó matematikai előképzettséggel rendelkezve éri el a sikereket az új nyelven matematikából. Egészen más a helyzet, ha a kisdiák úgy kerül be az iskolába, hogy sem a hétköznapi beszélgetés szavait, szituációit nem érti pontosan, sem azokat a matematikailag releváns tapasztalatokat nem szerezte meg a családjában - pl. táblás társasjátékokat játszva, szüleinek a kertben, konyhában, bevásárláskor segítve, közben sokat beszélgetve - amelyekben szerencsésebb társai részesülhettek. Szakszerű iskolai segítség nélkül még az okos gyerekek is iskolai kudarcokra vannak ítélve. A tanítók és a tanárok nagy része talál megoldást a problémákra. A pedagógiai kutatás feladata az, hogy ne kelljen mindenkinek saját megának kitalálnia eszközöket és módszereket, hanem álljon a pedagógusok rendelkezésére a lehetőségek széles választéka, amit az adott körülményeknek megfelelően lehet alkalmazni.

Cél

Kutatásunkban azt vizsgáljuk, hogyan jelentkeznek a kommunikációs zavarok a matematikatanulásban és hogyan lehet azokat csökkenteni.

Módszer

Kutatási módszerünk a résztvevő megfigyelés. A hagyományos, megfigyelésen alapuló, diagnosztizáló helyzetfelmérés ebben a témában nem vált be, mivel az óramegfigyelések éppen a kommunikációs folyamatokat zavarják leginkább (Tuveng, 2005). A hospitálások során a zavarok nehezen vehetők észre, valamint az esetleg mégis felmerülő zavarok megoldását a pedagógusok későbbre, a vendégek távozását követő időre halasztják. Ezért a pedagógusokkal együttműködve dolgozunk a tanulási-tanítási folyamat fejlesztésén és a munka közben felmerülő tapasztalatokra reflektálva tervezzük meg a következő lépéseket. Így egyszerre kapunk képet a problémákról és a lehetséges megoldási módokról.

Tanuláselméleti háttér

Hátrányos helyzetben a leggyakrabban alkalmazott didaktikai megoldás a tananyag és a követelmények csökkentése, ami érthető, hiszen sok, a többségitől eltérő családi helyzetből származó tanuló nem, vagy alig tud írni, olvasni, számolni még 8-10 éves kora után sem.

Mi ennek ellenére igényes, a tantervi követelmények optimumát megcélzó tananyagot kívánunk a gyerekeknek nyújtani. A feladatok megértéséhez jelentős segítséget adunk. Az elsődlegesen megoldandó feladatunk, hogy úgy tegyük világossá a feladat kontextusát, hogy közben a problémákat ne oldjuk meg a tanulók helyett. A kutatásunk során Bruner (1965, 1966) tanuláselméletét alkalmazzuk, pontosabban arra, valamint a továbbfejlesztéséből eredő egyéb tanuláselméleti, és különösen a speciális matematikatanítási eredményekre építünk (Tall, 2005).

A magyar matematikatanítás újabb kori hagyományaiban igen nagy jelentőségű Varga Tamás életműve. Eredményei beépülnek a magyar egyetemeken és főiskolákon tanított matematika tantárgypedagógiákba (Ambrus A. /2004/., Czeglédy /2005/, Szendrei /2005/)

Munkánk alapgondolata, hogy a tanulási folyamat egészét be kell vinnünk az iskolába, nemcsak annak harmadik, szimbolikus szintjét, mert a gyerekek hétköznapi tapasztalatai és az iskolákban alkalmazott középosztálybeli, bonyolult nyelvhasználat között a hidat aktív pedagógiai beavatkozással kell kiépíteni. A gyerekek számára ismerős tevékenységeket a tanteremben is elvégezve azokat úgy alakítjuk, hogy azokból könnyen lehessen matematikailag releváns tapasztalatokat szerezni, majd következtetéseket levonni.

Újdonságot jelent munkánkban, hogy a nyelvi problémák csökkentése érdekében egyrészt képekkel, a gyerekek számára ismerős szituációk felelevenítésével segítjük a matematikai feladatok megértését, úgy, hogy a szöveg szerepét minimálisra csökkentjük, ugyanakkor az órák fontos részét jelentik a beszélgetések, amelyek keretében a tanulók elmesélhetik, sőt még azt is megkérjük, hogy írják le a matematikatanulás során keletkezett élményeiket. Meggyőződésünk, hogy amit Bruner általában a tanulás narratív jellegéről mond, az a matematikatanulásra is jellemző (Bruner, 1991) A tanulás hatására matematikai ismeretek szerveződése természetesen döntően logikai lesz, de a tanulási folyamatban, a matematikában éppen úgy, mint az egyéb területeken igen nagy szerepe van a történeteknek és történéseknek

A hátrányos helyzetű gyerekek között a matematikatanulásban, akár a nyelvi hátrányt, akár az egyéb társadalmi hátrányokat tekintjük, egyaránt vannak nagyon, átlagosan és annál kevésbé tehetséges tanulók, így a tehetséggondozás modern elveinek (Balogh, 2004, 2007) megfelelően egyszerre fordítunk figyelmet a gyerekek közös munkájára, egységes foglalkoztatására és a kiemelkedően tehetségesek speciális fejlesztésére (Czeglédy, 1992). A matematikatanuláson belül Magyarországon ma már többféle tehetséggondozó program működik, feladatunk, hogy a 6-10 éves tanulóknak olyan indítást adjunk, ami lehetővé teszi számukra a felső tagozattól kezdődően a bekapcsolódást ezekbe a programokba.

Az empirikus vizsgálat

A vizsgálatokat az ELTE TTK **Multimédiapedagógiai és Oktatástechnológiai Központja és a Matematikatanítási és Módszertani Központ végzi, a program irányítója Kárpáti Andrea, a matematikai részprogramot Munkácsy Katalin vezeti. A kutatók és a pedagógusok együttműködésének keretét a** mentorált innovációs program jelenti. A program fő területei az Integrált esztétikai nevelés, a Matematika, a komplex tanulmányi verseny, az Aranysüveg verseny, valamint sok iskolán kívüli tevékenység voltak.

Populáció

Vizsgálatainkat összevont tanulócsoportos iskolákban végezzük.

Itt a kutatások szociológiai szempontból aktuálisak, mert az oktatás gazdaságosságának igénye és a települések népességmegtartó képességének megtartása ellentétes irányokba mozdítaná a kistelepülések kisiskoláinak sorsát.

Pedagógiai szempontból egyszerűbbé, jobban átláthatóvá teszi a helyzetet, hogy ezekben a kisiskolákban a gyerekek többsége hátrányos helyzetű, ezért ennek az iskolán belül nincs stigmatizáló hatása, a gyerekek nem kényszerülnek be a rossztanuló szerepébe, ahogyan ez a többségi, “normális” iskolákban gyakran előfordul. Így az érzelmi konfliktusok, önértékelési zavarok előfordulása ritkább, több lehetőség van a kognitív folyamatok megfigyelésére és alakítására.

Minta

Az ország minden nagyobb régiója képviselve van, a **részt vevő iskolák a következő településeken találhatók**: Alsószentiván, Debrecen (SNI), Felsőpetény, Hasznos, Keszeg, Kolontár, Nógrádkövesd, Nógrádsipek, Pogány, Sárhida, Somogyhárságy, Szárföld, Székesfehérvár (SNI), Vezseny, Vadna, Felsőkelecsény. Az iskolák egy felkérő levélre, amit közel 800 iskolába küldtünk ki válaszolva léptek be a programba, a későbbi beszélgetésekből megtudtuk, hogy a tanítók motivációja igen eltérő volt, néhányan közülük gyakorlott pályázók voltak, másokat az iskolák vezetői küldtek, hogy ezáltal is csökkentsék hiányosságaikat, voltak, akik jó informatikai előképzettségüket akarták az iskolai gyakorlatban is kamatoztatni, de többségük most akart megismerkedni az informatika alapjaival, ezért mintánkat - az önkéntes jelentkezés ellenére is - kvázi véletlennek tekinthetjük.

A két nagyvárosi iskolából sajátos nevelési igényű (SNI) csoportok kapcsolódtak be a munkába, így bár a minta nem reprezentatív ebből a szempontból sem, de képet kaphattunk az eltérő képességű tanulók eltérő lehetőségeiről is, de arról is, hogy a feladatok egy része minden tanuló számára élményt jelentett, más részébe pedig az egyes tanulók nagyon különböző szinten tudtak bekapcsolódni.

Matematikatanulási vizsgálat és fejlesztő program

Cél

A kommunikációs akadályok meglétének igazolása.

Fejlesztő program kialakítása és kipróbálása.

Lebonyolítás

A 2006–2007-es tanév második félévében négy kéthetes szakaszban tartottak rendhagyó matematikaórákat a tanítók.

A 2007–2008-as tanévben a tanulási folyamat mikrótörténéseit feltáró vizsgálat folyt, valamint megkezdődött a tapasztalatok alapján kiegészítő tantervi anyagok és terjesztésre oktatási eszközök készítése.

Elvek

- A tanulás során a gyerekek olyan problémákkal és úgy találkoznak, amelyek problémamegoldást várnak el tőlük (Pólya) és elvezetnek a magasabb matematikai összefüggések felismerése felé (Varga Tamás)
- Szerepet kapnak a történeti aspektusok
- A matematika széles köréből válogatjuk a témákat. Kapcsolódunk a tantervi anyaghoz, de a témákat úgy építjük fel, hogy azáltal kitekintést nyerhessenek a tanulók a mélyebb matematikai összefüggésekre. Az alapkészségek alacsonyabb színvonala miatt az aritmetikai, számelméleti és logikai jellegű feladatokat későbbre helyezzük, és térgeometriai, függvényekkel összefüggő, adatkezelési problémákkal vezetjük el a gyerekeket a matematikai összefüggések megértéséhez.
- Képekkel irányított tanulás: az órákon a gyerekek diasorozatot néznek, és a módszertani útmutató is a képekre épít, visszahat a tevékenységre, előre mutat a szimbolikus szintre
- Számítógéphasználat: kommunikációs eszköz és információforrás
- Csoportmunka
- Gazdag kommunikációs hatások

A közös munka szerkezete (online és face-to-face találkozások keretében):

— A kísérleti tananyag kiválasztása és elemzése

— A kiválasztott tananyag módszertani feldolgozása PowerPoint prezentációk formájában, módszertani útmutatók elkészítése

— Az iskolai alkalmazás után a tanítók beszámolója az órai tapasztalatokról, azok megvitatása

— Előkészületek az új szakaszra.

Tananyagkiválasztás

Olyan tantervi anyagokat választottunk ki, amelyek a tanítók szerint a tanulók számára nagy nehézséget jelentenek, ezek

Mérés, mértékegységváltás

Térbeli tájékozódás

Számolás, tizes-átlépés

Szöveges feladatok

Érvelés, állítások megvizsgálása, a döntések indoklása

A felsorolás tapasztalati lista, igen különböző jellegű tanulási problémákat takar, ennek ellenére alkalmasnak látszott a négy szakasz tananyagának megtervezésére.

A PowerPoint bemutatók diafilmszerűek voltak. E műfaj kiválasztásában a gyerekpszichológusuk javaslataira támaszkodtunk, akik a gyerek képzelőereje és gondolkodási folyamatainak intenzívebb fejlesztése érdekében a mozgóképeket csak kivételes esetekben javasolják.

A diafilmek egy macicsalád egyszerű kalandjai révén mutatták meg azokat a kontextusokat, amelyek érthetővé teszik, jelentéssel telítik a gyerekeknek szánt feladatokat. A diafilmek mintákat nyújtottak a tárgyi tevékenységre, pl az űrtartalom mérése során először a macik mérték meg a kancsókat és a poharakat, majd a gyerekeket biztatták ugyanerre, aminek a feltételeit a pedagógusok meg is teremtették. A diák ügyességi játékokra, pl. az egyensúlyi játékra is példát mutattak. A képek által kiváltott élmények alapján a tanítók beszélgettek a tanulókkal, sőt rövid, pár szavas írásos beszámolókat is kértek tőlük.

A forgatókönyvek MK állította össze, a szeretnivaló, ugyanakkor erős, karakteres macifigurákat egy főiskolai hallgató, Schlosszer Ákos tervezte meg, és rajzolta meg a diák különböző változatait. Ezeket CD-re gyűjtöttük és be fogom mutatni az előadás keretében.

A prezentációk

A diafilmek tartalmát a kutatáshoz később kapcsolódó pedagógus így írta le:

- *A mackó család az étkezőasztalnál teázik. Az asztalon van egy kancsó, a mackó apuka nagy pohara, és a kismackó kis pohara. Mackó apuka zavart tekintettel széttárja a kezét: "Vajon hány kis pohár víz fér a kancsóba? Tippeljétek meg!" A mackók teletöltik a kancsót. "Tíz pohár víz fér a kancsóba. És szerintetek?" Méricskélés, majd ösztönzés a saját kezű kísérletezésre. Ezután összegezzük a mért adatokat: "10 kispohár víz = 1 kancsó víz, 5 nagypohár víz = 1 kancsó víz", majd összefüggéseket állapítunk meg." "A kismackók szerint pontosan két kispohár víz fér a nagyba." "Más poharakkal 18 kicsi, vagy pedig 6 nagy pohárral lehetett megtölteni a kancsót. Most hány kispohárral lehet teletölteni a nagyot?"*
- *Maciék Erdélyben, Kézdivásárhelyen nyaraltak és elutaztak a legközelebbi nagyvárosba, Brassóba. Vonattal mentek. Maciéknak "nagyon tetszett az utazás, ezért elhatározták, hogy utazási irodát fognak játszani. Játsszatok Ti is! Nektek mennyibe kerül a jegy? Kik utaznak együtt?" A mackócsalád dobókockával, véletlenszerűen dönti el, hogy kik fogják alkotni a turistacsoportokat. A neveket és a jegyárakat táblázatba írják. Tippelnek, hogy kinek mennyibe kerülnek a teljes, félárú és egyéb kedvezményes jegyek. Ezután ki is számolják.*
- *A mackócsalád kirándulni megy, ezúttal gyalog. Egymás szórakoztatására mackó apuka különböző furmányos kérdéseket eszel ki. "Az út melyik oldalán van a fa?" Kismaci töpreng, majd így szól: "A menetirány szerinti bal oldalon". Egy útkereszteződéshez érnek. "Macipapa új feladványt eszelt ki. Ti meg tudjátok oldani?" Felülről látjuk a kereszteződést. Két autó érkezik egymással szemben és egy irányban kanyarodnak el. Vajon melyik kanyarodik jobbra?*

- *Mi sem természetes, a mackócsalád ezúttal a szomszéd erdőben lévő időkerék felé veszi az irányt, hogy "régmúlt időkbe kiránduljanak". "Az ókori Egyiptomban érdekes jeleket vettek észre". Szerencséjükre "segítségükre sietett egy írnok, aki elmagyarázta a jeleket". Beavatnak az ókori egyiptomi számírásba, majd az írnok teszteli, hogy le tudjuk-e írni jól megszokott arab számainkat hieroglifákkal. Szerencsére a megoldást is elárulja, majd az érdeklődőknek a szorzást is megmutatja.*
- *Mackóék az időkerékbe pattanva Görögországba is elutaztak. Egy nagy és két kis kocka csokiról mackó apukának eszébe jutott a Pitagorasz-tétel. "Ha van ez a két kicsi csoki meg ez a nagy, akkor mindegy, melyiket választom, ugyanannyi csokit fogok kapni. Elhiszitek?" "A kismaci saját szemével akarta látni, hogy tényleg így van. Kapott macipapától papírnégyzeteket. Mackó apuka egyik átlója mentén félbevágta a négyzeteket. "A macigyerek meg addig ügyeskedett, amíg kirakta a nagyot. Ti is megpróbáljátok?"*

Ez utóbbi feladathoz kapcsolva mutatok példát egy tanítói beszámolóra:

> "Együtt néztük tovább a ppt-t a négyzetekről. Nem akarták elhinni, hogy igaz az állítás. Csoportokban ültek, megkapták a 14 cm-es és a 10 cm-es négyzeteket. Tették-vették, forgatták, majd egyre többen állították, hogy el kellene vágni a kicsiket. Kaptak ollót, persze így sem sokra mentek, mert senki sem vágta átlósan. Könyörögtek, hogy 'szabad a gazda', áruljam már el a megoldást. Megmutattam."

A gyerekek beszámolóikat igen tömören fogalmazták meg, most néhány olyan példát idézek, amelyek a csoportban végzett matematikatanulás élményét elevenítik föl, az örömökkel és a konfliktusokkal együtt.

> "Amikor Erik tönkretette a háromszögemet, akkor mérges voltam. Nekem sem volt jó az Erikkel játszani, hanem a Petivel."
>
> "Szégyen volt a csapat, mert Nilla és Zsolti veszekedtek."
>
> "Ügyesek és türelmesek voltak a többiek."
>
> "Jó volt Z-val dolgozni. Megbeszéltük, amit csináltunk."
>
> "Jó volt, hogy nem kellett egyedül dolgoznom."

Matematikából tehetséges tanulók megtalálása

A program második részében a poliéderfogalom építését kezdtük meg. Minden tanuló sokat fejlődött, de a legtehetségesebbek képessé váltak igazán nehéz inverz feladatok megoldására is. Ezekre példát az előadás keretében fogok mutatni.

Eredmények, tapasztalatok

- Tanulási készültség

  Meg kellett különböztetni az iskolakészültséget és a tanulási készültséget.

Az iskolakészültség elvár elég gazdag szókincset, elemi számolási ismereteket, az írás-olvasás funkciójának ismeretét (olyan tapasztalatokat, ami alapján a gyerekek tudják, hogy írni-olvasni jó és hasznos), monotóniatűrést – hiszen a közös munkának ezek jó feltételeit jelentik és a tanulók jelentős része ezeknek az igényeknek képes is megfelelni.

A tanulási készültség Bruner szerint mást jelent, jelenti a kíváncsiságot, a figyelem képességét, az értelmi képességek elégséges fejlettségét az írás megtanulásához, a saját élmények megfogalmazásának képességét (Bruner, J. Kenney, H., 1965). A nyelvi hátrányokkal küzdő gyerekek e téren általában nincsenek lemaradva, ezért tudták a cselekvésbe ágyazott, az e korosztály számára egyébként nehéz problémákat is megoldani, pl. KRESZ vizsgához hasonló kérdéseket az útkereszteződésekben kanyarodó autókról.

Az alkalmazott motivációs vizsgálat, az OMT (Kuhl, 1999) teszt alapján az egyébként gyengén teljesítő tanulók nagy része is rendelkeznek a tanuláshoz szükséges belső feltételekkel, de az intellektuális technikákban, írás-olvasás-számolás gyakorlatlanok

- A tanulók gyakran nem értik a pedagógusok szavait, mégsem kérdeznek. A norvég kutatással, amelyben norvég és bevándorló gyerekek, 9 évesek matematikatanulását vizsgálták, egyező tapasztalat, a problémák megfogalmazásához és a kérés kimondásához az ebben az életkorban elvárhatónál jóval fejlettebb kommunikációs képességre volna szükség, itt pedig éppen annak alacsonyabb színvonala bizonyosodott be.
- A gyerekek szó- és fogalomkészlete a tantervekben feltételezettnél jóval alacsonyabb, ez külön segítség nélkül tanulási kudarchoz vezet, viszont támogatás esetén a gyerekek kíváncsiak és nagy örömmel tanulnak új szavakat, építenek fel új fogalmakat.
- A tanulók nagy többsége örömmel fogadta a matematikatörténetet. Szívesen ismerkedtek az egyiptomi hieroglifákkal, nagyon büszkék voltak rá, hogy ők ilyen különleges dolgokat is tanulnak. A tanítók egy része a felkészülést az átlagosnál nehezebbnek tartotta, de a gyerekek aktivitásával mindenki meg volt elégedve.
- A gyerekek szívesen végezték a cselekvéses problémamegoldást a játékos, meseszerű keretben, ennek fő segédeszköze a ppt volt, pl. a mértékegységek szívesen váltották át, mértek, úgy, hogy előtte megbecsülték az eredményt, majd összehasonlították a mért és a becsült értéket. A csoportmunka csökkentette a feszültségeket.
- A tanítóik átlátták az oktatási folyamat matematikai tartalmát, és reflektív módon más területeken is alkalmazták tapasztalataikat, pl. az anyanyelv órák után is megkérték a gyerekeket, hogy saját szavaikkal meséljék el, mit tanultak az órán..
- A matematika és a hétköznapi élet határterületén található szavak, pl. él-lap-csúcs esetében szinte minden vizsgált tanuló esetében nagy volt a bizonytalanság. A cselekvésbe ágyazott problémamegoldás esetében esettanulmányunkban, végül minden tanuló eljutott a jó leszámolási algoritmushoz, a legtehetségesebbek pedig nehéz inverz feladatot is képesek voltak

önállóan, jól megoldani. A tehetséges tanulók számára a pedagógusokkal együtt kerestük a fejlesztés lehetőségét az adott feltételek között. (Iskolán belül érdemes a kötelező tananyag keretein belül maradni és közben alkalmat találni a felmerülő problémák alaposabb megbeszélésére, ennek igényének fejlesztésére és keresni kell a lehetőséget iskolán kívüli programokban való részvételre.)

- Bevált az IKT, vagyis az információs és kommunikációs eszközök alkalmazása. Számítógép és internet nélkül nem is tudtunk volna együttműködni, de a kísérleti órákon is nélkülözhetetlen volt legalább egy kisteljesítményű gép.
- Kialakult és működött a tanítók virtuális és közvetlen kapcsolatokon alapuló közössége.

Következtetések

- A gyerekek szívesen fogadták a programot. Konkrétabbá vált a motivációjuk. Míg korábban is tapasztalható volt a személyiség gazdag motivációs rendszere és a jó tanulmányi eredmény iránti vágyuk, most ez kiterjedt a tanórai tanulásra is, örömteli élményt jelentett többségük számára is várják a folytatást.
- A pedagógusok szívesen vették részt a fejlesztési folyamatban, kreatív módon bekapcsolódtak, a tapasztalataik sok elemét képesek alkalmazni munkájuk más területein is. A korábban csak ünnepinek tartott módszertani megoldásokat, IKT, csoportmunka, részben kötött beszélgetése az adott nehéz körülmények között is alkalmazhatónak tartják és sikeresen alkalmazzák is.
- hatások tartóssága és elterjeszthetősége megkívánja, hogy a tananyag jól kiválasztott részeit külső szakemberek dolgozzák fel és a tanítók folyamatos konzultációs lehetőséget kapjanak a munka közben felmerülő problémák megbeszélésére.

