# Reforming Mathematics Classroom Pedagogy: Evidence-Based Findings and Recommendations for the Developmental Math Classroom

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**Abstract**

For developmental education students, rates of developmental math course completion and persistence into required college-level math courses are particularly low. This literature review examines the evidence base on one potential means for improving the course completion and learning outcomes of developmental mathematics students: reforming mathematics classroom pedagogy. Each study examined for this review was classified into one of six sets according to the main instructional approach focused on in the study. The six sets are: *student collaboration*, *metacognition*, *problem representation*, *application*, *understanding student thinking,* and *computer-based learning.* Because most of the studies across the sets did not employ rigorous methods, the evidence regarding the impact of these instructional practices on student outcomes is inconclusive. An analysis of the studies that did employ rigorous designs suggests that structured forms of student collaboration and instructional approaches that focus on problem representation may improve math learning and understanding. This paper concludes by making a number of methodological recommendations, proposing several needed areas of research related to developmental math pedagogy, and suggesting instructional practices that may improve the outcomes of developmental math students.

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## 1. Introduction

A majority of community college students enroll in developmental education (Bailey, Jeong, & Cho, 2010), but evidence of its effectiveness in promoting student progression and degree completion is mixed. While some studies have found that remediation reduces students' probability of dropping out (Bettinger & Long, 2009; Lesik, 2007), other studies find that students in remediation accumulate fewer college credits and are less likely to complete a degree (Boatman & Long, 2010; Calcagno & Long, 2008; Martorell & McFarlin, 2008). Equally concerning are the low levels of developmental education course completion, especially for developmental math. At the 57 community colleges participating in the Achieving the Dream: Community Colleges Count initiative (see www.achievingthedream.org), only one third of students who were referred to developmental education completed the recommended sequence of math courses (Bailey et al., 2010). Of the students who enrolled in developmental education courses, only 20% eventually completed a required college-level math course (Bailey et al., 2010).

Failing to complete developmental math and required college-level math not only prevents individuals from earning a college degree and pursuing certain professions but also has consequences for a young adult's likelihood of employment. Young adults with low levels of quantitative literacy skills, including the types of arithmetic operations and applications typically covered in developmental math courses, are more likely to be unemployed—and, moreover, low levels of quantitative literacy partially account for the lower employment rates for African Americans compared to Whites (Rivera-Batiz, 1992). Given the negative consequences of failing to complete developmental mathematics, it is critical to identify potential ways to improve developmental students' math success. Other working papers in the CCRC Assessment of Evidence Series discuss potential ways to improve developmental course completion through improved entry assessment (Hughes & Scott-Clayton, 2011), accelerated course structures (Edgecombe, 2011), and contextualized curricula (Perin, 2011). This paper examines the evidence base on another potential means for improving learning outcomes and course completion among developmental mathematics students: reforms to mathematics classroom pedagogy.

While national studies that document the common features of developmental education classroom instruction do not exist (Levin & Calcagno, 2008), typical developmental math pedagogy is thought to rely largely on procedural skill-building (Goldrick-Rab, 2007; Hammerman & Goldberg, 2003). Observational studies at community colleges in California found that mathematics instruction was characterized by review, lecture, independent seat-work, and math problems devoid of application to the real world (Grubb, 2010; Grubb & Worthen, 1999). Although traditional features of math instruction have been linked to better performance on standardized tests and much of the mathematics we encounter in our lives requires the ability to use algorithms to quickly and accurately solve computations, in order to understand mathematics, students need much more than procedural fluency (Hiebert & Grouws, 2007; Kilpatrick, Swafford, & Findell, 2001).

Kilpatrick et al. (2001) identified five interdependent strands of mathematical learning that instructional practices must address to build mathematical proficiency:

1. *conceptual understanding*—the understanding of why and when a mathematical idea is important or useful,
2. *procedural fluency*—the ability to use procedures in the right way and for the right purpose,
3. *strategic competence*—problem formulation and representation,
4. *adaptive reasoning*—logical reasoning about mathematical relationships, and
5. *productive disposition*—the belief that a sustained effort in learning mathematics will lead to greater understanding and benefit one's life.

This paper discusses forms of instruction that are thought to support components of mathematical learning beyond procedural fluency and should, therefore, develop mathematical proficiency more effectively than traditional instruction.

## 1.1 Studies Included in the Review

Although the purpose of this literature review is to identify promising developmental math pedagogy, there is very little empirical research on this topic and population specifically; thus, I also reviewed the literature on mathematics pedagogy in elementary and secondary schools and in college-level courses, focusing on empirical studies that evaluate the impact of an instructional practice on student outcomes.[1] The elementary and secondary school math pedagogy literature is included not only because it is more prevalent than empirical research on math instruction in higher education but also because recommended best practices in teaching are often similar across grades and even subjects. For example, important works that outline best practices for developmental education (e.g., Blair, 2006; Boroch et al., 2007; Boylan, 2002) and K-12 mathematics teaching (e.g., Donovan & Bransford, 2005) provide similar recommendations: that educators connect new knowledge to prior learning; use a variety of instructional methods, including learner-centered activities; and provide students with feedback through the use of ongoing assessment. Reviewing the K-12 empirical literature illustrates how these general recommendations are implemented in the classroom in ways that are effective or ineffective for primary and secondary students. These findings can then be used to make recommendations regarding how pedagogy that works in the K-12 math classroom could perhaps be adapted for adult students in the developmental education classroom in an effort to apply commonly accepted best practices in teaching.

It is important to note that articles on curriculum reforms that may also lead to or require changes to pedagogy are not included in this review. Curriculum reforms typically can involve so many interconnected instructional changes that it is difficult to isolate the effects of the individual reforms (Hamilton et al., 2003). For example, this

[1] The inclusion criteria for this study encompass qualitative and quantitative (excluding single-subject design) studies published from 1990 to the present whose target population includes students in K-12 schools and institutions of higher education in the United States. To find publications within the topic scope, major databases and websites were searched using the term *mathematics* in combination with *instruction* or *pedagogy* or in combination with *instructional* (or *pedagogical*) *practices* or *strategies.* Databases and websites included: EBSCO's Academic Search Premier, Education Research Complete, Education Full Text (Wilson), ERIC, JSTOR, ProQuest, state higher education websites, community college system websites, Mathematical Association of America (MAA), American Mathematical Association of Two-Year Colleges (AMATYC), and Google Scholar. I also conducted a manual search of the following pertinent publications: *Journal of Developmental Education*, *Remedial and Special Education*, *Community College Journal of Research & Practice*, *Community College Review*, and *Community College Journal*.

paper does not review a study that evaluated the impact of a reform-based undergraduate calculus textbook (Darken, Wynegar, & Kuhn, 2000), which includes exercises that use group work and technology and emphasize conceptual understanding. The authors did not report the extent to which instructors who volunteered to use these texts incorporated these changes to pedagogy in their classrooms, and it is impossible to understand the discrete impact of each pedagogical change on the overall results. In contrast, while many studies included in this review required changes to content (e.g., using multi-step word problems rather than problems that require only a few steps to solve), they focus on the methods teachers use to teach the content.

### 1.2 Assessing the Quality of Evidence

In reviewing papers, I critiqued the findings according to the strength and rigor of the research designs of the studies in order to evaluate the direction and quality of evidence on math pedagogy. Table 1 (found in the appendix) provides a concise summary of each reviewed article, including the author’s findings, effect size of the results, and methodological problems that weaken the internal validity of the results.[2] This review describes the results of rigorous studies and draws conclusions about effective pedagogy. In general, to be considered rigorous, a study had to be transparent about the comparability of the treatment and control conditions, providing some confidence that any outcomes are the result of the instructional intervention, not differences between the treatment and control groups. Most importantly, rigorous studies either demonstrated that there were no pretreatment ability differences between students who received the instructional intervention and students in the comparison group or statistically controlled for pretreatment ability, and they made an effort to assign similar instructors to the treatment and comparison groups. The results of the remaining studies are inconclusive because they could be due to differences between the treatment and control group rather than the instructional intervention.[3]

[2] There were four methodological flaws commonly found across studies: student design non-equivalency, teacher design non-equivalency, possible attrition issues, and lack of a comparison group. The notes for Table 1 include a detailed explanation of each of the four main flaws.

[3] Except for a few higher education studies, the results of the non-rigorous studies are not described in this review, but they can be found in Table 1.

### 1.3 Organizational Framework

There are several ways to categorize instructional practices. Grubb (2010) organizes practices along a behaviorist–constructivist continuum. Commonly accepted groupings of instructional practices include broad categories, such as student-centered versus teacher-directed instruction or direct instruction versus inquiry-based teaching (Hiebert & Grouws, 2007). The current paper uses a more detailed classification system driven by specific theories of learning, which provide concrete explanations of particular mechanisms underlying different pedagogical approaches. Based on an inductive approach, studies were organized into six sets, each containing similar types of practices that share a clearly identifiable (although not always explicitly identified by each study's authors) theory of learning. Each set of practices is therefore supported by a particular theory that explains why these practices should lead to improved math learning and understanding. The six sets are *student collaboration*, *metacognition*, *problem representation*, *application*, *understanding student thinking,* and *computer-based learning.*[4]

In the first section, for each of the six sets, I explain the relevant theories that support the pedagogical practices in the set, provide an example of how the theoretical concepts are applied in the developmental math classroom today, and evaluate the empirical evidence from the articles in the set. Subsequent sections describe the recommendations based on the review of this literature, including methodological recommendations and new directions for developmental math pedagogy research. The paper concludes with examples of instructional changes to the developmental math classroom that may contribute to improved outcomes for developmental math students.

