Intentional Experiences: Teaching|Learning Mathematics with Young Children

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Abstract
A recent emphasis on mathematics learning in the preschool years has sparked a flurry of research in children's thinking; yet, the same attention has not been paid to teaching mathematics in preschools. This paper examines the reciprocal relationship of teaching|learning mathematics with young children and attempts to move discussions beyond teacher as facilitator or director and learner as explorer or incompetent/incomplete receiver. Through an analysis and discussion of excerpts from two adult/child interactions (one of whom is the researcher), I attend to intentional experiences of teaching and learning through an enactive lens. What is revealed is the shared responsibility for intentional (teaching) acts to disclose intentional (mathematical) objects such that the experiences expand possibilities for acting in a mathematical space.

Paper presented at the Special Interest Group, Critical Perspectives in Early Childhood Education: New Perspectives on Early Childhood Education: Part 2. Annual Meeting of the American Educational Research Association, Denver, CO, May 1st, 2010.

Intentional Experiences: Teaching|Learning Mathematics with Young Children[1]

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A recent emphasis on mathematics learning in the preschool years has sparked a flurry of research investigating and revealing young children's spontaneous use of mathematics and their capacity to understand ideas across mathematical domains (e.g., Clements, Sarama & DiBiase, 2004; English, 2004; Sarama & Clements, 2009; Seo & Ginsburg, 2004). Research and other political agendas have also spawned curriculum programs for these early years (e.g., Clements & Sarama, 2007; Ginsburg, Greenes & Balfanz, 2003; Sophian, 2004). Yet, limited research exists that attempts to address and understand what it might mean to teach mathematics to preschoolers (Anderson, Anderson & Thauberger, 2008). This paper examines and articulates a position on the reciprocal and ethical relationship of teaching|learning mathematics with young children. It attempts to move discussions beyond teacher as facilitator or director, and learner as explorer or incompetent/incomplete receiver. Instead, I attend to the intentional experiences of teaching and learning through an enactive and phenomenological lens.

Intentionality: Acts and Ethics

Teaching, necessarily, must be explicated from a perspective on learning. The work here begins with a discussion of contemporary perspectives in cognitive science which reveals cognition to be a dynamic, contextually contingent, and body-centred phenomenon (e.g., Lakoff & Johnson, 1999; Maturana & Varela, 1992; Varela, Thomson & Rosch, 1991). In particular, an *enactive* perspective suggests that personal learning is not an accumulation or appropriation of knowledge, nor is it a matter of processing information, or even problem solving; instead, learning is an embodied, sense-making activity in the course of living. From this perspective, learning is intelligent action in the continual process of bringing forth (disclosing) a world through the ideas and questions that arise for us in our conversations with others and otherness (Varela et al., 1991).

Enactivism suggests that a dynamic system brings objects, ideas, images, and memories to awareness and enacts meaning through reciprocal relationships of *intentional acts* and *intentional objects* (Thompson & Zahavi, 2007). Drawing on phenomenology, intentional experiences are not events entered into with a particular purpose in mind (although that is one possible form), but they are experiences that direct our attention, stretch us towards something, and afford us an opportunity "to being sensuously affected and solicited by the world through the medium of our living body" (Thompson, 2007, p. 30). What is disclosed to a person through

[1] Teaching|learning signifies an inextricable relationship between acts of teaching and acts of learning that cannot be understood in isolation.

intentional acts is determined by his or her biological and phenomenological structure and the social and cultural context in which one's body is embedded.

From this perspective, teachers and learners enter into not only a reciprocal relationship, but also an ethical relationship following the assertion that all interactions "ride on an underground river of ethics" (von Foerster, 2003, p. 291). von Foerster's (2003) ethical imperative to "always try to act so as to increase the number of choices" (p. 295) encourages us to recognize that we always have freedom to choose. However, that freedom also increases our responsibilities to others. As pedagogues, we stand in ethical relationship with children. It is through our intentional experiences alongside children that there is a potential for children and educators to expand possibilities of intelligent action: "whenever I act, I'm changing myself and the universe as well" (p. 293).