Összefoglalva

Szívesebben és jobban dolgoztak a diákok, mint korábban – ez szinte bármilyen oktatási kísérletben így van, ami a munkánkat mégis megkülönbözteti: felszínre kerültek a kommunikációs problémák és világossá vált, hogy a társadalmi hátrányokból származó akadályok csökkentésére nincsenek kidolgozott matematikatanítási módszerek. Az értelmi sérültek számára összeállított fejlesztő programok nem alkalmazhatóak, hiszen a probléma alapvetően más. Az egészségesen fejlődő, de tanulási akadályokkal küzdő gyerekeknek sajátos, a szöveg és a szituációk megértését segítő, de intellektuálisan nagy kihívást jelentő feladatokra van szükségük. Az órai kommunikációs folyamatok elemzésével és az arra épülő módszertanok kidolgozásával segíthetjük, hogy a tanulók eljussanak a matematikai fogalmak és összefüggések mély megértéséhez, a legtehetségesebbek pedig a nehéz problémák megoldásához is.

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Z igro v svet verjetnosti

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Povzetek. Verjetnost je matematična vsebina, ki so se jo učenci v Sloveniji glede na večino drugih držav začeli učiti razmeroma pozno (v srednji šoli), in še to v večini srednjih šol le na formalni ravni. V novem učnem načrtu matematike za devetletno osnovno šolo je ta vsebina vpeljana že v osnovni šoli.

Današnji človek živi v hitro spreminjajočem se svetu in se mora vedno pogosteje soočati z novimi in negotovimi situacijami. Otroka, bodočega odraslega, moramo pripraviti na ta svet tako, da ga bo znal kritično interpretirati in zavestno delovati v njem. Zato je potrebna abeceda verjetnosti, ki zahteva poseben način mišljenja, tuj determinističnemu načinu mišljenja, prevladujočemu v naših šolah. Tudi odrasli, ki so bili deležni takšne šolske izobrazbe, velikokrat naletijo na težave pri dojemanju osnovnih pojmov iz verjetnosti, saj dvovalentna logika (narobe/prav, je res/ni res) odpove. Z raziskavo smo prišli do zaključka, da učenec že na razredni stopnji na intuitivnem oziroma izkustvenem nivoju preko igre dobro sprejema in usvaja najelementarnejše koncepte verjetnosti.

Učenje verjetnosti v osnovni šoli ni eksplicitno in formalno, ampak je zgolj sistematično pridobivanje izkušenj, na podlagi katerih bomo kasneje (v srednji šoli) učinkoviteje poučevali verjetnost. Zato v osnovnošolskem programu pri pouku matematike ne govorimo o formalni definiciji verjetnosti, ampak učenca pripravljamo na kasnejšo matematično analizo slučajnih dogodkov z didaktičnimi igrami in s smiselno stopnjevanimi drugimi dejavnostmi. Učenec opiše, kaj se mu zdi mogoče oziroma nemogoče; razlikuje med gotovim, slučajnim in nemogočim dogodkom; primerja med seboj verjetnosti slučajnih dogodkov; pri preprostih igrah na srečo postavlja smiselne hipoteze in jih skuša podpreti z izkušnjami... Učenec naj bi namreč pridobival izkušnje s slučajnimi dogodki in razvijal sposobnosti predvidevanja leteh predvsem preko igre. Z nivoja sprejemanja negotovosti naj bi preko nivoja predvidevanja dogodka prešel do nivoja primerjanja verjetnosti, preden bi kasneje (v srednji šoli) dojel statistično in klasično definicijo verjetnosti.

Ključne besede: devetletna osnovna šola, didaktična igra, dejavnosti, učenje verjetnosti, gotov, nemogoč in slučajen dogodek.

Uvod

V osnovni šoli ne poučujemo verjetnosti v eksplicitnem in formalnem smislu, pač pa je postopno učenje verjetnosti zgolj sistematično pridobivanje izkušenj, ki bodo otroku koristile zlasti v srednji šoli, ko bo spoznaval formalne vsebine verjetnosti. Področje verjetnosti je namreč z vidika poučevanja in učenja zelo zahtevno. Kljub dejansko neoporečnemu pouku imajo srednješolci in študentje pogosto izkrivljene predstave o verjetnosti. V osnovnošolskem programu pri pouku matematike ne govorimo o formalni definiciji verjetnosti, ne omenjamo ne klasične ne statistične definicije verjetnosti. Verjetnosti niti ne računamo, ampak učence na podlagi intuicije in ludizma pripravljamo, da bodo na kasnejši stopnji šolanja znali matematično analizirati slučajne dogodke. Podobno kot pri kombinatoriki torej tudi pri teh vsebinah ne presežemo izkustvenega nivoja. Učenec naj bi s smiselno stopnjevanimi aktivnostmi pridobival izkušnje s slučajnimi dogodki in si pri tem pridobil koncepte, principe in sposobnosti predvidevanja pri slučajnih dogodkih, kar je v današnjem svetu, ki je poln negotovosti in nepredvidljivosti, zelo pomemben cilj. človek se mora namreč znati "spopasti" z negotovimi situacijami, jih znati predvideti in se znati odločiti med različnimi alternativami ter na koncu znati rešiti probleme, ki ne morejo biti rešeni z dvovalentno logiko, oziroma kot pravi Fischbein: "razviti mišljenje, ki je drugačno od determinističnega mišljenja" (Fischbein, 1984, str. 35).

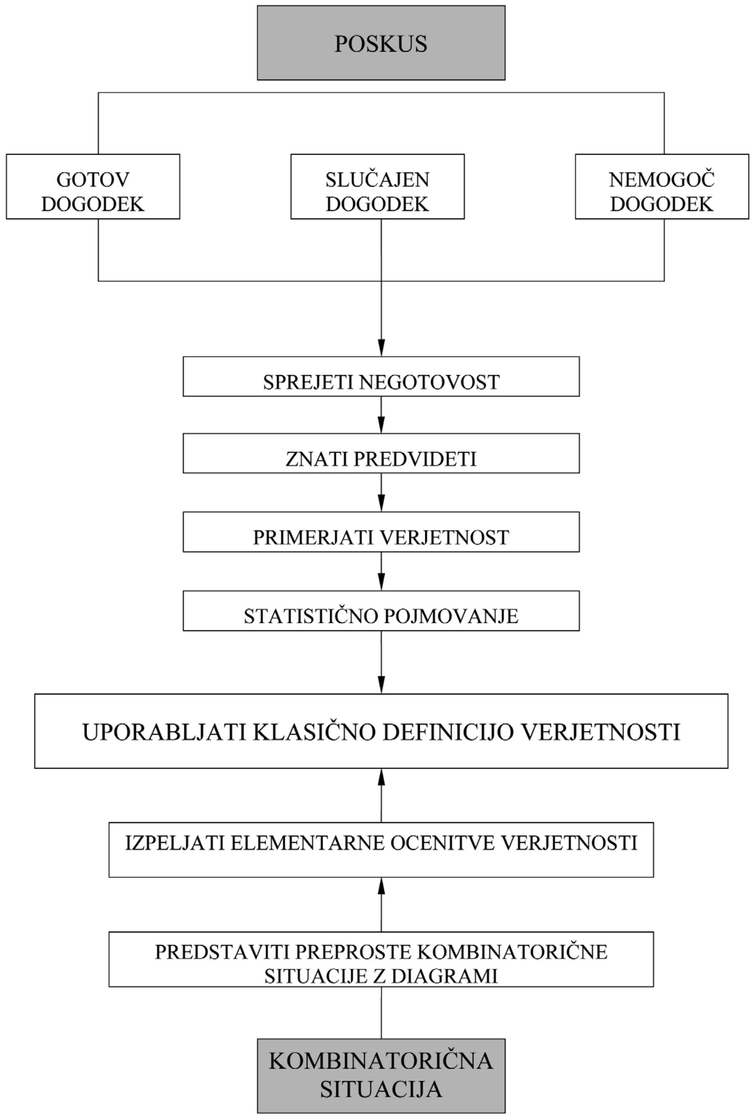
Učni cilji pri verjetnosti v prvem triletju

Učenec naj bi na začetku šolanja (v prvem triletju) preko igre in različnih drugih dejavnosti dosegel naslednje cilje:

— opiše, kaj je zanj mogoče oziroma nemogoče;
— razlikuje med gotovim, slučajnim in nemogočim dogodkom;
— v okviru praktičnih aktivnosti (met kocke, žreb, met kovanca) smiselno in dosledno uporablja izraze: mogoče, nemogoče, ne vem, morda, je možno, ni možno, slučajno, manj verjetno, enako verjetno, bolj verjetno;
— primerja med seboj verjetnosti raznih dogodkov;
— pri preprostih igrah na srečo postavlja smiselne hipoteze in jih skuša podpreti z izkušnjami;
— zapisuje izide slučajnih dogodkov (pri metu kocke, kovanca) v preglednico in s histogramom (Cotič, 1998).

Nivoji, ki vodijo do klasične definicije verjetnosti

Ni dovolj samo razložiti, zakaj mora "verjetnost" dobiti prostor v novih programih za matematiko. Pokazati moramo tudi, na kakšen način jo vpeljemo. Spodnja shema nam prikazuje, katere nivoje naj bi učenec prehodil, da bi v srednji šoli dojel klasično in statistično definicijo verjetnosti.



Slika 1. Nivoji pri verjetnosti.

Sprejeti negotovost

V prvem koraku v svet verjetnosti vodimo otroka, da sprejme brez vznemirjenja negotove situacije: torej h konceptu slučajnega dogodka. Po Piagetovi razvojni teoriji 6- do 7-letni otrok ne samo da nima jasne predstave o verjetnosti nekega dogodka, ampak tudi ne razlikuje med slučajnimi in neslučajnimi dogodki, četudi jih je sam doživel. Otrok teži k temu, da pripisuje namernost tudi predmetom in pojavom, ne samo ljudem; na primer: “Sonce je na nebu zato, da me greje; morje valovi zato, da bi premikalo ladje. . . " Nič se ne zgodi slučajno, vse je hoteno, načrtovano, določeno. Sprejeti slučajen dogodek ni torej samo kognitiven, ampak za otroka tudi afektiven problem, saj vsako negotovost sprejme s tesnobo. Otrok naj bi se počasi začel zavedati, da neki dogodek ni samo gotov ali nemogoč, ampak da je lahko slučajen. Najbolj postopno lahko ta naredimo z različnimi igrami, pri katerih je pomembna "sreča" (ćlovek ne jezi se, Tombola. . .) in sposobnost znati izbrati med različnimi možnostmi tisto, ki nudi največjo verjetnost za zmago (Vrtavka, Igralne karte. . .).

Znati predvideti

Pojme "gotovo", "slučajno" in "nemogoče" pri učencu najprej gradimo z različnimi dejavnostmi, v katerih je sam udeležen. Veliko didaktikov matematike to opisuje kot subjektivno verjetnost, ki je izhodišče za kasnejše razumevanje empirične in matematične verjetnosti. Otrok pri vsakdanjih dogodkih pravilno uporablja besede zagotovo, ni mogoče, nemogoče, mogoče, slučajno: "Popoldne bom šel *mogoče* v kino. Prestolnica Slovenije je *zagotovo* Ljubljana. *Nemogoče* je, da bi bil čez eno uro na planetu Jupiter."

Ti koncepti so tako preprosti, da je odrasel človek prepričan, da jih tudi otrok razume, vendar pa pri tem pozablja, da otrok največkrat *nemogoče* enači z *narobe*, *mogoče* pa z *zagotovo* in *prav* (Fischbain, 1975).

Slika 2. Mogoče, nemogoče.

Ni dovolj, da učencu ponudimo samo take situacije, ki se zgodijo neodvisno od njega (npr. opazovanje vremena, štetje določenih znamk avtomobilov. . .), ampak predvsem situacije, ki jih učenec obvladuje, in ki nudijo možnost, da jih lahko večkrat ponovimo pod enakimi pogoji (met kocke, žreb, met kovanca. . .), da bo lahko po večkratnih izkušnjah npr. zaključil (Fischbain, 1985):

— če vržem kocko, zagotovo "pade" število, manjše od sedem.
— Nemogoče je, da vržem deset.
— Mogoče vržem ena.

Slika 3. Mogoče, nemogoče, zagotovo.

Sprejeti negotovost pomeni tudi sprejeti dejstvo, da se predvideni dogodek ne zgodi. Zato je nujno z učencem izvajati take aktivnosti, ki mu nudijo možnost, da napoveduje verjetnost dogodkov v negotovih slučajih. Svoje napovedi nato preveri ter ugotovi, da ni nujno, da se njegova napoved uresniči. Kot smo omenili, naj bi učenec sprejel izid brez razburjenja, ne glede na to, ali se je zgodilo, kar je predvidel, ali ne. Povejmo primera:

Primer 1: *V vrečki imaš* 15 *črnih in* 6 *rumenih žogic. Kakšne barve je lahko izvlečena žogica? Katere barve žogico si izvlekel?*

Primer 2: *V vrečki je* 1 *rumena,* 1 *rdeča in* 1 *modra kocka. Napovej, kakšne barve je lahko kocka, ko vlečeš prvič; kakšne barve ne more biti kocka, ko vlečeš drugič; in kakšne barve bo zagotovo kocka, ko vlečeš tretjič. Pri tem izvlečene kocke ne vračaš v vrečko. V kakšnem vrstnem redu si izvlekel kocke?*

Po Piagetu in Inhelderjevi (1951) otrok, ki se nahaja v operativno-konkretnem obdobju, ni sposoben ločiti med gotovimi in slučajnimi dogodki in niti formulirati napovedi upoštevajoč izkušnje prejšnjih analognih situacij. Njegovi kriteriji velikokrat temeljijo na kriteriju ponavljanja (če je zadnja izvlečena kocka rdeča, bo tudi naslednja rdeča) oziroma na kriteriju kompenzacije (barva izvlečene kocke mora biti take barve, ki še ni bila izvlečena). To otrokovo ravnanje je determinirano z afektivno motivacijo, z zaupanjem v pravilnost njegove izbire, s potrebo po pravilnosti in redu in z dodelitvijo namernosti elementom (v našem primeru kock)

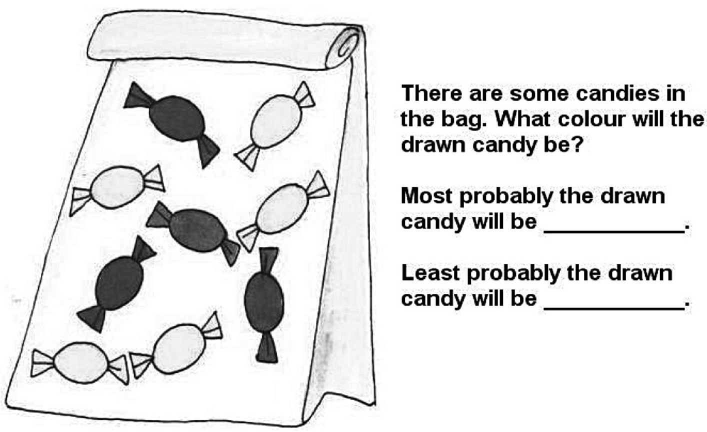
(Piaget, Inhelder, 1951). Zelo pomembno je torej, da učenec ponovi poskus večkrat pod enakimi pogoji, saj se bo samo na tak način zavedal, da je možno formulirati napovedi, ki niso subjektivne (Cotič, Hodnik, 1995).

Primerjati verjetnosti

Pojem verjetnosti se torej na začetku pri učencu gradi kot sposobnost obvladovanja slučajnih dogodkov. Seveda ta razvoj bazira na izkušnji, saj bo učenec prav preko izkušnje najprej ločil med gotovim, slučajnim in nemogočim dogodkom. Nato pa se bo začel zavedati, da so med slučajnimi dogodki nekateri bolj verjetni, drugi manj verjetni ali pa enako verjetni. Učenec je tako vpeljan v kvalitativno ocenitev verjetnosti slučajnega dogodka.

V primeru vrečke, v kateri je 20 rdečih, 5 belih in 1 črna kroglica, obstajajo dogodki, ki so malo verjetni, ampak verjetni (izvleka črne kroglice) in zelo verjetni, ampak ne gotovi (izvleka rdeče kroglice). Nujno je torej učencem predlagati igre, ki nudijo možnost, da primerjajo take slučajne dogodke, ki imajo različne verjetnosti, da se zgodijo. To so, kot že vemo, igre s kockami, kartami, kovanci... Učenec seveda kriterije za postavljanje svojih hipotez gradi počasi in postopno, tako da poskus velikokrat ponovi pod enakimi pogoji. Poudariti moramo, da je pri tovrstnih aktivnostih (poskusih) poudarek na delu v skupinah (dogovarjanje, razdelitev dela, koordinacija v skupini, komuniciranje v skupini).

Tako ga počasi vpeljujemo v statistično pojmovanje verjetnosti.



Slika 4. Učenec primerja verjetnosti.

Statistično pojmovanje verjetnosti

Učitelj naj v razred prinese na primer posodo z bonboni (žetoni, žogicami...) različnih barv. Učenci naj preštejejo bonbone posameznih barv, preden jih dajo v posodo. Na osnovi vedenja, koliko bonbonov določene barve je v posodi, naj učenci postavljajo hipoteze glede verjetnosti različnih dogodkov. Učitelj naj nato z učenci na empiričen način preveri odgovore. Ne da bi gledal, naj vsak učenec izvleče iz posode bonbon, in izid zapiše v preglednico. Nato bonbon vrne v posodo. Poskus naj učenci pod enakimi pogoji velikokrat (na primer 100 krat) ponovijo. Učenci bodo z učiteljevo pomočjo ugotovili, da se pri neprestanem povečevanju števila poskusov relativna frekvenca dogodka čedalje bolj bliža nekemu številu. S tem številom merimo verjetnost dogodka. Seveda bodo učenci to ugotovitev povedali s svojimi besedami.

Sklep

V prvem triletju torej pri verjetnosti izhajamo iz subjektivne verjetnosti. O subjektivnem doživljanju verjetnosti je potrebno z otroki govoriti ter uskladiti opisne načine njenega izražanja. Pri pouku matematike v prvem triletju morajo učenci z reflektiranimi izkušnjami priti do spoznanja, da tudi za slučajne pojave veljajo določene zakonitosti. Tako preidemo na empirično pojmovanje verjetnosti. V kasnejših letih šolanja pa naj bi se učenci počasi in postopoma srečali že z nekoliko zahtevnejšimi ocenitvami verjetnosti, kjer moramo najprej sistematično rešiti kombinatorični problem, da lahko na podlagi analize kombinatoričnih prikazov napovemo izid oziroma izpeljemo elementarno ocenitev verjetnosti, ki že vodi h klasični definiciji verjetnosti. Učenec tako počasi, ampak progresivno, spoznava preproste zakonitosti iz sveta verjetnosti, ki se mu je zdel na začetku popolnoma nepredvidljiv in celo negotov. Pri poučevanju verjetnosti se moramo zavedati, da je pojem verjetnosti v matematiki vedno zavzemal posebno mesto, saj je zelo težko opredeljiv s strogostjo, ki jo zahtevajo vse druge matematične discipline. Celo veliki matematik Laplace je o verjetnosti zapisal: "Zares je neverjetno, da je disciplina, katere korenine izhajajo iz proučevanja hazarda oziroma iger na srečo postala ena najpomembnejših matematičnih disciplin."

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Síkidomok alkotása, válogatása 4. osztályban

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(Poszter)

Összefoglaló. A magyar geometriaoktatás az első négy évfolyamon alapozó jellegű, célja az általános iskola kezdő szakaszán azon képességek fejlesztése, melyek segítségével a tanulók felkészülnek az önálló ismeretszerzésre. A geometria tanulásának alapja a tapasztalatszerzésből kiinduló induktív megismerés. A konkrétból való kiindulás, a sokféle tevékenységből származó tapasztalat összegyűjtése vezet el az általánosabb összefüggések megfogalmazásáig.

Néhány geometriai fogalom (téglalap, négyzet, párhuzamosság, merőlegesség, szimmetria) fejlődését vizsgáltuk egy negyedik osztályos fejlesztő oktatási kísérlet során, amelynek célja a van Hiele-modell szerinti geometriaoktatás megvalósítása. Az órák tervezésekor azt tartottuk szem előtt, hogy a gyerekek előbb konkrét tapasztalatok alapján, valóságos játékok keretében, tárgyi tevékenykedés közben, majd vizuális síkon (rajzolás), végül absztrakt, nyelvi síkon fedezzék fel az elsajátítandó geometriai fogalmakat.

Az előadásban különböző síkidomok alkotásával, válogatásával kapcsolatos feladatokat mutatunk be. Vázoljuk ezek feldolgozását, ismertetünk egyéni gondolatokat, megoldási módokat. A tanítási kísérlet alatt ebben a témában felmerülő néhány tipikus problémára, gondolkodási hibára is rávilágítunk.

Kulcsszavak: matematikatanítás, síkidomok alkotása, síkidomok válogatása

A magyar geometriaoktatás az első négy évfolyamon alapozó jellegű, célja az általános iskola kezdő szakaszán azon képességek fejlesztése, melyek segítségével a tanulók felkészülnek az önálló ismeretszerzésre. A geometria tanulásának alapja a tapasztalatszerzésből kiinduló induktív megismerés. A konkrétból való kiindulás, a sokféle tevékenységből származó tapasztalat összegyűjtése vezet el az általánosabb összefüggések megfogalmazásáig. Bolyai Farkas harmadik nevelési főelve is a konkréttal való kezdés fontosságát emeli ki: „mindég azokon kezdje, a'mit láthat, foghat, nem generalis definitiókon (nem grammaticán kezdődik az első szollás); 's ne kinozzon idő előtt hiába, hosszú soru okmútatással. ... Geometriai formákon s az olvasáson kell kezdeni ... ki kell a lapból is menni ...”

Egy negyedik osztályos fejlesztő oktatási kísérlet során foglalkoztunk - többek között - különböző síkidomok alkotásával, amelynek célja a *van Hiele*-modell szerinti geometriaoktatás megvalósítása. Az órák tervezésekor azt tartottuk szem előtt, hogy a gyerekek előbb konkrét tapasztalatok alapján, valóságos játékok keretében, tárgyi tevékenykedés közben, majd vizuális síkon (rajzolás), végül absztrakt, nyelvi síkon fedezzék fel az elsajátítandó geometriai fogalmakat.

P. H. van Hiele a geometriai ismeretszerzés folyamatát 5 szintre tagolta.

Az *alakzatok globális megismerésének szintjén* (1. szint) a gyerek a geometriai alakzatokat mint egységes egészet fogja fel. Könnyen felismeri a különböző alakzatokat a formájuk alapján, megtanulja az alakzatok nevét, nem fogja fel azonban az alakzatnak és részeinek kapcsolatát. Nem ismeri fel a kockában a téglatestet, a négyzetben a téglalapot, mert számára ezek egészen különböző dolgok.

