# Meeting Mathematics Standards with a Deck of Playing Cards

Deborah A. McAllister College of Health, Education, and Professional Studies The University of Tennessee at Chattanooga

> Shirley A. McDonald Ringgold Middle School, Ringgold, GA

Susan M. Bothman Ooltewah High School, Ooltewah, TN

March 3, 2010

#### Introduction

The activities contained in this document have been presented at several conference sessions. Participants engaged in hands-on activities for Grades 3 through 8 that utilized a deck of playing cards to meet mathematics content and process standards. Activities included variations of card games, but with a mathematical twist, and other card-based tasks. Games and activities include the following: Integer War, Fraction War, Salute, 24 with Playing Cards, Set with Playing Cards, Area, Quiz Time, and Attribute Eights.

## **Objectives for Conference Sessions**

- 1. To use a deck of playing cards to illustrate concepts in mathematics.
- 2. To correlate activities to both content and process standards.
- 3. To discuss the place of this activity in the classroom and in preservice education.
- 4. To provide participants with a set of low-cost activities which have been prepared for classroom implementation.

#### **Conference Sessions**

The activities have been presented at the following conferences:

- 2009 NCTM regional conference, Nashville, TN.
- 2008 Georgia Council of Teachers of Mathematics, Rock Eagle, GA.
- 2008 NCTM national conference, Salt Lake City, UT.
- 2008 Chattanooga Area Mathematics Teachers Association, Chattanooga, TN.
- 2007 Alabama Council of Teachers of Mathematics, Montgomery, AL.
- 2007 Georgia Council of Teachers of Mathematics, Rock Eagle, GA.
- 2007 Tennessee Mathematics Teachers' Association, Knoxville, TN.

The activities are also presented at the following Web address:

http://oneweb.utc.edu/~deborah-mcallister/deckofcards.html.

#### **Standards References**

Alabama Learning Exchange. (2010). *Mathematics*. Retrieved from http://alex.state.al.us/browseMath.php

Georgia Department of Education. (2008). Georgia standards – mathematics standards.

Retrieved from

https://www.georgiastandards.org/Standards/Pages/BrowseStandards/MathStandards.aspx

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author. (See http://standards.nctm.org/.)

Tennessee Department of Education. (n.d.). *Curriculum standards, mathematics*. Retrieved from http://state.tn.us/education/ci/math/index.shtml (Standard 1, Mathematical Processes, grade level expectations are present in all activities.)

### **Integer War**

Using a standard deck of cards, or the cards from 1 (ace) to 10, cards are dealt to each student. Each student turns over a card. The first student to correctly add the numbers on the cards wins the hand and receives the cards. For integers, the given color code will be black is positive and red is negative. After all cards are played, the student who has the most cards wins the game.

#### **Variations**

- ❖ Addition (positive only).
- Multiplication (positive only).
- ❖ Integers (black is positive, red is negative).
- The jack, queen, and king cards can be counted as 10, or as 11, 12, and 13, respectively, or removed from the deck.
- Exponent War could be played, with the assistance of a calculator. Use a base (first card) and an exponent (second card), or a base (my card) and exponent (your card).

## **NCTM Standard 1, Number and Operations**

Understand meanings of operations and how they relate to one another.

Grades 3-5

Understand various meanings of multiplication and division.

Understand the effects of multiplying and dividing whole numbers.

Compute fluently and make reasonable estimates.

Grades 3-5

Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.

## Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0306.2.9 - Solve contextual problems involving the addition (with and without regrouping) and subtraction (with and without regrouping) of two- and three digit whole numbers.

## **Georgia Performance Standards**

Grade 3 – M3N2 a, b, c; M3N3 b, f, g

Grade 4 – M4N3; M4N7 d

Grade 6 – M6N1 g

Grade 7 - M7N1 c, d

Grade 8 - M8N1 i

#### **Alabama Standards**

Grade 7 –

Demonstrate computational fluency with addition, subtraction, and multiplication of integers.

Solve problems requiring the use of operations on rational numbers.

Grade 8 -

Use various strategies and operations to solve problems involving real numbers.

Simplify expressions containing natural number exponents by applying one or more of the laws of exponents.

#### **Fraction War**

Using a standard deck of cards, or the cards from 1 (ace) to 10, cards are dealt to each student. Each student turns over two cards: a numerator (first card) and a denominator (second card). The student who has the larger fraction receives the cards. After all cards are played, the student with the most cards wins the game.

#### **Variations**

The jack, queen, and king cards can be counted as 10, or removed from the deck.

#### **NCTM Standard 1, Number and Operations**

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Grades 6-8

Work flexibly with fractions, decimals, and percents to solve problems.

Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line.