In mathematics teaching and learning, perhaps more so than any other discipline, is an emphasis on outcome based assessment and achievement emphasizing quality and effectiveness of early learning (e.g., NAEYC/NCTM, 2002). Yet, many of the reasons for emphasizing early mathematics learning, such as achievement gaps and future success (Sarama & Clements, 2009) promote that mathematics is needed for some future purpose. Given the focus on achievement it is understandable that the emphasis on early learning promotes "technical practice, finding the most efficient methods to achieve predetermined ends" (Dahlberg & Moss, 2005, p. 35) rather than reciprocal and ethical relationships. But mathematics teaching and learning, particularly in preschools, need not be directed solely towards achievement of predetermined outcomes. There are other reasons why is mathematics learning is important for young children. The perspective emphasized here is that learning is intelligent action in the continual process of disclosing a world with others. Mathematics, as is language, is a primary instrument of thought. Mathematics provides us with a way to understand, interpret, describe, and create our world (McGarvey, 2008). Mathematics simultaneously expands and constrains our ways of knowing, seeing, doing and being. Early learning of mathematics need not be in service of some future purpose or outcome; instead, mathematics provides a way of thinking and a set of tools (that include, but are not limited to, pre-established outcomes) that allow children and educators to make sense of present experiences using a mathematical lens.

Exemplars of Practice

The following discussion draws upon excerpts from two data sets involving interactions between an educator and child in which significant mathematics content (e.g., shape identification, composition of shapes, and transformations) arose in conversation. Both excerpts are analyzed through an enactivist lens that allow for descriptive analysis and interpretation of the lived experience of teaching|learning.

Exemplars from the video data were selected from a year long project in which the researcher, three teachers and 16 children in a preschool participated. The exemplars selected are useful examples for illustrating and discussing intentional acts and objects. They are not assumed to be ideal. In fact, there are both content and pedagogical issues in both of them.

The first episode involves one of the preschool teachers (Carol) and a student (Jared); the second episode involves myself as teacher-researcher and another child (Martin) in the class. I investigated intentional acts and objects leading to potential opportunities for teaching and learning.

Episode 1: Jared (5 years) and Carol (Pre-School Educator)

A group of five children came into the room where Carol and I welcomed them. We had set out tubs of polygons in a variety of shapes and sizes. The materials were selected because we assumed that they would lead to mathematical discussion, but we did not prescribe what the content of discussion or activity would be. The polygons were available four times over the course of two weeks. This interaction occurred on the first day the tubs were available.

Jared (5 years old) came to the polygon centre and immediately sat on the floor and selected some 2D shapes. He fit two large red right triangles together to form a square (see Figure 1) and continued to build a design.

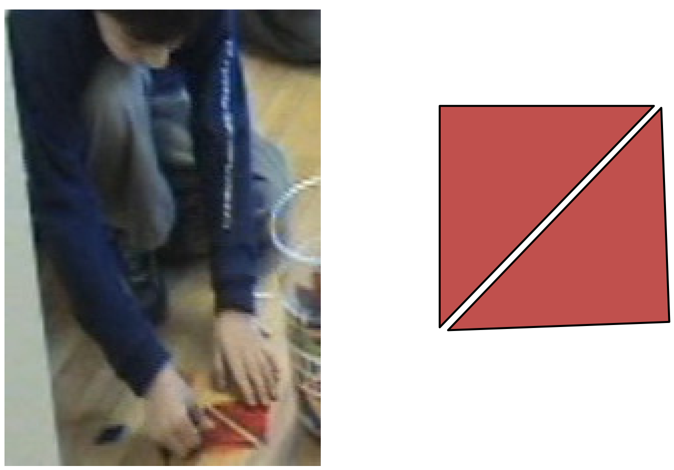

Figure 1: *Composition of two right isosceles triangles.*

He asked Carol sitting beside him on the floor, “What do we do with these?”

“Well, you can make different shapes…. They are kind of like puzzle pieces and you can try to fit them together. You can make pictures or shapes or patterns.”

Carol sat quietly (not touching the shapes) while Jared and the other children worked. After a minute, Carol pointed to the part of Jared’s composition with the two triangles. “Two triangles make a square. That’s so cool. Did you know it would do that?” He nodded yes.