Az *alakzatok elemzésének szintjén* (2. szint) a gyermek az alakzatokat részeire bontja, majd összerakja. Felismeri a mértani testek lapjait, éleit, csúcsait. A mértani testek lapjaiként a síkidomokat, amelyeket görbék, szakaszok, pontok határolnak. Fontos szerepet kap ezen a szinten a megfigyelés, a mérés, a hajtogatás, a ragasztás, a rajzolás, a modellezés, a parkettázás, a tükörhasználat stb. Ezen konkrét tevékenységek segítségével a tanuló megállapítja, felsorolja az alakzat tulajdonságait (lapok, illetve oldalak párhuzamossága, merőlegessége, szimmetriatulajdonságok, van derékszöge stb.), de nem definiál és a tulajdonságok közötti logikai kapcsolatokat még nem ismeri fel. Ezen a szinten még nem veszi észre a gyerek az alakzatok közötti kapcsolatokat.

A *lokális logikai rendezés szintjén* (3. szint) a tanuló már összefüggéseket lát meg egy adott alakzat tulajdonságai között illetve különböző alakzatok között. Megjelenik a következtetés lehetősége az alakzatok egyik tulajdonságáról a másikra. Megérti a meghatározás, a definíció szerepét. A logikai következtetések menetét azonban a tankönyv (illetve a tanár) határozza meg. Megkezdődik a bizonyítási igény kialakítása, de ez csak az alakzatokra terjed ki.

A negyedik (*törekvés a teljes logikai felépítésre*) és ötödik (*axiomatikus felépítés*) szinteknek megfelelő oktatás a középiskola és a felsőoktatás feladata.

Az órák tervezése során azt tartottuk szem előtt, hogy a gyerekek előbb konkrét tapasztalatok alapján, valóságos játékok keretében, tárgyi tevékenykedés közben, majd vizuális síkon (rajzolás), végül absztrakt, nyelvi síkon fedezzék fel az elsajátítandó geometriai fogalmakat.

Feladatok konkrét tárgyi tevékenységre:

1. Téglalap kettévágása az átlója mentén. Az így kapott háromszögek összeillesztéséből újabb síkidomok alkotása, megnevezésük. Tapasztalatgyűjtés a síkidomokról, megfigyelések megfogalmazása.

2. Papírból szabadon kivágott síkidomok előállítása. Tulajdonságaik megfogalmazása.
3. A logikai készlet 2, 3, 4, illetve 6 szabályos háromszögéből síkidomok alkotása.
4. Papírcsíkból egy-egy vágással különböző síkidomok előállítása. Megnevezésük. Tulajdonságok, közös tulajdonságok megfogalmazása.
5. Téglalapból általános rombusz kivágása. Tulajdonságai.
6. Téglalapból általános deltoid kivágása. Tulajdonságai.
7. 6 gyufaszálból téglalap, majd általános paralelogramma „keretének" elkészítése. A téglalap és a paralelogramma tulajdonságainak összehasonlítása.
8. 4 gyufaszálból négyzet, majd általános rombusz „keretének" létrehozása. A négyzet és a rombusz tulajdonságainak összehasonlítása.
9. Téglalapból egy vágással 2 téglalap, egy ötszög és egy háromszög, egy háromszög és egy négyszög, 2 négyszög, 2 háromszög előállítása.
10. Síkidomok válogatása adott tulajdonságok alapján.

Feladatok vizuális síkon:

1. Pontrácson négyzetek rajzolása.
2. Pontrácson különböző négyszögek rajzolása.
3. Pontrácson különböző háromszögek rajzolása.
4. Adott tulajdonságú négyszögek rajzolása.
5. Síkidomok rajzolása, amelyeknek nincs, pontosan 1, 2, 3, illetve 4 szimmetriatengelye van.
6. Adott feltételeknek megfelelő síkidomok rajzolása.

Egy sokszögekkel kapcsolatos feladat során tette fel Kornél a következő kérdést:

> *„A sokszögekhez tartoznak azok is, amelyeknek két szöge van?"*
> A tanító megkérdezte:
> *„Tudsz olyant rajzolni?"*
> *„Igen tudok."* - válaszolta Kornél és rajzolt egy félkört.
> A tanító a rajz láttán a következőt kérdezte.
> *„A sokszögeket határolhatja görbe vonal?"*
> A tanító ezen felvetésére elgondolkodott, majd azt válaszolta:
> *„Hát nem! Csak egyenes vonal, akkor ez nem is sokszög."*

Absztrakt, nyelvi sík

A gyerekek kedvelt játéka a barkóba, amely - többek között - a testek, síkidomok tulajdonságainak gyakorlására, rögzítésére is alkalmas. Egy ilyen játék során a szimmetrikus trapézt kellett kitalálniuk. A tanulók kérdései és a játékvezetői válaszok így követték egymást:

> — *Négyszög?*
> — *Igen.*
> — *Szemben levő oldalai párhuzamosak?*

— *Nem.*
Itt a tanítónő érezte, hogy segítenie kell:
— *Ezt a kérdést másképpen is megfogalmazhatjuk: mindegyik szemben levő oldalpárja párhuzamos? Erre mondtam, hogy nem.*
— *Vannak párhuzamos oldalai?*
— *Igen.*
— *Van derékszöge?*
— *Nincs.*
— *Oldalai egyformák?*
— *Ezt úgy is kérdezhetjük* — segített ismét a tanítónő —, *hogy mindegyik oldala egyenlő? Nem.*
— *Vannak egyenlő hosszú oldalai?*
— *Igen. Aki már tudja, rajzolja le! Aki még nem, kérdezzen!*
— *Szimmetrikus?*
— *Igen.*
— *1 tükörtengelye van?*
— *Igen.*

Ezek után a táblára is felrajzoltuk a keresett síkidomot.

Az alsó tagozatos (1–4. osztály) geometriaoktatásban a geometriai gondolkodás van Hiele-féle szintjeinek első két fázisa valósítható meg. A harmadik szintre nem lehet átlépni az alsó tagozat végére. Kialakulnak ugyan fogalomosztályok, de nincs különösebb kapcsolat köztük. Egy alakzat tulajdonságai közötti logikai kapcsolatokat még nem ismerik fel a gyerekek. Nem tudnak következtetni az alakzatok egyik tulajdonságáról a másikra.

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A nulla fogalma 7-12 éves korban

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Összefoglaló. Az elmúlt években több alkalommal vizsgálódtam a nulla fogalmával kapcsolatban a bajai tanítóképzős hallgatók körében. Jelentkeztek típushibák.

Az idei tanévben általános iskolás gyermekek körében vizsgálódtam. Arra voltam kíváncsi, hogyan fejlődik az iskolai tanítás keretében a nulla fogalma. Ezt összevetem a korábbi információkkal.

Írásbeli felmérést végeztettem a bajai Eötvös József Főiskola Gyakorló Általános Iskolájának egy-egy második, harmadik, negyedik, ötödik és hatodik osztályában. A feladatlapokon különböző alapműveletek szerepeltek, amelyekben valamelyik tag vagy tényező nulla volt., azonos számok közé műveleti jel kirakása, hogy az eredmény nulla legyen, nullával kapcsolatos állítások igaz – hamis voltának eldöntése, esszé a nulláról. Ennek tapasztalatairól szeretnék beszámolni.

Kulcsszavak: matematikatanítás, a nulla fogalma

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Matematički zahtjevi u PISA zadacima

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Sažetak. U radu su prikazani rezultati metodičke analize PISA zadataka iz područja matematičke pismenosti. Analiza je provedena na 26 objavljenih PISA zadataka iz područja matematičke pismenosti, koji predstavljaju ogledni presjek testnih PISA matematičkih zadataka. Rezultati analize nadalje su povezani s ishodom PISA testiranja hrvatskih učenika 2006. godine.

Istraživanjem su se nastojali utvrditi matematički zahtjevi u objavljenim PISA zadacima te u kojoj mjeri ti zahtjevi odgovaraju zahtjevima nastave matematike u Hrvatskoj. U svjetlu ove tematike cilj je također bio rastumačiti ishod PISA testiranja hrvatskih učenika iz područja matematičke pismenosti 2006. godine. Analiza podataka pokazala je da se PISA zahtjevi u mnogočemu razlikuju od zahtjeva u nastavi matematike u Hrvatskoj, i to u sadržajnom smislu, sposobnostima, stupnju složenosti te formulaciji pitanja. Sadržajno, PISA zadaci naglašavaju područja matematike poput statistike i vjerojatnosti, a hrvatski učenici koji su 2006. godine pristupili PISA testiranju nisu uopće imali ovo gradivo u programu svojeg matematičkog školovanja. Gledano po sposobnostima, PISA zadaci naglašavaju interpretaciju i argumentaciju dok u hrvatskoj nastavi matematike viših razreda osnovne škole dominira operacionalizacija do automatizacije. To se vidi i u obliku pitanja à u našoj nastavi dominiraju zadaci zatvorenog tipa koji zahtijevaju samo točan rezultat bez ikakve interpretacije i argumentacije dok su u PISA programu česti i zadaci otvorenog tipa.

Ovakve razlike između PISA zadataka i hrvatske nastavne prakse mogu se povezati s lošim rezultatima hrvatskih učenika na PISA testiranju iz područja matematičke pismenosti.

U radu se predlažu smjernice koje bi mogle dovesti do poboljšanja u matematičkom obrazovanju hrvatskih učenika, a proizlaze iz određenih komponenata PISA testiranja.

Ključne riječi: analiza, PISA zadaci, PISA testiranje, matematička pismenost, matematički zahtjevi, nastava matematike

Uvod

U posljednjih nekoliko godina u javnosti i stručnim krugovima često se spominje PISA istraživanje, uglavnom u kontekstu nacionalnog plasmana na međunarodnoj rang-listi. PISA (Programme for International Student Assessment) predstavlja program međunarodnog procjenjivanja znanja i vještina učenika koji su zajednički razvile zemlje članice organizacije OECD (Organisation for Economic Cooperation and Development). Prvo međunarodno PISA istraživanje organizirano je 2000. godine, a nakon toga se provodi svake treće godine i ispituje čitalačku, matematičku i prirodoslovnu pismenost kod petnaestogodišnjaka (OECD, 2003.). Dvije trećine svakog ispitivanja posvećuju se tzv. "glavnoj" domeni. Tako je glavna domena u 2000. godini bila čitalačka pismenost, u 2003. to je bila matematička pismenost, a u 2006. godini prirodoslovna pismenost. Zatim se kreće opet s čitalačkom pismenošću kao glavnim područjem, i tako redom. Hrvatska je prvi puta nastupila na PISA testiranju 2006. godine kada je rezultat naših učenika iz područja matematičke pismenosti smješten na 36. mjesto u rangiranju svih 57 zemalja (Braš Roth i dr., 2008.), što Hrvatsku smješta statistički značajno ispod prosjeka OECD-a. Detaljnije informacije o PISA natjecanju mogu se naći u knjizi M. Braš Roth i dr. (2008.).

OECD opisuje matematičku pismenost kao *"sposobnost pojedinca da prepozna i razumije ulogu koju matematika ima u svijetu, da donosi dobro utemeljene odluke i da primjenjuje matematiku na načine koji odgovaraju potrebama života tog pojedinca kao konstruktivnog, zainteresiranog i promišljajućeg građanina"* (Braš Roth i dr., 2008., str. 124.). Da bi se matematička pismenost što preciznije "izmjerila", razvijena je teoretska osnova PISA matematičke pismenosti koja se sastoji od triju komponenata: matematičkog sadržaja koji petnaestogodišnji učenik treba poznavati, skupine kompetencija koje učenik treba imati razvijene te situacija (konteksta) u koje je smješten zadatak. O komponentama matematičke pismenosti detaljnije se može čitati u Braš Roth i dr. (2008.) te Glasnović Gracin (2007. a, 2007. b).

Velika pozornost medija diljem svijeta skrenuta je na PISA istraživanje i na PISA rezultate upravo zbog međunarodnog uspoređivanja i natjecanja među nacijama. Stručnjaci, međutim, naglašavaju (De Lange, 2005., Peschek, 2006.) da je potrebno činiti dublje metodičke analize kako PISA zadataka tako i nacionalnih obrazovnih sustava sa svrhom poboljšanja kvalitete suvremene nastave matematike. Također se u istim izvorima naglašava da bi se takvim analizama trebalo posvetiti više pažnje od međunarodne kompeticije, koja danas, očito, zaokuplja najveći dio pažnje javnosti.

Metodička analiza PISA zadataka

Ovaj rad se ne bavi pitanjima vezanim uz međusobna natjecanja među nacijama kod PISA testiranja, već se bavi metodičkom analizom PISA zadataka iz područja matematičke pismenosti. Cilj istraživanja bila je analiza matematičkih zahtjeva objavljenih PISA zadataka te njihovo uspoređivanje sa zahtjevima osnovnoškolske nastave matematike u Hrvatskoj. Analiza je obavljena na skupini od 26 objavljenih PISA zadataka (OECD, 2007.) koji s podzadacima čine ukupno 42 matematička problema. Oni obuhvaćaju sve objavljene zadatke iz PISA testova iz 2000., 2003. i 2006. godine i ogledno prikazuju neobjavljene PISA zadatke.

Osnova za analizu po sadržaju, vještinama i kompleksnosti uzeta je iz sistematizacije Austrijskog obrazovnog standarda (Institut für Didaktik der Mathematik, 2007.). Svaki zadatak u ovom radu klasificiran je prema potrebnom matematičkom sadržaju, vještinama i svojoj kompleksnosti. Osim toga, analiza je obuhvaćala pitanja je li pojedini zadatak uobičajen za matematičku nastavnu praksu u Hrvatskoj po istim komponentama te je li svojim sadržajem, kontekstom i traženim kompetencijama uobičajen u svakodnevnom životu, tj. u prirodnom, društvenom i kulturalnom okruženju u kojem pojedinci žive. *Sadržajna područja* analize su podijeljena na: brojeve i mjerne jedinice, varijable i funkcijske ovisnosti, geometrijske oblike i tijela te na statističke prikaze i veličine. *Vještine* su podijeljene na: prikazivanje i modeliranje, računanje i operiranje, interpretiranje te na argumentiranje i dokazivanje. *Kompleksnost zadatka* u analizi podijeljena je na primjenu osnovnih znanja i vještina, izradu veza te na primjenu refleksivnog mišljenja.

Matematički zahtjevi u PISA zadacima

Rezultati analize pokazuju da se analizirani PISA zadaci uvelike razlikuju od zadataka koji su uobičajeni za matematičku nastavnu praksu u Hrvatskoj. Od 42 pitanja njih čak 22 procjenjujemo kao potpuno neuobičajenima za hrvatske matematičke udžbenike. Dakle, više od polovice analiziranih PISA zadataka potpuno se razlikuje od zadataka zastupljenih u nastavi matematike. Nadalje, dodatnih 26% zadataka samo je djelomično uobičajeno za nastavu matematike (tj. samo po nekim komponentama). To znači da je tek petina danih PISA zadataka uobičajena za nastavu matematike u Hrvatskoj. Dubljom analizom uočeno je da neki od zadataka koji nisu tipični za nastavu matematike pripadaju više nastavi fizike, geografije, tehničke kulture, zabavne matematike i drugih područja. To ukazuje da matematička pismenost kako je PISA opisuje prelazi okvire samo nastave matematike te obuhvaća i druge predmete i područja.

S druge strane, visoki postotak od 95% analiziranih PISA zadataka procijenjen je kao vrlo uobičajen ili uobičajen za svakodnevne životne situacije. Taj podatak podudara se s opisom matematičke pismenosti kao sposobnošću *"pojedinca da analizira, logički zaključuje i učinkovito komunicira prilikom postavljanja, rješavanja i interpretiranja matematičkih problema u mnoštvu različitih situacija"* (Braš Roth i dr., 2008., str. 122). U danom se opisu očito naglašava primjena matematike u raznim životnim situacijama. Na ovom mjestu dolazimo do razilaženja u glavnim ciljevima PISA ispitivanja u odnosu na ciljeve nastave matematike. Cilj nastave matematike u školi nije samo primjena matematike u svakodnevici već i stjecanje matematičkog znanja i sposobnosti vezanih uz apstraktne pojmove, dok je cilj PISA matematičke pismenosti usmjeren samo na ispitivanje primjene matematike u raznim životnim situacijama. Tako je, primjerice, na PISA natjecanju 2003. kada je glavna domena bila matematika, od 84 matematička zadatka bilo njih samo 3 iz područja algebre (Schneider i Peschek, 2006.).

Kako je rečeno, PISA zadaci analizirani su s obzirom na sadržaj, vještine i kompleksnost. Gledano po sadržaju, zanimljivo je da se 21% objavljenih zadataka odnosilo na sadržaje iz *statistike i vjerojatnosti*. Štoviše, u PISA testiranju provedenom 2006. godine bilo je u opticaju čak 17 zadataka od ukupno 48 iz matematičkog područja Neizvjesnost, koje odgovara statistici i vjerojatnosti (Braš Roth i dr., 2008., str. 161). To znači da je na PISA testiranju 2006. u više od trećine zadataka trebalo koristiti znanje iz statistike ili vjerojatnosti. PISA naglašava ovo matematičko područje kao vrlo važno. *"Ovakav tip statističkog mišljenja trebao bi koristiti svaki građanin"* (Braš Roth i dr., 2008., str. 133). No, statistika i vjerojatnost su uvedene u Hrvatski nastavni program tek od jeseni 2006. godine (MZOS, 2006.), i to u vrlo malom opsegu. To znači da naši učenici koji su u proljeće 2006. godine pristupili PISA testiranju u svom matematičkom obrazovanju uopće u školi nisu učili ove sadržaje.

Drugo veliko matematičko područje kojem PISA pridaje dosta pažnje i prostora, a koje se na potpuno drugačiji način i u mnogo manjem opsegu poučava na nastavi matematike, odnosi se na *funkcije*. Prema nastavnom planu i programu u nastavi matematike funkcije se uče u 7. razredu, i to samo linearna funkcija u vrlo naglašenom okruženju analitičke geometrije. U osmom razredu spomene se i kvadratna funkcija te eventualno funkcija korjenovanja. Ovakvo poučavanje funkcija u nastavi matematike u osnovnoj školi trebalo bi preispitati, kako sadržajno tako i u određenju pravog trenutka i zrelosti učenika, jer se pokazalo da naši učenici na inicijalnim testovima u prvom razredu srednje škole vrlo loše rezultate pokazuju baš iz područja funkcija (Dakić, 2000., Rac Marinić Kragić, 2007.). S druge strane, PISA nudi zadatke s posve drugačijim pristupom funkcijama nego što se to radi na nastavi matematike. Iz analiziranih PISA zadataka vidljivo je da grafovi funkcija dolaze iz realnih situacija, što znači da nisu prikazani isključivo linearni odnosi. Uz ovakve sadržaje često se od učenika traže vještine interpretiranja grafičkog prikaza. U školskoj praksi ovakvi su zadaci bliži problemima s nastave fizike nego matematike. *"Pojam funkcije se u našim školama spominje samo na primjerima u analitičkoj geometriji i ni u kakvom drugom praktičnom kontekstu, a upravo takav kontekst se nalazi u mnogim PISA zadacima"*, napisala je u izvješću stručna radna skupina za matematiku za PISA-u 2006 u Hrvatskoj (Braš Roth i dr., 2008., str. 161).

Znatna razlika u PISA zahtjevima i zahtjevima naše nastave matematike može se primijetiti i u području očekivanih učeničkih sposobnosti. U više od trećine analiziranih zadataka od učenika se očekivala vještina interpretiranja, tj. tumačenja danih matematičkih odnosa u određenom kontekstu. Ovu vještinu možemo povezati s interpretacijom brojnih nelinearnih funkcijskih prikaza, zatim danih grafičkih prikaza statističkih podataka i sl. Polovica analiziranih zadataka (53%) tražila je vještine lakšeg računanja i operiranja dok se u drugoj polovici (47%) tražilo modeliranje, argumentiranje i dokazivanje. U usporedbi s našom nastavom matematike u osnovnoj školi, u kojoj uvelike dominira operiranje i automatizacija, a modeliranje, argumentiranje, dokazivanje i interpretiranje susreću se u mnogo manjoj mjeri, mnoštvo PISA zadataka u kojima je nešto trebalo obrazložiti ili argumentirati našim je učenicima stvaralo probleme prilikom rješavanja PISA testa 2006.

Matematički sadržaj i posebice vještine koje su se tražile od učenika kako bi se riješio pojedini zadatak utjecale su na kompleksnost pojedinog zadatka. Čak 27% PISA zadataka u testiranju 2006. pripadalo je skupini kompetencija refleksije, koja obuhvaća kompleksno postavljanje i rješavanje problema, promišljanje, originalni matematički pristup, poopćavanje i sl. Takav pristup rijedak je na redovnoj nastavi matematike osnovne škole i potpuno je stjeran u kut pred operacionalizacijom i rješavanjem šablonskih zadataka. Stoga ne čudi što su naši učenici 2006. godine ove posebno zahtjevne zadatke vrlo slabo riješili na testiranju, a neke od njih čak i vrlo značajno ispod svjetskog prosjeka. Uz to, zadaci otvorenog tipa su činili otprilike četvrtinu analiziranih zadataka, što se slaže s udjelom otvorenih zadataka u redovnom PISA testiranju (Schneider, Peschek, 2006.). Takav tip zadatka našim je učenicima predstavljao problem na PISA natjecanju jer takvi zadaci traže kako dobro poznavanje sadržaja tako i dobro razvijene sposobnosti, poput interpretacije, argumentiranja i dokazivanja, te refleksiju.

Uza sve to važno je napomenuti da su PISA zadaci često tekstualno vrlo dugački i opsežni zadaci te se uz navedene zahtjeve od učenika traže sposobnosti razumijevanja teksta, izvlačenja bitnog od nebitnog u tekstu, koncentraciju čitanja te verbalne sposobnosti.

Zaključak

Rezultati provedene analize ukazuju na velike razlike između PISA zadataka i zadataka iz naše nastavne prakse u osnovnoj školi. Te razlike sigurno rezultiraju slabim ishodom naših učenika na međunarodnoj PISA ljestvici. Analiza objavljenih PISA zadataka pokazuje da OECD stavlja naglasak na tekstualne zadatke smještene u svakodnevne situacije, s naglaskom na područje interpretacije grafičkih prikaza, najčešće funkcijskih ovisnosti ili statističkih podataka. Također se od učenika traže vještine modeliranja problema, interpretacije raznih matematičkih sadržaja te argumentacije i objašnjavanja kompleksnijih tvrdnji u zadacima otvorenog tipa. Navedena obilježja PISA zadataka nisu toliko naglašena u nastavi matematike u Hrvatskoj. Izvješće PISA-ine radne skupine za matematiku u Hrvatskoj (Braš Roth i dr., 2008., str. 163) sugerira da bi bilo mudro uzeti one PISA zahtjeve koji mogu unaprijediti našu nastavu matematike.

