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Congruence Function for Measuring the Extent of Coverage of the Enacted Curriculum and District Pacing Calendars to State Assessments

John R. Walkup*
Ben S. Jones*
John Pilipenko*
David P. Davenport*

* The Standards Company LLC, 575 W. Alluvial Ave., Clovis, CA 93611

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This article presents a mathematical function for measuring the congruence among the strand representation in the enacted curriculum, state assessments, and district pacing guides. Using the Manhattan distance as its basis, this method allows researchers to incorporate the strand coverage of the enacted curriculum (i.e., the curriculum actually experienced by students) into alignment studies even when the analysis of student work is limited to short-term collections. The result is a mathematical instrument involving just basic arithmetic that even those without specialized knowledge of mathematics can understand and apply. Using this instrument, educators should monitor the adherence of the enacted curriculum and district pacing guides to the topics encompassed on state and district assessments. (Contains 4 figures, 2 tables, and 13 references.)

Keywords: Manhattan distance, congruence, enacted curriculum, pacing calendar, pacing guide, Walberg productivity model, breadth of coverage, curriculum alignment, standards alignment

1. INTRODUCTION

The content taught to students on a daily basis (the enacted curriculum) significantly influences how much they learn [Gamoran *et al*. 1997; Wiley *et al*. 1995]. Even in cases where a district adopts a fully standards-based curriculum, students can struggle on state assessments if the enacted curriculum diverges significantly from the adopted curriculum [Firestone *et al*. 2000; Bernauer 1999; Desmond *et al*. 1998].

Using a combination of surveys and collected student assignments, Firestone and colleagues [2000] appear to have performed the first large-scale effort to analyze the enacted curriculum. Other efforts relied on teacher logs, a form of self-reporting survey [Rowan *et al*. 2002; Porter 1997; Porter 1989]. Although more logistically taxing than surveys, private companies have collected and directly examined completed student assignments on a large-scale basis [The Standards Company LLC 2008].

2. NOMENCLATURE

State content standards typically categorize among three or four levels. A typical hierarchy comprises (from the most general to the most specific):

1. Subject (e.g., ″English language arts”)
2. Strand (e.g., “grammar and punctuation”)
3. Standard (e.g., “Students will use commas properly.”)

Unfortunately, this nomenclature varies from state to state, even to the point that a “strand” in one state corresponds to a “standard” in another state. This article adheres to the categorization scheme listed above, focusing mostly on level 2 (the strand).

3. EXTENT OF COVERAGE

Even when the enacted curriculum perfectly aligns to standards, students can still perform poorly on assessments if the curricular content they receive in class fails to adequately cover the range of strands they encounter on the state assessment. For example, students taught content predominantly from grammar/punctuation strands will likely struggle on sections of the state assessment that address reading comprehension of informational text. Similarly, students who overly concentrate on basic number facts at the expense of algebra will likely struggle on the state mathematics assessment. Therefore, educational agencies seeking to raise state test scores must ensure that the enacted curriculum adequately samples every tested strand before state testing begins.

To strengthen adherence between the content taught by teachers on a daily basis and state and district guidelines, district staff often create pacing calendars (sometimes called pacing guides) that provide a timeline for teaching particular objectives. However, district pacing calendars and state assessments do not always match tightly, nor do teachers always follow the pacing calendars.

4. APPLICATION

As a worthwhile study, researchers and school administrators can compare the strand sampling in the enacted curriculum with that found in state assessments to uncover underemphasis and overemphasis of particular strands of the standards. Fortunately, many states release blueprints describing the percentage they weight each strand on their state assessments. In most cases the state releases the blueprints in advance of the assessments to help guide lesson planning.

Accurate analysis of the enacted curriculum, however, requires the collection of completed student assignments. Typically, such short-term studies span a few days or weeks. Unlike alignment and rigor studies, however, researchers would waste time and effort comparing the extent-of-coverage of a short-term student assignment collection to that found on the state assessment—teachers often, and some may argue *should*, focus on particular strands for any given short period of time.