“I’m going to try that ... two triangles. Here’s two triangles. Do you think we could make a square with this?” Jared moved closer to see. Carol put two small blue equilateral triangles together (see Figure 2). “Oh! That didn’t work. How come mine didn’t make a square but yours did?”

“Because you need big ones like that” referring to the red triangles he had used.

Figure 2: *Composition of two equilateral triangles.*

“Okay. The big ones make squares and the small ones don’t. Let’s see. Let’s see if it works,” and she found two large red triangles. “Oh there!” she said as she successfully made a square.

“Hmmm, maybe because this is a different shaped triangle” pointing to the sides of the equilateral. “This one” pointing to the hypotenuse of the red triangle (see Figure 2) “has a longer side than this, hey?” Jared watched intently and nodded his head.

Carol found two small yellow obtuse, isosceles triangles. “Let’s see if these triangles make a square.” She put them together while Jared watched. “Hmm … nope. Interesting. It made like a diamond [rhombus] (see Figure 3). So did this one,” pointing to the blue equilateral triangles. Jared and Carol also tried the large equilateral triangles without success in making a square.

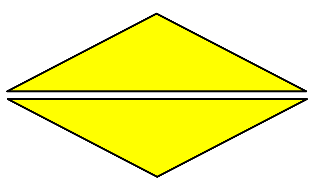

Figure 3: *Composition of two obtuse isosceles triangles*

Jared pointed to a yellow hexagon in his design that was the same colour as the yellow triangles.

Carol responded to shape he pointed to. “Do you know what this one looks like? … A stop sign. Don’t you think? Is this the same shape as a stop sign?”

Jared shrugged and returned to his design and continued building.

Carol held up two large yellow squares. “What if we put two of these together? Two squares makes a …?”

“Long square.”

“Hmm...it’s called a rectangle.”

Jared got two more yellow squares and joined them with the other two to form a large square (see Figure 4).

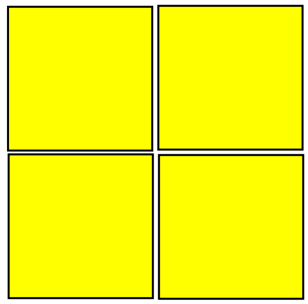

Figure 4: *Composition of four squares*

“Four little squares made a big ...?”

“One!”

“Square. Cool. That’s fun.”

She sat back and watched quietly as the children continued to build. Jared changed his design slightly to contain a long column of yellow squares aligned with a column of green rhombuses filling the space.

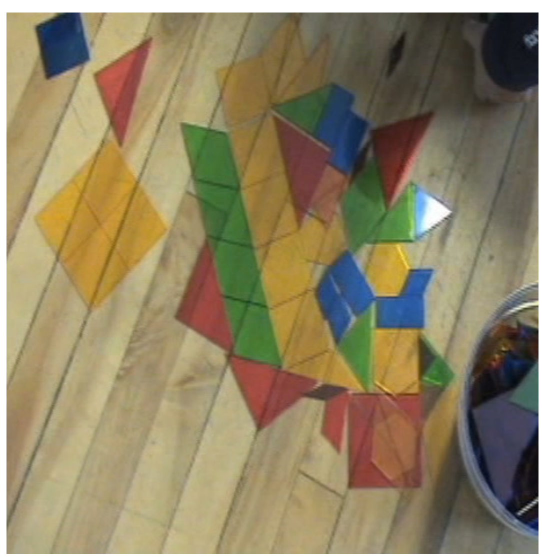

Figure 5: *Jared’s completed design.*

After a minute working independently (without interaction from the teacher), Jared said, “I’m all done” (see Figure 5).

"Look at this, it is so interesting. Do you want to tell me a bit about your creation?"

"It's all different shapes."