Sadržajno gledajući, PISA prikazuje novi pristup funkcijama koji bi se mogao uklopiti u nastavu matematike, ali i u nastavu fizike te ostalih predmeta u kojima se koristi interpretacija grafičkih prikaza. Uz to, područje vjerojatnosti i statistike trebalo bi opširnije i kvalitetnije uklopiti u cjelokupno matematičko školovanje, počevši od nižih razreda osnovne škole.

U području potrebnih sposobnosti i kompleksnosti zadataka PISA ne zadaje samo klasične zadatke koji naglašavaju operacionalizaciju, već nudi problemske zadatke koji zahtijevaju vještine modeliranja situacije ili interpretiranja problema te često i refleksivno mišljenje. Takvi zadaci često se kombiniraju sa zadacima otvorenog tipa jer zahtijevaju argumentaciju, objašnjavanje, verbalne sposobnosti i sl. Ovaj nalaz možemo usporediti s procjenom hrvatske PISA-ine stručne radne skupine za matematiku iz 2006. godine: *"PISA rezultati iz područja matematičke pismenosti ukazuju da bi u nastavi trebalo više stavljati naglasak na interpretacije, refleksivne kompetencije, argumentiranje i verbalno izražavanje matematičkih sadržaja."* (Braš Roth i dr., 2008., str. 163).

Provedena analiza i opisane razlike između PISA zadataka i zadataka iz naše nastavne prakse mogu pomoći u konkretnim diskusijama i zaključcima o sadržajnom poboljšanju kvalitete nastave matematike u Hrvatskoj.

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3.
O ishodima učenja iz nastave matematike za studente

Ishodi učenja u matematici: primjer implementacije i evaluacije uz pomoć e-učenja

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Sažetak. Ishodi učenja predstavljaju osnovni alat za poučavanje u čijem je središtu student. Implementacija ishoda učenja treba kombinirati pristup odozgo počevši od studijske razine, konkretnog studijskog programa s pristupom odozdo s razine nastavne cjeline i predmeta. Pri nastavi matematike za studente kojima matematika nije glavni cilj studija, važno je uočiti ulogu matematičkog aparata i postupaka u studiju struke, ali i uzeti u obzir ulazne kompetencije studenata.

U našem radu želimo prikazati način implementacije ishoda učenja u predmeta matematike na studiju Informacijskih i poslovnih sustava na Fakultetu organizacije i informatike Sveučilišta u Zagrebu. U prvoj fazi, implementacija ishoda učenja usklaena je s ulaznim kompetencijama, nastavnim metodama, opterećenjem studenata (ECTS), modelom praćenja i ocjenjivanja studenta, a uzeti su u obzir različiti stilovi učenja studenata te motivacija za rad na predmetu. U drugoj fazi se radi na modelu evaluacije ishoda učenja te provjera vezi izmeu različitih elemenata procesa učenja i poučavanja.

Cjelokupni proces je značajno poduprt informacijsko-komunikacijskom tehnologijom i provodi se uz uporabu mješovitog e-učenja te društvenog softvera. Takav pristup podiže motivaciju studenta za učenje matematike, ali i dostupnost materijala i komunikaciju. S druge strane, nastavniku je omogućena pohrana informacija o svim artefaktima koje studenti naprave tijekom semestra, pa se na taj način otvaraju mogućnosti evaluacije ishoda učenja.

Ključne riječi: ishodi učenja, matematika, ICT, e-učenje, taksonomija, e-portfolio

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Matematičke kompetencije mladih zainteresiranih za učiteljski studij
(analiza jednog prijemnog ispita iz matematike)

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Sažetak. Nezadovoljavajući rezultati učenika u matematici na svim razinama obrazovanja potiču nas na stalno preispitivanje svih čimbenika koji na taj rezultat mogu imati utjecaja. Učitelji razredne nastave prve su stručne osobe koje kod djece izgrađuju temeljni sustav matematičkih koncepata, procesa i terminologije kao i cjelokupni odnos djeteta prema matematici. Kompetencije koje se očekuju od učitelja možemo definirati kao duboko, konceptualno poznavanje i razumijevanje matematičkih sadržaja, poznavanje pedagoških spoznaja i psiholoških karakteristika djeteta te pozitivna uvjerenja o matematici. Neke od ovih kompetencija izgrađuju se i dopunjavaju na učiteljskim fakultetima, ali mnoge od njih očekuju se od budućih učitelja već pri dolasku na studij. Na učiteljskim se fakultetima u okviru matematičkih kolegija ne uče elementarni sadržaji početne nastave matematike, već se proširuju i produbljuju matematičke kompetencije s kojima studenti dolaze. Upravo iz tog razloga proučili smo matematičke kompetencije mladih zainteresiranih za učiteljski poziv. U tu smo svrhu analizirali rezultate jednog izrazito laganog prijemnog ispita iz matematike, kojemu su pristupila 123 kandidata u Splitu, 2008. godine. Rezultati ispita pokazali su duboko nerazumijevanje i nepoznavanje temeljnih matematičkih koncepata od kojih su mnogi sadržajno bili upravo iz početne nastave matematike.

Ključne riječi: matematika, učitelji, matematičke kompetencije, učiteljski studij

Uvod

Današnje je vrijeme karakteristično po stalnim i ubrzanim promjenama na svim poljima ljudskih djelatnosti u kojem se nova znanja, tehnologije, alati i načini djelovanja i komuniciranja stalno razvijaju i usavršavaju. Takav razvoj suvremenog društva usko je povezan s razvitkom znanosti, tehnike, tehnologije i ekonomije. "U tom razvoju matematici pripada posebno mjesto, jer su sve znanosti u svom razvoju prožete matematičkim načinom mišljenja i uopće matematikom, koja ih može povezati u jedinstvenu cjelinu, formirati njihova načela i utjecati na njihov napredak. Matematičke discipline vrlo su aktivne komponente razvijanja znanstvenih istraživanja u svim područjima znanosti" (Kadum, 2002, 137). Važnost poznavanja matematike, posebno njenog logičkog načina mišljenja i važnost sposobnosti primjene matematičkih spoznaja, kako za pojedinca, tako i za društvo u cjelini, nikada u povijesti nije bila veća.

Nažalost, potrebe pojedinaca i društva u cjelini za matematikom nisu proporcionalne rezultatima učenika u nastavi matematike. "I pored velikog značenja i uloge koje imaju matematika i matematičko obrazovanje u razvoju drugih znanstvenih disciplina i razvoju tehnike, tehnologije i obrazovanja uopće, učinkovitost (uspješnost) nastave matematike u osnovnoj i srednjoj školi, kod nas i u svijetu je nezadovoljavajuća" (Pejić, 2006, 19). Brojni rezultati mjerenja matematičkih znanja i sposobnosti učenika u osnovnim i srednjim školama, stavovi učenika prema matematici, interes studenata za matematičke studije i uspješnost primjene matematičkih znanja i sposobnosti u konkretnim situacijama ne ostavljaju mnogo razloga za zadovoljstvo. Učenici često posjeduju samo formalna matematička znanja, a "takva znanja nisu usvajana svjesno, u njima se nepravilno izražavaju matematičke zakonitosti, učenici ih teško ili nikako ne primjenjuju u praksi, ne pomažu daljnjem matematičkom obrazovanju" (Markovac, 1978, 21). Zbog takvog načina usvajanja matematike, ti učenici nisu u stanju primijeniti naučeno u konkretnim životnim, školskim ili profesionalnim okolnostima. "Prema svjedočenju velikog broja matematičara, mnogi maturanti ne znaju samostalno rasuđivati, a na prijemnim ispitima za fakultete oslanjaju se na pamćenje, a ne na mišljenje" (Ovčar, 1990, 16). To potvrđuju i brojna međunarodna istraživanja (PISA, TIMSS) koja uspoređuju rezultate učenika u matematici u različitim odgojno-obrazovnim sustavima širom svijeta.

Kao uzroci neuspjeha, u relevantnoj se metodičkoj literaturi navode neprilagođenost sadržaja individualnim mogućnostima učenika, nedostatak predznanja na koji bi se nastava trebala nadograđivati, neadekvatno obrazovanje učitelja, ali i nedovoljna metodička pripremljenost predmetnih nastavnika, neadekvatne metode i načini rada u nastavi, sustav obrazovanja općenito, nastavni programi te udžbenici (Markovac, 1978; Kadum, 2002; Pejić, 2006). Zbog svega navedenog, potrebno je stalno preispitivati parametre koji na ovaj ili onaj način utječu ili mogu utjecati na matematičke kompetencije učenika i to na svim razinama.

Kompetencije učitelja razredne nastave za poučavanje matematike

Učitelj razredne nastave prva je stručna osoba koja kod djeteta sustavno izgrađuje sustav matematičkih znanja, vještina, navika i interesa. Kao takav, njegov je utjecaj na učenika, njegov uspjeh, razumijevanje, stav i motivaciju za matematiku ogroman. U razrednoj nastavi obrađuju se najelementarniji matematički sadržaji koji postaju temelj za nadogradnju u budućem školovanju, pa je time značaj ovog perioda u matematičkom obrazovanju itekako velik. Kao i u građevinarstvu, ukoliko temelj buduće konstrukcije nije dobro postavljen, ni nadgradnja ne može biti kvalitetna. Svoj utjecaj učitelj ostvaruje kroz način na koji podučava matematiku, način na koji komunicira s učenicima o matematici, ali i kroz neverbalnu komunikaciju u kojoj kroz četiri godine učenicima svjesno ili nesvjesno prenosi vlastite stavove, asocijacije i strahove (Mišurac Zorica, 2007). Upravo zato, učitelj razredne nastave, baš kao i predmetni nastavnik matematike, mora biti stručan i kompetentan za poučavanje matematike.

Stručnost i kompetentnost učitelja nije uvijek jednostavno definirati i klasificirati, budući da mnoge studije koje su se bavile tim kompetencijama na raznolike načine određuju znanja i vještine koje kvalitetan učitelj matematike mora imati. Tako Verschaffel, Janssens i Janssen (2005) koriste model koji kompetencije učitelja za poučavanje matematike dijeli u tri kategorije. Prva kategorija su matematičke kompetencije, a podrazumijevaju poznavanje matematičkih sadržaja, što znači dobro poznavanje i duboko razumijevanje ključnih činjenica, koncepata, obrazaca pravila i dokaza, procedura i strategija rješavanja problema u domeni matematičkog sadržaja kojeg podučavaju. Drugu kategoriju čine specifična metodičko pedagoška znanja, koja podrazumijevaju umijeće prikazivanja matematičkih sadržaja djeci različitih sposobnosti i interesa, odabir optimalnih strategija i oblika rada, znanje o vrstama matematičkih zadataka, poznavanje udžbenika i drugih nastavnih materijala i slično. Treću kategoriju kompetencija čine učiteljeva psihološka znanja o tome kako učenici misle i uče matematiku, koliko poznaju njihova predznanja i razvojne karakteristike. Leou (1998) predlaže model koji postavlja četiri kategorije koje određuju nastavnikove kompetencije za poučavanje matematike, a to su (a) umijeće podučavanja, (b) sposobnost kvalitetne materijalne organizacije i prezentacije sadržaja, (c) umijeće stvaranja poticajne atmosfere za učenje stvorene između nastavnika i učenika i (d) nastavnički stavovi ili uvjerenja. Treći model kojeg su predložili Fennema i Franke (1992) na sličan način ukazuje na interaktivnu i dinamičku prirodu učiteljeva matematičkog znanja, pedagoškog znanja, znanja o učenikovom kognitivnom razvoju i učiteljevih uvjerenja. Ipak valja naglasiti da se u svim navedenim studijama naglašava da samo posjedovanje navedenih kompetencija nije garancija uspješnosti učitelja.

U svim navedenim kategorizacijama na prvo se mjesto postavljaju učiteljeve matematičke kompetencije, dakle njegovo matematičko znanje i razumijevanje koje je temelj znanja i razumijevanja njegovih učenika. Dobro poznavanje matematičkih sadržaja je "conditio sine qua non" u matematičkom poučavanju, snalaženju u nastavnom programu i udžbeniku te u pripremanju i realiziranju nastave matematike i njenoj učinkovitosti. "Stručno osposobljen učitelj uvijek će se lakše snaći i ukloniti eventualne nedostatke u programu ili u udžbeniku, a nedovoljno obrazovan ili nepripremljen učitelj može svojom nestručnom interpretacijom upropastiti uspješnost najbrižljivije pripremljenog programa ili udžbenika" (Pavleković, 1997, 271). Duboko i konceptualno razumijevanje matematike od strane učitelje nužan je preduvjet za poticanje i razvijanje učenikova konceptualnog mišljenja. Rezultati "Međunarodnog ispitivanja postignuća u nastavi matematike" koje je još 1964. proveo UNESCO potvrdili su da između matematičkog obrazovanja učitelja i njegovog rada u nastavi postoji konstantna i izravna proporcionalna veza; što je njegovo matematičko obrazovanje bolje i kvalitetnije bolja su i kvalitetnija matematička znanja, vještine i navike koje izgrađuje kod svojih učenika (Pejić, 2003).

Osim matematičkih znanja, učitelji svojim stavovima i pogledima na matematiku također bitno utječu na učenike. Mnoga su istraživanja (npr. Sun Lee i Ginsburg, 2007; MacNab, 2000) pokazala kako se stavovi i shvaćanja o ciljevima poučavanja i prirodi učenja odražavaju u ponašanju učitelja tijekom poučavanja, ali i na rezultatima učenika u matematici. Ranije provedeno istraživanje sa studentima razredne nastave (Mišurac Zorica, 2007) pokazalo je da mnogi budući učitelji imaju negativne stavove i mišljenja o matematici, kao i negativne asocijacije na samu riječ matematika. Uz to, većina ih nije voljela matematiku ni u školi, a budući su učitelji u najmanjem postotku izabrali matematiku kao predmet kojeg će najradije predavati. Iz svega navedenog izveden je zaključak da je za buduće učitelje podučavanje matematike samo segment učiteljskog posla kojega prihvaćaju kao nužno zlo, a ne kao vlastiti odabir.

Istraživanje

U ovom smo radu željeli istražiti matematičke kompetencije mladih koji su završili četverogodišnje srednjoškolsko obrazovanje, a zainteresirani su za učiteljski poziv. Riječ je o punoljetnim osobama od kojih očekujemo da imaju (barem donekle) definiranu sliku svog budućeg profesionalnog života. Cilj nam je otkriti kolike su njihove matematičke kompetencije, odnosno kolike su im sposobnosti i vještine primjene najelementarnijih matematičkih znanja od kojih bi mnoga jednog dana i sami razvijali kod svojih učenike. U tu smo svrhu analizirali jedan razredbeni ispit iz matematike za upis kandidata na učiteljski studij u Splitu 2007. godine.

Krenuli smo od nekoliko polazišnih pretpostavki koje su odredile tijek i način našeg istraživanja. Prva pretpostavka (dobivena našim dugogodišnjim iskustvom na učiteljskim studijima) je bila da brojni studenti učiteljskih studija imaju problema s polaganjem ispita iz matematike. Uz to, smatrali smo da ocjena ili postotak prolaznosti iz kolegija "Matematika" na pojedinim učiteljskim fakultetima ne mogu sami za sebe dati potpuno objektivnu sliku njihovih elementarnih matematičkih znanja i vještina, s obzirom da sadržaji tih kolegija obuhvaćaju mnogo šira, dublja i kompleksnija područja matematike. Druga je pretpostavka da temeljna matematička znanja koja se stječu u razrednoj nastavi, budući učitelji donose sa sobom na fakultet i tu ih produbljuju, proširuju i nadograđuju. Oni su, naime, prije dolaska na fakultet, matematiku učili 12 godina u kontinuitetu. Drugim riječima, oni neće tek na fakultetu upoznati matematičke strukture, naučiti osnovne matematičke termine, simbole, koncepte i procese, već će ih tu bitno produbljivati, širiti i nadograđivati te dalje razvijati sustav apstraktnog matematičkog mišljenja. Slične stavove iskazuju i sami studenti učiteljskog studija, koji smatraju da im matematički sadržaji naučeni na fakultetu nisu pretjerano korisni za njihov budući učiteljski rad, a najveći broj ih smatra da su matematiku najbolje naučili u osnovnoj školi (Mišurac Zorica, 2007). Upravo zato smo se odlučili istražiti njihove elementarne matematičke kompetencije u sadržajima koji se ne uče na fakultetu, već spadaju u osnovnoškolsku matematiku. Treća je pretpostavka da su osobe koje odabiru učiteljski fakultet svjesne da će veliki dio njihovog nastavnog rada (točnije četvrtina) biti upravo poučavanje matematike. Stoga smo pretpostavili da se osobe koje nemaju temeljna matematička znanja i kompetencije neće ni odlučiti za učiteljski poziv.

Zbog svih navedenih pretpostavki, smatrali smo važnim ispitati kakvi nam se kandidati interesiraju i upisuju na učiteljske fakultete. U svrhu istraživanja iskoristili smo razredbeni ispit na učiteljskom fakultetu u Splitu, gdje su se (između ostalog) testirale matematičke kompetencije kandidata.

Uzorak istraživanja

Uzorak istraživanja činilo je 123 kandidata koji su u ljeto 2008. godine pristupili razredbenom ispitu za upis na Učiteljski studij Filozofskoga fakulteta u Splitu. Obzirom da su se kandidati prijavljivali slobodno, da se radilo o prvom ispitnom roku, da se pristup prijemnom ispitu plaćao i da se većina pristupnika koji su prošli razredbeni prag (kojeg je činilo nekoliko prijemnih ispita iz područja matematike, hrvatskoga jezika, književnosti i test općih sposobnosti) i upisalo na fakultet, možemo zaključiti da su ispitanici bili visoko motivirani za upis na učiteljski studij. Drugim riječima, radi se o kandidatima koji zaista u budućnosti žele biti učitelji razredne nastave te poučavati i matematiku. Svi su ispitanici bile žene.

Instrument istraživanja

Instrument istraživanja bio je test matematičkih znanja i vještina kojeg smo sami osmislili i napravili (Test u Prilogu) u svrhu selekcije kandidata za učiteljski studij. Cilj nam nije bio dublje ispitivati matematička znanja (osim eventualno poznavanja osnovne matematičke terminologije što je preduvjet za razumijevanje teksta zadatka), već se bazirati na razumijevanje osnovnih matematičkih struktura i koncepata te logičko zaključivanje i primjenu matematičkih koncepata i procesa koje su kandidati u prethodnom školovanju usvojili u zadacima testa posebno pripremljenog za tu svrhu. Kao pilot istraživanje test je dan na rješavanje nekolicini srednjoškolaca te nekolicini asistenata s fakulteta iz društvenog i humanističkog područja. Zaključak učenika i nematematičara je bio da je test izrazito lagan, razumljiv i prilagođen potrebi selekcije na razredbenom ispitu.

Test se sastojao od 19 matematičkih zadataka koje bi svrstali u klasu jednostavnih zadataka. Pri tome su dva zadatka iz područja mjerenja i traže elementarno poznavanje mjernih jedinica za obujam i masu, a oba po sadržaju i po složenosti spadaju u područje razredne nastave (dakle mogli bi ga riješiti učenici u 4. razredu osnovne škole). Tri su zadatka matematički problemski zadaci, ali sva se tri mogu kontekstualno i strukturalno svrstati u zadatke za dodatnu nastavu matematike u prva četiri razreda osnovne škole. Tri zadatka su geometrijskog sadržaja i traže poznavanje i primjenu najelementarnijih geometrijskih činjenica. Dva među njima također spadaju (po sadržaju i složenosti) u područje razredne nastave. Pet aritmetičkih zadataka najjednostavnijeg su tipa, a od toga su tri iz početne nastave matematike dok je jedan numerički zadatak s razlomcima. Četiri algebarska zadatka spadaju u više razrede osnovne škole, ali riječ je o najtrivijalnijim primjerima sadržaja na koji se odnose (algebarski razlomak, jednadžba, nejednadžba i linearna funkcija). Ista je stvar i s jedinim zadatkom iz analitičke geometrije koji traži povezivanje jednadžbe za pravac i koordinata točke. Jedan zadatak s nizovima također je tipičan za dodatne zadatke iz matematike u periodu razredne nastave.

Možemo rezimirati da je od 19 zadataka, njih 12 prilagođeno sadržajima matematike iz prva četiri razreda osnovne škole, dok su preostalih 7 zadataka najjednostavniji i najosnovniji primjeri zadataka u sadržajima na koje se odnose, a koji po nastavnom programu spadaju u više razrede osnovne škole. Stoga test smatramo izrazito laganim, razumljivim i sadržajno prilagođen (čak i nedovoljno zahtjevnim) za buduće studente učiteljskog studija.

Rezultati i rasprava

Usprkos jednostavnost testa, dobili smo sljedeće rezultate prikazane u Tablici 1. Za svaki je zadatak naznačen broj i postotak kandidata koji su došli do točnog rješenja, broj i postotak kandidata koji su zadatak riješili pogrešno odnosno koji zadatak (barem na papiru) nisu ni pokušali riješiti.

zadatak	bez pokušaja		pogrešno		točno	
	N	%	N	%	N	%
1.	28	23	56	46	39	32
2.	9	7	60	49	54	44
3.	50	41	55	45	18	15
4.	42	34	55	45	26	21
5.	16	13	24	20	83	67
6.	29	24	34	28	60	49
7.	57	46	40	33	26	21
8.	17	14	21	17	85	69
9.	20	16	70	57	33	27
10.	57	46	43	35	23	19
11.	20	16	32	26	71	58
12.	8	7	43	35	72	59
13.	26	21	78	63	19	15
14.	13	11	32	26	78	63
15.	26	21	67	54	30	24
16.	28	23	65	53	30	24
17.	15	12	9	7	99	80
18.	32	26	27	22	64	52
19.	17	14	26	21	80	65

Tablica 1.

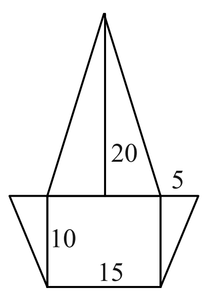
Najbolji rezultat postignut je u 17. zadatku, koji je glasio "Koji je broj sljedeći u nizu 1, 4, 9, 16, 25, 36, 49,. . . ", gdje je čak 80% ispitanika točno riješilo zadatak.