## Tennessee Mathematics Standards, Standard 1, Mathematical Processes

SPI 0306.1.4 - Match the spoken, written, concrete, and pictorial representations of fractions with denominators up to ten.

## Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0306.2.13 - Recognize, compare, and order fractions (benchmark fractions, common numerators, or common denominators).

SPI 0406.2.7 - Convert improper fractions into mixed numbers and/or decimals.

## **Georgia Performance Standards**

Grade 3 - M3N5 b, d, e, f, g

Grade 4 – M4N6 a, b, c

Grade 5 - M5N4 a, b, c, f

Grade 6 – M6N1 c, g

# Alabama Standards

Grade 7 –

Solve problems requiring the use of operations on rational numbers.

Grade 8 –

Use various strategies and operations to solve problems involving real numbers.

#### Salute

One general and two privates are needed for this game. Each private has half of a deck of cards, using 1 (ace) through 10. When the general says, "salute," each private deals one card away from the deck and holds it face-up on his or her forehead. The general computes and states the product of the two numbers (positive only). Each private must find the value of the card on his or her forehead. After all cards are played, the student who has the most cards wins the game. (Activity provided through personal communication, March 2007.)

#### **Variations**

- Use integers (black is positive, red is negative).
- The jack, queen, and king cards can be counted as 11, 12, and 13, respectively, or removed from the deck.

## **NCTM Standard 1, Number and Operations**

Understand meanings of operations and how they relate to one another.

Grades 3-5

Understand various meanings of multiplication and division.

Understand the effects of multiplying and dividing whole numbers.

Compute fluently and make reasonable estimates.

Grades 3-5

Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.

#### Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0306.2.5 - Identify various representations of multiplication and division.

SPI 0306.2.6 - Recall basic multiplication facts through 10 times 10 and the related division facts.

SPI 0306.2.8 - Solve problems that involve the inverse relationship between multiplication and division.

## Tennessee Mathematics Standards, Standard 3, Algebra

SPI 0306.3.3 - Find the missing values in simple multiplication and division equations.

## **Georgia Performance Standards**

Grade 3 – M3N3 b, f, g; M3N4 a, c, e, f

Grade 4 – M4N3; M4N4 a, c; M4N7 d

Grade 6 – M6N1 g

#### **Alabama Standards**

Grade 3 -

Multiply whole numbers with and without regrouping using single-digit multipliers.

Divide whole numbers using two-digit dividends and one-digit divisors.

Grade 4 -

Solve problems, including word problems, involving the basic operations of multiplication and division on whole numbers through two-digit multipliers and one-digit divisors.

Grade 7 -

Solve problems requiring the use of operations on rational numbers.

Grade 8 -

Use various strategies and operations to solve problems involving real numbers.

### 24 with Playing Cards

Using a standard deck of cards, or cards from 1 (ace) to 10, place four cards face-up on the table. Each student, in turn, will use the cards to find a value of 24 using the correct order of operations for addition, subtraction, multiplication, and division. The student takes the cards used. Empty slots are filled with new cards. Play continues until the deck is used. If a value of 24 cannot be reached, add another card to the table. After all cards have been played, the winner is the student with the most cards.

#### **Variations**

- The jack, queen, and king cards can be counted as 10 and/or as 11, 12, and 13, respectively, or removed from the deck.
- ❖ Use a fifth card for the target number, rather than 24. Place four cards in a row, with the target card beneath the row.
- Exponents can be used.
- ❖ Buy the game! See http://www.math24.com/

## **NCTM Standard 1, Number and Operations**

Understand meanings of operations and how they relate to one another.

Grades 3-5

Understand various meanings of multiplication and division.

Understand the effects of multiplying and dividing whole numbers.

Compute fluently and make reasonable estimates.

Grades 3-5

Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.

#### NCTM Standard 6, Problem Solving, Grades P-12

Apply and adapt a variety of appropriate strategies to solve problems.

Monitor and reflect on the process of mathematical problem solving.

## NCTM Standard 7, Reasoning and Proof, Grades P-12

Select and use various types of reasoning and methods of proof.

### NCTM Standard 8, Communication, Grades P-12

Use the language of mathematics to express mathematical ideas precisely.

## Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0306.2.6 - Recall basic multiplication facts through 10 times 10 and the related division facts.

SPI 0406.2.12 - Solve problems using whole number division with one- or two-digit divisors.

## **Georgia Performance Standards**

Grade 3 – M3N2 a, b, c; M3N3 f, g

Grade 4 – M4N4 a, c; M4N7 a, b, c, d

Grade 5 - M5N1 a, b, c

Grade 6 - M6N1 a

Grade 7 – M7A1 b

Grade 8 - M8N1 g, i

#### **Alabama Standards**

Grade 3 –

Solve addition and subtraction problems, including word problems, involving two- and threedigit numbers with and without regrouping.