Discussion of Episode 1

The exemplar with Carol and Jared is useful for discussing intentionality—both its acts and objects—of the teaching-learning interaction. To begin, the selection of the materials, in this case polygon shapes, was purposeful in that we assumed that forms of mathematical thinking and discussion would emerge. However, implications from enactivism suggest that while no act of teaching or set of materials can *cause* learning to occur (Davis, 2004; Kieren 1995), the possibilities for acting are not infinite, nor are they random. The polygons created a play-space that was not perceptually bounded; that is, we did not and cannot specify in advance the modes of action in which young children will engage. However, the modes of action are not boundless. Based on previous experience, the mathematics that may occur includes topics in geometry, number, pattern and measurement. In particular, a range of two-dimensional geometric topics such as labelling and comparing shapes, composition of two of more shapes to form new shapes, congruence of simple shapes with composition, symmetric designs, tessellations, and transformations (e.g., translation, rotation, and reflection). The challenge for the educator is to be aware of the potential mathematical objects that could arise, understand children's learning in relation to those objects, and also be attuned to new possibilities of acting in the space.

In this episode the materials were new to both Carol and Jared. It is not surprising that both of them engaged in actions that were familiar, although not predictable. On many occasions throughout the session Carol seems to shift back and forth between being an observer and a participant. It is when she is a participant she attends to Jared's composition in a way that reflects her own history of interest and experience. We see her actively trying to direct his attention to the composition of two triangles. Her actions are not uni-directional 'towards' Jared, but her actions simply point beyond herself and have the potential of bringing intentional objects to awareness for both Jared and herself. Carol's topic of concern in this moment is, "Which triangles form a square?" This question is not a prespecified problem in the typical mathematical sense. Instead, the question and intentional acts can be considered as "skillful know-how" since the dynamic system which includes the inextricable interactions of Carol-Jared-environment "both poses the problems and specifies what actions need to be taken for their solution" (Thompson, 2007, p. 11). The episode becomes an intentional experience, not because it is an interaction with mathematical content, but because it is an act of directedness in which there is an effort for both Carol and Jared to stretch towards something such that an idea is disclosed through their questions and inter-actions.

Jared appears acutely interested through much of the episode. Observing the triangles forming different shapes does not (at least in this session) provide an occasion for him to act differently in the space. Interestingly, the second half of the episode which may be considered a direct teaching approach does have an immediate impact on Jared's actions. Not in the labelling of shapes (which was one aspect that Carol was pointing to), but in the composition of squares which was the intentional object of action. In his final composition Jared alters his design to include a long column of squares.

Although possibly less obvious but equally important is that Jared's actions intentionally attract the teacher to the action of combining triangles to form a square. What shape is made when two triangles are combined along their hypotenuse? Carol's own curiosity is at play and the intentional experience becomes a reciprocal act of teaching and learning. Learning is said to have occurred when the possibilities for acting within the (play) space expand. Both Jared and Carol as a dynamic system have expanded their possibilities for acting within the play-space through their interactions with each other and with the materials.

Episode 2: Martin (4 years) and Lynn (Teacher-Researcher)

On the third day in which the tub of polygon shapes was made available to the children, Martin (4 years old) sat at a table next to me (teacher-researcher). A collection of polygon shapes were available onto a table.

We independently began selecting shapes and putting them together. I looked at Martin's design, "What have you got going there?"

"I started off with a traffic light and I'm just adding more colours" (see Figure 6). I copied the same design so that I also had a traffic light.

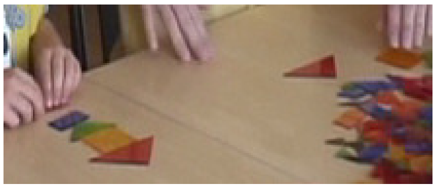

Figure 6: *Martin's initial image of a traffic light.*

Martin cleared away his traffic light and I suggested we play a matching game in which we each copied the other person's shape design (see Figure 7).

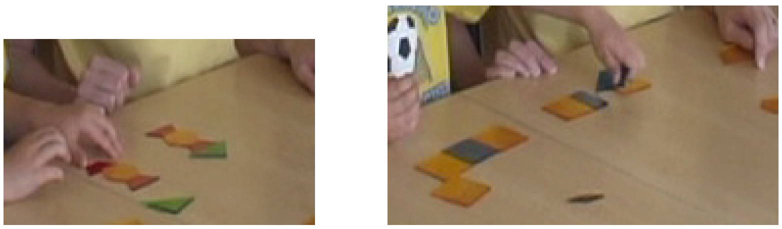

Figure 7: *Game of copying designs.*

After we each had a turn, Martin continued putting two or three shapes together to see what they made. Martin pushed two red trapezoids together. "This is a stop sign."