Lengthening the collection period would improve this situation, but long-term collections can impose tremendous logistical burdens on district staff. The approach used in this article offers a remedy—a means of gauging the congruence between the state assessments and enacted curriculum with a short-term assignment collection.

The method presented in this article involves two distinct phases:

1. A broad-scope comparison between the extent of coverage of the state assessments and the district pacing calendar.
2. A narrow-scope comparison between the enacted curriculum and the portion of the pacing calendar that spans the collection period.

The next section discusses a mathematical means of gauging the strand congruence for both steps and a way to summarize the results mathematically into an overall congruence function. We must first, however, discuss the mathematics that underlie this approach to measuring congruence.

5. MANHATTAN DISTANCE

The Manhattan distance (sometimes called the taxicab distance) obtains its name from the shortest distance a vehicle can travel between two points in a city with streets arranged in a rectangular grid [Black 2006]. Some readers with military experience will remember the act of "squaring corners," which is also directly analogous to traveling along Manhattan paths. Others will liken the paths to the possible moves of a rook on a chessboard.

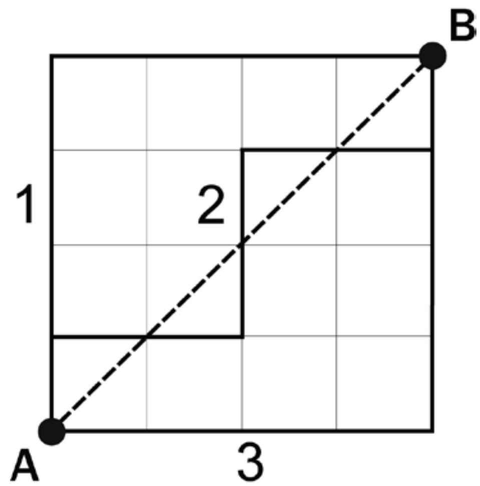

Fig. 1. All three paths from Point A to Point B in this figure feature the same travel distance, termed the Manhattan distance. The Euclidean distance ("as the crow flies") corresponds to the dashed line.

Multiple paths will feature the same Manhattan distance S between two points. Figure 1 identifies three paths between two points (more exist) that feature the same Manhattan distance. The mathematical expression for S for each path all simplify to the same expression

$$S = d_1 + d_2 + \cdots + d_N = \left(\left|A_1 - B_1\right| + \left|A_2 - B_2\right| + \cdots + \left|A_N - B_N\right|\right), \tag{1}$$

where N represents the dimensionality of the system ($N = 2$ in Figure 1). This expression translates to "S is the sum of individual perpendicular-path lengths needed to move from Point A to Point B in N dimensions."

From here on we will scale the lengths of each side of the grid to unity, producing a maximum Manhattan distance of 2. (This holds even in higher dimensions, since the distance needed to travel to the origin is never larger than 1 and the distance needed to travel from there to any other point is also never larger than 1.) As a result, our approach relies on the scaled Manhattan distance $s = (1/2)S$ to measure congruence.

6. BROAD-SCOPE INDEX

This article defines the broad-scope index, C_{PT}, as the congruence between the state assessment and the district pacing calendar. First, let $T = (T_1, T_2, T_3, \ldots T_N)$ denote the percentage of questions on the state assessment that correspond to each of the N strands of the state content standards. For example, $T = (0.45, 0.50, 0.05)$ indicates that 45% of the state assessment corresponded to the first strand, 50% to the second strand, and 5% to the third strand. Likewise, $P = (P_1, P_2, P_3, \ldots P_N)$

represents the percentage of questions found on that portion of the pacing calendar that extends from the start of the school year to the state testing date. Using the scaled Manhattan distance s_{PT} between Sets T and P, the congruence index

$$C_{PT} = 1 - s_{PT} = 1 - \frac{1}{2}\left(\left|P_1 - T_1\right| + \left|P_2 - T_2\right| + \left|P_3 - T_3\right| + \cdots + \left|P_N - T_N\right|\right) \tag{2}$$

measures the congruence between this "tested-year" pacing calendar and the state assessment. Perfect congruence correlates to $CPT = 1$. At the other extreme, $CPT = 0$ represents complete misalignment.

