A Journey of Learning Mathematics through Inside-out Approach

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Abstract

In this paper I discussed an episode of inside-out approach of learning mathematics that started from my deeply held values, feelings, thoughts, and experiences making them starting point of learning process. I was engaged in a brief (one hour) learning session with a geometrical problem as context within the session. Data from video record of the learning process during the session and a written experiential anecdote were analyzed and interpreted. Some important aspects of learning process were identified as self-confrontation, mediation, dialogue, and transformation in the learning.

Context of Study

What is learning? When does learning occur? What are characteristics of learning? How do we know we have learned something? What happens to us during learning? What do we do during learning? How can we manifest our learning? Why learning is important to us? What motivates us learning? I was always pondering with these fundamental questions in relation to learning mathematics. For me these are some general questions and these questions are important for us to discuss in order to develop an understanding of what learning is. This paper is an attempt to discuss learning mathematics with inside-out approach which portrays an example of how learning takes place.

At first, it would be reasonable to review learning theories in brief. Behaviorist learning theories (e.g., classical conditioning and operant conditioning) assume that a learner is essentially passive responding to environmental stimuli. This theory considers learner as a clean slate (tabula rasa) and behavior is shaped through positive or negative reinforcement. “As an objective natural science, psychology was to make no sharp distinction between human and animal behavior; and its goal was to develop principles by which behavior could be predicted and controlled" (Wozniak, 1994). The behaviorist theories concentrate on the study of overt behaviors that can be observed and measured (Good & Brophy, 1990) which views the mind as a black box in the sense that response to a stimulus can be observed quantitatively ignoring the possibility of thought processes occurring in the mind (Mergel, 1998). Some of the key players in the development of behaviorist theory were Pavlov, Watson, Thorndike, and Skinner.

Next development in the learning theory was Gestalt theory which was a holistic approach and rejected the mechanistic approach of stimulus-response model of behaviorism. According to Gestalt theory learning consists of grasping of structural whole and not just a mechanistic response to a stimulus. Learning depends upon various factors such as factor of closure, factor of proximity, factor of similarity, factor of background effect, etc (Kantz, 1950).

Cognitive theories of learning revolutionized the theories of learning. According to cognitive theory of learning, a student actively learns when he or she makes an effort to organize, store, and find the relationship between old and new information, script, and schema. Its focus is in how the mind processes information. "Cognitive theorists recognize that much learning involves associations established through contiguity and repetition. They also acknowledge the importance of reinforcement, although they stress its role in providing feedback about the correctness of responses over its role as a motivator. However, even while accepting such behaviorist concepts, cognitive theorists view learning as involving the acquisition or reorganization of the cognitive structures through which humans process and store information." (Good & Brophy, 1990, pp. 187).

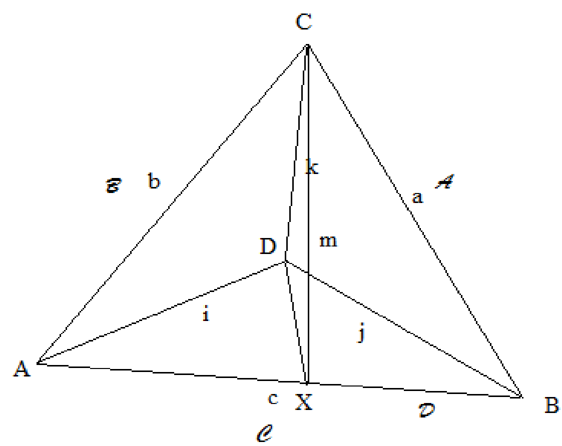
Constructivist theory of learning posits that learning is an active, constructive process. Learning is not transferring information from one person to another but learning is active construction of own subjective representation of objective reality. This theory assumes that knowledge is constructed from experience and learning is a personal interpretation of the world (von Glasersfeld, 1987). It further assumes that learning is an active process in which meaning is developed on the basis of experience and conceptual growth comes from the negotiation of meaning, the sharing of multiple perspectives and the changing of our internal representations through collaborative learning (Merrill, 1991). Some of the key players of constructivist theories are Piaget, Vygotsky, von Glasersfeld, Schoenfeld, Ernest, Jaworski, etc.

