Title:

Designing professional development around key principles and formative assessments to improve teachers' knowledge to teach mathematics

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Abstract Body

Background/context:
Proficiency in algebra correlates with future academic and economic achievement (Atanda, 1999; Berkner & Chavez, 1997), yet a great many students find success in algebra elusive (Ball, 2003; Helfand, 2006; Kollars, 2008; Meehan & Huntsman, 2004; Rubin, 2007; Silver, Saunders, & Zarate, 2008). Although it seems that developing this proficiency sooner rather than later is important, for many students this does not seem to be an option, in part, because they lack the understanding of the basic arithmetic concepts on which algebra will build (National Mathematics Advisory Panel, 2008).

At least one of the reasons cited for this lack of understanding and the difficulty students often encounter when transitioning from arithmetic to algebra is that many teachers of mathematics seldom build on students' prior pre-conceptions, understanding, intuition, or innate problem solving strategies (Donovan & Bransford, 2005). Instead they often present math as a set of rules, procedures and facts (Bransford, Brown, & Cocking, 1999); present mathematics in a seemingly random or disorganized manner; and divorce procedural knowledge from what the processes or results *mean* (Fuson, Kalchman, & Bransford, 2005). In fact, our experience and a good deal of research suggest that, in the courses that precede algebra, instruction in the United States primarily focuses on the learning and application of algorithms or "shortcuts" that students can use to solve a (*particular*) problem (Ma, 1999; Novak, Mintzes, & Wandersee, 2000; Saxe, Gearhart, & Seltzer, 1999; Sternberg et al., 2000).

Increasingly, research in three areas – expert versus novice learning, learning research in cognitive science, and research into the use of formative assessment – suggests how we might improve the way mathematics is taught.

Work as far back as deGroot (1965), and continuing through Chi and her colleagues (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi & Slotta, 1993) into the present (Fuson et al., 2005; Kaput, 2004; VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003), hints at the importance of helping teachers present a more "expert-like" conceptualization of domains like arithmetic. Expert-novice research emphasizes that a common distinction between experts and novices is that experts have well connected webs of knowledge formulated around key foundational ideas (e.g. natural selection in biology and plate tectonics in geology). Novices, on the other hand, organize domains around superficial aspects of a problem and approach problem solving by trying to recall a memorized algorithm based on those superficialities (Chi & Roscoe, 2002). It is apparent from our experiences that many elementary and middle school math teachers (and their students) organize their understanding as novices. Coupled with the literature, this suggests to us that organizing the primary and middle school mathematics domain around key foundational algebraic concepts, especially concepts already understood by students, would improve both teacher pedagogy and student success in algebra and in mathematics as a whole.

Cognitive science research also suggests the importance of conceptual understanding before learning algorithms or procedures if we want students to build on existing knowledge in order to abstract key principles (Kester, Kirschner, & vanMerriënboer, 2005; Rittle-Johnson & Alibali, 1999; Siegler, 2003; Vendlinski, 2009). Research suggests that many students have little understanding of what procedures mean or why they work (Fuson et al., 2005; Kilpatrick, Swafford, & Findell, 2001) and, consequently, struggle when required to abstract concrete representations to broader applications or situations. The literature and our research suggests that, in the United States, instruction prior to algebra focuses largely on the teacher transmission and student application of problem solving routines (American Educational Research Association, 2006; Zevenbergen, 2005). Often neither student nor teacher is aware of how or why the algorithm works, what the limitations of the algorithm might be, and how the algorithm is connected to concepts or knowledge the student has previously mastered (examples include "combine like terms" , "invert and multiply", and "whatever you do to one side, you do to the other") (Fuson et al., 2005; Ma, 1999; Vendlinski et al., In press). Here again, connecting the development and meaning of such procedures with prior knowledge seems key to improving both teacher and student understanding.

Finally, knowing what to teach the students in front of them requires that teachers accurately assess current student knowledge and understanding, provide appropriate feedback to the student and create or modify instruction to help move the student to the desired goal. The research literature is clear that the use of formative assessment can significantly improve student performance (Black, Harrison, Lee, Marshall, & Wiliam, 2003; Black & Wiliam, 2004; Black & William, 1998; Wiliam & Thompson, 2007). Yet our research just as clearly suggests that teachers have difficulty providing appropriate feedback to students and, depending on the topic, many teachers don't know how to use formative assessment results to modify instruction to promote student learning or proficiency in mathematics (Heritage, Kim, Vendlinski, & Herman, In press). Clearly, improving teacher understanding of student learning is key to improving student mathematical proficiency.