Ovaj bi zadatak mogli svrstati u (dodatni) zadatak za drugi ili treći razred osnovne škole. Zadatak nije tražio nikakva posebna matematička znanja, već samo sposobnost prepoznavanja i razumijevanja obrazca ponavljanja brojeva u nizu. Najmanji je postotak točnih rješenja u 3. i 13. zadatku, gdje je svega 15% kandidata točno riješilo zadatak. Treći je zadatak tražio da se broj 699 napiše rimskim znamenkama, što sadržajno spada u četvrti razred osnovne škole. Rimske se znamenke uglavnom ne obrađuju u fakultetskoj matematici, pa možemo samo pretpostaviti s kolikim matematičkim znanjem ovih sadržaja budući učitelji dolaze u razred. Trinaesti zadatak tražio je zbroj prva tri prosta, dvoznamenkasta broja, po čemu i on sadržajno spada u početnu nastavu matematike. U ovom je zadatku čak 57% ispitanika došlo do pogrešnog rješenja, a ako pretpostavimo da nisu pogriješili u samom zbrajanju, možemo pretpostaviti da do rješenja nisu došli jer nisu poznavali pojam prostog broja.

Zadaci koje ispitanici u najvećem broju (njih 57 ili 46%) nisu ni pokušali riješiti su zadaci 7. i 10. Sedmi je zadatak problemski i glasi "Koliko litara goriva troši automobil na svakih 100 prijeđenih kilometara puta, ako je na putu od 3800 km potrošio 361 litru?".

Rješenje ovog zadatka je decimalni broj iako zadatak koncepcijski i kontekstualno liči na problemske zadatke koji se daju i učenicima u početnoj nastavi matematike. Usprkos činjenici da se radi o izrazito životnom zadatku i potpuno poznatom i bliskom kontekstu, svega je 21% ispitanika riješio ovaj zadatak.

Deseti je zadatak geometrijski i tražio je izračunavanje površine lika nacrtanog na Slici 1. Ovaj zadatak spada u početno izračunavanje površine, a može se izračunati poznavajući samo formulu za izračunavanje površine pravokutnika koja se uči u četvrtom razredu osnovne škole. Naime, po dva simetrično postavljena pravokutna trokuta zajedno daju pravokutnik čije su nam sve stranice poznate duljine. Usprkos tome, svega je 19% ispitanika riješilo točno zadatak. U istom je zadatku i najveći broj pogrešnih rješenja, čak 63%.



Slika 1.

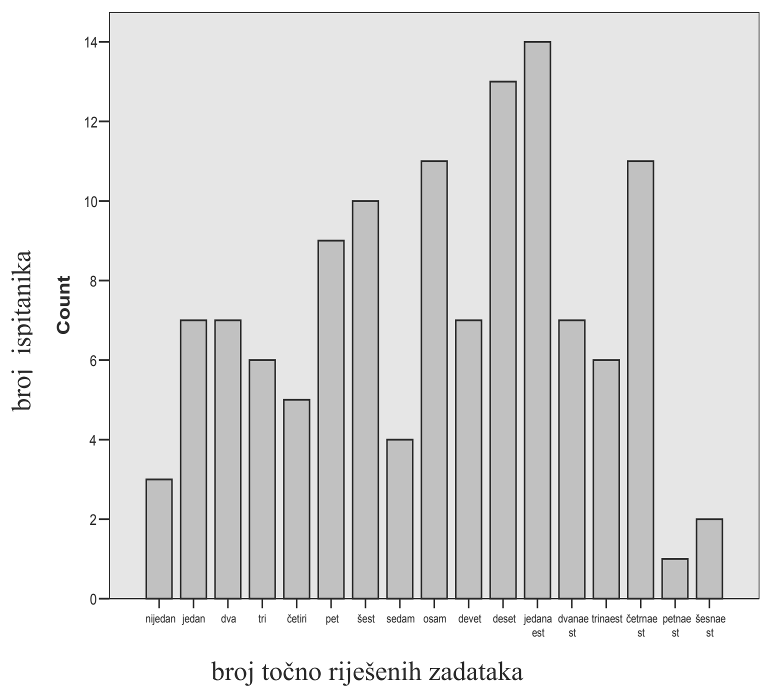
Veliki je broj pogrešnih rješenja i u devetom zadatku (57%), koji je glasio "Nađi sve prirodne brojeve koji su rješenja nejednadžbe $5 - x \geq 2$". Najveći je problem u ovom zadatku bio traženje samo prirodnih brojeva kao rješenja. Naime, mnogi su kao rješenje nejednadžbe ponudili nejednakost $x \leq 3$, što nije ispravno rješenje.

U analizi rezultata istaknuli bi i predzadnji zadatak koji je tražio prepoznavanje geometrijskog tijela kojemu je baza kvadrat, a pobočke jednakokračni trokuti. Ovaj je zadatak točno riješilo tek 52% ispitanika iako se radi o zadatku iz prvog razreda osnovne škole! Od 22% ispitanika koji su pogrešno riješili zadatak dobili smo odgovore kvadar, trapez, stožac, prizma i slično.

Zadatke s veličinama (mjernim jedinicama), 1. i 16. riješilo je manje od trećine ispitanika, a radilo se o najjednostavnijim zadacima (vidi Prilog 1) koji sadržajno također spadaju u razrednu nastavu. Ispitanici ne poznaju odnose među mjernim jedinicama i ne znaju izračunati obujam kocke poznate duljine brida.

Sveukupno gledano, ni u jednom području matematike koja su se nalazila u ispitnom testu ne možemo biti zadovoljni rezultatima. Posebno su nas iznenadili jako loši rezultati u zadacima koji u najvećoj mjeri sadržajno spadaju u početnu nastavu matematike. Tu smo, naime očekivali da će ispitanici prepoznati njihovu jednostavnost i da će rezultati u tim zadacima biti bolji od, na primjer, algebarskih zadataka.

Iz Tablice 1. također je vidljivo da je tek u osam zadataka više od polovice ispitanika došlo do točnog rješenja. U deset je zadataka više pogrešnih, nego točnih rješenja. Sve navedeno ukazuje na jako loše matematičko znanje i razumijevanje temeljnih matematičkih struktura i koncepata te slabu sposobnost primjene ranije naučenih matematičkih sadržaja.



Slika 2.

Gledajući ukupan broj riješenih zadataka pojedinog ispitanika (Slika 2), uočavamo da nitko od ispitanika nije točno riješio sve zadatke, dok troje ispitanika nije riješilo niti jedan zadatak. Najveći je broj ispitanika (njih 14 od 123) riješio jedanaest zadataka, a prosječan broj točno riješenih zadataka je 8 zadataka od ukupno 19, dakle 42%. Ovakav rezultat smatramo poraznim s obzirom na činjenicu da se radi o najelementarnijim zadacima iz područja osnovnoškolske matematike. Treba još jednom naglasiti da se radi o temeljnim matematičkim sadržajima koji se uče u kontinuitetu kroz dugi niz godina u osnovnoj i srednjoj školi, što ovakav rezultat čini još više zabrinjavajućim.

Ispitni je test bio izuzetno jednostavan i sadržavao je zadatke koji su tražili izvođenje najjednostavnijih matematičkih radnji. U njima su se tražila minimalna matematička znanja i to prije svega poznavanje osnovnih matematičkih termina koji se provlače kroz cijelu osnovnoškolsku matematiku, kao što su umanjenik ili umanjitelj, prost broj, djelitelj i slično. Rezultate koje smo dobili u ovom istraživanju smatramo izuzetno lošima. Kao prvo, ispitanici su pokazali izuzetno slabo poznavanje matematičkih pojmova i simbola, ali i slabo razumijevanje i poznavanje matematičkog jezika, što je posebice došlo do izražaja u zadacima koji su tražili razumijevanje uputa uz izvođenje elementarnih računa. Nadalje, sadržaje početne nastave matematike oni jako slabo poznaju, a ne pokazuju ni da razlikuju složenost tih zadataka od na primjer algebarskih zadataka koji se rade na višim razinama matematike (viši razredi osnovne škole). Zabrinjava i činjenica da je u većini zadataka broj pogrešnih rješenja veći od broja točnih rješenja, te da je prosječan broj točno riješenih zadataka samo osam od devetnaest, što iznosi svega 42%. Ovakvi rezultati ukazuju na činjenicu da mladi ljudi zainteresirani za učiteljski poziv nemaju razvijeno konceptualno razumijevanje matematike te nemaju sposobnost povezivanja matematičkih sadržaja koje su tijekom prethodnih dvanaest godina školovanja učili.

Zaključak

Iz dobivenih istraživačkih rezultata možemo zaključiti da se na studij učitelja prijavljuju kandidati kojima su matematičke kompetencije, u smislu vještina primjene naučenih matematičkih sadržaja, izuzetno slabe. Uzmemo li u obzir i ranije dobivenu činjenicu da su njihovi stavovi prema matematici negativni i da im je matematika najmanje drag nastavni predmet za poučavanje (Mišurac Zorica, 2007), možemo sa žalošću utvrditi da kandidati koji se prijavljuju i upisuju na učiteljske studije imaju veoma slabe kompetencije za buduće poučavanje matematike.

Dobiveni su rezultati u skladu i s rezultatima nekih ranijih istraživanja slične problematike. "Ispitivanje koje je u 1987/1988. školskoj godini provedeno analiziranjem slučajnog uzorka od 100 pismenih radova sa klasifikacijskog ispita, od ukupno 320 prijavljenih kandidata iz raznih srednjih škola za studij razredne nastave u Zagrebu, pokazalo je ove rezultate: od 10 osrednje teških ispitnih zadataka iz srednjoškolskog programa matematike, prosječni rezultat je bio 34% riješenih zadataka. Samo 3% kandidata riješilo je sve zadatke, 4% nije riješilo niti jedan zadatak, a 11% kandidata riješilo je samo dva (najlakša) zadatka" (Pejić, 2006, 20).

Budući da je osnovni cilj ispitnog instrumenta bio utvrditi kod kandidata razinu razumijevanja i poznavanja temeljnih matematičkih struktura i koncepata te sposobnost logičkog zaključivanja i praktične primjene matematike koju su oni u prethodnom školovanju usvojili, možemo utvrditi da su rezultati izrazito loši. Mladi ljudi zainteresirani za učiteljski poziv pokazuju veoma nisku razinu matematičkih kompetencija u svim ispitivanim segmentima i u svim sadržajnim područjima matematike. Možemo zaključiti da mladi koji odabiru učiteljski poziv u velikoj mjeri nemaju za taj poziv relevantna matematička znanja, razinu razumijevanja i vještinu primjene matematike u rješavanju različitih problema i zadataka u svakodnevnoj profesionalnoj i životnoj praksi. To naravno može biti i odraz činjenice da mladi kod nas inače imaju nisku razinu matematičke pismenosti, što je potvrdilo i međunarodno istraživanje petnaestogodišnjaka PISA 2006 (Braš-Roth i dr., 2007), pa su tako i oni koji dolaze na učiteljske studije dio takve populacije.

Ipak, ostaje činjenica da će oni koji završe učiteljski studij jednoga dana poučavati matematiku djeci u najosjetljivijim godinama, godinama u kojima se razvijaju i stječu temelji njihovoga matematičkoga obrazovanja, logičkog mišljenja i rasuđivanja te njihovog razumijevanja matematičkih struktura i koncepata kao i općenito odnosa prema matematici. Štoviše, četvrtina njihova budućeg nastavnog rada biti će pripremanje, organiziranje i izvođenje nastave matematike.

S obzirom da se na fakultetu u poučavanju matematike (matematičkim kolegijima) neće krenuti sa studentima od najelementarnijih matematičkih koncepata i procesa, to jest od sadržaja početne ili osnovnoškolske nastave matematike, pitamo se kada bi ti budući učitelji nadoknadili ovakav nedostatak matematičkih kompetencija s kojim ulaze u sustav visokog obrazovanja. Naime, oni će na fakultetu u najvećoj mjeri steći nužna metodičko – didaktička znanja i umijeća, odnosno osposobiti se i pripraviti za uspješno izvođenje početne nastave matematike. U budućem nastavnom radu čekaju ih preopterećeni nastavni program u periodu razredne nastave i komercijalni, nestalni, konfuzni i uglavnom nezadovoljavajući udžbenici koji sigurno ne mogu biti korektiv koji bi kompenzirao ove nedostatke. Kao jedini izlaz vidimo period njihova studiranja na učiteljskim studijima u kojem bi se matematici i metodici nastave matematike trebalo posvetiti mnogo više vremena i pažnje. Mnogo više pažnje treba posvetiti i selekciji kadrova na učiteljskim studijima, kao i osvještavanju studenata, budućih učitelja o kompetencijama koje se od njih očekuju. Bez kvalitetnih učitelja nema ni kvalitetnog temelja matematičkom obrazovanju, bez dobrih matematičkih temelja nema ni uspjeha u matematici u višim razredima, a bez tog uspjeha iz škole će nam opet izlaziti matematički nepismena mladež koja će odabirati učiteljski poziv. Na taj način će se nastaviti lanac neuspjeha matematičke nastave, iracionalnog i pogubeljnog straha od matematike te stigme matematike kao "bauk" predmeta.

Prilog – Test s razredbenog ispita

MATEMATIKA

Dragi pristupnici, u pravokutnike s desne strane upišite samo rezultat zadatka.

Prije nego započnete rješavati zadatke, PAŽLJIVO PROČITAJTE TEKST. Zadataka ima 19, a svaki točno riješeni nosi 1 bod. SRETNO!!!

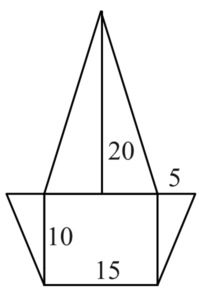
ZADATAK	PROSTOR ZA REZULTAT
1. Koliko kubičnih decimetara ima obujam kocke s bridom 0,5 m?	
2. Rak se kreće pravocrtno 2 koraka naprijed pa 1 korak natrag. Svaki mu je korak dug 1 cm. Koliko se centimetara od početne točke udaljio rak nakon 15 koraka?	
3. Napiši broj 699 rimskim znamenkama.	
4. Skrati razlomak $\frac{a^2+a-6}{a+3}$.	
5. Ako je umanjenik najveći parni troznamenkasti broj, a umanjitelj najmanji neparni troznamenkasti broj, kolika je razlika?	
6. Riješi jednadžbu $\frac{x^2}{x+2} = x - 3$.	

7. Koliko litara goriva troši automobil na svakih 100 prijeđenih kilometara puta, ako je na putu od 3800 km potrošio 361 litru?

8. Ako je $f(x) = 2x - 3$, koliko je $f(-1)$?

9. Nađi sve prirodne brojeve koji su rješenja nejednadžbe $5 - x \geq 2$.

10. Kolika je površina skiciranog lika?



11. Ako je radnik za 12 dana obavio dvije trećine posla, koliko mu treba da obavi cijeli posao ukoliko uvijek radi istim tempom?

12. Izračunaj $-2 \cdot \left(\frac{3}{2} + \frac{2}{7}\right) - \frac{3}{7}$.

13. Koliki je zbroj prva tri prosta, dvoznamenkasta broja?

14. Ako jedan kut u trokutu ima 35°, a drugi je dvostruko veći od njega, koliki je treći kut tog trokuta?

15. Nađi najvećeg zajedničkog djelitelja brojeva 168 i 264.

16. Kako se zove stoti dio osnovne jedinice za mjerenje mase?

17. Koji je broj sljedeći u nizu 1, 4, 9, 16, 25, 36, 49, ...

18. Ako geometrijsko tijelo ima jednu bazu koja je kvadrat i pobočke koje su jednakokračni trokuti, o kojem je tijelu riječ?

19. Kolika je prva koordinata točke $T(x, -7)$ ako znamo da ona leži na pravcu $2x - y - 3 = 0$?

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Istraživanje prostornog zora studenata nastavničkih smjerova matematike na PMF – Matematičkom odjelu Sveučilišta u Zagrebu

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Sažetak. Prostorni se zor može definirati kao intuicija o oblicima i njihovim međusobnim odnosima, a obuhvaća sposobnost stvaranja, pamćenja, rekonstruiranja i transformiranja dobro strukturiranih vizualnih (mentalnih) slika. Osobe s razvijenim prostornim zorom imaju osjećaj za geometrijske aspekte svoga okruženja i oblike koje tvore objekti koji se u njemu nalaze. Prostorni je zor važan aspekt geometrijskog mišljenja, čije razvojne razine opisuje općeprihvaćena Van Hieleova teorija (P. Van Hiele i D. Van Hiele-Geldorf, 1959). Njome se razlikuje pet hijerarhijskih razina razumijevanja prostornih ideja, redom: vizualizacija, analiza, neformalna dedukcija, dedukcija i strogost. Budući da je razvoj geometrijskog mišljenja, a osobito prostornog zora, jedan od ključnih ciljeva nastave matematike u osnovnoj i srednjoj školi, provedeno je početno istraživanje prostornog zora i Van Hieleovih razina na populaciji studenata nastavničkih studija matematike na PMF – Matematičkog odjela Sveučilišta u Zagrebu. U ovome ćemo izlaganju prezentirati njegove rezultate i diskutirati ih u kontekstu postavljenih ishoda učenja geometrije na razini osnovne i srednje škole te sveučilišnog obrazovanja nastavnika matematike. Uz to, rezultati će biti analizirani i iz perspektive rodnih razlika.

Ključne riječi: geometrijsko mišljenje, prostorni zor, Van Hieleova teorija, početno istraživanje, studenti – budući nastavnici matematike

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Primjena osnovnih matematičkih koncepata i vještina u kontekstu fizike

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Sažetak. U edukacijskim istraživanjima u svijetu identificirani su neki temeljni matematički koncepti i vještine, koji predstavljaju značajan problem učenicima, a važni su i za matematiku i za fiziku. To su svakako interpretacija grafova, vektori, proporcionalnost, te interpretacija matematičkih izraza.

U ovom ćemo izlaganju diskutirati postavljene ishode učenja u nastavi matematike i fizike, a koji se tiču navedenih koncepata i vještina. Nadalje, osvrnut ćemo se i na rezultate početnog istraživanja učeničkih razumijevanja grafova u matematici i fizici provedenog na uzorku gimnazijalaca.

Ključne riječi: edukacije matematike, edukacije fizike, interpretacija grafova

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4.
Utjecaj strategija učenja i poučavanja na ishode učenja

Számelméleti játék sakkfigurákkal

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(Édesapám, Molnár Ernő emlékére,
aki első igazi matematika tanárom volt)

Összefoglaló. Jól tudjuk, hogy a sakkjátékban $N = 32$ figura van. (De az alábbi játékot nemcsak sakk-figurákkal, hanem kövekkel és más játékeszközökkel, továbbá papírral és ceruzával, stb. is játszhatjuk). Két játékos, a kezdő **B** és a második **S** felváltva veszik el a sakkfigurákat, mindegyik legalább $a = 1$ darabot, de legfeljebb $A = 4$ darabot.

Az nyer, aki az utolsó sakkfigurát elveszi.

Több kérdést tehetünk fel: Például, igaz-e, hogy a kezdő **B** mindig nyer? Ha igen, hogyan kell játszania? Hogyan módosítsuk a játékszabályokat, pl. A vagy N értékét, hogy a második **S** nyerjen?

Hasonló kérdések talán jobban megvilágítják a játékelmélet és párhuzamosan az elemi
számelmélet néhány fontos fogalmának a szerepét, remélhetően szórakoztató módon.

Kulcsszavak: számjáték, kivonás adott szabállyal, maradékos osztás, prímszámok

A játék átfogalmazása

Mindjárt az elején átfogalmazzuk játékunkat úgy, hogy papírral és ceruzával két gyermek is játszhassa, vagy még inkább a táblán krétával játszhassa az előadó a hallgatóság képviselőjével, mint itt most a konferencián. Vagy egy általános iskolai osztályban a tanár játszhatta valamelyik tanulóval, mint ahogy az velem is megtörtént, amikor tanárjelölt voltam (több mint 40 évvel ezelőtt).

A játék elején felírjuk a 32 kezdőszámot (ezt N-nel jelöljük) a táblára. A tanulónak, mint kezdőjátékosnak (**B**-vel jelöljük) kell felírnia a következő számot úgy, hogy az legalább $a = 1$-gyel, de legfeljebb $A = 4$-gyel kisebb legyen a $N = 32$ kezdőszámnál. Ezután jön a tanár, mint második (**S**-sel jelöljük), akinek az előbb rögzített szabályok szerint kell kisebbet írnia a táblán lévő új számnál, és így tovább. *Az nyer, aki a* 0*-t (nullát) felírja a táblára a játék végén.*

Mondjuk, az elején a **B** tanuló felírja a 29-es számot, aztán az **S** tanár következik 28-cal, és így tovább, **B**: 24, **S**: 20, **B**: 16, **S**: 15, **B**: 12, **S**: 10, **B**:?

1. *Közeledik a végjáték*, és a tanuló rájön, hogy veszíteni fog. Hiszen bármelyiket is írja fel **B** a 9, 8, 7, 6 számok közül, **S** az 5-öst írja fel, és legközelebb a 0-t, bármit is választ **B** a 4, 3, 2, 1 közül. Ezután elemezhetjük a játék táblai *jegyzőkönyvét*. Hol vesztette el **B**, illetve nyerte meg **S** a játékot?
2. Tehát *a játék elemzését ellenkező irányban végezzük, a végén kezdve haladunk az elejéig*. Hamar kiderül, hogy **S** már akkor megnyerte a játékot, amikor a 20-at felírta, hiszen utána – a szabályok szerint – következetesen írhatta a 15, 10, 5, 0 számokat. Természetesen **B** nyerhetett volna, ha ő írja fel a 25-öt és korábban a 30-as számot a játék elején.
3. *Ezután a tanuló, mint kezdő* **B**, *meg tudja fogalmazni nyerőstratégiáját*. Először a 30-as számot írja fel a táblára, majd a 25, 20, 15, 10, 5, 0 következik. És ezt megteheti a játék szabályai szerint, hiszen $a = 1$, $A = 4$ miatt $A + a = 5$-tel tudja előző számát csökkenteni, és $32 = 6 \times 5 + 2$ a maradékos osztás szerint. A maradék 2 lesz, ha az $N = 32$ kezdőszámból egymásután elveszünk $A + a = 5$-öt.
4. Természetesen *a második játékos*, **S** *nyerhetett volna ha*, $N = 30$ lett volna a kezdőszám. Vagy $N = 32$ esetében, ha a maximálisan kivonható szám $A = 7$ (vagy 3, 15, 31) lett volna, mivel $32 = 4 \times 8$ $(8 \times 4, 2 \times 16, 1 \times 32)$.
5. *Összefoglalhatjuk a játék lényegét a játékszabályok, vagyis a* N, A, $a \in \mathbf{N}$ (a természetes számok halmazának jele) $N > A > a > 0$ *ismeretében.* Tegyük fel, hogy
$$N = k \times (A + a) + r \quad (r, k \in \mathbf{N},\ 0 \le r < A + a)$$
a maradékos osztás (vagy egymásutáni kivonások) eredménye.
   - i) **B** nyer, ha $a \le r \le A$. Ebben az esetben először az $N - r = k \times (A + a)$ számot írja fel, majd így tovább a $\ldots$, $A + a$ számot és a 0-t, ez a **B** kezdőjátékos nyerőstratégiája.
   - ii) **S** *nyer* (vagyis **B** veszít), ha $r = 0$.
   - iii) Ha $0 < r < a$ *vagy* $A < r < A + a$, *akkor sem* **B***-nek, sem* **S***-nek nincs nyerőstratégiája, de mindkét játékos elérheti a döntetlent (remis, franciául, az utolsónak a-nál kevesebb,* 0*-nál nagyobb marad) bármilyen jól is játszik a partner*. Ugyanis, **B** a-val csökkent, ha $0 < r < a$ (és $A + a < N$); és A-val csökkent ha $A < r < A + a$. Ugyanígy tesz **S**, amikor ő következik. *Ha valamelyikük nagyot hibázik, a másik nyerhet, de több döntetlen helyzet is van, ha* $1 < a$ *elég nagy*.

