# Comparing the Common Core State Standards and Singapore's Mathematics Syllabus



## Introduction

Through the Common Core State Standards (CCSS) Initiative, states and territories have collaborated in the development of a common core of standards in English Language Arts and mathematics for grades kindergarten through twelve that are now being adopted by states. Designed not only for the purpose of providing strong, shared expectations, the Common Core State Standards will also allow adopting states to collectively create and share high-quality tools such as assessments, curricula, instructional materials (such as textbooks and software), and professional development programs.

As educators and policymakers review the CCSS in mathematics, they will want to consider the way these new standards compare to, and build on, existing standards in mathematics. This brief describes the comparison between the CCSS and Singapore's Mathematics Syllabus.

## Common Core State Standards in Mathematics

The K-5 standards provide students with a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions and decimals—which help young students build the foundation to apply more demanding math concepts and procedures successfully, and move into applications. They also provide detailed guidance to teachers on how to navigate their way through knotty topics such as fractions, negative numbers, and geometry, and do so by maintaining a continuous progression from grade to grade. Having built a strong foundation in K-5, students can move to more complex work in geometry, algebra and probability and statistics in the middle grades to gain a rich preparation for high school mathematics. Students who have completed 7th grade and mastered the content and skills through the 7th grade will be well prepared for algebra in grade 8. The high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically across the major strands of mathematics, including number, algebra, geometry, probability and statistics. Note that the CCSS promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible.

The CCSS in mathematics lay out a vision for what all students need to master to be ready for credit-bearing college mathematics courses without remediation. Some of the high school standards are designated by a (+), indicating that they are above the college- and career-ready requirement but necessary for students to take advanced mathematics courses in high school such as calculus, advanced statistics, or discrete mathematics, and to be prepared for Science, Technology, Engineering, and Mathematics (STEM) coursework in college.

## Singapore's Mathematics Syllabus

Singapore's students have consistently been high performers, ranking first in the world in mathematics on the Trends in International Mathematics and Science Study (TIMSS) in 2003. As a result, researchers have explored Singapore's successful approach to mathematics instruction to identify features that could work in U.S. schools.[1] There are several components that account for Singapore's success, including "a highly logical national mathematics framework, mathematically rich problem-based textbooks, challenging mathematics assessments, and highly-qualified mathematics teachers."[2]

1 *What the United States Can Learn From Singapore's World-Class Mathematics System (and What Singapore Can Learn from the United States):* An Exploratory Study: American Institutes for Research, 2005

2 Ibid



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Singapore's mathematics framework identifies five key aspects of mathematics learning as essential:

- Skills, such as estimation, mental calculation, arithmetic and algebraic manipulation, and handling of data;
- Numerical, geometrical, algebraic, and statistical concepts;
- Processes, or thinking skills;
- Metacognition, the monitoring of one's own thinking; and
- Attitudes regarding mathematics, including interest, confidence, and perseverance.

The Mathematics Syllabus, which is a part of Singapore's mathematics framework, describes the primary content expectations for each level through topic lists, which are similar to standards. Details are differentiated for students in different streams, or academic tracks. *All* students will study content through what is called the O-Level. Students who wish to pursue university coursework study A-Level mathematics. Because of its quality, the Singapore Syllabus was an important resource for the developers of the CCSS.

**More about Singapore's O- and A-Levels:** *The O-Level syllabi are titled Secondary 1, Secondary 2, and Secondary 3/4, which are equivalent to Grades 7, 8, and 9-10 in the U.S. Virtually all students in Singapore will take the O-Level exams, and 82% do well enough to qualify for junior college, which culminates in the A-Level exams. About 30% of students go on to junior college, while about 40% go on to polytechnic schools after completing the O-Level exams.*

## Achieve's Analysis

Achieve has analyzed the CCSS and the 2007 version of Singapore's Mathematics Syllabus for the primary and secondary levels to determine how they compare in terms of **rigor, coherence, and focus**. **Rigor** refers to the degree that sets of standards address key content that prepares students for success beyond high school. **Coherence** refers to whether the standards reflect a meaningful structure, revealing significant relationships among topics and suggest a logical progression of content and skills over the years. **Focus** refers to whether the standards suggest an appropriate balance in conceptual understanding, procedural skill, and problem solving with an emphasis on application and modeling; the standards should be teachable within a school year (or across four years of high school), and key ideas in a given grade or topic area should be clear. Standards that are rigorous, coherent, and focused provide better guidance to educators, students, and parents about desired learning outcomes than those that are not. Expert mathematics content analysts conducted a side-by-side comparison of the CCSS and the Singapore Mathematics Syllabus, looking particularly at the inclusion and treatment of mathematics topics at each grade level. This brief describes their findings.