"Hmmm … well … not quite. You've made a hexagon. Like this." I covered the two trapezoids with a yellow hexagon. "A hexagon has six sides. See ... one, two, three, four, five, six. A stop sign has eight sides" (see Figure 8).

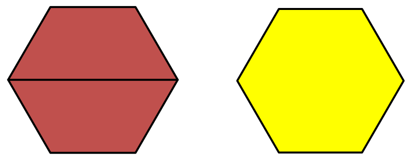

Figure 8: *Two trapezoids congruent to a hexagon.*

"Where's a stop sign?"

"I don't think there is a stop sign here." We searched briefly through the polygons. "We could try making one out of paper." I left the table and returned with red construction paper scraps, scissors and a pencil. I took a square of the paper. I looked at the paper for a few seconds. I cut the corners off. "Here, Martin! This is an octagon ... a stop sign. Here, you try one. You just need to cut the corners off."

I handed Martin a square piece and he cut off the corners. Frustrated he said, "That's not a stop sign" (see Figure 9a).

"Oh, I guess you have to be careful to cut them the same way. Here, try again ... uh ...." I marked two dots on each edge of the paper. "Cut dot to dot."

Martin cut of the corners close to the dots drawn and smiled.

"Great job! That's terrific, Martin" (see Figure 9b). We counted the eight sides together.

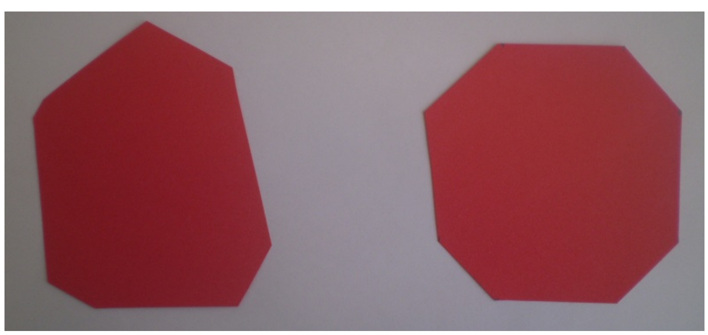

Figure 9 *(a) Martin's first attempt followed by (b) his second attempt to make an octagon.*

Martin took the pencil from me and a piece of paper. He looked up at the stop sign he cut out and tried to draw one. Again he became frustrated. "That's no good" (see Figure 10a).

"Oh gosh, Martin. Wow, you almost had it. Hmmm …. You could do it the same way from a square." Independently, he sketched a square, placed dots, and drew the outline. "It's just like connect the dots." I helped him correct the top right corner, then I erased the corners of the square and he coloured it in red felt pen (see Figure 10b).

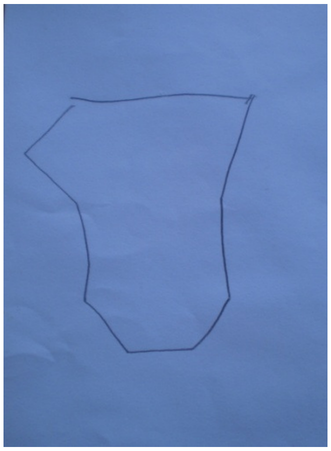

Figure 10a: *Martin's first attempt at drawing an octagon.*

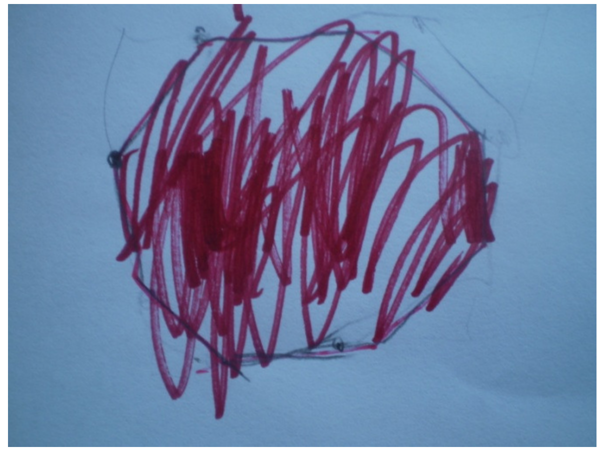

Figure 10b: *Martin's second attempt at drawing an octagon.*

Martin and I continued for several more minutes creating road signs with the polygons and drawing signs on paper (e.g., yield, no parking, one-way sign).