Számelméleti megjegyzések

Nyilvánvaló, hogy a játékszabályokat megváltoztathatjuk úgy, hogy valamelyik játékosnak, a **B** (kezdő) tanulónak, vagy a (második) **S** tanárnak kedvezzen.

Ha a kezdő **B** mondja meg a $N > A > a > 0$ számokat, akkor ő nyer. Ha először **B** rögzíti N-et, **S** választja A-t és $a - t$, akkor **S** fog nyerni.

Hogy a döntetlent kizárjuk, tegyük fel, hogy $a = 1$. Hogy a könnyű játékot is kizárjuk, legyen $N >> A$, vagyis a kezdőszám sokkal nagyobb mint a maximális kivonható szám. *Ekkor a prímszámok is szerephez jutnak*. Ha **B** a N kezdőszámot *prímszámnak* választja, például 31-nek, akkor **S** *nem tud olyan A számot mondani, hogy ő nyerjen*, hiszen $A = 30$ nyilván nem lenne *sportszerű szabály*.

És így tovább, *ezt a játékot teljesen kielemeztük*.

Más játékok, a sakkjáték

Néhány általános megállapítást is tehetünk a *kétszemélyes játékokra*, például a sakkra, és bizonyos *egyszemélyes játékokra* (rejtvényekre, feladványokra), például a *Rubik-kocka* visszarendezésére, de az utóbbiakkal most nem foglalkozunk.

A sakkjáték jól ismert szabályai már több mint ezer évesek. Ezekhez bizonyos, elég absztrakt előírások kellenek, melyek a *sakktáblára és a sakkfigurák lépésmódjára* vonatkoznak, éppen úgy, ahogy fenti játékunkban a N, A, a számokra és a velük történő műveletekre (a kivonásra) tettünk kikötéseket. Ezek a szabályok ugyanazok a kezdő **B**-re (fehér sakkfigurákkal játszik) és az **S** másodhúzóra (fekete figurákkal játszik).

1. *A végjáték a játék legfontosabb része*, így van ez a sakkban is.
2. *A játék lényegét a végétől visszafelé haladva kell megértenünk*. A sakkban alapvető szerepük van a matt-adási eljárásoknak (például királlyal és vezérrel a másik színű királlyal szemben, stb). Általában *a végjátékbeli ismert nyerőállások*, meghatározzák a korábbi stratégiákat.
3. *A sakkban a játék kezdetétől induló nyerőstratégia*, ha ilyen van egyáltalán, nem ismert, és nem is reményteljes erre törekedni. Ugyanez igaz a döntetlen-stratégiákra. Ez jelenti éppen a sakkjáték csodáját, szépségét, művészetét, tudományát, és még sok mindent, amiért érdemes sakkozni.
4. *De az* 5*-figurás végjátékokat már számítógépekkel megoldották* (ez a legutóbbi információm). Ez azt jelenti, hogy bármely 5-bábos végjáték-állást teszünk fel, a számítógép eldönti, vajon a kezdő nyer, veszít, vagy a játszma döntetlenül végződik. Természetesen feltesszük, hogy mindketten a legjobban játszanak, vagyis két tökéletes számítógép-program küzd egymás ellen.

Az axiomatikus módszer, mint játék, záró megjegyzések

A modern matematika bizonyos részeit, melyeket *már elég jól axiomatizáltak*, *individuális játékoknak* is tekinthetünk. A matematikus(ok), mint játékos(ok) az emberiséget is képviselik. Az alapfogalmakat, mint a számok, geometriai alakzatok (pontok, egyenesek, síkok, vagy a tér maga), a rájuk vonatkozó kapcsolatokkal, műveletekkel, logikai, gondolkodási szabályokkal együtt úgy tekinthetjük, mint játékszabályokat. Ezek az emberi társadalom kiemelkedő személyiségeinek, tudósoknak, ... tapasztalatain, felismerésein, egyezményein alapulnak. Ezeket a játékszabályokat gyűjtik össze az axiómákban, melyek a matematika bizonyos szűk területeire vonatkoznak, de a fizikára és más tudományokra, sőt talán még társadalom-tudományokra is vonatkozhatnak.

Egy matematikai eredmény a bizonyításával együtt úgy is tekinthető, mint egy játékszabályokon alapuló eljárás terméke. A lépések hasonlók azokhoz melyeket eddig érzékeltettünk. A végjáték elemzésével kezdjük visszafelé haladva, hogy nyerőstratégiát találjunk, ha ilyen egyáltalán létezik. *Már tudjuk, hogy ilyen stratégia általában nem létezik*. *David Hilbert, Kurt Gödel*, a magyar *Neumann János*, csak példaként említve, tevékenysége kapcsolódik témánkhoz. Az emberi történelem, kultúra, tudomány és művészet alátámasztja sokunk megállapítását: *"Az élet – játék"*.

A kiindulási játékunktól most már messzire jutottunk. Sakkozni édesanyámtól tanultam először, majd apámtól, és ma is sakkozom. Apám, *Molnár Ernő* (1912-1994) tanított meg arra a játékra, melyet ebben az előadásban elemeztem, éppen sakkfigurák segítségével. Akkoriban éppen ennek a játéknak a problémáját oldotta meg, melyet a *Matematika Tanítása* című folyóiratban tűztek ki tanárok számára. A Bolyai János Matematikai Társulat és a Magyar Oktatási (és Kulturális) Minisztérium ezen folyóirata ma is létezik. Lelkes tanárok oldják meg a feladatokat képességeik és ismereteik gyarapítására és a saját gyönyörűségükre.

Itt is megköszönöm kedves kollégáimnak *Molnár Sáska Gábornak és Lángi Zsoltnak* segítő javaslataikat.

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Zabavna matematika u nastavi matematike

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Sažetak. U svim dosadašnjim reformama na području matematičkog obrazovanja učenika tražio se suvremeniji put stjecanja znanja, bolji način učenja i prenošenja određenih novih spoznaja znanosti u nastavni predmet te znanosti. Osuvremenjivanje nastave matematike I rasterećenje učenika može se postići na razne načine. U članku se opisuje jedan od novih oblika rada koji vodi u tome pravcu – zabavna matematika.

Ključne riječi: matematika, zabavnost, zabavna matematika, rasterećenje

Prvi dojam koji kod učenika i nastavnika matematike izaziva gornji naslov jest: matematika i zabava! O matematici se uobičajilo mišljenje kao o teškom nastavnom predmetu, za učenje kojega je potrebno uložiti dosta vremena, napora i truda. To je točno. Matematika je bez sumnje teži nastavni predmet, pa pri usvajanju novog gradiva učenici često osjećaju stanovit psihološki pritisak. U nastavi matematike još uvijek prevladavaju tradicionalni oblici rada i nastavne metode, a njezin osnovni cilj je puko usvajanje gradiva propisanog nastavnim programom i stjecanje znanja koja se temelje na nizu pravila, formula i umijeća rješavanja standardnih zadataka. Uspješno svladavanje nastavnog gradiva često znači samo usvajanje još više novih informacija i činjenica i može rezultirati daljnjim opterećenjem učenika. To nije dobro. Umjesto preopterećenja pamćenja učenika velikim brojem činjenica treba pobuđivati i pokretati njihovo mišljenje i nastojati da dobar dio novih znanja stječu vlastitim snagama i sposobnostima. Da bi se to postiglo, nužno je neprestano osuvremenjivati nastavu matematike.

U svim dosadašnjim reformama na području matematičkog obrazovanja učenika tražio se suvremeniji put stjecanja znanja, bolji način učenja i prenošenja određenih novih spoznaja znanosti u nastavni predmet te znanosti. Osuvremenjivanje, posebno u nastavi matematike u osnovnoj školi, može se u prvom koraku postići češćom izmjenom poznatih nastavnih metoda i njihovim poboljšanjem. Ali to nije dovoljno. Potrebno je uvoditi **nove oblike rada**, promijeniti uvriježenu predodžbu o matematici i pokazati da matematika može biti lagana i zabavna. Naravno, uvođenje novih oblika rada zahtijeva od nastavnika matematike ozbiljnu pripremu i dodatni napor. Međutim, sve to ne bi trebalo biti ništa prema zadovoljstvu koje bi trebao osjećati nastavnik matematike kad vidi interes učenika i njihovo usvajanje novog gradiva bez psihološkog opterećenja i prisile.

1.1.1.1 Učenici su se s neobičnim matematičkim zadacima već susretali i susreću se u matematičkim časopisima za učenike i drugdje, a nastavnici matematike u nekim školama već povremeno drugačije rade, uvode nove oblike rada sa zanimljivim i zabavnim matematičkim sadržajima koji ublažuju težinu nastavnog predmeta. Tako dolazimo do spoznaje da matematika uistinu može biti lakša i zabavna. Učenicima koji su skloni matematici ona je dovoljno zanimljiva i zabavna već sama po sebi. Matematika može postati zanimljivija i drugim učenicima ako se nastava matematike prožme drukčijim sadržajima.

Evo nekih suvremenih mogućnosti: Izrada panoa sa zanimljivim i zabavnim matematičkim sadržajima, izrada modela geometrijskih tijela, matematički kvizovi, matematičke igre, matematičke križaljke, matematički projekti, matematička otkrića pomoću računala, matematika u prirodi, školski matematički časopis, zabavni matematički zadaci, zabavni sati i dr.

U svakom od ovih novih oblika rada sadržan je element zabavnosti. Tako je izrada panoa kreativni čin koji učenicima otkriva zadovoljstvo stvaralačkog rada. Taj rad osim određenog obrazovnog učinka ima i zabavni karakter: stjecanje znanja kroz razonodu! Matematičke križaljke su zanimljiv oblik rada koji omogućuje da se učenici u određenom trenutku opuste, odmore i pripreme za nastavak nekog novog napornog rada. Kao i sve križaljke, i matematičke križaljke su zabavne, ali imaju i obrazovni karakter, najviše kao sredstvo provjere stečenog znanja. Matematička otkrića pomoću računala mogu u učenika izazvati oduševljenje, a to je najviši stupanj u načelu interesa. Oduševljenog učenika matematika više ne opterećuje, već zabavlja! Na sličan način mogli bi ukratko navesti i ulogu i obrazovni učinak ostalih navedenih novih oblika rada.

Svi ti elementi zabavnosti mogu se objediniti u jedinstvenu cjelinu koju danas nazivamo – **zabavna matematika**. U ovom članku predmet našeg interesa je upravo ta matematička disciplina. Zabavna matematike najlakše se može prepoznati po zabavnim zadacima. Izbor zabavnih zadataka je velik. Gotovo za svaku nastavnu temu, za svaku nastavnu jedinicu, za svaki matematički pojam može se pronaći niz zadataka koji o tome čpričajuć na zabavan način. Zabavni zadatak može biti lijep uvodni motivirajući primjer prije obrade nekog matematičkog pojma ili kao osvježenje ponekad krute nastavne situacije.

Što su zapravo zabavni zadaci? Evo glavnih značajki kojima se odlikuje takva vrsta zadataka i po kojima ih možemo prepoznati:

1) Zabavni zadaci su matematičke minijature za čije je rješavanje dovoljno najosnovnije znanje iz aritmetike, algebre i geometrije.
2) Formulacije zadataka su jednostavne i svakome razumljive.
3) Tekstovi su pisani u obliku malih duhovitih pričica iz svakidašnjeg života.
4) Veće matematičko predznanje nije uvijek garancija bržeg rješavanja.
5) Problemi nisu uvijek lagani, mnogi od njih zahtijevaju priličan umni napor, logičko rasuđivanje, a posebno domišljatost u pronalaženju puta njihovog rješavanja.

6) Važnu ulogu u ljepoti takvih zadataka često igraju i duhovite ilustracije.
7) Osnovne vrijednosti: razvijanje logičkog rasuđivanja i domišljatosti, pobuđivanje interesa za matematiku, popularizacija matematike.

Područja zabavne matematike: *neobična svojstva brojeva, brojevi i slova u likovima, igre s brojevima, geometrijska tijela, iste znamenke, jednim potezom olovke, kombinatorni problemi, likovi, logičke minijature, magični kvadrati, matematičke igre, pokrivanje likova, računski kriptogrami, razrezivanje likova, sastavljanje likova, šibice i štapići, testovi i dr.*

Recimo par riječi i o neposrednoj namjeni zabavnih zadataka. Zabavna matematika namijenjena je SVIMA: učenicima, učiteljima i profesorima matematike, roditeljima i svim ostalim ljubiteljima matematike! Jer, za svakoga ima ponešto zanimljivo i zabavno.

Problemi iz zabavne matematike namijenjeni su ipak prije svega učenicima osnovne škole predmetne nastave, ali i učenicima nižih razreda srednje škole zbog različitog stupnja težine problema.

Zabavni zadaci nisu uvijek samo zabava. Oni mogu biti i dopuna nastavnog programa matematike u tom smislu da u umni rad učenika i nastavu unose one elemente za koje u redovnoj nastavi nema dovoljno vremena: duhovita kratka priča, humor, domišljatost, psihološko rasterećenje.

Nastavnik matematike treba posebnu pozornost obratiti na težinu zadataka, jer nije svejedno, primjerice, govoriti o prostim brojevima u osnovnoj školi ili u srednjoj školi. Pa čak ni na zabavan način. Zabavan zadatak ne znači uvijek i lagan zadatak. Također rješenja zabavnih zadataka ne trebaju uvijek biti iscrpna. Trebaju biti dovoljna za razumijevanje, ali treba ostaviti i dovoljno prostora za promišljanje učenika. U zabavnoj matematici nisu cilj mukotrpna izračunavanja, već ideja i zadovoljstvo otkrivanja nečeg zanimljivog i neobičnog. Ponekad već pogled na rješenje neobičnog problema može osvježiti i zabaviti.

Pogodni trenuci za primjenu:

Početak školske godine (nastava matematike još nije "ozbiljno" krenula; zabavni zadaci vrlo su pogodno sredstvo da nastavi daju početni zamah i potaknu interes učenika za učenje matematike; za tu svrhu posebno su pogodni zadaci iz područja: MJERENJA, VAGANJA, PRETAKANJA).

Redovna nastava (zabavni zadaci mogu poslužiti kao uvodni motivirajući primjeri, osvježenje, igra).

Domaća zadaća (izbor zadataka za domaću zadaću treba povremeno obogatiti ponekim zadatkom neobičnog sadržaja i zabavnog karaktera; takva domaća zadaća mogla bi poslužiti za "okupljanje" i "sudjelovanje" cijele obitelji!).

Ispravljanje slabih ocjena (zamorna i iscrpljujuća atmosfera; posebno su zapostavljeni naprednije učenici).

Zabavni nastavni sati.

Kraj školske godine (posljednji dani školske godine trebali bi biti prirodan završetak jednog obrazovnog razdoblja i bez prisile, napetosti, većih ispitivanja znanja; završetak bi trebao biti zabavan, ali još uvijek i poučan; za tu svrhu posebno su pogodne MATEMATIČKE KRIŽALJKE).

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Metakognicija i samoregulacija u učenju i nastavi matematike

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Sažetak. U današnje vrijeme velikih ekonomskih i socijalnih promjena zahtjevi za novim pristupima učenju i obrazovanju postaju sve jači. Sve se više govori o metakogniciji u procesima učenja i nastave, koja zajedno s kognicijom i motivacijom predstavlja osnovu za samoregulaciju, odnosno samostalnom, svrhovitom i trajnom učenju s ciljem poboljšanja znanja vještina i sposobnosti.

U ovom radu raspravlja se o utjecaju, strategiji i trendovima upotrebe metakognicije i samoregulacije u učenju i nastavi matematike.

Ključne riječi: metakognicija, samoregulacija, matematika, učenje i nastava

Uvod

Učenje i razumijevanje: 7 načela

NCTM (National Council of Teachers of Mathematics) je u svojim poznatim standardima 2000. godine predložio sljedeća načela za matematičko obrazovanje:

1. Jednakost
2. Kurikulum
3. Nastava
4. Učenje
5. Ocjenjivanje
6. Tehnologija

Također je istaknuto 7 načela za učenje i razumijevanje:

1. *Princip konceptualnog znanja*
   Učenje s razumijevanjem olakšano je kada se novo i postojeće znanje strukturira oko glavnih koncepata i načela određene discipline.

2. *Prethodno znanje*
   Učenici koriste postojeće znanje da izgrade novo s razumijevanjem.
3. *Metakognicija*
   Učenje se bitno olakšava korištenjem metakognitivnih strategija koje identificiraju, prate i reguliraju kognitivne procese.
4. *Razlike između učenika*
   Učenici imaju različite strategije, pristupe, vrste sposobnosti i stilove učenja koji su funkcija interakcija između njihova naslijeđa i prethodnog iskustva.
5. *Motivacija* Motivacija učenika da uči i njegova samosvijest utječu na ono što će biti učeno, koliko će biti učeno i koliko će se napora uložiti u proces učenja.
6. *Situirano učenje*
   Čine ga praksa i aktivnosti u kojima učenici, dok uče, oblikuju ono što je naučeno.
7. *Zajednice učenja*
   Učenje se obogaćuje kroz procese socijalno podržanih interakcija.

Metakognicija je koncept koji se odnosi na učenje, a prvi ga je put opisao John Flavell 1976. godine. Ukratko, metakognicija je "mišljenje o mišljenju" ili "znanje o znanju". Postoje mnoge definicije metakognicije, od kojih je možda najprikladnija, a i najkraća, Brownova iz 1987. godine – "Metakognicija se odnosi na znanje i upravljanje kognitivnim sustavom pojedinca".

Metakognicija i samoregulacija

Nadalje, Brown 1987. te Garner i Alexander 1989. godine definiraju metakogniciju kao skup znanja i izvršnih kontrola o procesu učenja.

Znanje

Relevantan je način na koji pojedinci procesuiraju informaciju, pri čemu razlikujemo tri vrste znanja:

1. *Osobno znanje* ili znanje o tome kako svaki čovjek ponaosob uči ili obrađuje informaciju, npr.:
   - "ja najbolje učim ujutro"
   - "ljudi trebaju povratnu informaciju (engl. *feedback*) o adekvatnosti njihova razumijevanja da bi učili efektivno.
2. *Znanje o zadatku*, odnosno znanje o različitim tipovima zadataka koji se uče.
   - "lakše je prepoznati ili ne prepoznati da je nešto točno nego se prisjetiti točne informacije".
3. *Strateško znanje* ili znanje o efektivnosti odgovarajućih strategija učenja.

Upravljanje kognitivnim sustavom (izvršne kontrole)

1. *Prognoziranje*
   Na primjer, koliko je težak određeni zadatak, tj. određeno poglavlje u knjizi i sl.
2. *Planiranje* se odnosi na ono što će se raditi za vrijeme učenja pri izboru odgovarajuće strategije itd.
3. *Monitoring* (nadgledanje) procesa učenja koji uključuje:
   a) što mi znamo o materijalu koji se uči
   b) što mi ne znamo, a trebamo znati kako bi postigli ciljeve učenja
   c) koliko razumijemo materijal koji učimo.
4. *Evaluacija* (vrednovanje) rezultata različitih prethodno navedenih aktivnosti.

Metakognicija je "maglovit koncept" jer nema definicije koja bi bila općeprihvaćena, ali se gotovo svi slažu da ona uključuje znanje i upravljanje kognitivnim procesima. Često je veoma teško razlikovati kogniciju od metakognicije.

Samoregulirajuće učenje odnosi se na našu sposobnost da razumijemo i kontroliramo okruženje u kojem učimo. Kako bismo to mogli, moramo postaviti ciljeve, odabrati strategije koje nam pomažu postići te ciljeve, implementirati te strategije i promatrati naš napredak u postizanju ciljeva (Schunk, 1996.). Malo je učenika/studenata koji se mogu potpuno samoregulirati. Oni s razvijenijim vještinama samoreguliranja uče više s manje napora i postižu bolje rezultate (Pintrich, 2000.; Zimmerman, 2000.).

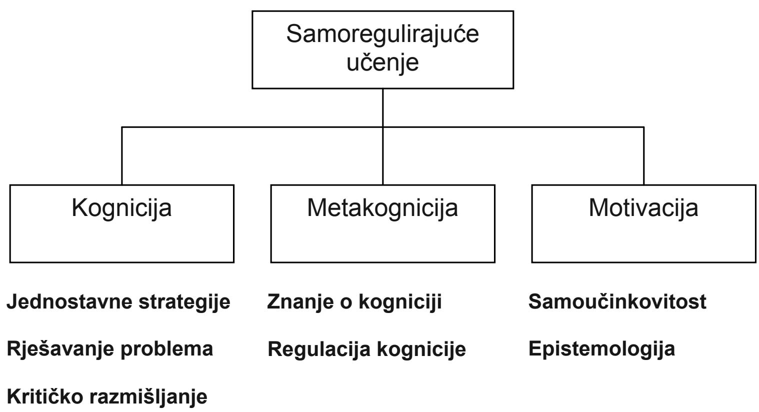
Samoregulirajuće učenje ima korijene u teoriji socio-kognitivnog učenja Alberta Bandure (Bandura, 1997.). Osnova Bandurove teorije jest da je učenje rezultat faktora okruženja i ponašanja te osobnih faktora.

Osobni faktori uključuju učenikova uvjerenja i stavove koji utječu na učenje i ponašanje. Faktori okruženja uključuju kvalitetu instrukcije, učiteljeve povratne informacije, pristup informacijama i pomoć roditelja. Faktori ponašanja uključuju učinke prijašnje izvedbe. Svaki od tih triju faktora utječe na preostala dva.