Learning can be viewed as outside-in or inside-out process. Based on these two processes all these learning theories can be divided into two groups: outside-in learning theories and inside-out learning theories. Outside-in approach of learning leaves human affairs to the experts and the inside-out perspective of learning is rooted in our experiences (Baker and Kolb, 1993). Inside-out learning refers to the internal subjectivities of a person built upon experiences that lays foundation for learning. Outside-in learning refers to the external ideas and events that act upon us and shape our knowing. Often, we find ourselves caught between the conflicting demands of the external world and the need to follow our true voices from within (Baker, Jensen, & Kolb, 2002). Outside-in learning is rooted on behaviorist (classical conditioning, operant conditioning) and cognitive theories of learning whereas inside-out learning is rooted on experiential learning (of Dewey, Piaget, Kolb, etc.) and constructivist learning (of von Glasersfeld, Schoenfeld, Steffe, etc).

I discussed inside-out learning of mathematics in a brief session in which I was engaged in learning of extension of Pythagorean Theorem from two dimensions to three dimensions. This discussion includes the description of the learning session in which I was actively engaged in learning based upon my prior experiences of mathematics, some nodal moments during the session, drawing out some themes from the session, my reflection on the learning process, and post session reflexivity.

Evoking the Session

My professor and I started the session at 3:45 of November 10, 2010 with a geometry problem that was based on the very famous Pythagorean Theorem in a right angled triangle.



We had a *tetrahedron* with triangular faces ABC (area denoted as $\mathcal{D}$), ABD (area denoted as $\mathcal{C}$), ACD (area denoted as $\mathcal{B}$), and BCD (area denoted as $\mathcal{A}$), and sides were named as AB= c, BC=a, AC=b, AD=i, BD=j, and CD=k. AD, BD, and CD were perpendicular to each other at point D.

"What might be the relation of among the triangles ABD, BCD, ACD, and ABC? What can we conjecture from these triangles?" This question led me to think about the triangles in relation to each other. Then I thought there should be some relations in terms of their areas. I was thinking whether the sum of area of triangles ABD, BCD, and ACD would be equal to the area of triangle ABC. I mentioned that the sum of the area of the first three triangles should be equal to the area of the fourth triangle. He wrote the relationship (triangle ABD + Triangle BCD + Triangle ACD = Triangle ABC) that I was guessing at the moment. This was just an initial guess and still I was thinking whether this relation holds. "You are fairly close", my professor said to me. Oh, what might be the correct relationship then? His question that if this relationship holds true made me thoughtful for a while but I could not reach to correct relation at the moment. Then, the professor said, “Let’s square each of them”. After squaring the triangles now the problem turned to be analogous of the Pythagorean Theorem. He mentioned that sum of the squares of the area of the first three triangle is equal to the square of the area of the fourth triangle. The problem was not to discover this relationship but to prove this relationship.

“How can we show that the relationship is true with square of the area of the triangles? Is there any way that we do it?” The professor continued his questions to me. I was thinking about the relationship of sides of triangle with their corresponding areas. I remembered that I had once proved the relation that area of triangle ABC is equal to the square root of s(s-a)(s-b)(s-c) where s is semi perimeter of the triangle. I was not aware about who introduced this formula. The professor mentioned that it was Heron’s formula and it was found some centuries B.C. This could be a possible way to solve the problem but there being too many variables it would require a long algebraic simplifications by solving many linear equations. We accepted that this could help us to reach the solution; however, being long process, it was left for me to see at home. Then we changed our strategy to use area of triangle as half the product of base and height (or altitude). We then proceeded to use this strategy thinking that it would be shorter than earlier strategy to use semi-perimeter (s). We constructed altitude m from vertex C to side AB at point X of triangle ABC. We joined DX. Let the area of triangle BCD be $\mathcal{A}$, the area of triangle ACD be $\mathcal{B}$, area of triangle ABD be $\mathcal{C}$, and area of triangle ABC be $\mathcal{D}$.