[4] The practices across the six sets can interact in the classroom and overlap in myriad ways. Hiebert and Grouws (2007) emphasize that "teaching is a system of interacting features" (p. 374); different features do not fit easily into categories. For this paper, studies were categorized according to the feature by which they were most clearly defined. For example, in the application set, instructional practices always involve students working together and often promote metacognition and problem representation skills. Studies in the application set are distinct from studies in the other five sets because instruction is characterized by the use of elaborate problem-solving activities that use real-world contexts. This type of pedagogy is supported by distinct theories of learning, suggesting that student learning in classrooms that use real-world problem solving is enhanced in different ways than learning in classrooms that use student collaboration, promote metacognition, and/or build problem representation but do not use real-world problem solving.

# 2. Theory, Practice, and Empirical Evidence

## 2.1 Student Collaboration

This first set of studies evaluates different models of student collaboration in the math classroom. Some studies examine the impact of informal forms of student collaboration, while others evaluate the effectiveness of more structured forms of peer collaboration, such as Math Excel, Peer-Led Team Learning, the Learning Together model of cooperative learning, peer-assisted learning strategies (PALS), peer-mediated instruction (PMI), and Team Accelerated Instruction.

**Theory.** Springer, Stanne, and Donovan (1999) organize the theories that support student collaboration into three theoretical perspectives that identify the different mechanisms that link small-group instruction to improved student outcomes. The motivational perspective supports the notion that in competitive learning environments, the probability of success decreases at an increasing rate as other students succeed, but in small-group learning environments, success is dependent on students working together to achieve a common goal. The affective or humanist perspective explains that interaction among students leads to a nonthreatening environment in which underrepresented students have more opportunities to participate and learn. Finally, according to the cognitive perspective, student collaboration on open-ended questions leads to greater cognitive growth, and the act of explaining material to another student is one method of cognitive elaboration, which facilitates the retention of information.

Johnson and Johnson's social interdependence theory, which is cited in a number of empirical and instructional publications on cooperative learning in higher education math courses (e.g., Arendale, 2004; Dees, 1991; Norwood, 1995; Summers & Svinicki, 2007; Zachry, 2008), draws from all three theoretical perspectives (motivational, affective, and cognitive). Negative interdependence, which is characterized by a competitive learning environment, can potentially be demotivating because students can only succeed if others are failing (Johnson, Johnson, & Smith, 1991). In addition, in both classrooms with no interdependence (where students work on their own) and classrooms with negative interdependence, students do not benefit from an improvement in social skills and cognitive growth that is thought to result from positive social interactions and the exchange of information between peers (Johnson et al., 1991). On the other hand, positive interdependence, where the success of one individual is dependent on the success of others, is found in cooperative learning environments characterized by face-to-face interaction, personal responsibility in working toward a shared goal, the use of interpersonal skills, and group processing through the exchange of feedback, explanations, and other information (Johnson et al., 1991). In structured cooperative learning situations with these elements, motivational, affective, and cognitive mechanisms are thought to lead to improvements in learning outcomes.

Student collaboration methods are also supported by Lev Vygotsky's theory of the zone of proximal development and the theory of constructivism. The zone of proximal development is the cognitive space between what a learner can do independently (i.e., their ability) and what a learner can do with the aid of a teacher or through peer collaboration (i.e., their potential) (Safford-Ramus, 2008). Constructivism describes learning as a process in which knowledge is constructed through building on prior experiences, engaging in self-discovery, and collaborating with peers (Yilmaz, 2008). Collaborative learning in developmental math education is a way to both expand students' zone of proximal development and apply the principles of constructivist learning (Casazza, 1998; Norwood, 1995; Safford-Ramus, 2008).

**Empirical evidence.** For the most part, the 15 studies in this set (see Table 1) found that student collaboration has a positive impact on math learning. Most of these studies did not ensure treatment and control group equivalency; thus, their results could be due to factors other than the student collaboration method. However, five rigorous elementary school student collaboration studies demonstrate that highly structured forms of student collaboration are especially effective for low-achieving math students.[5]

With PALS and PMI, teachers train students to follow a routine that involves students taking turns, acting either as the tutor who asks questions and provides feedback at every step of the problem-solving process or as the tutee who answers questions at every step (Fuchs, Fuchs, Phillips, Hamlett, & Karns, 1995; Fuchs et al., 1997; Fuchs, Fuchs, & Karns, 2001). Lower-achieving students are usually paired with higher- or

[5] The Calhoon and Fuchs (2003) study is also rigorous but evaluates the effects of PALS and curriculum-based measurement (CBM) in combination and, therefore, the outcomes cannot be attributed strictly to student collaboration.

average-achieving students. Across the three studies of PALS and PMI (Fuchs et al., 1995; Fuchs et al., 1997; Fuchs et al., 2001),[6] results varied for students with different levels of math aptitude. Students with learning disabilities and average-achieving students usually made small gains in achievement compared to their counterparts in the control classrooms. Lower-achieving students benefited the most from the different student collaboration treatments, making small to moderate gains on all tests but one that measured application skills in one study. Higher-achieving students only benefited from paired work when more complex math tasks were included in the PMI treatment. Similarly, a randomized study by Ginsburg-Block and Fantuzzo (1998) found that low-income, low-achieving third- and fourth-grade students who used a highly structured peer-tutoring format to review and reinforce math skills outperformed comparable students who received traditional instruction. In contrast, Karper and Melnick's randomized study (1993) in a wealthy school district found that the cooperative learning technique Team Accelerated Instruction had no impact on the math achievement of students in the treatment group. Taken together, the five studies suggest that structured student collaboration may be more beneficial for low-achieving elementary school students struggling with math.

Finally, the highest quality developmental education study in this review (Dees, 1991)[7] suggests that an instructor-designed, structured student collaboration method may be a promising practice for the developmental math classroom. Dees (1991) randomized over 70 students in her developmental math course into four laboratory sections taught by graduate assistants: two that used small-group instruction and two that used teacher-directed instruction. Students in the cooperative learning lab outperformed students in the control group on teacher-made tests and a standardized final exam; however, the internal validity of the study's findings is undermined by the lack of detail about the research design, such as how graduate assistants were assigned to sections.

[6] One concern with these studies is that although teachers were randomized to the treatment and control classrooms and implemented the treatment with the whole class, due to resource constraints only a subset of students were chosen to take the posttest. However, these studies show that there are no statistical differences along observable characteristics between the treatment and control students in the whole class and subset of students.

[7] The Dees article meets the inclusion criteria because it was published after 1990; however, it is important to note that the study took place in 1982, so it is much older than most of the studies included in this review that were conducted during the past 20 years.

**Student Collaboration in Practice**

Peer-Led Team Learning (PLTL) was first established at the City University of New York and has since been used in math and science courses at colleges across the country (Arendale, 2004). Through the PLTL program, advanced students are trained in teaching and cooperative learning techniques, becoming "peer leaders" who lead weekly collaborative workshops with small groups of students (Arendale, 2004). The developmental math faculty at Mountain Empire Community College (MECC) in Virginia have implemented PLTL in the developmental math course Algebra I.* Peer leaders are assigned to sections of Algebra I, which they attend for one hour on Mondays and Wednesdays. Students from these courses who have chosen to participate in PLTL meet with their peer leaders during the PLTL session, named "Power Hour," held on Fridays directly before their Algebra I course. During the PLTL sessions, the peer leaders and students work cooperatively in small groups on areas of Algebra the students are struggling with. A PLTL leader who was a former tutee in the program spoke about her experience with PLTL: "I love math. I'm a math tutor. But after being out of school for so long, it was like reading Hebrew the first semester, and the Power Hour made me feel more comfortable. I could understand mostly what my instructor told me, but it was just a different language coming from my PLTL leader."

*Information on PLTL is from a Community College Research Center site visit to MECC in spring 2010.

## 2.2 Metacognition

The second set of studies includes instructional practices that promote metacognition, or an awareness of one's own thought processes, through comprehension monitoring, cognitive strategy instruction, or using writing and questioning during the problem-solving process to foster self-reflection.

**Theory.** The connection between metacognition and math learning is supported by a number of theories. Garofalo and Lester (1985) extended Flavell's (1979) theory about the role of metacognition to identify its two main aspects, "knowledge of cognition" and "regulation of cognition" (p. 164), and apply them to the mathematical problem-solving process. In Garofalo and Lester's (1985) cognitive–metacognitive framework, the knowledge of cognition begins in the orientation phase, in which the problem solver assesses the information given about the mathematical task, his or her level of familiarity with the task, the difficulty of the task, and possible strategies to use. Next, in the organization or planning phase, the regulation of cognition aids the problem solver in connecting his or her understanding of the concept to an understanding of the appropriate strategies to use to solve the problem. In the execution stage, the problem solver solves the problem using the necessary strategies and procedures. Finally, during the verification stage, the problem solver confirms that the solution is correct by checking the computations and problem-solving process for errors. Garofalo and Lester (1985) argue that mathematical problem solving can be improved by training students to incorporate the stages of metacognition into their problem-solving process.

The theory of information processing provides another justification for the importance of metacognition, contending that learning occurs as individuals think about their own thinking as they retrieve stored information from memory (i.e., prior knowledge) and use it to process new information (Safford-Ramus, 2008). Metacognition has also been integrated into forms of constructivism. For example, in Narode's (1989) description of instruction in a constructivist developmental math program for students at the University of Massachusetts, Amherst, he writes, "the method of instruction incorporates two key notions: constructivism, the idea that students must construct knowledge for themselves, and metacognition, the supposition that the vehicle for the construction of knowledge is self-reflection" (p. 6). In the math classroom, instruction that uses the cognitive–metacognitive framework would emphasize not only each student's ability to solve problems but also each student's capacity to assess a problem's difficulty, choose the appropriate strategy or strategies to solve a problem, engage in self-monitoring during the problem-solving process, and evaluate the final solution for its accuracy.