Multiply whole numbers with and without regrouping using single-digit multipliers.

Divide whole numbers using two-digit dividends and one-digit divisors.

Grade 4 –

Solve problems, including word problems, involving the basic operations of multiplication and division on whole numbers through two-digit multipliers and one-digit divisors.

Grade 5 -

Solve problems involving basic operations on whole numbers, including addition and subtraction of seven-digit numbers, multiplication with two-digit multipliers, and division with two-digit divisors.

Grade 6 -

Demonstrate computational fluency with addition, subtraction, multiplication, and division of decimals and fractions.

Grade 7 -

Use order of operations to evaluate numerical expressions.

Solve problems requiring the use of operations on rational numbers.

Grade 8 -

Use various strategies and operations to solve problems involving real numbers.

Use order of operations to evaluate and simplify algebraic expressions.

## **Set with Playing Cards**

Place 12 cards, face up, on the table as a 3 by 4 array. When a set can be seen, the student says, "set," and removes the cards. Empty slots are filled with new cards. Play continues until the deck is used. If a set cannot be found, add three cards to the table. After all cards have been played, the winner is the student with the most cards.

Sets include the following:

- Three of a kind, e.g., 7 of hearts, 7 of diamonds, 7 of clubs.
- Three in a row, all of the same suit, e.g., 3 of hearts, 4 of hearts, 5 of hearts.
- Three in a row, all different suits, e.g., 8 of clubs, 9 of hearts, 10 of diamonds.
- Ace can precede 2 or follow king.

#### **Variations**

- ❖ Buy a deck of Set cards!
- Solve the daily puzzle, online, with six solutions. See http://www.setgame.com/

## NCTM Standard 2, Algebra

Understand patterns, relations, and functions

Grades 3-5

Describe, extend, and make generalizations about geometric and numeric patterns.

Represent and analyze patterns and functions, using words, tables, and graphs.

Grades 6-8

Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.

## NCTM Standard 7, Reasoning and Proof, Grades P-12

Select and use various types of reasoning and methods of proof.

# NCTM Standard 10, Representation, Grades P-12

Use representations to model and interpret physical, social, and mathematical phenomena.

# **Georgia Performance Standards**

Grade 4 – M4N3; M4A1 a

#### Area

- Find the length, width, perimeter, and area of the playing card using U.S. customary units.
- 2. Find the length, width, perimeter, and area of the playing card using metric units.
- 3. If the playing card is an irregular shape, trace it on a piece of graph paper (square-inch, square-half-inch, or square-centimeter) to estimate the area.
- 4. Place one vertex of the playing card at the origin on a coordinate plane of square-inch paper (or square-half-inch paper, or square-centimeter paper). To the nearest inch (or unit used), label all vertices. Use both portrait and landscape orientations in each of the four quadrants.

#### **Variations**

- Adjust the unit size of the paper, and number of sheets of paper needed, based on the size of the card.
- In "portrait" orientation, draw one diagonal of the card, from the origin to the upper-right corner. Determine the slope of the line. Repeat with the other diagonal.
- Repeat the previous variation when the card is drawn in "landscape" orientation.
- Use the Pythagorean theorem to find the length of the diagonal.
- ❖ Use the distance formula to find the length of the diagonal.
- Find the point of intersection of the two diagonals using a system of two linear equations in two variables.

## **NCTM Standard 3, Geometry**

Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Grades 3-5

Make and use coordinate systems to specify locations and to describe paths.

Apply transformations and use symmetry to analyze mathematical situations.

Grades 3-5

Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.

Identify and describe line and rotational symmetry in two- and three-dimensional shapes

and designs.

Grades 6-8

Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.

Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

## **NCTM Standard 4, Measurement**

Understand measurable attributes of objects and the units, systems, and processes of measurement.

Grades 3-5

Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute.

Understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems.

Grades 6-8

Understand both metric and customary systems of measurement.

Apply appropriate techniques, tools, and formulas to determine measurements.

Grades 3-5

Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes.

### Tennessee Mathematics Standards, Standard 1, Mathematical Processes

SPI 0306.1.5 - Represent problems mathematically using diagrams, numbers, and symbolic expressions.

SPI 0306.1.6 - Identify and use vocabulary to describe attributes of two- and three-dimensional shapes.

SPI 0306.1.7 - Select appropriate units and tools to solve problems involving measures.

## Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0606.2.7 - Locate positive rational numbers on the number line.

SPI 0606.2.8 - Locate integers on the number line.

SPI 0706.2.5 - Solve contextual problems that involve operations with integers.

## Tennessee Mathematics Standards, Standard 3, Algebra

SPI 0606.3.9 - Graph ordered pairs of integers in all four quadrants of the Cartesian coordinate system.

SPI 0706.3.4 - Interpret the slope of a line as a unit rate given the graph of a proportional relationship.