We had to prove condition that: $\mathcal{A}^2 + \mathcal{B}^2 + \mathcal{C}^2 = \mathcal{D}^2$

I was thinking of area of triangle as half the product of base and altitude. Then area of triangle BCD = (½ jk), triangle ACD = (½ ik), triangle ABD =(½ ij), triangle ABC = (½ mc)

Then we proceeded the further steps as:

$$(\tfrac{1}{2}\,jk)^2+(\tfrac{1}{2}\,ik)^2+(\tfrac{1}{2}\,ij)^2 = (\tfrac{1}{2}\,mc)^2$$
$$(jk)^2+(ik)^2+(ij)^2 = (mc)^2$$
$$(jk)^2+(ik)^2+(ij)^2 = m^2c^2$$
$$(jk)^2+(ik)^2+(ij)^2 = (l^2+k^2)\,(i^2+j^2) \text{ where } c^2= i^2+j^2,\ l=DX$$
$$j^2k^2+i^2k^2+i^2j^2 = l^2\,i^2+l^2j^2+k^2i^2+k^2j^2$$
$$i^2j^2 = l^2\,i^2+l^2j^2$$
$$i^2j^2= l^2(i^2+j^2)$$
$$i^2j^2= l^2\,c^2$$
$$ij= lc$$

This relation was possible only when the two triangles (triangles AXD and ABD) were similar. This confirmed that DX was perpendicular to AB. Then multiplying both sides by 1/2 would give the area of the same triangle ABD. That would be Area of triangle ABD = Area of triangle ABD and this proved the statement. The professor asked me whether I was convinced that it proved the relationship. I believed that the relationship was proved. "What would be the situation if the triangle ABC was an equilateral triangle?" professor asked me. I was thinking that the square area of triangle ABC would be three times square of area of each triangle. Later I tried this at home. I thought that when sides AB, BC and CA were equal, then sides AD, BD and CD also would be equal to each other and then we would have $\mathcal{D}^2= 3\mathcal{A}^2= 3\mathcal{B}^2=3\mathcal{C}^2$.

We further came up with idea that the relation might hold true for volumes of tetrahedrons based on the respective areas and having equal heights (say x). Also our thought reached to the possibilities of relationships of areas and volumes when the formation of right tetrahedrons continued on each faces up to n levels. We agreed that it would produce a complex fractural structure and might be a researchable question. The session was over after one hour. I realized that the session was very useful to learn application of the Pythagorean Theorem in three dimensional right tetrahedron. I packed up my video record and came home. On the way the same tetrahedron was in my mind and I was thinking about vivid possibilities.

I used Heron's formula to solve the problem at home. I followed the following procedures:

$$(½\ jk)^2+(½\ ik)^2+(½\ ij)^2 = s(s-a)(s-b)(s-c)$$

Right Hand Side = $\frac{(a+b+c)}{2}\ (\frac{a+b+c}{2}-a)\ (\frac{a+b+c}{2}-b)\ (\frac{a+b+c}{2}-c)$

$$= \frac{(a+b+c)}{2}\ (\frac{-a+b+c}{2})\ (\frac{a-b+c}{2})\ (\frac{a+b-c}{2})$$

$$= \frac{((a+b)^2-c^2)}{4}\ \frac{(c^2-(a-b)^2)}{4}$$

$$= \frac{(a^2+2ab+b^2-c^2)}{4}\ \frac{(c^2-a^2-b^2+2ab)}{4}$$

$$= \frac{1}{16}\ (j^2+k^2+2ab+i^2+k^2-i^2-j^2)(\ i^2+j^2-j^2-k^2-i^2-k^2+2ab)$$

$$= \frac{1}{16}(2k^2+2ab)(-2k^2+2ab) = \frac{1}{4}((ab)^2-(k^2)^2)$$

$$=\frac{1}{4}\,(a^2b^2-k^4)$$

$$=\frac{1}{4}\,((j^2+k^2)(i^2+k^2)-k^4)$$

$$=\frac{1}{4}\,(i^2j^2+j^2k^2+i^2k^2+k^4-k^4)$$

$$= (½\ ij)^2+(½\ jk)^2+(½\ ik)^2$$

Starting form right hand side (Heron's formula), I simplified the relationship to get the left hand side representing sum of areas of the three triangles equal to the fourth triangle. It did not take much time to simplify because there were straight forward algebraic relationships. I was then thinking about relationship with volume of tetrahedrons. I thought there should be a common altitude. I multiplied sides of the relation $\mathcal{A}^2+\mathcal{B}^2+\mathcal{C}^2=\mathcal{D}^2$ with square of common altitude x and then formed the formula of volume of pyramids.