Tijekom posljednja dva desetljeća istraživači su primijenili Bandurovu socio-kognitivnu teoriju i na učenje. To je dovelo do razvoja teorije samoregulirajućeg učenja, prema kojem na učenje utječe skup različitih interaktivnih kognitivnih, metakognitivnih i motivacijskih komponenti (Butler i Winne, 1995.; Zimmerman, 2000.). Socio-kognitivni pravci samoregulirajućeg učenja tvrde da pojedinci postaju sposobni samoregulirati učenje napretkom kroz četiri stupnja razvoja: stupanj promatranja, oponašanja, samokontrole i samoregulacije (Schunk, 1996.; Zimmerman, 2000.). Učenje na stupnju promatranja usredotočuje se na modeliranje, dok se učenje na stupnju oponašanja usredotočuje na društveno vođenje i davanje povratnih informacija. Oba stupnja naglašavaju vanjske socijalne faktore. Za razliku od navedenih faza učenja, kako se učenici razvijaju, sve se više oslanjaju na unutarnje, samoregulirajuće vještine. Na stupnju samokontrole, učenici izgrađuju unutarnje standarde za prihvaćanje izvedbi i poticanje samih sebe kroz pozitivno razmišljanje i povratne informacije. Na stupnju samoregulacije, pojedinci imaju snažna uvjerenja o svojoj učinkovitosti te širok spektar kognitivnih strategija koji im omogućuju da samoreguliraju svoje učenje.

Samoregulirajuće učenje sastoji se od tri glavne komponente: kognicije, metakognicije i motivacije. Kognicija uključuje vještine potrebne za kodiranje, pamćenje i prisjećanje informacija. Metakognicija uključuje vještine koje omogućuju učenicima da razumiju i promatraju svoje kognitivne procese. Motivacija uključuje uvjerenja i stavove koji utječu na upotrebu i razvoj kognitivnih i metakognitivnih vještina. Svaka od tih triju komponenti jest potrebna, ali nije dovoljna za samoregulaciju. Primjerice, oni koji posjeduju kognitivne vještine, a nisu motivirani da ih upotrebljavaju, ne postižu onoliko koliko oni koji posjeduju vještine i motivirani su da ih upotrebljavaju (Zimmerman, 2000.). Slično, oni koji su motivirani, a ne posjeduju potrebne kognitivne i metakognitivne vještine, često ne uspijevaju dosegnuti visok stupanj samoregulacije.

Tri osnovne komponente mogu se dalje podijeliti na podkomponente koje su prikazane na Slici 2.1. Ovdje nećemo (osim metakognicije) detaljnije raspravljati o ovim podkomponentama.



Slika 1. Samoregulacija.

Mjerenje metakognicije

Brojna istraživanja pokazala su da su metakognitivno svjesni učenici bolji u učenju od drugih. Planiranje, procjenjivanje i razmišljanje o učenju popravlja njihovu izvedbu. Postoji snažna potpora modela metakognicije koji uključuje dvije glavne komponente – znanje i regulaciju kognicije. Znanje o kogniciji jest znanje koje učenici imaju o sebi, o strategijama i uvjetima pod kojima su metode koje koriste najučinkovitije. Deklarativno, proceduralno i kondicionalno znanje mogu se smatrati "kamenim blokovima" od kojih je sagrađeno konceptualno znanje.

Deklarativno znanje

- činjenično znanje koje učenik treba prije nego što procesira ili kritički razmišlja o temi
- znanje o nečemu, znati nešto
- znanje o vlastitim vještinama, intelektualnim, kao i drugim učeničkim sposobnostima
- učenici/studenti mogu steći takvo znanje kroz prezentacije, demonstracije i rasprave

Proceduralno znanje

- primjena znanja u svrhu izvršavanja postupka ili procesa
- znanje o tome kako implementirati postupke učenja (primjerice, strategije)
- zahtijeva od učenika/studenata da poznaju postupak te kada ga primijeniti u određenoj situaciji
- učenici/studenti mogu stei takvo znanje kroz otkrivanje, suradničko učenje i rješavanje problema

Kondicionalno (uvjetno) znanje

- odlučivanje o tome u kojim okolnostima treba primijeniti određeni postupak ili vještinu
- znanje o tome kada i zašto koristiti određeni postupak učenja
- primjena deklarativnog i proceduralnog znanja pod određenim uvjetima
- učenici/studenti mogu steći takvo znanje kroz primjenu (simulaciju)

Regulacija kognicije odgovara znanju o načinu na koji učenici *planiraju*, *implementiraju metode*, *promatraju*, *ispravljaju pogreške* i *procjenjuju svoje učenje*. Jaka povezanost između tih faktora indicira da znanje i regulacija kognicije zajednički pomažu učenicima da se prilagode učenju na njima najbolji način. Upitnik o svijesti o metakogniciji (MAI, Metacognitive Inventory Awareness) dan u nastavku može se koristiti na način da se u razredu započne rasprava postavljajući sljedeća pitanja:

Razmislite o vlastitim metakognitivnim procesima.
Koje metode upotrebljavate za promatranje i procjenu vlastita učenja?
Od deklarativnog, proceduralnog ili kondicionalnog znanja, u kojemu ste vi najbolji, a u kojima najneučinkovitiji?
Od strategija planiranja, upravljanja informacijama, promatranja, otklanjanja pogrešaka i procjene, u kojima ste najbolji, a u kojima baš i niste?

Označi tvrdnju kao točnu ili netočnu.

Tvrdnja	**T**	**N**
1. Povremeno se pitam ostvarujem li svoje ciljeve.		
2. Prije nego što odgovorim na pitanje, sagledam nekoliko mogućih odgovora.		
3. Pokušavam upotrebljavati načine koji su se u prošlosti pokazali uspješnima.		

4. Odabirem tempo učenja da bih imao/la dovoljno vremena za obavljanje svih zadataka.		
5. Znam koje su moje intelektualne jakosti i slabosti.		
6. Razmišljam o tome što zaista trebam naučiti prije nego započnem sa zadatkom.		
7. Znam koliko sam bio/la uspješan/na kad završim s ispitom.		
8. Postavljam si konkretne ciljeve prije nego započnem sa zadatkom.		
9. Usporim kad naiđem na važnu informaciju.		
10. Znam koju je vrstu informacija najvažnije naučiti.		
11. Pitam se jesam li uzeo/la u obzir sve mogućnosti prilikom rješavanja problema.		
12. Dobar/a sam u organiziranju informacija.		
13. Svjesno se usredotočujem na važnu informaciju.		
14. Postoji konkretan razlog za svaku metodu koju koristim.		
15. Najbolje učim kad mi je več nešto poznato o temi.		
16. Znam što profesor/učitelj očekuje od mene da znam.		
17. Dobar/ra sam u pamćenju informacija.		
18. Koristim različite strategije učenja, ovisno o situaciji.		
19. Pitam se postoji li i lakši način izvršenja zadatka nakon što završim sa zadatkom.		
20. Imam kontrolu nad time koliko uspješno učim.		
21. Povremeno ponavljam, što mi pomaže pri povezivanju znanja.		
22. Postavljam si pitanja o gradivu prije nego što započnem.		
23. Sagledam nekoliko načina za rješavanje problema prije nego odaberem najbolji.		
24. Sažmem ono što naučim nakon što završim s učenjem.		
25. Pitam druge za pomoć kad nešto ne razumijem.		
26. Mogu se motivirati za učenje kad je potrebno.		
27. Svjestan/na sam strategija koje upotrebljavam kad učim.		
28. Dok učim, analiziram korisnost strategija koje koristim.		
29. Koristim svoje intelektualne jakosti kao kompenzaciju za svoje slabosti.		
30. Usredotočujem se na značenje i važnost novih informacija.		
31. Stvaram vlastite primjere kako bi mi informacije bile što razumljivije.		
32. Dobro prosuđujem koliko nešto dobro znam.		
33. Primjećujem da automatski koristim korisne strategije učenja.		
34. Primjećujem da redovito zastajem kako bih provjerio/la razumijem li nešto.		
35. Znam kad će svaka strategija koju koristim biti najučinkovitija.		
36. Pitam se koliko sam uspješno ostvario/la svoje ciljeve kad nešto završim.		
37. Crtam slike ili dijagrame koje mi pomažu razumjeti što učim.		
38. Pitam se jesam li sagledao/la sve mogućnosti nakon što sam riješio/la problem.		
39. Pokušavam izreći nove informacije vlastitim riječima.		
40. Mijenjam strategiju kad ne uspijevam razumjeti.		
41. Koristim organizacijsku strukturu teksta kao pomoć pri učenju.		
42. Pažljivo čitam upute prije nego započnem sa zadatkom.		
43. Pitam se je li ono što čitam povezano s onim što već znam.		
44. Nastojim procijeniti svoje pretpostavke kad se zbunim.		
45. Organiziram svoje vrijeme kako bih što uspješnije ostvario/la svoje ciljeve.		

46. Bolje učim kad me tema zanima.		
47. Pokušavam učenje podijeliti na manje korake.		
48. Usredotočujem se na cjelovito značenje, a ne na specifično.		
49. Pitam se kako mi dobro ide dok učim nešto novo.		
50. Pitam se jesam li naučio/la najviše što sam mogao/la nakon što završim s učenjem.		
51. Stanem i ponovno se vratim na informacije koje mi nisu jasne.		
52. Stanem i ponovno pročitam nešto što mi nije jasno.		

Izvor: G. Schraw, R. S. Dennison, Assessing Metacognitive Awareness, Contemporary Educational Psychology, 19, 1994., str. 460–475

Tablica 1. Upitnik o svjesnosti metakognicije (MAI – Metacognitive Awareness Inventory).

Metakognicija i matematika

Matematičar Pólya je 1957., davno prije Flavella (1979.), uveo u područje matematike pojam metakognicije (iako ga nije tako nazvao) kroz poučavanje kako riješiti matematički zadatak (*How to solve it?*). Kroz postupke razumijevanja, planiranja, izvršavanja plana i osvrta na učinjeno, on je zapravo istaknuo važnost metakognitivnog obrazovanja u matematici. Gotovo 30 godina kasnije Schoenfeld (1985.) je videosnimkama pratio kako studenti rješavaju matematičke probleme. On je vježbao studente da nakon izvjesnog razdoblja rješavanja matematičkih zadataka stanu i zapitaju se: *Što sada radim?*, *Zašto to radim?*, *Kako mi to pomaže?* itd. Studenti koji su se tako uvježbavali postizali su bolje rezultate. Na temelju tih studija Mevarech i Kramarski (1997.) izgradili su metodu IMPROVE, u kojoj se koristi niz metakognitivnih pitanja koje studenti/učenici postavljaju sami sebi. IMPROVE je akronim za sljedeće nastavne korake:

Introducing the new concepts
- Meta-cognitive questioning
- Practicing
- Reviewing
- Obtaining mastery
- Verification
- Enrichment and remedial.

Provedene su brojne studije koje su upotrebljavale metodu IMPROVE i rezultati su uvijek bili bolji za one koji su se koristili tim pristupom. U tablici 2 prikazan je rezultat jedne takve studije Mevarecha i Fridkina (2006.), u kojoj su "IMPROVE" učenici bili značajno bolji od onih u kontrolnoj skupini, kako na ispitivanju matematičkog znanja, tako i na ispitivanju matematičkog rezoniranja.

		IMPROVE	Control
Total Score			
Pretest	M	63.55	66.70
	SD	23.14	23.58
Posttest	M	75.08	66.91
	SD	18.74	24.18
Mathematical knowledge			
Pretest	M	63.42	66.61
	SD	23.20	24.19
Posttest	M	74.50	62.62
	SD	19.20	24.72
Mathematical reasoning			
Pretest	M	67.26	67.93
	SD	21.85	25.07
Posttest	M	75.08	66.91
	SD	18.74	24.18

Tablica 2. Rezultati ispitivanja metode IMPROVE.

U svom radu o prirodi veze matematike i metakognicije Lucangeli i Cornoldi (1997.) navode, među ostalim, i rezultate provedene studije na 781 školske djece, 397 trećeg i 394 četvrtog razreda osnovne škole u Italiji. Koristeći razne testove, došli su do rezultata u kojima pokazuju pozitivne rezultate metakognitivnih treninga u aritmetici, geometriji i rješavanju problema (Tablica 2).

	Arithmetic				*Geometry*				*Problem Solving*			
	Prediction	*Planning*	*Monitoring*	*Evaluation*	*Prediction*	*Planning*	*Monitoring*	*Evaluation*	*Prediction*	*Planning*	*Monitoring*	*Evaluation*
Grade 3												
Poor	1.5 (0.99)	1.3 (0.7)	3.7 (2.4)	0.57 (0.77)	1.9 (0.92)	1.4 (0.57)	2.6 (1.4)	0.65 (0.45)	3.0 (1.7)	1.7 (0.65)	3.0 (1.7)	0.74 (0.9)
Good	2.5 (1.2)	1.7 (0.6)	9.0 (2.4)	1.2 (0.77)	2.6 (1.0)	2.7 (0.59)	5.6 (1.1)	1.2 (0.08)	4.6 (1.5)	2.4 (0.7)	6.0 (2.7)	1.1 (0.1)
	**	**	**	**	**	**	**	**	**	**	**	**
Grade 4												
Poor	4 .0 (2.1)	1.8 (0.65)	5.3 (2.0)	6.3 (2.5)	1.5 (0.6)	1. 9 (0.7)	1.4 (1.3)	1.3 (1.0)	1.4 (0.99)	1.3 (0.64)	1.8 (1.4)	1.1 (0.36)
Good	4.7 (2.8)	1.7 (0.65)	5.1 (2.2)	6.8 (2.8)	2.8 (0.9)	2.5 (0.7)	2.6 (1.6)	2.8 (1.3)	2.5 (0.98)	2.4 (0.75)	3.6 (1.7)	2.7 (0.63)
	n.s.	n.s.	n.s.	n.s.	**	**	**	**	**	**	**	**

**$p < .01$.

Note: The scores of the third-graders (top) and fourth-graders (bottom) were compared using the Student's t-test (the asterisks indicate the significant effects obtained with Bonferroni's corrections).

Tablica 3. Metakognicija u aritmetici, geometriji i rješavanju problema.

Razvijanje metakognitivnih vještina i samoregulacije

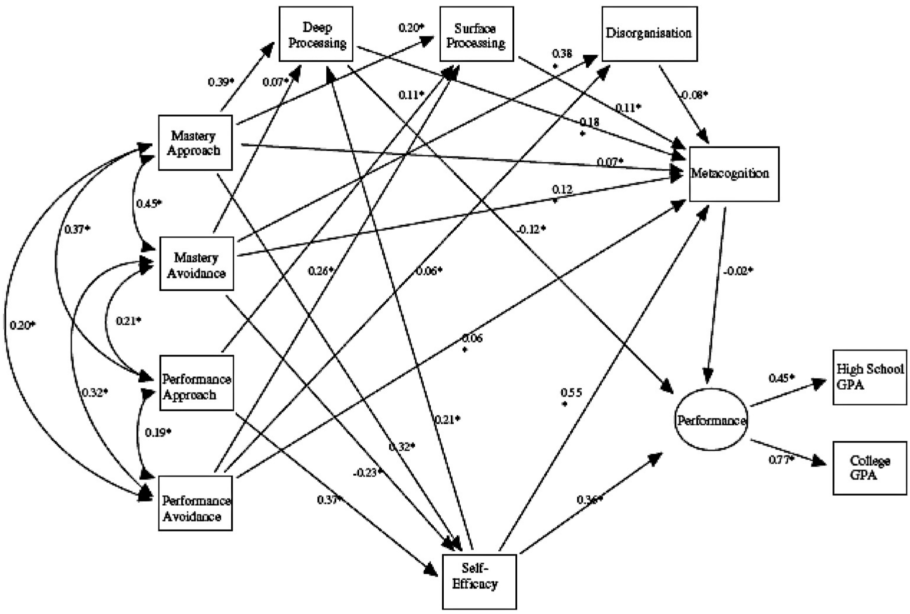
Suvremena samoregulirajuća teorija učenja usredotočava se na prelazak od ovisnog na samostalnog učenika. Samoregulirajući učenici imaju integrirani repertoar kognitivnih, metakognitivnih i motivacijskih vještina. Oni upotrebljavaju te vještine za planiranje, postavljanje ciljeva, implementaciju i promatranje korištenja strategija i evaluaciju njihovih ciljeva. Takvi učenici koriste širok spektar strategija na fleksibilan način, pojačavajući te strategije različitim motivacijskim uvjerenjima, kao što su samoučinkovitost i epistemološki pogledi na svijet. Ovdje je ukratko opisano šest instrukcijskih strategija koje potiču samoregulaciju pomažući učenicima razviti repertoar kognitivnih vještina, metakognitivnu svijest i čvrsta motivacijska uvjerenja. Postoji mnogo načina na koje se kognitivne, metakognitivne i motivacijske vještine mogu poboljšati upotrebom tih strategija. U Tablici 5.1. dan je sažetak osnovnih načina na koji svaka od šest instrukcijskih strategija poboljšava kogniciju, metakogniciju i motivaciju. Samoregulacija je od velike važnosti za učenike. Škole trebaju pripremiti učenike za cjeloživotno učenje u znanosti i u drugim akademskim domenama. U tome im uvelike mogu pomoći instrukcijske strategije koje poboljšavaju učenje i povećavaju uspjehe u matematici i općenito u znanosti.

	Kognitivni procesi	**Metakognitivni procesi**	**Motivacijski procesi**
Istraživanje	Potiče kritičko razmišljanje kroz eksperimentiranje i refleksiju	Poboljšava eksplicitno planiranje, promatranje i evaluaciju	Omogućuje ekspertno modeliranje
Suradnja	Modelira strategije za početnike	Modelira samorefleksiju	Omogućava socijalnu podršku
Strategije	Omogućuju niz strategija	Pomažu učenicima razviti kondicionalno znanje	Povećavaju samoučinkovitost u učenju
Mentalni modeli	Omogućuju analiziranje eksplicitnih modela	Potiču eksplicitnu refleksiju i evaluaciju predloženog modela	Potiču rekonstrukciju i konceptualnu promjenu
Tehnologija	Ilustrira vještine povratnim informacijama, omogućuje modele i simulira podatke	Pomaže učenicima testirati i evaluirati modele	Daju izvore informacija i omogućuju suradničku potporu
Osobna uvjerenja	Povećavaju uključivanje i ustrajnost među učenicima	Potiču konceptualnu promjenu i refleksiju	Potiču modeliranje epistemološke karakteristike ekspertnih znanstvenika

Tablica 4. Načini na koje šest instrukcijskih strategija poboljšavaju kognitivne, metakognitivne i motivacijske procese.

Istraživanja i budući razvoj

U ovom području danas vrše se brojna istraživanja čiji rezultati obećavaju i mogućnost značajnije primjene u praksi. Slika 6.1. prikazuje rezultate svojih istraživanja koji obuhvaćaju veći broj varijabli u području metakognicije i samoregulacije.



Slika 2. Model metakognicije, orijentacije na postizanje ciljeva, stilovi učenja i samo-efikasnost.

Zaključak

Matematika se smatra jednim od najznačajnijih predmeta koji se poučava u školi. Unatoč tome, prisutne su značajne poteškoće u proučavanju matematike (npr. rezultati TIMSS-a, PISA-e). Ova činjenica ukazuje na potrebu izrade efektivnih nastavnih metoda koje imaju potencijal da unaprijede matematičko znanje i mišljenje. NCTM (National Council of Teachers of Mathematics) naglašava da kvalitetna matematička nastava treba proširiti načine izgradnje znanja pomoću rješavanja problema, stvaranjem veza, razvojem matematičke komunikacije i upotrebom različitih vrsta prikazivanja matematičkog znanja. NCTM (2000.) osobito naglašava važnost razvijanja metakognitivnih sposobnosti učenika/studenata kao načina za poboljšavanja njihovih sposobnosti rješavanja matematičkih problema i rezoniranje. Cilj ovog rada jest isticanje važnosti metakognicije i samoregulacije u nastavi i učenju.

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5.
Razmatranja o studiju i nastavi matematike

"Hoćemo li studirati matematiku?" — nakon pedeset godina (o tekstu akademika S. Bilinskog iz 1959.)

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Sažetak. Navodimo, komentiramo i analiziramo članak "Hoćemo li studirati matematiku?" Stanka Bilinskog (1909.-1998.) objavljen 1959. u Matematičko-fizičkom listu, kojim se potiču učenici na upis studija matematike.

Ključne riječi: matematičko obrazovanje, motivacija

Ove, 2009., godine obilježavamo stogodišnjicu rođenja akademika Stanka Bilinskog (Našice, 22.04.1909. – Varaždin, 06.04.1998.). Svojim dugogodišnjim marljivim i plodonosnim djelovanjem višestruko je zadužio našu matematičku znanost i zajednicu. O tome se čitatelji mogu detaljnije upoznati kroz *Spomenicu* br. 88 (46 str.) Hrvatske akademije znanosti i umjetnosti, posvećenu akademiku S. Bilinskom, izdanu 1999. u povodu prve obljetnice njegove smrti. Istaknimo da je akademik Bilinski radio u Geometrijskom zavodu Matematičkog odjela od stvaranja Prirodoslovno-matematičkog fakulteta 1946. pa do umirovljenja 1978.

Profesor Bilinski (takvo oslovljavanje je bilo uobičajeno i bliže njegovim suradnicima i učenicima) jedan je od utemeljitelja Društva matematičara i fizičara Hrvatske. Ono je osnovano 1949. (nakon što je 1945. formirana matematičko-fizička sekcija Hrvatskog prirodoslovnog društva). Razdvajanjem 1990. nastaju današnja društva: Hrvatsko matematičko društvo i Hrvatsko fizikalno društvo.

Profesor Bilinski je dvije godine bio predsjednik Društva matematičara i fizičara (22.01.1959.-25.1.1961.) i iz toga doba su sačuvani njegovi predsjednički govori koje je održao na godišnjim skupštinama Društva (*Ekonomsko i kulturno značenje matematike* (1960.), *Utjecaj otkrića neeuklidske geometrije na savremeni razvoj nauke* (1961.)). Oba su govora i danas zanimljiva, izražavajući poglede i stavove profesora Bilinskog o matematici, njezinoj ulozi i značenju te položaju u društvu.

Pored ova dva teksta postoji još jedan (a drugih sličnih pisanih tragova nemamo) iz kojeg možemo djelomice isčitati mišljenje profesora Bilinskog vezano uz matematiku, točnije matematičko obrazovanje. ćlanak je pisan sa svrhom poticanja upisa na studij matematike i nastao je nakon što je profesor Bilinski obnašao dužnost dekana fakulteta (1956./57.). Objavljen je prije pedeset godina u časopisu za učenike srednjih škola *Matematičko-fizički list* (kraće MFL), vol. 10 (1959./60.), str. 115–117.