Discussion of Episode 2

In the second episode, we see that Martin and I act in ways that are both similar to and different from Carol and Jared. Our actions are initially familiar, but not pre-planned. Interestingly, our initial efforts—building a traffic light, playing a matching game—had directedness, but they were sporadic and did little to stretch us towards ideas beyond ourselves. However, through the interaction, Martin's interest in traffic signs continued to arise first with constructing a traffic light using coloured shapes (attending to colour) to constructing a stop sign (attending to colour and shape). I hesitated in correcting Martin's shape identification. That is, his composition of two trapezoids formed a hexagon while a stop sign is an octagon. Many other children (and even adults such as Carol in the previous episode) make a similar error. Perhaps it might be seen as an unnecessary attempt to correct an error of a four year old who was not expected to consistently count the sides of the shape, let alone properly label it. However, attending to Martin's labelling of the shape and his strong interest in road signs prompted a search for octagons. As we searched through the polygon shapes I was actively thinking whether it was possible to create a standard octagon with the polygons. When I didn't think it was possible, I suggested making stop signs on paper. The incidental act of making a distinction between a hexagon and an octagon (which was not done in the previous episode) triggered many new actions.

Although I had a square piece of paper in my hand, I had not until that moment imagined how to create an octagon in such a way so that it was accessible for a four-year-old child. In fact, it prompted even further thinking and new action after the episode was over as I wanted to consider approximately where the dots on the edges should be placed if a regular octagon were to be drawn. I had to recall trigonometry objectives that had not been applied in a number of years, but it brought forth new awareness for me as I learned that the side length needed to be segmented in a ratio of approximately 30:40:30 (see Figure 11).

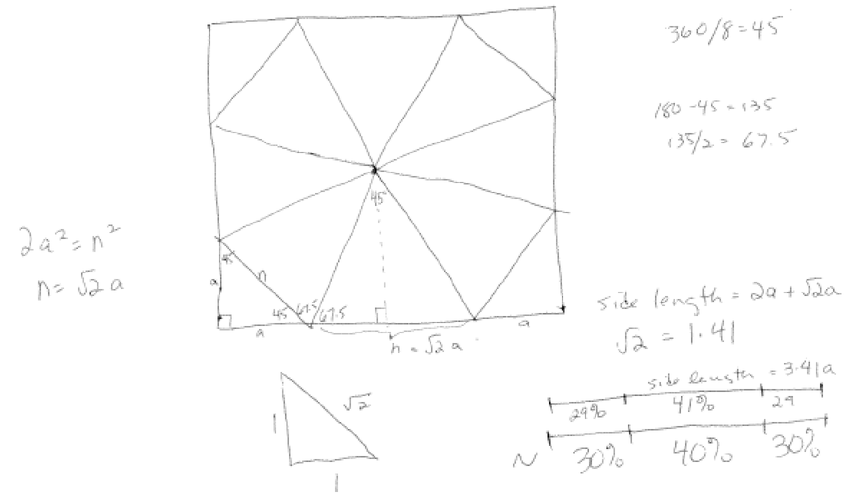

Figure 11: *Teacher-Researcher inquiry into ratio of side lengths for regular octagon.*

Certainly, having a four year old learn to create and draw an octagon will not be an expected outcome in any preschool curriculum. Yet, the intentional object and act expanded both Martin's and my ways acting in the space. Martin was able to draw octagons and octagons began appearing repeatedly for several weeks as his fascination with road signs continued. Similarly, reviewing trigonometry for a preschool educator's professional training is an unlikely objective; yet, the interaction with Martin became an occasion for me to utilize past experience and bring forth new mathematics.