$$\mathcal{A}^2+\mathcal{B}^2+\mathcal{C}^2=\mathcal{D}^2$$
$$x^2.\ \mathcal{A}^2+x^2.\ \mathcal{B}^2+x^2.\mathcal{C}^2=x^2\ .\mathcal{D}^2$$
$$((\frac{1}{3}x\mathcal{A})^2+(\frac{1}{3}x\mathcal{B})^2+(\frac{1}{3}x\mathcal{C})^2=(\frac{1}{3}x\mathcal{D})^2$$

$$V_1^2+V_2^2+V_3^2=V_4^2$$

Where V_1, V_2, V_3, and V_4 are volumes of tetrahedron with base triangles BCD, ACD, ABD, and ABC respectively and same altitude of x. I thought whether relationship would hold with the total surface areas of right tetrahedrons on each faces of the original tetrahedron. I thought it would be interesting to see further with a progression of new right tetrahedrons in each faces up to n levels of progression as we discussed during our session. May be we could develop some algebraic sequence with n number of progressions of right tetrahedrons.

Reflexive Thinking: Self Interviewing

I thought it would be helpful to develop an understanding of how I went through the learning process and it was not possible through only describing the session but it needed a deep and thoughtful reflection over my role as a participant in the learning session, engaging in dialogue with my professor and progressing through processes of learning journey during the session. I devised five questions to ask anybody (here me) to reflect on his or her (here my) experiences of learning mathematics during the session (or similar sessions). Here I am playing the dual roles: role of participant during the session (as self me) and role of interviewer after the session (as other me). The other me interviews the self me in at five levels: *experience level*, *value level*, *perception level*, *knowledge level*, and *background level*. The interview proceeded with open ended questions:

Other me: What did you experience when you went through the session?

Self me: I was excited about the learning session from very beginning when I knew that I would be engaged in a learning session for my class project. But I did not know what the session would look like or be until I was ready for the session on the final day. My professor asked me about the Pythagorean Theorem in three dimensions. I did not have any idea *about the problem in three dimensions though it was very common to me in two dimensions since high school. I was puzzled with the question at first. When my professor illustrated the problem on the board and explained the conditions given, then I felt more comfortable about the problem. I thought that the three dimensional analogy of the Pythagorean Theorem would be related to the area of the triangles formed in a right tetrahedron. I went through the stages of confusion, connection to my past experiences, confrontation, and then resolution. These stages were the prime characteristics of my experiences so far I could notice from the video and my personal written anecdote.*

Other me: Will you please share your experience of confusion, confrontation, connection, and resolution while solving the problem?

Self me: Well, confusion started with the very first question of my professor. He asked me the number of triangles in a right tetrahedron and I could not say exactly at the beginning. There was picture on the board and I said three triangles at first and after a while I realized that there was one big triangle at the front and altogether there were four triangles. Second time I was confused when I was asked to say about relationship of triangles. I said, "The sum of the area of first three triangles should be equal to the area of the fourth (largest) triangle". I was confused whether the relation would hold with just area or square of their areas. Third time I was confused when I could not say exactly whether the side DX was perpendicular to the side AB. These confusions actually led me to dilemma and confrontation with my belief about geometrical properties of triangles, areas, and Pythagorean Theorem. The confrontation stage was not long.

Then dilemma ended with probing of my professor. When he asked me questions to think over the relations where I was in confusion, I deeply thought from what I knew earlier *(area of triangle in terms of Heron's formula or area of triangle as half the product of base and altitude) that helped me to connect to the problem situation. This connection really comforted me to move ahead and reach to the solution that convinced me that the relationships (DX perpendicular to AB, Sum of squares of areas of first three triangles is equal to square of the fourth triangle) were true. When I was fully satisfied or convinced with the solution it was my state of resolution.*

Other me: How this session was helpful to you?