Based on this literature and experience, we see our professional development challenge as improving the ability of teachers to "help students build and consolidate prerequisite competencies, understand new concepts in depth, and organize both concepts and competencies in a network of knowledge." (Donovan & Bransford, 2005, p.232). While the ultimate measure of change in teacher quality for these efforts is improved student proficiency in algebra, the fact that such improvements will occur two or three years in the future and is mediated by a number of intervening teachers suggests a more proximal measure of teacher quality is required to determine the immediate effects of our professional development efforts.

Our conceptualization of what makes an effective teacher has important implications for how we improve and assess teacher quality. Among other things, a model based on the research described above suggests that merely evaluating a teacher's ability to recall procedures is likely to lead to inadequate or inaccurate (invalid) inferences about teacher quality. In fact, Nathan, Koedinger, and Martha (2001) and others (e.g. Shulman, 1986) have concluded that both content knowledge and pedagogical content knowledge must be assessed in order to make quality inferences about teachers. Still, such assessments of teacher knowledge have proven difficult (Tittle, 2006), although not impossible to develop (Hill & Ball, 2004). As Tittle (2006) points out, "assessment methods [in studies of teacher learning and development] are evolving from work within the expert / novice paradigm … and using analyses of content…" (p. 956). With this in mind, reserachers at the National Center for Research on Evaluation, Standards, and Student

Testing (CRESST) have built on the work of Leinhardt and Ball and their colleagues (see Ball, Lubienshi, & Mewborn, 2001; Leinhardt & Greeno, 1984; Leinhardt & Smith, 1985) to developed a means to measure teacher ability to conceptualize foundational ideas and to use information from formative assessment to plan or modify instruction (Heritage et al., In press).

Purpose/objective/research question/focus of study:
Our hypothesis is that if teachers (as experts) understand and teach concepts from the position of expertise teacher quality will improve. We believe that focusing on the key ideas will deepen both teacher and student understanding and allow learners to build the concepts necessary to form solid foundations for the application of mathematics both in and out of school. But such methods of instruction often require that teachers learn to reorganize and teach fundamental concepts in more expert-like ways. Because many teachers were taught in a system that stressed procedure rather than conceptual understanding, and the dearth of guidance on how teachers might change their teaching (Kieran, 2003) in this manner, such change may be difficult.

Building on the expert-novice literature (Chi et al., 1989), and the findings by Carpenter, Fenamya, Levi, Franke, and Empson (2000) using cognitively guided instruction, and research on formative assessment (Black et al., 2003; Herman et al., 2008; Marzano, 2006; Wiliam, 2006), we developed a program of professional development designed to help teachers conceptualize mathematics instruction and improve their ability to effectively use formative assessment data in the 6^{th} grade math classroom. Both the formative assessments and the professional development were designed around a collection of big ideas developed by content experts and math educators (Niemi, Vallone, & Vendlinski, 2006). The program of professional development supports a larger formative assessment effort as part of a US Department of Education sponsored program called PowerSource.

In particular, the professional development builds on three of the approximately twenty foundational concepts experts suggest are the most important aspects of the algebra domain. These three concepts are: 1) the multiplicative identity and its application to rational equivalency; 2) the understanding of multiplication as specifying a number of identical groups as applied in the case of distribution; and 3) the understanding of the equal sign as an equivalence operator in solving equations.

We combined the knowledge map of Leinhardt as refined by Chung and colleagues (Chung et al., 2006; Leinhardt & Smith, 1985) and a validated inferences from a student work task developed by Heritage and Vendlinski (Heritage et al., In press; Heritage & Vendlinski, 2006) to measure teacher quality (content organization and pedagogical ability). We used these as pre-post measures to determine changes in the teachers between the start and end of professional development in order to test our hypotheses and to measure the effects of professional development. These combined measures were designed to assess how each teacher organized a limited part of the domain of algebra and that teacher's ability to use student feedback in planning instruction.

Our ultimate goal is to demonstrate that the professional development methods we advocate improve student proficiency in algebra. Initially, however, we must demonstrate that our efforts actually improve near-term teacher content knowledge organization and have demonstrable effects on teacher ability to use formative assessment results. The results from these efforts are reported here. As next steps, we will observe actual teacher pedagogical practice and evaluate the proficiency of students in the classrooms of teachers who have actively participated in the PowerSource program.