**Empirical evidence.** The eight reviewed studies (see Table 1) describe unique pedagogical practices that seek to improve students' ability to monitor their problem-solving process, but only one study employed a rigorous experimental design. In Tournaki's (2003) study, second-grade students with and without disabilities were randomized into a control group and two treatment groups in which, in addition to their regular class time, students received either explicit instruction in either strategy instruction (verbalizing problem-solving steps), or drill-and-practice strategies. Students with learning disabilities who received strategy instruction experienced large gains in addition facts performance compared to students with learning disabilities who received drill-and-practice instruction, but there were no differences in addition facts achievement between students without disabilities who received the strategy instruction versus drill-and-practice instruction. Tournaki concludes that explicit instruction in problem-solving strategies, even for tasks as simple as adding, may be especially important for students with learning disabilities.

**Metacognition in Practice**

The College Transition Initiative (CTI) at the CUNY community colleges prepares GED graduates to take the COMPASS placement math exam, with the related aims of reducing their need for remediation, deepening their understanding of algebra, and preparing them for their first college-level math course (Hinds, 2009). In addition to a supportive structure in which the CTI math course is part of a learning community and an innovative math curriculum, CTI math instruction employs a number of non-traditional pedagogical practices. Procedural rules are not taught; in their place, students build their conceptual understanding by discussing mathematical relationships, and rules that emerge through these activities are discussed at the end of lessons so that students can also build their procedural fluency. The instructor does not lecture; instead, the instructor asks students higher order thinking questions about contextualized functions that help students transition to more abstract work. For most of class time, students work in groups to solve problems and are constantly involved in discussions sparked by metacognitive questioning. Common questions asked by the instructor include:

*What did you do?*
*Why did you do that?*
*Do you agree with what she/he just said? Why?*
*Did any of you do it differently? Why?*
*What do you see?*
*Does this remind you of anything?*

## 2.3 Problem Representation

The third set of articles encompasses pedagogy that improves students' problem representation skills, such as the use of multiple representations and strategies during the problem-solving process, the learning/teaching approach, concept-based instruction, the concrete-to-representational-to-abstract (CRA) instructional sequence, schema-based instruction, and the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project.[8]

**Theory.** The cognitive phases of problem solving are analogous to the metacognitive framework except that in the first phase, called the problem representation phase, the focus is on building a mental representation of the problem rather than assessing the problem's difficulty, strategies to use, and other aspects of the problem-solving process (Brenner et al., 1997). The importance of problem representation is based on the cognitive theory that problem solvers must understand the connection between the problem and its different representations before they can move on to planning and executing the procedural steps to solve a problem (Brenner et al., 1997; Chappell, 2006). By focusing on the procedures needed to execute or solve problems, teachers and instructors neglect this critical first phase of problem solving. Higher levels of math, such as algebra, require an understanding of how algebraic (or symbolic) representations can be represented by graphs and other forms, so students who did not learn problem representation skills in lower-level math courses, such as pre-algebra, may experience increasing difficulties as they progress in math (Brenner et al., 1997; Zawaiza & Gerber, 1993).

The cognitive phases of problem solving suggest that problem representation skills will lead to more success in the use of procedures during the problem execution phase, and Rittle-Johnson, Siegler, and Alibali (2001) find empirical evidence suggesting that, in fact, problem representation is the mechanism underlying the process by which conceptual knowledge can lead to improved procedural fluency, which in turn can lead to improvements in conceptual knowledge. In the Rittle-Johnson et al. study, fifth-grade students were randomly assigned to receive assignments on placing decimals on a number line that provided different levels of representational supports (e.g., a zero-to-one number line with no markings versus a zero-to-one number line with the tenths place marked). The level of representational supports determined students' problem representation skills, measured by their ability to explain why they chose a spot on the number line to represent each decimal, on an intervention test, and their procedural

[8] The QUASAR project includes many more instructional components than strategies that improve students' problem representation skills (Silver & Stein, 1996), but the QUASAR articles in this review focus on instructional tasks that use multiple representations or solution strategies.

fluency, measured by marking the position of a decimal on a number line, on a posttest. Procedural skill performance was then a significant predictor of conceptual knowledge, measured by student performance on new tasks that required an understanding of decimal fractions, on a posttest.

The Rittle-Johnson et al. study (2001) does not suggest a specific instructional strategy, such as providing students with worksheets that include representational supports. Rather, it supports the theory that students who develop an understanding of how concepts can be represented in different, connected ways will be able to link these representations to the procedures that are necessary to solve a problem. This will lead to more accurate procedural fluency (or solution execution), and solving problems with understanding will enhance conceptual knowledge.

**Empirical evidence.** Five of the nine studies in this set (see Table 1) are among the strongest studies in this review and provide compelling evidence that improving students' problem representation skills has a small to moderate positive effect on math learning. The four high-quality studies that took place in elementary and middle school classrooms (Brenner et al., 1997; Jitendra et al., 1998; Jitendra et al., 2009; Witzel, Mercer, & Miller, 2003) found support for the routine use of multiple representations during problem solving by teachers and students.

In the only higher education study in this set, Chappell (2006) employed a number of methods to ensure that even though students self-selected into concept-based calculus and traditional calculus sections (unaware of the instructor or instructional method of each section), the faculty and students across both groups were comparable. In addition, frequent, unannounced classroom observations by faculty not directly involved in the study confirmed that in the concept-based sections, faculty taught students how to solve problems using numerical, graphical, and algebraic methods while constantly connecting new ideas to prior knowledge. In the control sections, faculty moved through the textbook teaching definitions and formulas in a linear manner. Students in the concept-based sections performed significantly better on the midterm and final and were better able to transfer their understanding to unfamiliar concepts. For example, on a final exam problem that had never been introduced in any of the classes, 88% of the students in the concept-based classrooms answered this question correctly, and only 3% did not support their answer with an explanation. Only 54% of the students in the traditional sections answered this question correctly, with most of them providing textbook definitions to explain their answer and 31% not providing any explanation at all.

**Problem Representation in Practice**

The traditional approach to introducing radical equations teaches the procedural skills related to solving them, as in the following exercise*:

Solve $2\sqrt{x+2} = 10$

First, isolate the radical by dividing both sides of the equation by 2:
$\sqrt{x+2} = 5$

Next, square both sides of the equation:
$\left(\sqrt{x+2}\right)^2 = 5^2$
$x + 2 = 25$
$x = 23$

Now check for extraneous solutions.

The following exercise is thought to better promote conceptual understanding of radical equations:

| ***X*** | ***f* (*x*) = √*x*** |
|---|---|
| 0 | √0 = 0 |
| 1 | √1 = 1 |
| 2 | √2 =1.414 |
| 3 | √3 = 1.732 |
| 4 | √4 = 2 |
| 5 | √5 = 2.236 |
| 7 | √7 = 2.646 |
| 9 | √9 = 3 |

Graph the basic square root function using the data from the input/output table.
*Drawing the graph by hand helps students remember concepts about domain.*

Now draw the graphs of $f(x) = 1$, $f(x) = 2$, $f(x) = 3$, and $f(x) = -1$ all on the same axes.
*This allows students to see the points of intersections (the solutions) and where there are no points of intersection (the extraneous solutions). Then, they will be more prepared to solve the radical equation below.*

Solve $2\sqrt{x+2} = 10$.

**Problem Representation in Practice, Continued**

Students should continue to connect procedures and solutions to the graph of the equation by answering the following questions:

*What is the basic function?*
*What is the translation or the transformation of the function?*
*Solve $f(x)$ by labeling all important points on a graph.*

*This example is adapted from Gowribalan A. Vamadeva's presentation "Fostering Conceptual Understanding in a Developmental Algebra Classroom" at the 2009 AMATYC Conference proceedings and included with her permission.

## 2.4 Application

Application-oriented instructional approaches include project-based learning, the modeling-based approach, the functional approach, enhanced anchored instruction, the algorithmic instructional technique, and culturally responsive pedagogy, all of which are designed to teach math concepts and skills through real-world problem-solving. Many of these instructional approaches may be referred to as "problem-based learning" (Boroch et al., 2007, p. 45), which is one form of contextualization (Perin, 2011).

**Theory.** The link between application of math concepts to students' everyday lives and improvements in math performance is supported by the theory of situated cognition (The Cognition and Technology Group at Vanderbilt, 1990). Situated cognition is based on the idea that since an important objective of schooling is the transfer of skills from the abstract to the concrete, skills and concepts should not be taught without reference to the real world; rather, they should be situated in authentic activities (Brown, Collins, & Duguid, 1989). Authentic activities value the experiences and knowledge students bring with them to the classroom and allow students to learn math in a context that is meaningful to them. For example, culturally responsive pedagogy, which utilizes authentic, culturally based activities, is believed to be more effective than traditional instruction because "it filters curriculum content and teaching strategies through their cultural frames of reference to make the content more personally meaningful and easier to master" (Gay, 2000, p. 24).