The broad-scope index performs a meaningful function on its own—low values of the broad-scope index indicate that the pacing calendar may need restructuring.

7. NARROW-SCOPE INDEX

Consider a typical week-long collection of student assignments. In a vein similar to the previous section, $p = (p_1, p_2, p_3, \ldots p_N)$ represents the percentage of questions that align to N strands of the state content standards found in the pacing calendar, but only for that portion of the pacing calendar spanning the collection period. Analogously, $e = (e_1, e_2, e_3, \ldots e_N)$ tabulates the percentage of questions in the enacted curriculum (that is, the assignments collected for analysis) that align to the same N strands. Therefore,

$$C_{ep} = 1 - s_{ep} = 1 - \frac{1}{2}\left(\left|e_1 - p_1\right| + \left|e_2 - p_2\right| + \left|e_3 - p_3\right| + \cdots \left|e_N - p_N\right|\right) \tag{3}$$

This narrow-scope index measures the extent to which the teaching staff adhered faithfully to the pacing calendar during the collection week.

Like the broad-scope index, the narrow-scope index serves a useful purpose in its own right—low values indicate that teachers may need to reassess the content they teach in light of the content suggested by the pacing calendar.

8. CONGRUENCE FUNCTION

We define the overall congruence function that measures the match between the enacted curriculum and the state content standards as the product of the broad-scope and narrow-scope indices,

$$C = BC_{PT}^{j} C_{ep}^{k}, \tag{4}$$

a variation of the familiar Walberg function used to model educational productivity [Walberg 1980]. The coefficient B normalizes the index, whereas the exponents j and k weight each contribution to the overall index. Although researchers could establish values for j and k using multiple regression, there is little reason to think that one index should dominate over the other, so this article assumes $j = k = 1$ for the sake of simplicity. Since both indices range from 0 to 1, then $B = 1$ as well. Therefore,

$$C = C_{PT} C_{ep} \ . \tag{5}$$

As a product of two terms, schools can perform well with respect to one index but still register a low congruence for the following reasons:

1. The pacing calendar could align closely to the state assessment, but the enacted curriculum does not align to the pacing calendar. In this case, teachers delivered the "wrong" content.
2. The enacted curriculum could align closely to the adopted pacing calendar, but the pacing calendar does not align to the state assessment. In this case, the pacing calendar compelled teachers to deliver the "wrong" content.

Both results would expose a severe misalignment between the content that teachers delivered to their students and the questions asked on state assessments. If so, low values of the congruence index make sense. One can strengthen this justification for using the product term using an analogy to probability theory: If students have (say) learned only half of what they need in preparation for a district test because of a misalignment of the enacted curriculum to the district pacing calendar, but the pacing calendar itself reflects only half of the strands coverage on the state assessment, then the product (50%)(50%) = 25% reasonably describes their chances of adequately responding to a problem on the state assessment.

The congruence function cannot model the situation in which the enacted curriculum aligns closely to state assessments, but the pacing calendar matches poorly to both the state assessments and the enacted curriculum. In such a situation, teachers apparently used the state content standards as their framework for teaching rather than the pacing calendar. (If so, the teaching staff could judge the pacing calendar worthless with much justification.) However, if one could accurately measure the congruence between the enacted curriculum and the state assessment (which again would require a long-term collection of student work), then the direct calculation

$$C = 1 - s_{eT} = 1 - \frac{1}{2}\left(\left|e_1 - T_1\right| + \left|e_2 - T_2\right| + \cdots + \left|e_N - T_N\right|\right) \tag{6}$$

would replace the expressions in Equations 1 and 2.

Note that forcing the exponents in Equation (4) to sum to 1 would allow the congruence function to follow the Law of Diminishing Returns [Cobb *et al*. 1928]—for a fixed value of one of the indices, the gains of C would diminish as the other index increases. Although this functionality seems reasonable from a dimensional analysis standpoint, the authors of this article have already argued that the probability model introduced earlier describes the effect of subsequent misalignment realistically, so the approach advocated in this article declines to force the sum of j and k to equal 1.