Self me: The session was helpful to me in three different ways: first, the session helped me to understand the learning session as a journey which had high pedagogical significance (to me); second, the session was a model to develop a reflective mathematical thinking or thinking mathematics reflectively; third, it was helpful to understand how experiential learning built up to new stage from our prior knowledge or experience. In other way, it helped me to understand how to design such learning sessions (technicality); how to help students go through such sessions (practicality); and how to help students to be reflective thinker through such sessions (emancipatory).

Other me: How do you feel about the session?

Self me: The session was really a great experience. The subject matter might be anything simple or complex but the process through which I went was really made a huge difference in my thinking and acting while solving such mathematical problem. I feel that it was an excellent opportunity to learn about self and learn from self in such guided sessions. Probing of my professor was really helpful to think deep and reason on various possibilities.

Other me: What kind of prior knowledge helped you in solving the problem?

Self me: I think my prior knowledge or experience in algebra and geometry helped me to solve the problem. The prior knowledge of properties of triangles and their areas were very helpful in reasoning about why DX was perpendicular to AB and how the Pythagorean Theorem was true for the areas of triangles in a right tetrahedron. Actually my prior knowledge or experience was the origin from where my thinking emerged to the contemporary problem situation.

Other me: What did you do to solve the problem?

Self me: I carefully watched to the picture, carefully listened to my professor, tried to find the given conditions (perpendicularity of sides), planned (mentally) for next possible steps, tested the steps if they were logical, confirmed them and looked back to see what would happen in other conditions (such as when triangle ABC was an equilateral triangle). I was actively engaged in thinking and doing (reasoning) through dialogue and action (writing), connection (linking to what I already knew) and construction (building new relationships).

This interview session shows a glimpse of my hermeneutic lived experience before the session, during the session and after the session. I think the context and subject matter of the session were very powerful to develop an understanding of meaning of learning, process of learning and post learning reflexivity.

Discussion on Nature of Learning

The vignettes of proofs progressed through steps when we were engaged in dialogues. The reasoning process was based on dialogic interpretations of relationship of parts of the tetrahedron. There was triangular mediation among the learner, the professor, the objects of our construction (a right tetrahedron, Heron's formula, and formula of area of triangles, relationship of sides with areas, volumes, etc.).

It was easy to write the area of the first three triangles in terms of half the product of base and altitude of the triangles and add their squares. But we did not have the altitude of the fourth triangle ABC at the beginning. Therefore, I thought the area of this triangle could be determined using the lengths of the three sides a, b, and c using the formula of area of triangle ABC equal to square root of [s(s-a)(s-b)(s-c)]. I was confident that we could simplify the relationship to prove the equality condition. But we chose a simpler way by constructing an altitude from C to AB at point X. We joined DX too. I thought DX should be perpendicular to AB but I did not have any proof. Therefore, I suspended this idea (DX perpendicular to AB) for a while and proceeded simplification. After working out on simplification of the relationship of sides (that started from areas), I came to a relation which helped me to see that DX was actually perpendicular to AB. Finally, it convinced me that the relationship that the sum of the squares of the area of the first three triangles was equal to the square of the area of the fourth triangle. I was thinking to prove the same by using Heron's formula and I did it at home after the session.

In order to analyze the data from the description of the session and interview between other me and self me, at first, I identified the following key concepts or categories and then I discussed those categories: self-confrontation, mediation, inter and intra personal dialogue, and self-transformation.

Self-confrontation in Learning

When I wrote my brief personal anecdote of lived experience in the session and watched the video of the session, I came to realize that I was not able to perform well enough as I was expecting during the session. Though I felt that the session went perfect but after watching the video again and again I came to understand that the session had a lot of room to be better than how it went. I think I was too much dependent on my professor's probing questions to think deeply. Though I thought that I was actively participating in the session with my professor, but careful analysis of the time duration of our speaking (dialogue) showed that I spoke much less than my professor. Also, from video analysis I came to realize that most the time duration I followed the instruction of my professor instead of acting on myself and getting help from him in extreme case only.

What lessons did I learn from these self-confrontations may be important question that I should discuss. Comparison of my written anecdote and video record put me in such self-confrontation through which realized some severe lacking in my participation during the session. I was not acting as much as I should have and I was not able to gain much independence to solve the problem, rather I was expecting probing questions from my professor. This is a great lesson for me and I think I should be more active in questioning, reasoning, and performing.