The cognitive apprenticeship model of learning explains how pedagogy and content (e.g., authentic activities) based on the theory of situated cognition are connected to student learning and transfer of skills. In this model, students are apprentices who through guided practice become independent learners, and the skills they learn are both physical and cognitive, since "knowing and doing" (Brown et al., 1989, p. 39) are inseparable. First, teachers introduce a new concept by modeling how to solve a problem that students are familiar with; then, as students become more comfortable with the concept, they are given authentic activities to work on in collaborative groups (Brown et al., 1989). Finally, as opposed to traditional instruction that begins with equation solving and ends with word problems that situate math in real-world contexts, instructional approaches based on situated cognition may end with students independently using algorithmic procedures to solve problems that assess the understanding of the same math ideas and skills embedded in the authentic activities (Brown, et al., 1989; Laughbaum, 2003; Vasquez, 2003). In other words, this learning process is thought to aid in the transfer of math skills to both procedural problem solving and open-ended, realistic problem-solving activities (Boaler, 1998).

Another mechanism that may explain why application-oriented instructional approaches in this set may improve math learning is motivation. Motivation may be the result of pedagogical techniques, such as collaborative learning (Johnson et al., 1991; Springer et al., 1999), and the contextualized curriculum (Perin, 2011). The practices in this set may improve student learning through different mechanisms because they are defined by a number of instructional and curricular supports, including instructor scaffolding, complex problem-solving embedded in real-world or culturally based situations, collaborative group work, and hands-on activities that use technology that helps students contextualize math concepts in real-world problems (Bottge, Heinrichs, Chan, & Serlin, 2001; Bottge, Heinrichs, Mehta, & Hung, 2002; Brenner, 1998; Ellington, 2005a, 2005b; Ganter & Jiroutek, 2000; Hickey, Moore, & Pelligrino, 2001; Hollar & Norwood, 1999; Kennedy, Vasquez, & Huber, 2003; Laughbaum, 2003; Lipka & Adams, 2004; O'Callaghan, 1998; Shore, Shore, & Boggs, 2004; Vasquez, 2003, 2004).

**Application in Practice**

At Maricopa community colleges, students in arithmetic review courses were given two weeks to solve the following problem in groups (Tannehill & Zeka, 1997):

> *You are interested in purchasing a new vehicle.*
> *What should your annual salary be to afford the car you want?*

Instructors acted as facilitators, providing students with web resources and formulas to help them calculate their dept-to-income ratio, car costs, loan payments, and other critical pieces of information that students needed to be able to calculate how much they would have to earn to finance the car they want.

**Empirical evidence.** The eleven studies in this set (see Table 1) consistently find a positive association, with trivial to large effect sizes, between teaching math through application and improved performance on tests of conceptual understanding. However, only the Hickey et al. (2001) study attempted to ensure that the treatment and control conditions and students and teachers in both groups were comparable, and the treatment effects were too small for the results to conclusively support the use of a video series that takes elementary students on complex math adventures. In many of the studies in this set, treatment students outperformed control students on tests of understanding, but there were no differences in performance on more traditional tests of procedural fluency (Bottge et al., 2001; Bottge et al., 2002; Hickey et al., 2001; Hollar & Norwood, 1999; O'Callaghan, 1998)—a finding that is consistent with studies in the problem representation set (Brenner et al., 1997; Chappell; 2006). These studies highlight the possible trade-offs that are made when reform-based instructional practices are used in the math classroom at both the K-12 and college levels. While these practices show promise for improving students' conceptual understanding, which is considered a necessary condition to be successful in math at any level (Bransford, Brown, & Cocking, 1999; Hiebert & Grouws, 2007; Katz, 2007; Kilpatrick et al., 2001), they may require more time and preparation and sometimes do not improve students' computational skills (Brenner et al., 1997).

### 2.5 Understanding Student Thinking

The fifth set of practices encompasses instructional methods that help teachers understand student thinking and adjust their instruction to meet the needs of their students. These pedagogical practices include assessment methods that are used during instruction to monitor student progress and guide instruction, such as frequent testing, classroom assessment techniques, classroom voting, the Keystone Method, progress monitoring, and curriculum-based measurement (CBM).

**Theory.** While cognitive and behavioral theories explain the relationship between assessment and *students'* understanding of their own thinking, theories and applications of cognitively guided instruction (CGI) explain the importance of *teachers'* understanding of student thinking (Bransford, Brown, & Cocking, 1999; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Villasenor & Kepner, 1993). CGI is the use of instructional practices that are informed by research on how students solve math problems and by knowledge of the prior math skills and misconceptions students bring to the math classroom (Carpenter et al., 1989; Villasenor & Kepner, 1993). CGI is based on a cognitive perspective of teaching that examines the interaction between teachers' instructional choices, their knowledge of student thinking, and student performance (Carpenter et al., 1989). The cognitive view of teaching hypothesizes that teachers who have an understanding of their students' thinking (as well as strong subject matter and pedagogical content knowledge) will make better instructional choices that result in improvements in student math achievement (Ball & Bass, 2000; Bransford et al., 1999; Carpenter et al., 1989; Kieran, 2007).

Applying the cognitive view of teaching can be challenging, however, since monitoring the progress of a classroom of students "create[s] an overwhelming demand on the cognitive resources of the teacher" (Carpenter et al., 1989, p. 501). As a result, teachers are only able to make small adjustments in their instruction based on their assessment of student understanding and thinking. However, today, formative assessment, or the adjustment of instruction based on performance of students, can be driven by technological tools that provide an efficient means of monitoring student progress through frequent assessment, especially at the college level (Blair, 2006; Boroch et al., 2007; Cline, 2006). Technological supports, such as clickers (see, e.g., Cline, 2006), are aligned with a cognitively based perspective on instruction that hypothesizes that teachers make more effective instructional choices that help students build their mathematical understanding when they are able to assess students' prior knowledge and current understanding of the material.

**Empirical evidence.** While it is generally accepted that college instructors should adapt instruction to meet the needs of their students through meaningful, ongoing assessment (Adams, 1997; Blair, 2006; Boroch et al., 2007; Boylan, 2002; Siadat et al., 2008), the four studies targeting college and developmental education students did not utilize comparison groups or compared non-equivalent groups (see Table 1), so their findings cannot confirm the positive impact of ongoing, formative assessment. Among the nine K-12 studies (see Table 1), the rigorous studies by Fuchs and Fuchs (1990), Fuchs, Fuchs, Hamlett, and Stecker (1991), and Fuchs, Fuchs, Hamlett, Phillips, and Bentz (1994) find strong support for using CBM that provides teachers with expert recommendations to make instructional changes with elementary school students with learning disabilities. However, given that all three studies were conducted by the same researchers in the same setting, confidence in the validity of these studies would be strengthened if their results were replicated elsewhere.

**Understanding Student Thinking in Practice**

Daley College implemented the Keystone Method in Elementary Algebra, Intermediate Algebra, and College Algebra (Siadat et al., 2008). This instructional method utilizes daily assessment of students to inform the instructor of the progress of the class and each student. The instructor can then adjust instruction accordingly and inform students of their individual progress. Before each class, the instructor conducts an item analysis of student answers on the quiz questions that tells the instructor which areas should be retaught or reviewed the next day and which problems to include on future quizzes. If the standard deviation of the quiz scores is greater than 0.25 (i.e., if there is considerable divergence in student understanding of specific skills), the instructor creates small, heterogeneous groups of students with one from each of the quiz performance quartiles. As a result, groups have weak, average, and high-performing students who can motivate and help each other learn the material. If quiz scores reflect a general understanding of skills across all students, the instructor uses more traditional modes of teaching to move on to new concepts.

## 2.6 Computer-Based Learning

This last set includes studies in which students work through technology-delivered mathematics content at their own pace during some or all of classroom time, with the instructor providing some face-to-face interaction through individualized attention, delivery of instruction, or technology support. Computer-based learning may also encompass many of the approaches previously discussed, such as using computer-based technology to provide students with real-world problem-solving opportunities and monitoring the progress of students in order to guide instruction and content. Computer-based learning includes course redesign models, a popular trend in developmental mathematics where some or all of face-to-face instruction is replaced with a set of self-paced, online curriculum modules (Epper & Baker, 2009; Twigg, 2005); hybrid or blended online learning; and forms of computer-based learning where the traditional course structure is maintained and the instructor still has a role in the classroom.

**Theory.** Originally, computer-based instruction was based on the theory of behaviorism (Hung, 2001; Safford-Ramus, 2008). According to behaviorism, responses to stimuli (e.g., questions) that are directly followed by positive or negative reinforcement will lead to the conditioning of consistent, correct responses representing the learning of material (Safford-Ramus, 2008). In one of the first educational applications of behaviorism, a “teaching machine” provided students with academic material followed by factual questions; students’ responses were then fed back into the machine (Skinner, 1960). The machine provided immediate responses to each answer: the student received new material for correct answers and the same question for incorrect answers. Skinner (1960) believed that this type of programmed instruction motivated learning by breaking down concepts into small, manageable pieces of information that students could work through at their own pace while continually receiving immediate feedback on their understanding.

Computer-based learning has evolved since the advent of Skinner’s teaching machine. First, there is a range of pedagogy inherent to instructional software programs. Some computer-based tutorial and learning programs deliver drill-and-practice exercises (Hung, 2001), while other instructional software programs provide problem-solving activities that emphasize deeper understanding of mathematical concepts (Epper &

Baker, 2009; Hung, 2001; Stillson & Alsup, 2003; Twigg, 2005). As a result, technology-mediated instructional content may determine the extent to which students experience a balanced learning environment where procedural skills, conceptual understanding, and other components of mathematical proficiency are all addressed. Second, computer-based learning can be designed to incorporate principles of constructivism (Kanuka & Anderson, 1999). For example, because of its self-paced learning component, computer-based instruction is called a student-centered model of learning (Trenholm, 2006; Zhu & Polianskaia, 2007). Computer-based learning utilizes an important constructivist principle—that students are active in the construction of knowledge rather than passive recipients of knowledge, and educators serve as their guides and helpers (Kanuka & Anderson, 1999). Other essential features of a constructivist classroom are discovery-based learning and meaningful interactions between students, which can also be incorporated into computer-based instructional models (Kanuka & Anderson, 1999). Finally, the Open Learning Initiative (OLI), which creates online courses for students at Carnegie Mellon University, has demonstrated how computer-based delivery of course content can incorporate instructional elements that traditional instruction cannot. For example, the structure of the OLI-Statistics course is influenced by the theory that instructional design should try to eliminate "extraneous cognitive load" (Lovett, Meyer, & Thille, 2008, p. 6), or tasks and information that are unnecessary to learn a concept or skill. Therefore, the course explains statistics concepts through animations that are accompanied by verbal explanations so that students do not have to process separate visual and text-based explanations of statistics concepts (Lovett et al., 2008).