9. ALTERNATIVE FORMULATIONS

One might consider using the average discrepancy

$$S_{ave} = \frac{1}{N}\left(\left|e_1 - p_1\right| + \left|e_2 - p_2\right| + \cdots + \left|e_N - p_N\right|\right) \tag{7}$$

rather than the Manhattan distance. However, the average discrepancy scales incorrectly and so produces considerable error, especially in the limit of complete misalignment. Consider a state that tests four strands of the standards: *Number sense*, *algebra*, *geometry*, and *measurement*. Suppose the pacing calendar at (fictitious) Gingko Middle School specifies that teachers should address only *algebra* during a certain week; therefore, $p = (0, 1, 0, 0)$. However, work collected from the school during this time features only *geometry*; so $e = (0, 0, 1, 0)$. In principle, this situation constitutes a complete misalignment, but the average discrepancy $C_{ep} = 0.50$, not 0.

The Euclidean "as the crow flies" distance (dashed line in Figure 1)

$$S_E = \sqrt{\frac{1}{2}\left(\left|e_1 - p_1\right|^2 + \left|e_2 - p_2\right|^2 + \cdots + \left|e_N - p_N\right|^2\right)} \tag{8}$$

functions considerably better than the average discrepancy. For Gingko Middle School, $C_{ep} = 0$, a reassuring result. However, if some of the work collected from the school aligned to *measurement* instead of *geometry*, C_{ep} no longer equals 0 even though the extent of coverage remains completely misaligned. This inconsistency arises because the shortest distance between two points does not truly correlate to congruence. Consider a target point located in three dimensions at (1, 0, 0). Physically, Point $A = (0, 0.5, 0.5)$ lies closer to the target than Point $B = (0, 1, 0)$ and so would generate a larger C_{ep} index. In terms of congruence, however, both represent complete misalignment.

10. VISUAL INTERPRETATION

The Manhattan distance *S* used to calculate the broad-scope and narrow-scope indices is directly analogous to the distance needed to travel along orthogonal (perpendicular) axes between two points in *N*-dimensional space, where *N* depicts the total number of strands appearing in the calculations.

Consider the two-dimensional case, tabulated in Table 1 and illustrated in Figure 2, where the state tests knowledge in just two strands of the state content standards. Here, the enacted curriculum reflected only Strand A (Point *e*), as opposed to the requirements of the pacing calendar for the assignment collection period (Point *p*), which specified that only 75% of the content should have sampled Strand A. This represents a total Manhattan distance of 0.25 + 0.25 = 0.5, so that $C_{ep} = 1 - (0.5/2) = 0.75$. On the other hand, the overall pacing calendar (Point *P*) specifies that only 25% should sample Strand A; however, 50% of the state test (Point *T*) is composed of Strand A. This also represents the same value for C_{PT}, that is, $C_{PT} = 0.75$. Therefore, the congruence $C = (0.75)(0.75) = 0.56$.

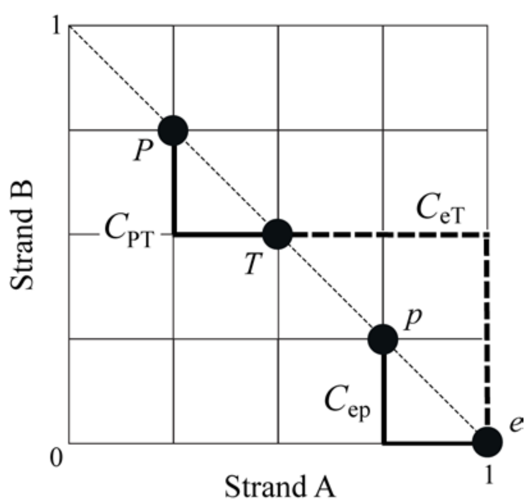

Fig. 2. Demonstration of the Manhattan distances and their relation to the broad-range and narrow-range indices for the simplistic case of only two strands. All points must lie on the $x + y = 1$ line (depicted with thin dashes) since all strand percentages must sum to 100%. Ideally, a long-term sampling of student work could directly establish C_{eT} (thick dashed line), eliminating the need for C_{PT} and C_{ep}.