Setting:
The 9.25 hours of professional development was conducted largely outside of school hours at the district office or at one of the school sites within each district. The initial four hours of PD was almost always done prior to the beginning of the academic year. In the two cases where this was not possible, substitutes were provided for teachers during the school day. All 90 minute follow-up sessions with the teachers were conducted in after school settings.

Population/Participants/Subjects:
Of the almost 125 teachers in our professional development program, 111 completed the knowledge mapping survey and 115 teachers completed and returned the evaluation of student work measure. The sixth grade teachers in this study worked in 25 public schools, in seven school districts in Southern California and Arizona. The teachers, on average, taught two classes of 25 students each.

Intervention/Program/Practice:
After an introductory professional development session, PowerSource© treatment teachers and researchers met for three more professional development sessions of approximately 90 minutes each. During the first 45 minutes of each session, teachers and researchers discussed student work on the formative assessments associated with a particular foundational concept, possible misconceptions identified by those assessments and possible instructional interventions to correct those misconceptions. The last 45 minutes of each session focused on another single “big idea” (the meaning of multiplication or the meaning of equality) and its application, how that big idea would be developed from its nascent form into abstract concepts in algebra, and how the big idea could be appropriately taught and applied to sixth grade subject matter. To aid teachers with their instruction, teachers were given an instructional handbook on each of the big ideas during the second half of each 90 minute session. The professional development integrated this instructional handbook (pedagogical content) with the conceptual development of each of the big ideas (content knowledge). The teachers then returned to their classrooms to develop their actual instructional plan and to instruct their students on the applicable big idea for two class periods of approximately 40 minutes each. After the initial presentation of a big idea to their students, teachers were encouraged to continue to use each big idea in other instructional units they developed during the year.

Research Design:
Our study involved a randomized field trial where subject teachers were approximately equally divided between treatment and control groups. Treatment teachers received PowerSource professional development. Control teachers were further divided into three groups. The first control (in two districts) received the usual district professional development; the second control group (in one district) received instruction in determining the technical quality of district benchmark assessments, and the final control (in four districts) received instruction in student self-efficacy and motivation.

Data Collection and Analysis:
All teachers in the treatment and control groups created a knowledge map and evaluated a sample of student work at the beginning of the study prior to any professional development. At the end of the school year, after the completion of treatment and control professional development activities, teachers again completed the same knowledge map and evaluation of student work activities.

Each teacher knowledge map was compared to an expert knowledge map created by the researchers conducting professional development. This "expert map" was created by combining the individual maps of those researchers. Although the individual maps were identical on more than 98% of the relationships and concepts, the researchers met to resolve remaining differences in the map prior its use as the "expert" standard.

The percentage of teacher agreement of the teacher map with the expert map was analyzed using a paired t-test to determine significance. More importantly, the mean agreement score of the treatment and control groups were analyzed using a t-test to determine the significance of the difference between these two groups.

All teacher evaluations of student work (both pre- and post-PD) were scored by two raters based on the rubrics developed by Heritage and Vendlinski (2006). Differences in ratings given by these two raters were discussed by three other raters to arrive at a "true" score.

The significance of change between the pre- and post-PD evaluations of student work were analyzed for each group using the Chi-square statistic. In addition, the significance of the differences between changes in the treatment and the control group were analyzed using this same statistic.

Findings/Results:
Initial results suggest that while there are differences between pre- and post-professional development knowledge maps and the ability of teachers to analyze the results of student work on formative assessments. While these trends are positive, the changes detected are small and are not significant at the $a = .05$ level. Initial analysis of the data from the evaluation of student work task are more positive and suggest that these improvements are correlated with participation in PowerSource professional development activities.

Conclusions:
Our results suggest that both the way teachers organize their content knowledge and the way they evaluate student work changes in important ways. Even with the positive trends, however, our findings suggest that nine hours of professional development may not be enough to produce the degree of change anticipated. As the literature suggests, producing change in teacher pedagogical practice requires sustained and intense efforts. These results suggest areas where important changes might be made such as a more intense focus on pedagogical methods, replacing in-situ curriculum with more conceptually appropriate instruction, and providing focused problems that teachers can use for sustained practice in their classrooms In addition, we intend to use more powerful Heirarchical Linear Models (HLM) to account for the degree of participation in professional development activities and to observe the degree of actual classroom implementation of PowerSource.

Appendix A. References

American Educational Research Association. (2006). *Do the Math: Cognitive Demand Makes a Difference*. Retrieved June 29, 2007, from http://www.aera.net/uploadedFiles/Journals_and_Publications/Research_Points/RP_Fall06.pdf

Atanda, R. (1999). Do gatekeeper courses expand education options? *Education Statistics Quarterly, 1*(1).