Computer-based learning is also a form of mastery learning (Hagerty & Smith, 2005; Trenholm, 2006). Under the mastery learning approach, course content is divided into small units, and students must demonstrate mastery of one unit before they can move on to the next unit (Kulik, Kulik, & Bangert-Drowns, 1990). The mastery learning approach is thought to be more effective than traditional instruction because it is tailored to each student's needs; students only work on material they are ready to learn (Kulik et al., 1990). Computer-based instructional programs allow instructors to efficiently provide their students with a mastery learning experience through individualized learning programs that monitor student progress and adjust the content delivered accordingly (Hagerty & Smith, 2005).

**Empirical evidence.** The evidence regarding the impact of computer-based learning is inconclusive because none of the 13 studies in this set (see Table 1) employed a rigorous research design. Lovett et al. (2008) randomized students in their study on the impact of the OLI-Statistics course, but because the treatment condition is an eight-week OLI-Statistics course and the comparison condition is a 15-week traditional course, it is impossible to disentangle the effects of the OLI pedagogy from the possible effects of acceleration. In other words, the eight-week structure of the course may have contributed to the outcomes either by motivating students or through another mechanism. The highest quality study used a quasi-experimental design and found that college algebra students in course sections using ALEKS (Assessment and Learning in Knowledge Spaces), a computer-adaptive, online assessment and learning program, experienced small gains in math learning compared to students in traditional algebra course sections (Hagerty & Smith, 2005). However, the final results only include students who took both the pretest and the posttest, not accounting for the test scores of students who enrolled late or withdrew from the course. As a result, differential attrition could have biased the results.

**Computer-Based Learning in Practice**

Cleveland State Community College redesigned its developmental and college math courses by replacing three hours per week of class time with one hour per week in a computer classroom with faculty and two hours per week in a large computer lab (Squires, Faulkner, & Hite, 2009). Students use MyMathLab to work through 10–12 curriculum modules on their own by watching an instructional video, completing homework, and then passing a quiz. Based on the mastery learning approach, to move to the next module, students must earn at least a 70% on the homework and quiz for each module. Faculty teach 10 math sections each and spend 10 hours per week in the computer lab providing students with one-on-one attention and individualized instruction. The Community College Futures Assembly awarded Cleveland State the Bellwether Award in the Instructional Programs and Services category for its redesigned math courses.

## 3. Research Recommendations

Evidence from this literature review supports the National Mathematics Advisory Panel’s (2008) conclusion that there is a crucial need for more methodologically rigorous scientific research in the area of effective math instructional practices. All of the pedagogical practices discussed in this review may have the potential to improve the outcomes of developmental math students, but given the poor internal validity of many of the studies, it is difficult to infer whether most of the pedagogy is effective in practice. The following are four methodological recommendations that could improve the internal validity of future research on developmental math instruction. The principal issues and patterns of findings that emerged from this review also highlight three directions for research on developmental math pedagogy.

### 3.1 Methodological Recommendations

Since it is difficult to conduct randomized experiments in K-12 and college settings, researchers should collect information on student abilities and demographics and control for any observable differences between groups using statistical methods. Doing so would provide more convincing evidence that differences in outcomes are due to the instructional intervention rather than preexisting differences between treatment and control students. Second, it is recommended that, if the treatment takes place across multiple classrooms, participating instructors teach both a treatment section and a control section, and that instructors be similar along observable measures of teacher quality. This would help ensure that the effects of an instructional practice can be more credibly disentangled from the impact of individual instructors.

Third, multiple-choice math tests and other standardized assessments are often used to study the effects of reform-based math instructional practices, but traditional assessments may neglect to measure the types of skills that reform-based pedagogy promotes (Hamilton et al., 2003). However, assessments that are designed specifically for a study to measure the impact of a single pedagogical practice may not be fair outcome measures if they only emphasize the skills and knowledge that were taught in the treatment classrooms. It may be most appropriate, therefore, to use multiple outcome measures that include standardized and thoughtfully designed alternative measures of math achievement and learning.[9]

Finally, in higher education research, it is important to consider that even if students are unaware of which course sections have been assigned to the treatment or control group, course scheduling can influence the types of students that register for each course, and the characteristics on which these students differ may be related to educational performance. Therefore, offering treatment and control sections at similar times reduces the likelihood that any outcomes are partly determined by student characteristics related to the time of day the course sections are offered.

### 3.2 Directions for Future Research

Conducting rigorous evaluations of computer-based learning in developmental math is essential to furthering our understanding of how this popular developmental mathematics reform affects student math learning, persistence, and other outcomes. Anecdotally, some community colleges have experienced improved pass rates and persistence for developmental math students after the introduction of computer-based instruction and course redesign (see, e.g., Speckler, 2008; Squires et al., 2009; Twigg, 2005). But many questions remain regarding exactly how these new models of developmental education are connected to observed outcomes. Course redesign models involve substantial changes to the way course content is delivered by replacing some or all of the traditional course structure with self-paced online learning modules (Epper & Baker, 2009; Twigg, 2003). As a result, outcomes may be due to any number of changes in how course content is delivered, when students can access course content, and the pedagogy utilized in each model. For example, in some models, students are able to access course content at any time from home or in a large computer lab, while in others, they must work through the content during structured lab times (Twigg, 2003, 2005). Regarding pedagogy, there is variation in how much time students spend working

[9] The midterm and final exams in Chappell's (2006) study are examples of well-designed outcome measures. A faculty member not involved in the study designed the midterm and final that tested knowledge and skills that were covered in both the control and treatment classrooms, and six other faculty members not involved in the study assigned each item to the procedural skill subscale or conceptual understanding subscale. Then, the four instructors in the study designed a rubric and graded the exams together, ensuring that the students in the treatment and control sections were assessed fairly.

through the course material on their own, and some models emphasize individualized attention from instructors or tutors and small-group work more than others (Twigg, 2003, 2005). Differences in pedagogy also extend to the type of instructional software used (Hung, 2001): some computer-based course content may allow for investigative problem solving or discovery-based learning, while other software programs rely on drill-and-practice problems.[10] Finally, all of these components of course redesign may impact students in different ways, and they may even have differential effects for different types of students. Future research should aim to isolate the effects of different components of course redesigns or to assess the possible differential effects of this reform on student subgroups.

Next, unlike the studies in the application and computer-based learning sets, which typically take place in colleges and universities, the studies in the other sets target diverse student populations, including elementary, secondary, developmental education, and college students as well as students with learning disabilities at each of these educational levels. Across all these sets, there are slightly larger effect sizes from high-quality studies whose target population is students with learning disabilities compared to high-quality studies whose target population is typical students, and within rigorous studies that compare outcomes between subgroups of students, effect sizes are slightly larger for lower-achieving students or students with learning disabilities. It is important to note that many of studies focusing on students with learning disabilities are by the same set of authors (i.e., Fuchs et al.). Nonetheless, this pattern may be indicative of the importance of pedagogy that utilizes cooperative learning, teaches students to monitor their understanding as they solve problems, improves problem representation, and involves ongoing assessment and the adjustment of instruction for students struggling with mathematics. Therefore, a direction for qualitative and quantitative research could

[10] For example, in a randomized study, Campuzano, Dynarski, Agodini, and Rall (2009) compared the effects of different software products (PLATO, Larson, and Cognitive Tutor) on middle school math achievement. PLATO is described as relatively behaviorist because it includes only independent practice on procedural skill building, while the Larson products address both skill building and problem solving, and Cognitive Tutor requires students to use graphs to represent and solve problem scenarios. None of the products had significant effects on student math achievement, and standard errors were large, such that one cannot say with confidence that their effects were negative, zero, or positive. However, this study still presents a model for research that should be performed at the higher education level.

be a more focused examination of how these types of pedagogy could be effective in building the foundational skills that adult students need to move beyond basic math and into more advanced math subjects.

A final priority for developmental education research is designing and investigating the impact of more balanced instructional approaches that promote all strands of mathematical learning. This is especially important since the Mathematical Association of America (MAA) and American Mathematical Association of Two-Year Colleges (AMATYC) recommend replacing traditional college algebra courses with modeling-based college algebra courses, in which students solve problems situated in real-world contexts by creating and interpreting mathematical models (Katz, 2007). However, while the studies in the application set do not suggest anything conclusive about the effects of this type of instruction, they do consistently suggest that application-oriented instructional approaches may support some strands of mathematical proficiency but do not improve procedural fluency. A challenge for researchers and practitioners is to develop modeling-based approaches that improve students' math understanding as well as their performance on traditional standardized tests of mathematics achievement.

## 4. Instructional Recommendations

There are a number of studies in the student collaboration and problem representation sets that employed rigorous designs with positive results, and, therefore, adaptation and evaluation of these pedagogical practices ought to be considered for the developmental mathematics classroom.