Table 1. Sample calculation of C_{PT} and C_{ep} for two strands.

	C_{PT}						C_{ep}				
Strand	T		P		$\lvert d \rvert$		E		P		$\lvert d \rvert$
A	T_1	0.50	P_1	0.25	0.25		e_1	1.00	P_1	0.75	0.25
B	T_2	0.50	P_2	0.75	0.25		e_2	0.00	P_2	0.25	0.25
				S_{PT}	0.50					S_{ep}	0.50
				s_{PT}	0.25					s_{ep}	0.25
				C_{PT}	0.75					C_{ep}	0.75

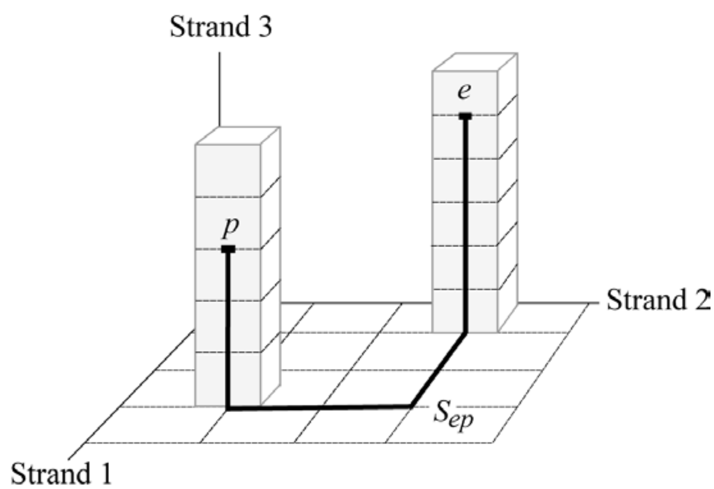

Fig. 3. A visual analogy between the Manhattan distance used to measure the congruence between curricular materials encompassing three strands of the state content standards and a metropolitan grid of skyscrapers. (For clariy, we only show the narrow-scope skyscrapers.)

Physically, the three-dimensional case (that is, three strands) mirrors a metropolitan grid of skyscrapers, with one strand aligned north, another strand aligned west, and the number of floors in each skyscraper corresponding to the third strand. The Manhattan distance between (say) the enacted curriculum and the weekly pacing calendar would then describe the distance needed to descend the elevator in one skyscraper, travel along perpendicular city streets to another skyscraper, and ascend the elevator to the target value. (See Figure 3.)

The 2007 Nevada English language arts standards serve as a good example of the three-dimensional model. Table 2 lists the Nevada state assessment blueprint coefficients for eighth-grade English language arts [Nevada Department of Education 2008], along with values obtained from a hypothetical school pacing calendar and data obtained from an analysis of student assignments over a one-week collection. Note that the broad-scope pacing calendar data, which reflect the (roughly) seven months preceding the state test date, do not coincide with the narrow-scope pacing calendar data, which reflect a collection period on the order of a few days or weeks. Again, the Manhattan distance S_{PT} equates to the perpendicular-path distance between Points P and T, illustrated with the thick line in the figure. Since $C_{PT} = 1 - (1/2)S_{PT}$, as the distance between the two points shrinks, C_{PT} increases. The same reasoning applies to the narrow-scope index.

Table 2. Sample calculation of C_{PT} and C_{ep} for a hypothetical Nevada school.

	C_{PT}						C_{ep}				
Strand	T		P		$\lvert d \rvert$		e		P		$\lvert d \rvert$
Reading 1.0	T_1	0.23	P_1	0.21	0.15		e_1	0.72	p_1	0.50	0.22
Reading 3.0	T_2	0.31	P_2	0.74	0.43		e_2	0.28	p_2	0.50	0.22
Reading 4.0	T_3	0.46	P_3	0.18	0.28		e_3	0.00	p_3	0.00	0.00
				S_{PT}	0.86					S_{ep}	0.44
				s_{PT}	0.43					s_{ep}	0.22
				C_{PT}	0.57					C_{ep}	0.78

From the data and calculations shown in Table 2, even though the extent of coverage of the student assignments looks nothing like that on the state assessment, the school achieves a reasonable value for the narrow-scope index because the enacted curriculum mimics to a large extent the adopted pacing calendar. However, the pacing calendar does not adequately reflect the state assessments, producing a small broad-scope index and, as a result, a small congruence ($C = 0.47$). In terms of curriculum coverage, the low extent of coverage of the enacted curriculum at this school did not occur during the collection period, but rather before the year started.