Ball, D. L. (2003). *Mathematical Proficiency for All Students*. Santa Monica, CA: RAND.

Ball, D. L., Lubienshi, S. T., & Mewborn, D. S. (2001). Research on Teaching Mathematics: the unsolved problem of teachers mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching* (4th ed., pp. 433-456). Washington D.C.: The American Educational Research Association.

Berkner, L., & Chavez, L. (1997). *Access to Postsecondary Education for the 1992 High School Graduates (NCES 98-105)*. Retrieved April 13, 2007, from http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=98105

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Buckingham, UK: Open University Press.

Black, P., & Wiliam, D. (2004). The formative purpose: assessment must first promote learning. In M. Wilson (Ed.), *Towards coherence between classroom assessment and accountability* (Vol. 2, pp. 20 - 50). Chicago, IL: University of Chicago Press.

Black, P., & William, D. (1998). Inside the black box: raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139 - 148.

Carpenter, T. P., Fennema, E. E., Levi, L., Franke, M. L., & Empson, S. B. (2000). *Children's Mathematics: Cognitively Guided Instruction: A Guide for Workshop Leaders*. Washington, DC: The National Council of Teachers of Mathematics, Inc.

Chi, M. T. H., Bassok, M., Lewis, M. W., Reimann, P., & Glaser, R. (1989). Self-explanation: how studetns study and use examples in learning to solve problems. *Cognitive science, 13*, 145 - 182.

Chi, M. T. H., & Roscoe, R. D. (2002). The processes and challenges of conceptual change. In M. Limon & L. Mason (Eds.), *Reconsidering Conceptual Change: Issues in Theory and Practice* (pp. 3-27). Netherlands: Kluwer Academic Publishers.

Chi, M. T. H., & Slotta, J. D. (1993). The ontological coherence of intuitive physics. *Cognition and Instruction, 10*, 249-260.

Chung, G. K. W. K., Baker, E. L., Brill, D. G., Sinha, R., Saadat, F., & Bewley, W. L. (2006). *Automated Assessment of Domain Knowledge With Online Knowledge Mapping* (No. 692). Los Angeles, CA: UCLA / CRESST.

Darling-Hammond, L. (2000). Teacher Quality and Student Achievement: A Review of State Policy Evidence. *Education Policy Analysis Archives, 8*(1).

deGroot, A. D. (1965). *Thought and choice in chess*. The Hague: Mouton.

Donovan, M. S., & Bransford, J. D. (Eds.). (2005). *How students learn: history, mathematics, and science in the classroom*. Washington D.C.: The National Academy Press.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical Understanding: An Introduction. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: history, mathemcatics, and science in the classroom*. Washington D.C.: The National Academy Press.

Hanushek, E. A. (2002). Teacher Quality. In L. T. Izumi & W. M. Evers (Eds.), *Teacher Quality* (pp. 1-12): Hoover Institution Press.

Helfand, D. (2006, January 30, 2006). A Formula for Failure in LA Schools. *Los Angeles Times*, pp. A1, A14-15.

Heritage, M., Kim, J., Vendlinski, T. P., & Herman, J. L. (In press). From evidence to action: a seamless process in formative assessment? *Educational Measurement: Issues and Practice*.

Heritage, M., & Vendlinski, T. (2006). *Measuring Teachers' Mathematical Knowledge* (Technical Report No. 696). Los Angeles: UCLA / CRESST.

Herman, R., Dawson, P., Dee, T., Greene, J., Maynard, R., Redding, S., et al. (2008). *Turning Around Chronically Low-Performing Schools: A practice guide* (No. NCEE #2008-4020). Washington, DC: Institute of Education Sciences, U.S. Department of Education.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: results from California's mathematics professional development institutes. *Journal for research in mathematics education, 35*(5), 330-351.

Kaput, J. (2004, May 5, 2004). Retrieved November 1, 2004, from http://neirtec.terc.edu/ma/experts/02_experts_p01.cfm

Kester, L., Kirschner, P. A., & vanMerriënboer, J. G. (2005). Timing of information presentation in learning statistics. *Instructional Science, 32*, 233-252.

Kieran, C. (2003). The Learning and Teaching of School Algebra. In D. A. Grouws (Ed.), *Handbook on Research on Mathematics Teaching and Learning* (pp. 390 - 419). Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding + it up: Helping children learn mathematics*. Washington D.C.: National Academy Press.