Once again we see this episode as an intentional experience that brings forth a world of new ideas, not as prespecified outcomes, and not as incoherent or random events, but as a way to expand the ways of acting within the space of possibilities. Focusing on *know-how* (different/new ways of acting) rather than *know-what* (fragments of knowledge) expands possibilities for learning for both children and teachers (c.f. Varela, 1999).

Concluding Remarks

The paper offers a view of preschool teaching|learning as a reciprocal relationship arising in intentional experiences. Through intentional acts teachers orient children's attention to particular mathematical objects; however, some acts may alter a child's perception and others will not. Similarly, children engage in intentional acts of which some are attended to by the teacher and classmates and others are not. The teacher's role then is an active one searching for and participating in intentional experiences that might shift patterns of acting, while recognizing that perception and attention are highly contingent on the context and structures of the participants at any given moment. In that active search is an ethical imperative to increase, rather than decrease, ways of acting in a space of possibility. Intentional experiences within a proscribed mathematical space (rather than prescribed content) offer an alternative to child-centred exploration or teacher-directed practices.

References

Anderson, A., Anderson, J. & Thauberger, C. (2008). Mathematics learning and teaching in the early years. In O. N. Saracho & B. Spodek (Eds.), *Contemporary perspectives on mathematics in early childhood education* (pp. 95 – 132). Charlotte, NC: Information Age Publishing, Inc.

Clements, D.H., Sarama, J., & Dibiase, A.-M., (Eds). (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Clements, D. H., & Sarama, J. (2007). *SRA Real Math, PreK-Building Blocks.* Columbus, OH: SRA/McGraw-Hill.

Dahlberg, G. & Moss, P. (2005). *Ethics and politics in early childhood education.* London: RoutledgeFalmer.

Davis, B. (2004). *Inventions of teaching: A genealogy.* Mahwah, JH: Lawrence Erlbaum Associates.

English, L. (Ed.) (2004). *Mathematical and analogical reasoning of young learners*. New Jersey: LEA.

von Foerster, H. (2003). *Understanding understanding: Essays on cybernetics and cognition.* New York: Springer.

Ginsburg, H., Greenes, C., & Balfanz, R. (2003). *Big math for little kids*. Parsippany, NJ: Dale Seymour Publications.

Kieren, T. E. (1995). Teaching mathematics (in-the-middle): Enactivist view on learning and teaching mathematics. Invited plenary at Queens/Gage Canadian National Leadership Conference: Queens University, Kingston, Ontario.

Lakoff, G. & Johnson, M. (1999). *Philosophy in the flesh: The embodied mind and its challenge to Western thought.* New York: Basic Books.

Maturana, H. & Varela, F. (1992). *The tree of knowledge: The biological roots of human understanding (Revised Edition)*. Boston, MA: Shambhala.

McGarvey, L.M. (2008). Rationale gone missing: A comparative and historical curriculum search. *Delta-K, 46,* 19-23.

National Association for the Education of Young Children (NAEYC)/National Council of Teachers of Mathematics (NCTM). (2002). Early childhood mathematics: Promoting good beginnings. Retrieved from: http://www.naeyc.org/about/positions/pdf/psmath.pdf

Sarama, J. & Clements, D.H. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* New York: Routledge.

Seo, K.-H. & Ginsburg, H.P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D.H. Clements, J.Sarama, & A.-M. Dibiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 91-104). Mahwah, NJ: Lawrence Erlbaum Associates.

Sophian, C. (2004). Mathematics for the future: developing a Head Start curriculum to support mathematics learning. *Early Childhood Research Quarterly, 19,* 59-81.

Thompson, E. (2007). *Mind in life: Biology, phenomenology, and the sciences of mind.* Cambridge, MA: Harvard University Press.

Thompson, E. & Zahavi, D. (2007). Philosophical issues: Phenomenology. In P.D. Zelazo, M. Moscovitch, and E. Thompson (eds.), *The Cambridge Handbook of Consciousness.* Cambridge University Press. Uncorrected proofs. Retrieved from http://individual.utoronto.ca/evant/ThompsonZahavi.pdf.

Varela, F. (1999). *Ethical know-how: Action, wisdom, and cognition*. Stanford, CA: Stanford University.

Varela, F.J., Thompson, E. & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge, MA: The MIT Press.