### 4.1 Structured Student Collaboration

Structured peer-collaboration methods for developmental and college math students are already taking place outside the classroom with programs like Peer-Led Team Learning (Hooker, 2010) and Math Excel, workshops where students work together collaboratively on math problems that reinforce or review material that is covered in their regular course (Dick, 2003). However, it is unknown how prevalent highly structured peer-collaboration methods are in the developmental math classroom.

Many instructors may use cooperative learning in informal ways, but theory and research suggest that cooperative learning may not be effective unless it is formally and systematically integrated into a course. According to social interdependence theory, positive educational outcomes are the result of engagement in frequent, meaningful interactions with others for the purpose of working toward a common goal (Johnson et al., 1991), and the rigorous student collaboration studies found that students benefited from cooperative learning methods in which all students played a role in working toward a shared goal.

Applications of structured student collaboration in developmental math include collaborative problem-solving activities that have a group grade tied to them. Part of the final grade in a course may even include group performance on collaborative activities. A specific example of a more formal student collaboration activity comes from Dees's (1991) study. In the developmental math lab sections, groups of four to six students received only parts of the instructions to a problem, and then students shared with their group the information they received. The group had to work together to understand the problem instructions and then solve the problem. At the end of the activity, one group member was randomly chosen to explain the group's solution, and the group's grade was based on this explanation, so group members had to collaborate to ensure everyone in the group understood the solution steps and final answer.

Activities like the one from Dees's classroom—those that ensure that each student has a role in accomplishing a task with a group grade—could be used to supplement more traditional instructional practices in the developmental math classroom and used in combination with other alternative pedagogical practices. For instance, the use of ongoing assessment can help determine the specific areas where students need more practice, so instead of conducting a whole-class review of the material, an instructor could group students together to work on only the specific areas with which they are struggling. Requiring students to explain out loud how they arrived at their solution may help them start thinking about their own mathematical thinking, thereby incorporating a metacognitive framework into their problem-solving process. Exercises could also require students to represent the problem situation in several ways in order to develop their problem representation skills.

### 4.2 Improving Problem Representation

There were no developmental education studies that focused on problem representation, but rigorous studies involving other student populations demonstrate how instructors can improve student learning by integrating problem representation instruction into their lectures, interactive board work, and other traditional modes of instruction. For example, in a study by Brenner et al. (1997), pre-algebra students were taught and asked to represent problems using graphs, diagrams, tables, pictures, and equations and to solve problems using multiple representations. Similarly, in a study by Chappell (2006), faculty routinely represented calculus concepts numerically, algebraically, and graphically and solved them using these representations in their lectures, and students were expected to do the same on homework and assessments. Evidence from both of these studies suggests that improving students' problem representation skills is a promising teaching strategy for improving math learning outcomes.

Developmental math instructors may want to consider modeling problem situations numerically, algebraically, and graphically in their lessons and expecting their students to represent and solve problems in multiple ways on homework and assessments. This would require more time devoted to lesson preparation, a change in the content of homework and assessments, and lessons that spend more time on each concept, which may reduce the time spent working on procedural fluency through solving equations (Brenner et al., 1997). It would be useful for researchers to design and evaluate balanced instructional approaches that are able to effectively teach all strands of mathematical proficiency, but individual instructors can also experiment with how to find ways to focus on problem representation while still providing students with practice on traditional equation solving.

## 5. Summary of Recommendations

To summarize, identifying effective pedagogical practices in developmental math could potentially lead to improved student learning outcomes and, ultimately, improved rates of course completion and persistence. However, more rigorous research in the area of developmental math education is needed in order to confirm that certain practices that seem promising are indeed effective in the classroom. First, evaluations of course redesign in developmental mathematics should attempt to separate the effects of the structural and instructional components of the various models of computer-based learning, examine how different types of students respond to reform, and consider the possible differential impact of instructional software that emphasizes procedural fluency and software that emphasizes mathematical understanding and application. Future research should also explore how cooperative learning, ongoing formative assessment, and strategies that encourage self-reflection and problem representation skills help developmental education students build foundational math skills. Finally, research should develop and test balanced instructional methods that have an impact on all strands of mathematical proficiency.

In the meantime, the literature yields support for a few recommendations that instructors may find immediately useful. For example, instructors should consider using structured collaborative problem-solving activities in which each group member has a role in working toward a group product or answer. Second, instructors should consider representing the same problem numerically, graphically, and algebraically in their lessons on a routine basis. Students can then be expected to represent problems in multiple ways on in-class exercises, homework, and assessments.

Currently, the academic outlook for students who enroll in developmental math courses is generally unfavorable. Improving outcomes for developmental math students will require the continued efforts of researchers and practitioners. The payoff for those efforts may be significant, since bringing more effective pedagogy to the developmental math classroom could have profound effects on academic outcomes and job attainment for developmental math students.