## Major Findings

- The CCSS and Singapore Mathematics Syllabus describe similar levels of rigor. Where grade placement discrepancies occur between the two documents, they are usually within one year of each other.
- The CCSS and the Singapore Mathematics Syllabus are comparable in their coherence and focus, but there are a few key differences, which result in the CCSS providing greater detail and specificity.



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## Detailed Findings

### *Rigor*

The CCSS and the Singapore Mathematics Syllabus describe expectations of comparable rigor at each grade level. The two documents contain similar expectations for what students should know and be able to do by the end of grades 4 and 8. In high school, though, the CCSS expect slightly more than the Singapore Mathematics Syllabus expects of all students.

- **Elementary grades:** Through the end of grade 4, the CCSS and the Singapore Mathematics Syllabus are comparable in rigor. For example, both documents require students completing grade 4 to be fluent at adding, subtracting, and multiplying with whole numbers; to understand and be able to apply place value; and to be able to classify simple two-dimensional geometric figures. These expectations form the basis for basic mathematical understanding in elementary school.

- **Middle grades:** There also are substantial similarities in the middle grades. Both documents include some of the major hallmarks of algebra—proportionality, linear expressions and inequalities, and using equations and inequalities to solve real-life and mathematical problems—which prepare students well for more advanced mathematics in high school. More specifically, the CCSS contain much of the same content in Grade 8 that is found in the Singapore Secondary 2 expectations.

- **High school:** The CCSS compare favorably with Singapore's secondary-level syllabus. The CCSS include nearly all of the topics found in the Singapore Mathematics Syllabus through the end of the O-Level, which are the requirements that all students must meet, including solving quadratic equations and the graphs of exponential functions, before advancing to further education and training. While the two documents describe similarly rigorous expectations, the CCSS cover some concepts not addressed in the Singapore Mathematics Syllabus for all students. When comparisons are made between the CCSS that are beyond the college- and career-ready minimum (designated with a (+) symbol), and the Singapore A-Level standards (which are beyond the minimum expectations of all students in Singapore), the Singapore standards are somewhat more rigorous, as they describe content typically found in a U.S. Calculus course, whereas the CCSS describe content up to that which is normally found in a U.S. Pre-Calculus course. As a result, advanced mathematical content is more comprehensive in the Singapore standards than in the CCSS.

### *Coherence and Focus*

Achieve's analysis indicates that the Common Core State Standards and the Singapore Mathematics Syllabus share some key traits of coherence and focus. For example, both documents place a heavy emphasis on number concepts and skills in the primary grades, in order to provide students with a solid foundation for work with more advanced content in later years. The expectations for each grade level are very similar in the two documents, and both describe coherent expectations that build gradually from the primary grades on. For example, both documents lay out a reasonable progression of addition, subtraction, multiplication, and division with fractions by connecting them to measures such as length, as well as by addressing them across grades four through seven. By grounding work with fractions in measurement, both sets of standards provide students with the opportunity to grasp an otherwise elusive topic through more concrete means. Furthermore, by situating the progression of content across four grades, the CCSS and Singapore Mathematics Syllabus emphasize depth of learning without unnecessarily repeating content from one year to the next.



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While they are substantially similar in terms of focus, the development of key learning trajectories is more detailed in the CCSS than in the Singapore Mathematics Syllabus. The CCSS present content expectations with greater detail and specificity, more clearly describing the conceptual understanding students should have than the Singapore Mathematics Syllabus does. For example, the CCSS require students to understand place value and then to apply it in computation as in following Grade 1 standard, "Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using…strategies based on place value, properties of operations, and/or the relationship between addition and subtraction…" Here, Singapore provides little or no information about the connection between place value and addition and subtraction, stating only "include addition and subtraction within 100 involving a two-digit number and ones, a two-digit number and tens, two two-digit numbers," in its first grade (Primary 1) standard. As a result, teachers who use the CCSS will have greater guidance regarding both the skills and the concepts students should learn.

In short, the Common Core State Standards and the Singapore Mathematics Syllabus share many key traits of coherence and focus. Significant differences are found in the greater detail and specificity provided by the CCSS, which make them more useful for teachers.

## Conclusion

Overall, the CCSS are well aligned to Singapore's Mathematics Syllabus. Policymakers can be assured that in adopting the CCSS, they will be setting learning expectations for students that are similar to those set by Singapore in terms of rigor, coherence and focus.

*Achieve is a bipartisan, nonprofit education reform organization that has worked with states, individually and through the 35-state American Diploma Project, for over a decade to ensure that state K-12 standards, graduation requirements, assessments and accountability systems are calibrated to graduate students from high school ready for college, careers and life. Achieve partnered with NGA and CCSSO on the Common Core State Standards Initiative and a number of its staff and consultants served on writing and review teams. Achieve thanks the Brookhill Foundation for its generous support in making this brief available, and providing educators and policymakers across the nation with a way to more deeply understand the CCSS through comparison to other well-known mathematics expectations. For more information about Achieve, visit www.achieve.org*



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