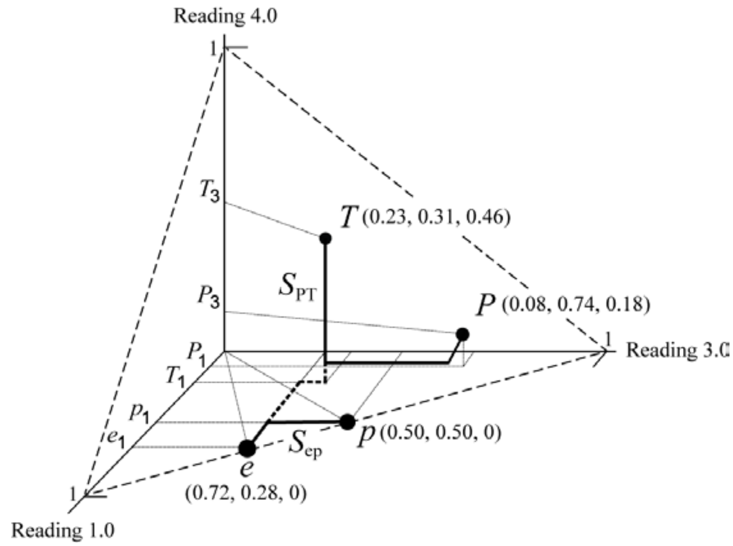

Fig. 4. A visual representation of the Manhattan distances S_{ep} and S_{PT} in three dimensions for the data in Table II, with each axis corresponding to a particular strand of the Nevada state content standards. (Some of the axis labels are omitted for clarity.) All points must lie on the $x + y + z = 1$ plane (depicted with dashes) since all strand percentages must sum to 100%. Note that points P and p do not necessarily coincide or even lie near each other. A long-term collection of student assignments could establish S_{eT} (the length of the heavy dotted line); however, short collection periods require measuring S_{ep} and S_{PT} instead.

11. DISCUSSION

The Manhattan distance provides a strong visual interpretation of congruence. Mathematically, the Manhattan distance involves nothing but basic arithmetic, making the approach accessible to those without special training in mathematics. As a result, the congruence formulation presented in this article offers a practical means for researchers to present research results to district coaches and, in turn, for district coaches to inform teachers.

In some situations, the state only assesses a limited portion of the state content standards. An example occurs in California, which does not explicitly test the *mathematical-reasoning* strand. However, a district would probably consider such a standard important to its academic mission and include it in its pacing calendar. In this case, the appearance of curriculum related to these strands, although within the goals of the district, will artificially lower the congruence. Researchers can remedy this situation by simply filtering out any appearance of such non-tested standards from the pacing calendar and enacted curriculum before calculating the indices.

Some districts establish benchmark assessments to identify deficiencies in learning before the state assesses their students. Researchers can incorporate the extent of coverage of district benchmarks by defining (1) a broad-scope index between the district pacing calendar and the district benchmark and (2) another broad-scope index between the district benchmark and the state assessment. Equation 5 then incorporates three indices instead of two.

Naturally, not all schools or districts provide pacing calendars. However, researchers could substitute the adopted textbook for the pacing calendar once they have established the broad-scope index describing the congruence between the textbook and the state assessment. (Final exams could potentially fill this role as well.) Since the enacted curriculum incorporates all curricular materials given to students during a specific collection period, researchers can measure the narrow-scope index by comparing the enacted curriculum to that portion of the textbook that encompassed the collection period.

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CONTACT
John R. Walkup, Ph.D.
President
The Standards Company LLC
575 W. Alluvial Ave.
Clovis, CA 93611
(888)777-1417
jwalkup@standardsco.com