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## Appendix

**Table 1**
**Review of Math Pedagogy Studies**

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| *Student Collaboration* | | | | | | | | | | | |
| Baxter, Woodward, & Olson (2001)<br>*Qualitative* | X | | | | Whole-class discussions were difficult for the low-achieving students to follow, but these students were more engaged during pair work, especially when they worked with an average or high-ability peer. However, close observation of pair work interactions revealed low-achieving students primarily copying their partner's work or managing the materials. | N/A | N/A | | | | X |
| Beirne-Smith (1991) | X | | | | Students with disabilities who were randomly assigned to work with peer tutors using two different instructional methods performed better on an addition facts assessment than the control group. | 0.75 | Moderate positive | X | X | | |
| Calhoon & Fuchs (2003) | X | | | | PALS and CBM show promise for improving the computational skills of secondary students with disabilities. However, since the PALS and CBM interventions were implemented in combination, it is impossible to attribute the outcomes strictly to CBM. | -0.29 to 0.40 | Trivial negative to small positive | | | X | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Dees (1991) | | | X | | Students in the cooperative learning sections performed significantly better on the algebra word problem and geometry proof-writing sections than students in the traditional sections. | 0.39 to 0.56 | Small positive | | X | | |
| DePree (1998) | | | X | | Latino and female students in cooperative learning classes had positive gains in self-reported math confidence relative to the control group, but there were no differences in the achievement gains of treatment and control groups. | 0.45 to 0.72 | Small to moderate positive | | X | X | |
| Duncan & Dick (2000) | | | | X | The Math Excel students attained significantly higher grades than the non-Math Excel students, and students in the Math Excel Program outperformed their predicted grades, as determined by their SAT Math score, by half a grade point. | N/A | Moderate positive | X | X | X | |
| Fuchs et al. (1995) | X | | | | Students with disabilities and average-achieving students in the PALS groups outperformed control group students on operations and application tests, while low-achieving students outperformed their counterparts only on the math operations test. | 0.07 to 0.95 | Trivial to moderate positive | | | | |
| Fuchs et al. (1997) | X | | | | Students with disabilities and low-achieving students in the peer mediated instruction (PMI) groups outperformed their peers in the control group, and when tasks that emphasized understanding were added to the PMI treatment, both average- and high-achieving students made greater gains than their counterparts in the control group. | 0.18 to 1.15 | Trivial to moderate positive | | | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Fuchs et al. (2001) | X | | | | Medium- and low-achieving students and students with disabilities in the peer-assisted learning strategies (PALS) group outperformed the control group, but growth was higher for high-achieving control group students than high-achieving PALS students. | -0.20 to 0.53 | Small negative to small positive | | | | |
| Ginsburg-Block & Fantuzzo (1998) | X | | | | Peer collaboration methods had a positive impact on the computational and word problem skills, academic motivation, and self-concept of third- and fourth-grade students. (Peer collaboration and problem solving methods were not significantly more effective in combination than implemented separately.) | 0.29 to 0.36 | Small positive | | | | |
| Hooker (2010) | | | X | | Students in pre-algebra classes that used peer-led team leader (PLTL) workshops had higher persistence and completion rates than control group students. | N/A | Trivial to small positive | X | X | X | |
| Karper & Melnick (1993) | X | | | | There were no significant differences between students using Team Accelerated Instruction and students in comparison classrooms at any grade level on scores of math aptitude, concepts, and computations. | 0.00 | Trivial positive | | | | |
| Keynes & Olson (2000)<br><br>*Descriptive* | | | | X | Students in the Calculus Initiative classrooms had higher GPAs, pass rates, and retention rates than students in traditional calculus. | N/A | Small positive | X | X | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Norwood (1995) | | | X | | Compared to students who took Developmental Algebra in the semester when instructors used traditional methods, a higher proportion of students who took Developmental Algebra in the semester that the instructors used the Learning Model of cooperative learning completed their first college-level math course. | N/A | Small positive | X | | | |
| Summers & Svinicki (2007) | | | | X | Students in the cooperative learning classrooms reported significantly more motivation for mastery and perceived more interactive learning and classroom community but reported significantly less performance motivation than students in the traditional classrooms. | -0.65 to 0.37 | Moderate negative to small positive | X | X | X | |
| *Metacognition* | | | | | | | | | | | |
| Abdalkhani & Menon (1998) | | | X | X | In earlier courses in which journal writing was not used in math, students had a mean score of 65% on quizzes, while studends in the course that incorporated journal writing had a mean quiz score of 72%. | N/A | Trivial positive | X | X | | |
| Hiebert & Wearne (1993) | X | | | | The opportunity for students to explain, describe, and question their learning contributed to higher gains in achievement in classrooms that used classroom discourse versus classrooms that did not. | 0.5 to 1.5 | Moderate to large positive | | X | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Hohn & Frey (2002) | X | | | | The SOLVED method, which stands for "state the problem, options to use, links to the past, visual aid, execute your answer, and do check back," had a positive impact on third-, fourth-, and fifth-grade students' math performance compared to control group students. | 0.45 to 0.93 | Small to moderate positive | | X | | |
| Porter (1996) | | | | X | There were no differences in the number of procedural errors made by students in a college calculus course that used writing to learn math and students in a comparison course on a final exam, but students in the treatment group made more conceptual errors. | -0.63 to 0.09 | Moderate negative to trivial positive | X | X | | |
| Pugalee (2001)<br>*Qualitative* | | X | | | A metacognitive framework emerged in high school students' writings about their problem-solving processes. | N/A | N/A | | | | X |
| Pugalee (2004)<br>*Qualitative* | | X | | | Writing their problem solving process was more beneficial for a group of 20 high school math students than verbalizing their problem-solving process. | N/A | N/A | | | | X |
| Schurter (2002) | | | X | | Students who received direct instruction in the use of comprehension monitoring or Polya's four-step problem-solving method performed better in mathematical problem solving than those who did not. | N/A | Small positive | | X | | |
| Tournaki (2003) | X | | | | Students with and without learning disabilities who received strategy instruction in verbalizing the problem-solving process improved much more on an addition facts test than students who received drill-and-practice instruction. | 0.10 to 1.58 | Trivial to large positive | | | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| *Problem Representation* | | | | | | | | | | | |
| Brenner et al. (1997) | X | | | | Pre-algebra students who received instruction in problem representation were better able to create and apply multiple representations than control students at significant levels, but the control students did significantly better on a test of symbol-manipulation skills. | -0.21 to 0.91 | Small negative to moderate positive | | | | |
| Chappell (2006) | | | | X | Students in the concept-based calculus sections scored significantly better on the midterm and final exams than the students in the traditional sections, except for on the final procedural skill section. | 0.34 to 0.64 | Small to moderate positive | | | | |
| Fuson & Briars (1990) | X | | | | The addition and subtraction performance of the second graders in learning/teaching approach classrooms was above that reported for typical third graders. | N/A | Very large positive | X | | X | |
| Jitendra et al. (1998) | X | | | | Elementary students with or at risk for mild learning disabilities who received schema-based instruction, direct instruction in using schematic diagrams and multiple solution strategies, improved their word problem performance more than students in the control group. | 0.57 to 0.81 | Small to moderate positive | | | | |
| Jitendra et al. (2009) | X | | | | A diverse group of high- and low-ability students who received schema-based instruction improved their understanding of ratio and proportion more than the control group. | 0.45 to 0.56 | Small positive | | | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Silver & Stein (1996)<br><br>*Descriptive* | X | | | | Test performance and number of students eligible for ninth-grade algebra increased, and QUASAR students performed significantly better on NAEP compared to a similar sample of urban, low-SES students. | N/A | Small positive | | | | X |
| Stein, Grover, & Henningsen (1996)<br><br>*Qualitative* | X | | | | QUASAR students were observed using multiple strategies and representations to solve and explain their solutions but had difficulty maintaining a high level of cognitive processing during many of the challenging tasks. | N/A | N/A | | | | X |
| Witzel et al. (2003) | X | | | | Students with disabilities and at-risk students who received concrete-representational-abstract instruction showed greater improvements in their performance on single- and multiple-variable algebra equations than similar students receiving traditional instruction. | 0.52 to 0.87 | Small to moderate positive | | | | |
| Zawaiza & Gerber (1993) | | | | X | Community college students with learning disabilities who received a schema-based intervention made greater gains on a word problem test than those who did not, and they performed at almost the same level as their math-competent peers. | 0.27 to 1.11 | Small to large positive | X | X | X | |
| *Application* | | | | | | | | | | | |
| Bottge et al. (2001) | X | | | | The students in the remedial Enhanced Anchored Instruction (EAI) class matched the performance of students in the pre-algebra classes on the problem-solving and maintenance test but not on the computation test. | N/A | Trivial positive | X | X | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Bottge et al. (2002) | X | | | | Students in classrooms using EAI performed better than the group receiving traditional instruction on a video problem test but not on a computation and word problem test. | 0.11 to 0.86 | Trivial to moderate positive | X | X | | |
| Brenner (1998) | X | | | | Native Hawaiian kindergarten students exposed to culturally responsive math instruction for a year performed better, on average, on the standardized math assessment than kindergarten students in the same class the year before. | N/A | Small | X | | | |
| Ellington (2005a) | | | | X | Students in modeling sections had higher levels of self-reported confidence, lower levels of anxiety, and lower withdrawal rates than students in traditional sections. | N/A | Trivial to small positive | X | X | | |
| Ellington (2005b) | | | | X | Students in modeling sections performed significantly better on an assessment and had higher pass rates than students in traditional sections. | 0.41 | Small positive | X | X | | |
| Ganter & Jiroutek (2000) | | | | X | Calculus sections that utilized long-term projects in the computer lab did not perform better than the control sections on the final exam. On the standardized exam the control group outperformed the treatment group. | N/A | Trivial negative | | X | | |
| Hickey, Moore, & Pelligrino (2001) | X | | | | From third to fifth grade, students in the reform-oriented Jasper videodisc classrooms had the largest gains on the problem-solving and conceptual sub-tests and the largest decline on the computation sub-test, compared to students in comparison classrooms and non-reform-oriented treatment classrooms. | N/A | Trivial negative to trivial positive | | | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Hollar & Norwood (1999) | | | | X | Scores on the function test were higher for the graphing-approach group than for the control group, but there were no significant differences in terms of final exam scores (scores not reported). | 1.02 | Large positive | X | X | | |
| Lipka & Adams (2004) | X | | | | Teachers were randomly assigned to use culturally relevant pedagogy or traditional instruction with their sixth-grade Yu'pik students, and students in the treatment classrooms had higher gains in understanding the concepts of perimeter and area than students in the control group. | 0.44 to 0.63 | Small to moderate positive | | X | | |
| O'Callaghan (1998) | | | | X | The Computer-Intensive Algebra students scored higher than students in the traditional algebra classrooms on the function test, and there were no significant differences in final exam performance (scores not reported). | 0.86 to 1.07 | Large positive | | X | X | |
| Vasquez (2004)<br>*Descriptive* | | | X | | A greater percentage of students passed higher level math courses after the introduction of the Algorithmic Instructional Technique. | N/A | Trivial to small positive | | X | | X |
| *Understanding Student Thinking* | | | | | | | | | | | |
| Adams (1997)<br>*Qualitative* | | | | X | The introduction of graphing calculators into the classroom improved an instructor's assessment methods and understanding of student learning. | N/A | N/A | | | | X |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Allinder et al. (2000) | X | | | | Students with disabilities performed better on a test of computational ability in a classroom where teachers used curriculum-based measurement (CBM) plus self-monitoring of their instructional practices than students of teachers who only used CBM, and both treatment groups performed better than the control group. | 0.35 to 0.92 | Small to moderate positive | X | | X | |
| Boylan & Saxon (1998) *Descriptive* | | | X | | Institutions with exceptional developmental education programs based on observation and developmental education pass rates reported using frequent testing in their developmental education classrooms. | N/A | N/A | | | | X |
| Calhoon & Fuchs (2003) | X | | | | PALS and CBM show promise for improving the computational skills of secondary students with disabilities. However, since the PALS and CBM interventions were implemented in combination, it is impossible to attribute the outcomes strictly to CBM. | -0.29 to 0.40 | Trivial negative to small positive | | | X | |
| Fabry et al. (1997) *Qualitative* | | | X | | Students self-reported that the classroom assessment technique improved their perceptions and attitudes about learning. | N/A | N/A | | | | X |
| Fuchs & Fuchs (1990) | X | | | | Students with disabilities who had teachers who used CBM and skills analysis (which allows teachers to analyze proficiency in specific skills) performed somewhat better on a computation test than students with disabilities whose teachers used only CBM, and both treatment groups outperformed the control group. | 0.28 to 0.67 | Small positive | | | | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Fuchs et al. (1991) | X | | | | Students with disabilities who had teachers who used CBM with expert instructional recommendations made greater gains on an operations test than students in the control group. | 0.84 to 0.94 | Moderate positive | | | | |
| Fuchs et al. (1994) | X | | | | Students with disabilities who had teachers who used CBM with instructional recommendations made slightly greater gains on an operations test than students with disabilities whose teachers used only CBM, and both treatment groups outperformed the control group. | 0.16 to 0.43 | Trivial to small positive | | | | |
| Nunnery & Ross (2007) | X | | | | Students in Accelerated Math (AM) classrooms did significantly better on standardized assessments than students in comparison classrooms. | 0.17 to 0.22 | Trivial to small positive | | X | | |
| Siadat et al. (2008) | | | X | X | Students in the Keystone Method classes had higher final exam scores and persistence rates than students in control classes. | N/A | Small to large positive | X | X | | |
| Villasenor & Kepner (1993) | X | | | | Students in the cognitively guided instruction classrooms outperformed students in the control classrooms. | 3.55 to 5.44 | Very large to nearly perfect positive | X | | | |
| Ysseldyke & Bolt (2007) | X | | | | AM classrooms performed significantly better on the STAR Math test compared to control classrooms, but there were no significant differences between the two groups in Terra Nova performance. | 0.37 | Small positive | | | X | |