Mediation in Learning

I consider and agree that learning is a higher mental process that takes place when context of learning is mediated either through dialogue between teacher and student, or self (monological) interaction of self with the object of learning. Vygotsky (1978) proposed three major classes of mediators: materials tools, psychological tools, and human beings (professors, teachers, parents, peers etc.). When I was actively engaged in the learning session, I was mediated with my professor, the object of learning (the problem situation), and the representation of the problem on the board (a diagram) through active interplay among us. My professor was asking me questions in relation to my thinking about the problem situation and possible strategy that I would undertake in order to reach the goal (final solution).

I was thinking and processing the ideas that I knew earlier and the idea that I did not know earlier and trying to seek connections (such as Heron's formula). When I was not able to figure out the exact relation of side DX to AB, I was trying to see how I can convince myself that DX is perpendicular to AB (material tool). This problem engaged me in further processes to develop relationship of CX (m), DX (l) and CD (k) to see if I would be able to find some new relations that could simplify the situation (psychological tool). Also, I was deeply engaged in communication and interaction with my professor at the time when I was trying to ease myself to see the relation (lc=ij) that could convince me that DX was actually perpendicular to AB (through dialogue, a mediation with professor).

Inter and Intra-personal Dialogue

There was dialogue between speaking and thinking, between social/individual (my professor) and individual (me), and within individual as a self and other (other me and self me). Through dialogue (inter-personal communication) between my professor and me, I was able to acquire knowledge and skill developing a mutual understanding of differences, and respectful environment in a cultural context. We created or developed an understanding of shared meaning of the perpendicularity and the Pythagorean Theorem in three dimensional space. I think we were trying to share the authority of creation of knowledge equally by both the learner (me) and the teacher (my professor).

When I was actively engaged in the process of developing proof of the problem, I was not only communicating with my professor but also I was thinking and speaking to me internally. There was ongoing dialogue between my thinking and speaking. After the session, when I was comparing my written anecdote of experience and video record, there was intra-personal dialogue between other me (as observer, interviewer) and self me (as participant) that made me more conscious about subtleties of my roles as an active and constructive learner during the session and after the session.

Transformative Learning

I think there was implicit element of transformative learning in the learning session. I can claim that the learning session went beyond just content knowledge acquisition (establishing the Pythagorean Theorem in right tetrahedron). The session provided me an opportunity to develop increased level of consciousness, understand function of feeling and thinking, soulful learning through deep feeling, intuition and imagination (that I was imagining about extension of the Pythagorean Theorem into volume level and beyond…up to n^{th} extension).

Also, the post session comparison of my written anecdote with video record created a dilemma (Did I really act as expected to act?), self examination with feeling of guilt or shame (Why I could not see four triangles at a glance to the right tetrahedron?), a critical assessment of assumptions (Why I did not understand the perpendicularity of DX to AB at the beginning?), and exploring options for new actions (Actually tested the proof by using Heron's formula and it was perfect.). To me, the learning session was helpful to transform my thinking from the level of pre-reflective to reflective, transform my practice from technicality to emancipatory (more criticality), and transform my role as a performer to a creator.

Epilogue

Most of the time learning starts with a problem that creates confusion, makes us to think creatively and critically about the problem situation, may invite confrontation between past experiential knowledge and current problem, may connect with possible ideas that we have, test our assumptions about what we know and what we might know, and finally may come to a resolution (temporary or permanent). Learning of mathematics or any subject matter connects our two sets of epistemological domains of understanding or consciousness through which we gain more maturity of our thinking and doing, problem solving, decision making, and acting is a situation. From the analysis of the learning experience I came to realize that learning is very complex process and it is difficult to trace a particular trajectory of one's learning, however, thick descriptions of learning and reflecting on experiences help us to develop and understanding of how people go through different stages while learning. Knowing these stages is an important aspect of teaching mathematics (or any other discipline). Mathematics educator, teachers and research should focus more on learning experiences of students in order to find out how they learn and how their learning can be enhanced. May be looking at 'how we learn' can be a good thing at first step before we start to think how students (or others) learn.

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