Stoga, možemo reći da je tekst bio namijenjen čitateljstvu sklonom matematici. Istodobno, ne zaboravimo da je u to vrijeme MFL imao visoku tiražu, pa je možda broj onih koji su iskreno bili skloni matematici bio ipak manji. Naslov članka je *Hoćemo li studirati matematiku?*, a autor se navodi kao prof. dr. STANKO BILINSKI, Zagreb.

Mišljenja smo kako je sadržaj članka i danas zanimljiv, informativan i poticajan. Naime, oslikava nam autora i vrijeme kada je pisan. Istodobno nudi mogućnost usporedbe današnjih upisa na studij matematike s vremenom prije pedeset godina. Stoga članak profesora Bilinskog u daljnjem tekstu navodimo u cjelini (u kurzivu), bez lektorskih i stilskih intervencija, prekidajući ga popratnim komentarima.

Hoćemo li studirati matematiku?

Prije dvije, tri godine u nekom gradu (nije važno kojem) u nekoj srednjoj školi (nije važno kojoj) kolektiv učenika jednog višeg razreda napisao je pismo sekretaru Savjeta za prosvjetu, u kojem traži, da se matematika kao nastavni predmet u školi ukine, jer "ona i onako nije ni za što potrebna".

Da li je doista tako?, zar zbilja matematika nije ni za što potrebna? Da li je mišljenje tog đačkog kolektiva bilo ispravno, odnosno, što je dovelo te đake do ovog zaključka? Možda je taj kolektiv bio sastavljen od iznimno loših đaka, koje je potsvjesna želja, da se riješe jednog teškog predmeta, dovela do ovakvog zaključka. No možda krivnja, da su đaci mogli ovako zaključiti, leži i na nastavniku tog predmeta.

Sigurno od vas nitko ne misli kao đaci ove škole, no ipak da vidimo, kako tu stvari uistinu stoje.

Naravno, već sam provokativni naslov članka krije informaciju o autorovom mišljenju kako je potrebno obrazlagati smislenost i opravdanost studiranja matematike, ali i donekle objasniti što se na tom studiju može očekivati.

Tekst započinje iznošenjem "vječne" slike o matematici kao školskom predmetu kojega bi se svi rado oslobodili, drastičnim naglašavanjem kako bi matematiku neki najradije ukinuli, pa makar to morali tražiti od vlasti. Učeničko obrazloženje je nategnuto: matematika nije potrebna, dakle, nije ju korisno učiti. Autor dopušta da bi pravi uzrok takvoj želji mogla biti težina samog predmeta, znači, i sam priznaje da matematika spada u tzv. teške predmete, ali odmah naglašava i važnost uloge nastavnika koji bitno utječu na izgradnju učeničke predodžbe o matematici.

S jednim i drugim ćemo se i danas složiti: matematika spada u teže školske predmete, a kako će je učenici doživjeti i iskusiti bitno ovisi o nastavniku.

Za vrijeme cvata antičke kulture na prijelazu iz petog u četvrto stoljeće prije naše ere, u staroj je Ateni živio Platon, najveći filozof svog vremena. On je naučavao u školi, koja se zvala Akademija, na uspomenu grčkog narodnog heroja Akadema, a ta je škola bila smještena u nekom javnom parku, nasadima platana i maslina. Pripovijeda se, da je Platon nad ulaz u tu školu stavio ovaj natpis: Μηδεις αγεωμετρητος εισιτω *što bismo mogli ovako slobodno prevesti: "Neka nitko ovamo ne ulazi, tko ne zna matematiku". Napominjem da Platon sam nije bio matematičar; on nije matematiku niti stvarao niti naučavao. No on je ipak dobro uočio opće kulturni značaj matematike i uvidio, da je vještina u matematičkom zaključivanju nužni preduvjet za spoznaju i dublje razumijevanje i općih filozofskih misli.*

I danas na ulazima na naše fakultete i visoke škole stoje natpisi sličnih sadržaja. Vi ih doduše niste vidjeli nad ulaznim vratima tih škola, jer stvarno oni tamo nisu ni napisani. Ali pogledajte programe propisane za prijamne ispite za te škole. Većina fakulteta, pa i takvih, koji inače nemaju matematiku kao predmet u nastavnim planovima, predviđa nju u programima prijamnih ispita. Očito je dakle, da se danas općenito smatra da je znanje matematike nužni preduvjet i osnova za studij mnogih nauka.

Stavljajući na stranu pitanje kako je to prije pedeset godina zvučalo kad je netko spominjao narodne heroje iz doba prije naše ere, obratimo pozornost na najvažnije. To je misao o značaju matematike. I tu profesor Bilinski spominje ono što je i drugdje napisao. Značaj matematike je opće kulturni, ona nije važna samo sebe radi, ona nije važna samo zbog svoje široke primjenjivosti, ona je važna za razumijevanje svijeta i čovjeka. A to je znao već i nematematičar Platon.

Usputno, uočivši da se rabio termin prijamni ispit, kao i danas, a ne prijemni, kao u međuvremenu, važnije je vidjeti da se i u to doba za upis na fakultete moralo prolaziti dodatne provjere (koje su u nekom periodu zvali klasifikacijski ispiti). Međutim, bilo je i godina kada nije bilo prijemnih ispita.

Danas sigurno nitko ne sumnja, da je matematika neophodno potrebna za razumijevanje fizike, kemije i svih tehničkih nauka. No ne vrijedi to samo za ove nauke. Mnoge moderne metode i u takvim naukama, kao što je to biologija, medicina, politička ekonomija i t. d., danas se zasnivaju na matematičkim disciplinama kao na pr. na matematičkoj statistici, diferencijalnim jednadžbama i slično. Prelistavajući naučne časopise ovih nauka od prije tridesetak godina jedva bismo mogli naći u njima koju matematičku formulu. Danas međutim ti časopisi, barem tamo, gdje naučni rad u tim naukama stoji na visokom nivou, obiluju matematičkim formulama i primjenom matematičkih teorija na rješavanje specijalnih problema iz područja tih nauka. Nije zato čudo, da je neki učenjak (imena mu se ne sjećam), koji po struci nije matematičar, rekao, da neka nauka toliko vrijedi, koliko je u njoj matematike. Ta je izjava o značenju matematike možda malo pretjerana, no ipak, ona nije jako daleko od istine.

Prije nekoliko mjeseci, kada su u Sovjetskom savezu uspjeli izbaciti raketu, koja je pala na Mjesec, a čak i takvu, koja ga je obišla i tom prilikom fotografirala njegovu stražnju stranu, mogli ste u našoj dnevnoj štampi čitati intervju jednog novinara s nekim engleskim stručnjakom za raketu. Bilo mu je postavljeno pitanje, *kako to, da su Rusi toliko pretekli Amerikance u izbacivanju umjetnih satelita. Taj je engleski stručnjak izjavio, da glavni razlog tog uspjeha ne leži u novom pogonskom gorivu, kako se to općenito misli. Njegovo je mišljenje ovo: Tehnički problem teledirigiranih projektila i umjetnih satelita u krajnjoj liniji vodi na izvjesne sisteme diferencijalnih jednadžbi. Ekipe matematičara, koje surađuju kod tih tehničkih problema na jednoj i na drugoj strani, nastoje što boljim metodama rješavati ove sisteme. Tajna sovjetskih uspjeha bila bi u tome, što oni imaju bolje ekipe matematičara, kojima je pošlo za rukom pronaći uspješnije metode za rješavanje ovih sistema diferencijalnih jednadžbi.*

Već iz ovih nekoliko nasumce nabačenih činjenica muže se zaključiti kakvu važnost ima matematika u suvremenom životu. Velike, opsežne i mnogostruke su njezine primjene. No najveće njezino značenje je u njezinoj unutarnjoj kulturnoj vrijednosti, a o ovoj vrijednosti nije lako steći jasniju predodžbu samo na osnovu poznavanja srednjoškolske matematike.

Dakako, ovdje već izbija matematička pristranost kad se kaže da nije daleko od istine izjava kako vrijednost neke znanosti pokazuje "količina" uključene matematike. Međutim, ta se tvrdnja potkrepljuje poznatim (i danas prihvaćenim) mišljenjem kako je baš bolje matematičko obrazovanje (pa posljedično i bolji matematičari) u Sovjetskom savezu donijelo primat pred Sjedinjenim državama u svemirskim letovima. Takve spoznaje potakle su velike školske reforme u Americi. No, odmah slijedi misao kako nije sve u primjeni, najveće je značenje u kulturnoj vrijednosti. Mislimo da su dva razloga tom ponavljanju: jedan jest da profesor Bilinski tako iskreno misli (pa sukladno djeluje), a drugi da je doba nastanka teksta još uvijek ono vrijeme kada se od znanosti poput matematike u prvom redu tražila brza, neposredna i evidentna primjenjivost. To nije obilježje samo toga doba, pa ni samo ovog podneblja.

Važno je podvući i onu zadnju rečenicu prethodnog citata. I danas matematičari u privlačenju mladih za matematiku ili obrazlaganju što jest matematika odraslim nematematičarima, moraju naglašavati kako dvanaest godina učenja školske matematike (do upisa na fakultet) ipak ne može dati jasnu predodžbu o (pravoj) matematici i njezinom pravom značenju.

Što se pak tiče tvrdnje o znatnoj promjeni u primjenjivosti matematike u odnosu na vrijeme od prije trideset godina (dakle, prije osamdeset godina u odnosu na danas), to je dakako stvar interpretacije. Relativno gledajući, primjenjivost matematike nije toliko značajno porasla. Riječ je o tome da su se u međuvremenu pojavile nove znanosti, a stere su se promjenile i razvile, pa se matematika u jednima i drugima pojavljuje/primjenjuje u novim područjima. Zato autor mudro spominje matematičke formule, pa njegova tvrdnja postaje "točnom".

Napretkom civilizacije uloga matematike naglo raste. Sve se više primjenjuju matematičke metode u raznim područjima ljudske djelatnosti. Pa ipak nastavni programi matematike u srednjoj školi, za razliku od programa iz drugih prirodnih nauka, mijenjaju se sporo i neznatno. Ono što se danas u srednjoj školi iz matematike uči, najvećim je dijelom bilo ljudima poznato još prije više stoljeća. To bi nekog moglo dovesti do pogrešnog zaključka, da je danas matematika mrtva nauka, *tj., da je na njenom području otkriveno već sve, što se otkriti može. No ovakav bi zaključak bio vrlo daleko od istine. Radi vrlo ograničenog broja sati matematike u nastavnim planovima srednjih škola nastavni programi matematike mogu obuhvatiti samo one najjednostavnije i najvažnije osnovne spoznaje matematičkih nauka, koje su neophodne za njene najobičnije primjene, a i za njezinu daljnju izgradnju. Ovi najosnovniji dijelovi matematike dakako da se i stoljećima gotovo i ne mijenjaju. Njezini viši dijelovi ne mogu ući u programe srednjoškolske matematike, jer radi složenosti svojih metoda ne bi bili općenito učenicima pristupačni. No i danas se u matematici pronalaze nove metode i otkrivaju bitno novi putovi, a poznate matematičke metode usavršavaju se i nadopunjuju. Da dobijemo približnu sliku o naučnoj produktivnosti u području matematičkih nauka, dovoljno je spomenuti da danas postoji više od šest stotina naučnih časopisa i publikacija iz područja tih nauka, da u njima izlazi godišnje oko osam hiljada originalnih naučnih članaka, i da taj broj iz godine u godinu raste.*

Važno je ovdje primijetiti stalnu "muku" matematičara: broj sati matematike je ograničen; zapravo se želi reći da je broj sati premalen, nedovoljan. A izbija (opravdana) bojazan kako drugi (dakako, neznalice) misle da je sve završeno/gotovo u matematici.

Usputno, uočimo da se riječ civilizacija rabi u jednini. I da se koristi množina govoreći o matematičkim znanostima.

Pritom postoji jedna bitna razlika izmedu naučnog rada u matematici i u ostalim naukama. Dok se u ostalim naukama vrlo često u isti mah gradi i razgrađuje, jer nove teorije nadomještaju one starije, koje se zabacuju, pa postaju suvišne, ili se znatno izmjenjuju, dotle se u matematici od samog njenog početka samo gradi. Što je jednom u njoj dokazano kao ispravno, ostaje tako zauvijek. Zato je današnja zgrada matematike golema. Još prije nekih sto i pedeset godina bilo je moguće, da jedan jedini čovjek upozna sva dotadanja otkrića u području matematike. No danas i najveći matematičari za cijelog svog života mogu upoznati samo jedan posve maleni njezin dio. Možda je zato i lakše razumjeti činjenicu, da se definicije same matematike, što su ih dali razni matematičari, toliko međusobno razlikuju, da nijedna nije općenito usvojena. No postoji veliki broj izjava o matematici kao nauci, koje nju manje ili više točno karakteriziraju. Tako je matematičar Ulam (koji se istakao i vrlo uspješnom matematičkom obradom teorije atomske bombe, na osnovu koje je ova tada bila i konstruirana) jednom prilikom, napola u šali, rekao, da je matematika metoda "činiti najbolje". Ako bi se naime jednoj skupini ljudi – a među njima i matematičaru – odredilo, da obave neki posao, a nitko od njih prije toga nešta sličnog nije radio, matematičar bi to učinio bolje od ostalih. To dakako treba shvatiti onako, kako je i rečeno, t. j. ne doslovice.

U početnom dijelu ovog odjeljka izbija činjenica da je proteklo pedeset godina od objave članka. Iznosi se donekle i danas rašireno, ali ipak pretjerano pojednostavljeno mišljenje o razlici u rastu i razvoju/izgradnji između matematike i drugih znanosti. Posljedično, kao da razvoj matematike ne bi imao obilježje koje sve druge znanosti imaju: mogućnost zastranjenja, zablude, krivih predodžbi.

Zanimljivo je kako se profesor Bilinski od (tobože) mnogih i raznih "definicija" matematike (naglašavajući kako opće prihvaćene definicije nema) odlučio za onu S. Ulama. Sigurno da nije bilo presudno, ali ipak navedimo, da je riječ o matematičaru istog imena, rođenom iste godine i istog mjeseca i u istoj državi kao profesor Bilinski (Stanislaw Ulam (Lavov, 13.04.1909 – Santa Fe, 13.05.1984.). Točnosti radi, dometnimo, spomenuta bomba je bila hidrogenska bomba. U svakom slučaju, odabranu "definiciju" je matematičaru ugodno čuti, istodobno provocirajući nematematičare. I kad je S. Ulam spomenut, budi nam slobodno navesti jedan drugi citat iz njegove knjige *Adventures of a Mathematician* (Pustolovine matematičara) (1976.): "In many cases, mathematics is an escape from reality. The mathematician finds his own monastic niche and happiness in pursuits that are disconnected from external affairs. Some practice it as if using a drug. Chess sometimes plays a similar role. In their unhapipiness over the events of this world, some immerse themselves in a kind of self-sufficiency in mathematics (često je matematika bijeg od stvarnosti. U traganju, daleko od vanjskoga svijeta, matematičar nalazi svoje osamljeno mjesto i osobnu sreću. Matematiku koristi poput lijeka (a ponekad i šah ima istu svrhu). Nezadovoljan i nesretan događanjima oko sebe, uranja u samodostatnosti svijet matematike)". A što se tiče definicije matematike S. Ulam je rekao i ovo: "What is really mathematics? Many have tried but nobody has really succeded in defining mathematics; it is always something else (Što je to zaista matematika? Mnogi su pokušali, ali nitko nije stvarno uspio definirati matematiku; ona je uvijek još nešto drugo)".

Radi važnosti, koju matematika ima za razvoj i napredak ljudskog društva, danas postoji i velika potreba za matematičarima, a ta će potreba s vremenom još i više rasti. Velika je potražnja matematičara na srednjim i stručnim školama, gdje danas matematiku u velikom broju predaju nekvalificirani stručnjaci. Matematičari su danas u znatnom broju potrebni u statističkim i osiguravajućim zavodima. Daljnjim razvojem i unapređivanjem industrijske proizvodnje kod nas, ukazat će se potreba, da se matematičari namještaju u velikim privrednim i industrijskim poduzećima i ustanovama, kako je to danas već uobičajeno u tehnički razvijenim zemljama. Oni najdarovitiji od diplomiranih studenata matematike, koji se budu posvetili i naučnom radu u području matematičkih nauka, danas će lako naći namještenje u naučnim institutima i zavodima na univerzitetima, visokim školama i akademijama nauka. Potražnja za odličnim stručnjacima u tim ustanovama u posljednje vrijeme je vrlo velika.

I danas tvrdimo da se matematičari lako zapošljavaju. Do jučer je to bilo točno.

Pored sveg ovog nedostatka u kadrovima matematičara, danas se javlja relativno maleni broj kandidata na natječaje za studij matematike na Prirodoslovno-matematičkim fakultetima. Istina je doduše, da taj studij nije lagan i da će matematiku s uspjehom moći studirati samo oni studenti; koji su već u srednjoj školi pokazali znatniji uspjeh u matematici, a nipošto takvi, koji su u tom predmetu jedva nekako izvlačili pozitivne ocjene. No u našim srednjim školama ipak ima dosta darovitih učenika, pa bi broj onih, koji se odlučuju za studij matematike, mogao biti i mnogo veći. Jedan od uzroka, što nije tako, leži i u nepoznavanju gore navedenih činjenica, pa je zato ovaj članačić i napisan. Kad budete dakle za koji mjesec ili *za koju godinu donosili odluku o izboru vašeg studija*[*]*, promislite malo i o tome, što je ovdje bilo rečeno.*

Ovaj zadnji odlomak i pripadna bilješka donose nekoliko zanimljivih informacija današnjem čitatelju o situaciji prije pedeset godina, što nameću usporedbe s današnjim stanjem: interes za studij matematike je relativno slab, a sam studij je relativno težak. Nastavni planovi se mijenjaju, struka će reći da se reduciraju, da se studij olakšava. A i režim studija se mijenja (spomenuto pooštrenje se odnosi na stroži upis više studijske godine s obzirom na nepoložene ispite iz prethodnih). A najnovija knjižica s nastavnim planovima još nije dostupna.

Ukratko, kao da nije prošlo pedeset godina od objave članka profesora Bilinskog.

A što se tiče pitanja *Hoćemo li studirati matematiku?* odgovor je dakako pozitivan. U takvu nas uvjerenju ostavlja i činjenica da je 1958. na zagrebačkom Prirodoslovno-matematičkom fakultetu diplomiralo matematiku 30 studenata, a broj diplomiranih matematičara na PMF – Matematičkom odjelu je 2008. bio 175, gotovo šest puta veći.

Danas nam ostaje pitanje *Tko će studirati matematiku?*.

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[*] Tko želi pobliže upoznati tijek studija na Prirodoslovno-matematičkom fakultetu u Zagrebu, može u Dekanatu tog fakulteta nabaviti knjižicu "Prirodoslovno-matematički fakultet — Nastavni programi, Zagreb 1957.". Osim opisa samog toga studija tu će čitalac naći Nastavne planove svih struka i programa svih kolegija, koji se slušaju na tom fakultetu. Doduše, od izlaska ove knjižice iz štampe studij na fakultetu nešto je izmijenjen. Pritom su u svrhu bržeg završavanja studija nastavni planovi pojedinih struka nešto reducirani, a režim studija na fakultetu nešto je pooštren. Ipak ta knjižica još i danas može poslužiti da dade dosta dobru sliku o tijeku studija na spomenutom fakultetu.

Mišljenje nastavnika matematike o nastavi matematike u osnovnim i srednjim školama u hrvatskoj — rezultati empirijskog istraživanja

Branislava Baranović i Marina Štibrić

Institut za društvena istraživanja, Zagrebu, Hrvatska

Sažetak. U prezentaciji su predstavljeni rezultati empirijskog istraživanja o mišljenjima nastavnika matematike o nastavi matematike u osnovnim i srednjim školama u Hrvatskoj. Istraživanje je provedeno na prigodnom uzorku od 292 sudionika Trećeg kongresa nastavnika matematike održanom u Zagrebu 2008. godine. Primijenjen je upitnik strukturiran od pitanja pretežno zatvorenog tipa u kojima su procjene nastavnika mjerene na Likertovim skalama od četiri stupnja te manjeg broja otvorenih pitanja.

Rezultati istraživanja pokazuju da u nastavi matematike u osnovnim i srednjim školama u Hrvatskoj dominiraju tradicionalan pristup i tradicionalne metode i postupci rada. Tako su napr. nastavnici u definiranju matematičke kompetencije najveći naglasak stavili na rješavanje matematičkih zadataka, usvajanje temeljnih znanja iz matematike i primjenu naučenog. Procijenili su da se u nastavi matematike najveći naglasak stavlja na rješavanje matematičkih zadataka i razvoj logičkog načina razmišljanja, a manje razvijaju upotrebu matematike u svakodnevnom životu i razvoj kritičkog promišljanja o matematičkim konceptima i postupcima. Nastavnici smatraju da kod učenika najviše razvijaju vještinu postavljanja pitanja i traženja informacija, razumijevanje koncepata i interes učenika za matematiku, a najmanje potiču razvoj vještine pisanog komuniciranja ideja i rezultata vlastitog rada te vještine učenja matematike. Ako izuzmemo domaće zadaće koje se redovito zadaju i pregledavaju, najčešće korištene nastavne metode i postupci su metoda dijaloga (94,8%), demonstracija postupaka rješavanja zadataka (93,6%) i predavačka metoda pri uvođenju novih matematičkih koncepata (86,2%). Prema procjeni samih nastavnika, za vrijeme nastave matematike učenici najčešće slušaju i prepisuju s ploče (87,8%), 'na svome mjestu' samostalno rješavaju zadatke (87,8%) i komentiraju prethodnu ili najavljenu provjeru znanja (87,8%). Aktivnosti koje zahtijevaju samostalan rad učenika kao što su zapisivanje vlastitih ideja o matematičkim konceptima (4,1%), samostalno istraživanje i eksperimentiranje (17,0%), te samostalno obrađivanje gradiva iz udžbenika (18,2%) najrjeđe se

koriste u nastavi matematike. Nastavnici navode da, u prosjeku, samo 34,8% nastave provode poučavajući matematičke koncepate, a najveći dio vremena odvajaju za rješavanje matematičkih zadataka.

Iz podataka je vidljivo da se u nastavi matematike dovoljno ne razvija kritičko mišljenje o matematičkim konceptima, samostalno promišljanje i zaključivanje o matematičkim konceptima i postupcima, te da se rijetko primjenjuje metoda otkrivanja i problemska nastava. Međutim, navedene vještine imaju veliku važnost za razvijanje matematičke kompetencije učenika.

Ključne riječi: nastava matematike, nastavnici matematike, matematička kompetencija, nastavne metode i postupci, učeničke aktivnosti na nastavi

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