| Study and Design[1] | Target Students: K-8 | Target Students: High School | Target Students: Developmental Education | Target Students: College Level | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4]: Student Design Non-Equivalent | Common Empirical Flaws[4]: Teacher Design Non-Equivalent | Common Empirical Flaws[4]: Possible Attrition Issues | Common Empirical Flaws[4]: No Comparison Group |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ysseldyke & Tardrew (2007) | X | X | | | AM third- to sixth-grade classrooms performed significantly better on standardized assessments than comparison classrooms. There were no significant findings for seventh- to tenth-grade classrooms. | 0.18 to 0.57 | Trivial to small positive | | X | | |
| *Computer-Based Learning* | | | | | | | | | | | |
| Garcia (2003) | | | X | | Pre- to post-test mean ACCUPLACER scores increased for students in an elementary algebra class with learning style and attitude surveys, workshops, computer-based instruction, self-assessment, and supplemental instruction. | N/A | Small positive | | | X | X |
| Hagerty & Smith (2005) | | | | X | The students using ALEKS had higher gains from pre- to post-test than the students in the traditional classrooms. | 0.49 | Small positive | | | X | |
| Lovett et al. (2008) | | | | X | Students randomly assigned to an eight-week accelerated OLI-Statistics hybrid course (in which students met with the instructor to review and reinforce material) performed better on the final exam than students randomly assigned to the traditional 15-week statistics course. | N/A | Small positive | X | X | | |
| McClendon & McArdle (2002) | | | X | | Retention was higher in the lecture mode of instruction versus ALEKS and Academic Systems. | N/A | Trivial to moderate negative | X | X | X | |
| O'Dwyer et al. (2007) | X | X | | | Algebra students in eighth and ninth grade using a hybrid online learning model scored slightly higher on an algebra test at the end of the year. | 0.13 | Trivial Positive | | | X | |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Speckler (2008)<br>*Descriptive* | | | X | X | At 18 colleges, retention and pass rates, course enrollments, and/or grades were generally higher for students in sections that used MyMathLab or MathXL, with a few exceptions. | N/A | Small to moderate positive | | | | X |
| Squires et al. (2009)<br>*Descriptive* | | | X | | At Cleveland State Community College, after the introduction of a course redesign model, course completion rates in developmental math and subsequent college-level math courses increased. | N/A | Small | | | | X |
| Stillson & Alsup (2003)<br>*Qualitative* | | | | X | A higher percentage of students failed the course in the semester that ALEKS was introduced than in previous semesters. Students in the study reported that ALEKS in combination with group work, the lectures, and individual assistance from the instructor was helpful in learning algebra. | N/A | Trivial to small negative | X | X | | |
| Taylor (2008) | | | X | X | The control group made larger gains from pre- to post-test than the ALEKS group on the algebra test, but self-reported math anxiety decreased more for the ALEKS group than for the control group, and self-reported attitudes about math improved for the ALEKS group and worsened for the control group. | -0.21 to -0.12 | Small negative | X | X | | |
| Twigg (2005)<br>*Qualitative/ Descriptive* | | | X | X | Different outcomes were examined at 30 selected institutions, with in-depth case studies on 15 institutions. Colleges that redesigned their math courses reported increases in retention, math learning, and course pass rates and decreased cost per student. | N/A | N/A | | | | X |

| Study and Design[1] | Target Students | | | | Summary of Findings | Effect Size[2] | Effect Category[3] | Common Empirical Flaws[4] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | K-8 | High School | Developmental Education | College Level | | | | Student Design Non-Equivalent | Teacher Design Non-Equivalent | Possible Attrition Issues | No Comparison Group |
| Waycaster (2001)<br><br>*Qualitative/ Descriptive* | | | X | X | The success rates in 15 developmental math courses were not related to the method of instruction (lecture or Computer-Aided Instruction). | N/A | Trivial negative | X | X | | |
| Zavarella & Ignash (2009) | | | X | | 20% of students in the lecture-based course, 42% of students in the hybrid course, and 39% of students in the online course withdrew. | N/A | Small negative | X | X | | |
| Zhu & Polianskaia (2007) | | | X | | Over most years in a ten year period, a higher percentage of students in lecture courses had higher pass rates, course completion rates, and final exam scores than students in computer -mediated courses. | N/A | Trivial to small negative | X | X | | |

Notes 1-4 appear on the next page.

## Table 1 Notes

### 1. Study and Design

Unless otherwise noted, the study is a quantitative study that used a quasi-experimental or randomized design. Descriptive studies usually did not utilize comparison groups but reported frequency outcomes of students in existing programs. Qualitative studies usually involved observation of students in some program or instructional environment, and most did not utilize a comparison group.

### 2. Effect Size

Effect sizes are used to compare results across studies with different outcome measures and are also important in measuring meaningful changes in outcomes that may not necessarily be statistically significant. The table reports Cohen's *d* effect sizes, which can be interpreted as the standardized difference between the treatment and control group means. A positive *d* indicates that the treatment group had superior outcomes; a negative *d* indicates that the comparison group had superior outcomes. When *d* was not reported in a given paper, the effect size was calculated using means and standard deviations for the treatment and control groups as provided in the article, using the following formula, where *x* is the mean, *n* is the sample size, *s* is the standard deviation, and the subscripts *t* and *c* denote treatment and control:

$$d = \frac{\overline{x}_t - \overline{x}_c}{\sqrt{\dfrac{(n_t - 1)s_t^{\,2} + (n_c - 1)s_c^{\,2}}{n_t + n_c}}}$$

If the means and/or standard deviations were not provided but the *F*-statistic was, Cohen's *d* was calculated using *F* and the sample sizes of the treatment and control groups:

$$d = \sqrt{F\left(\frac{n_t + n_c}{n_t\, n_c}\right)\left(\frac{n_t + n_c}{n_t + n_c - 2}\right)}$$

### 3. Effect Category

For articles that do not report the information to necessary calculate Cohen's *d*, "N/A" is written in the effect size column. However, some of these articles report group percentages or correlations that allow for an estimation of their effect size category. Hopkins (2009) proposes a scale that can be used to compare traditional effect size estimates to differences in percentages. The following scale was used to compare effect size estimates across various types of effects, in order to categorize them by size:

| Measure | Trivial | Small | Moderate | Large | Very Large | Nearly Perfect |
|---|---|---|---|---|---|---|
| Correlation coefficient (*r*) | 0–0.1 | 0.1–0.3 | 0.3–0.5 | 0.5–0.7 | 0.7–0.9 | 0.9–1.0 |
| Standardized difference in means (*d*) | 0–0.2 | 0.2–0.6 | 0.6–1.2 | 1.2–2.0 | 2.0–4.0 | 4.0–∞ |
| Percentage difference | 0–10 | 10–30 | 30–50 | 50–70 | 70–90 | 90–100 |

### 4. Common Empirical Flaws

Four common methodological flaws were identified across the studies. Studies that did not have any of these flaws are considered rigorous. Below is an explanation of each empirical flaw.

**Student design non-equivalent.** The most common weakness, especially of the quantitative studies whose target population is developmental or college-level students, is a non-equivalent student design. Most of these studies allowed students to self-select into treatment and control groups but neglected to collect pre-treatment ability and demographic information on the student participants. As a result, any significant differences in outcomes could be due to the preexisting differences between the treatment and control students rather than the instructional intervention. Another common research design is to compare the outcomes of students who received the treatment to the outcomes of students from previous years or semesters, when traditional instructional practices were in use. However, this comparison introduces time-varying characteristics that are not controlled for and, therefore, could explain any differences in outcomes. Other studies do collect pretreatment information but do not use it to conduct a rigorous analysis of group equivalence. For example, these studies do not adjust post-treatment test performance for pre-treatment test performance, or they do not test for statistical differences between treatment and control group pre-test scores. Finally, some articles do not report pre-test scores, but the author claims that they revealed no significant differences between the treatment and control groups. This lack of transparency raises doubts about the authors' claims and the internal validity of these studies.

**Teacher design non-equivalent.** The second flaw, teacher design non-equivalency, could arise from a number of different study features. First, most of the reviewed empirical studies do not describe how teachers were assigned to the treatment and comparison groups and do not report characteristics of teachers who volunteered for the treatment and control sections. These studies disregard the influence that the underlying characteristics of individual teachers have on educational outcomes; variability in teacher characteristics could confound the relationship between the intervention and any outcomes. Second, even in a few randomized and quasi-experimental studies, treatment teachers demonstrated intrinsic motivation to improve their teaching by volunteering for the treatment group. That they possessed intrinsic motivation can be inferred from the fact that the treatment required them to attend training. Therefore, differences in instructional quality or motivation may be responsible for the outcomes of the study rather than the treatment. Finally, in a number of studies, the researcher was also the instructor of the treatment and/or control group(s), implying an unconscious or maybe even a conscious investment in ensuring the classroom intervention is effective, which calls into question the researcher's impartiality in the implementation of the treatment and analysis of the results.

**Possible attrition issues.** Many studies do not address the substantial attrition that occurred over the course of the study. High attrition is common in studies at the developmental education level, where course dropout rates are high. If the attrition is more pronounced for students in a treatment or control group, it could bias the results of the study. For example, if lower-performing students were more likely to drop out of the control group, then this could have biased downward the impact of the treatment, but if lower-performing students dropped out of the treatment group at higher rates, then this could have inflated the impact of the treatment (for an illustration of differential attrition, see Figure 1 in Jaggars [2011]).

**No comparison group.** A comparison group provides information about what would have happened if the students did not receive the instructional intervention. It is not possible to attribute any observed outcomes to the instructional intervention when it is unknown if the student outcomes would have been similar, better, or worse in the absence of the treatment. Most of the quantitative studies employ a comparison group, while most of qualitative and descriptive studies do not. These studies provide important descriptive information about the outcomes of an instructional intervention or qualitative data about the challenges individual students may face when using alternative or reform-based math pedagogy, but they do not provide evidence of a causal link between the instructional practice or program and student outcomes.