Kollars, D. (2008, May 12). Despite high school algebra focus, more students need remedial college math. *Sacramento Bee*, p. A1.

Leinhardt, G., & Greeno, J. G. (1984). The cognitive skill of mathematics teaching. *Journal of Educational Psychology, 78*(2), 75-95.

Leinhardt, G., & Smith, D. A. (1985). Expertise in Mathematics Instruction: Subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247 - 271.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Earlbaum.

Marzano, R. J. (2006). *Classroom Assessment & Grading that Work*. Alexandria, VA: Association for Supervision and Curriculum Development.

Meehan, K., & Huntsman, H. (2004). *Pathways Through Algebra Project: Improving Student Success Rates in Elementary Algebra*. Retrieved April 11, 2007, from http://72.14.205.104/search?q=cache:z55AbRizN_AJ:www.ijournal.us/issue_09/ij_issue09_09_MeehanAndHuntsman_01.html+%22algebra%22+%26+success&hl=en&ct=clnk&cd=11&gl=us

Nathan, M., Koedinger, K., & Martha, W. (2001, April 2001). *The expert blindspot: when content knowledge and pedagogical content knowledge collide*. Paper presented at the 2001 AERA Annual Meeting, Seattle WA.

National Mathematics Advisory Panel. (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Washington DC: U.S. Department of Education.

Niemi, D., Vallone, J., & Vendlinski, T. (2006). *The Power of Big Ideas in Mathematics Education: Development and Pilot Testing of POWERSOURCE Assessments* (No. CSE Report 697). Los Angeles: CRESST / UCLA.

Novak, J. D., Mintzes, J. J., & Wandersee, J. H. (2000). Learning, teaching, and assessment: a human constructivist perspective. In J. J. Mintzes, J. H. Wandersee & J. D. Novak (Eds.), *Assessing science understanding* (pp. 1-13). New York: Academic Press.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: does one lead to the other/. *Journal of Educational Psychology, 91*(1), 175-189.

Rockoff, J. E. (2004). The Impact of Individual Teachers on Student Achievement: Evidence from Panel Data. *AEA Papers and Proceedings, 94*(2), 247-252.

Rubin, J. (2007, July 11, 2007). Locke High's weary teachers face a hard multiple-choice test. *Los Angeles Times*.

Saxe, G. B., Gearhart, M., & Seltzer, M. (1999). Relations between classroom practices and student learning in the domain of fractions. *Cognition and Instruction, 17,* 1 - 24.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15,* 4-14.

Siegler, R. S. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, W. B. Martin & D. E. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 219 - 233). Reston, VA: National Council of Teachers of Mathematics.

Silver, D., Saunders, M., & Zarate, E. (2008). *What factors predict high school graduation in the Los Angeles Unified School district?* Santa Barbara, CA: UC Santa Barbara.

Sternberg, R. J., Forsythe, G. B., Hedlund, J., Horvath, J. A., Wagner, R. K., Williams, W. M., et al. (2000). *Practical intelligence in everyday life*. New York: Cambridge University Press.

Tittle, C. K. (2006). Assessment of teacher learning and development. In P. Alexander & P. Winne (Eds.), *Handbook of Educational Psychology* (pp. 953-980). Mahwah, N.J.: Lawrence Erlbaum Associates.

VanLehn, K., Siler, S., Murray, C., Yamauchi, T., & Baggett, W. (2003). Why do only some events cause learning during human tutoring? *Cognition and Instruction, 21*(3), 209 - 249.

Vendlinski, T. P. (2009). *The importance of intention and order: teaching for conceptual understanding using handheld technology.* Paper presented at the Annual Meeting of the American Educational Research Association, San Diego.

Vendlinski, T. P., Hemberg, B. C., Howard, K. E., Vinyard, L., Martel, A., Kyriacou, E., et al. (In press). Connecting with Prior Knowledge: Distribution and the Meaning of Multiplication. *Journal for research in mathematics education*.

Wiliam, D. (2006). *Does Assessment Hinder Learning?* Retrieved December 21, 2007, from http://www.uk.etseurope.org/home-corpo-uk/news-home/?print=1&news=136&view=detail&no_cache=1

Wiliam, D., & Thompson, M. (2007). Integrating Assessment with Instruction: What Will It Take to Make It Work? In C. A. Dwyer (Ed.), *The Future of Assessment: Shaping Teaching and Learning*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Zevenbergen, R. (2005). Primary Preservice Teachers' Understandings of Volume: The Impact of Course and Practicum Experiences. *Mathematics Education Research Journal, 17*(1), 3-23.