SPI 0806.3.1 - Find solutions to systems of two linear equations in two variables.

SPI 0806.3.5 - Determine the slope of a line from an equation, two given points, a table or a graph.

## Tennessee Mathematics Standards, Standard 4, Geometry and Measurement

SPI 0306.4.4 - Calculate the perimeter of shapes made from polygons.

SPI 0306.4.5 - Choose reasonable units of measure, estimate common measurements using benchmarks, and use appropriate tools to make measurements.

SPI 0306.4.6 - Measure length to the nearest centimeter or half inch.

SPI 0306.4.7 - Solve problems requiring the addition and subtraction of lengths.

SPI 0406.4.3 - Construct geometric figures with vertices at points on a coordinate grid.

SPI 0406.4.7 - Determine appropriate size of unit of measurement in problem situations involving length, capacity or weight.

SPI 0406.4.9 - Solve problems involving area and/or perimeter of rectangular figures.

SPI 0406.4.10 - Identify images resulting from reflections, translations, or rotations.

SPI 0506.4.5 - Find the length of vertical or horizontal line segments in the first quadrant of the coordinate system, including problems that require the use of fractions and decimals.

SPI 0806.4.1 - Use the Pythagorean theorem to solve contextual problems.

SPI 0806.4.2 - Apply the Pythagorean theorem to find distances between points in the coordinate plane to measure lengths and analyze polygons and polyhedra.

## **Georgia Performance Standards**

Grade 3 – M3M2 b, g; M3M3 a, b, c; M3M4 a, b, c; M3A1 a

Grade 4 – M4M2 b; M4G3 a, b, c

Grade 5 – M5M1 a, f; M5G1; M5A1 c

Grade 6 – M6M2 a, b; M6G1 b; M6A2 d, e, f

Grade 7 – M7G2 a, b; M7G3 c; M7A3 a, c, d

Grade 8 – M8N1 e, f, h, k; M8G1 d; M8G2 a; M8A3 a, b, f, g; M8A4 a, b, f

#### Alabama Standards

Grade 3 -

Specify locations on a coordinate grid by using horizontal and vertical movements.

Measure length in metric units.

Grade 4 –

Find locations on a map or grid using ordered pairs.

Grade 5 -

Identify components of the Cartesian plane, including the x-axis, y-axis, origin, and quadrants.

Estimate perimeter and area of irregular shapes using unit squares and grid paper.

Calculate the perimeter of rectangles from measured dimensions.

Grade 6 -

Identify two-dimensional and three-dimensional figures based on attributes, properties, and component parts.

Plot coordinates on grids, graphs, and maps.

Solve problems involving perimeter and area of parallelograms and rectangles.

Grade 7 -

Express a pattern shown in a table, graph, or chart as an algebraic equation.

Determine the transformation(s), including translations, reflections, or rotations, used to alter the position of a polygon on the coordinate plane.

Grade 8 -

Use various strategies and operations to solve problems involving real numbers.

Solve problems using the Pythagorean Theorem.

## **Quiz Time**

For a 10-item, true-false quiz, a student decides to use a deck of playing cards to select answers. Red will represent "true" and black will represent "false." After randomly drawing a card and recording the answer, replace the card within the deck so that each card will be as likely to be drawn for each of the quiz items. What the student doesn't know is that the teacher made up the answer key in the same manner. Using two decks of cards, simulate the teacher's answer key and the student's answers. Approximately what percent of quiz items will be correct? Write this value as a fraction, a decimal, and a percent.

#### **Variations**

- ❖ Use the four card suits, clubs, diamonds, hearts, and spades, to represent multiple-choice answers, "a," "b," "c," and "d." Approximately what percent of quiz items will be correct? Write this value as a fraction, a decimal, and a percent.
- Conduct 10 trials. Report the mean, median, and mode.
- Conduct 10 trials. Display the results in a graph.

## NCTM Standard 5, Data Analysis and Probability

Understand and apply basic concepts of probability

Grades 3-5

Predict the probability of outcomes of simple experiments and test the predictions.

Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

#### NCTM Standard 9, Connections, Grades P-12

Recognize and apply mathematics in contexts outside of mathematics.

#### NCTM Standard 10, Representation, Grades P-12

Create and use representations to organize, record, and communicate mathematical ideas.

Use representations to model and interpret physical, social, and mathematical phenomena.

## Tennessee Mathematics Standards, Standard 1, Mathematical Processes

SPI 0606.1.1 - Make conjectures and predictions based on data.

## Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0606.2.5 - Transform numbers from one form to another (fractions, decimals, percents, and mixed numbers).

SPI 0706.2.6 - Express the ratio between two quantities as a percent, and a percent as a ratio or fraction.

## Tennessee Mathematics Standards, Standard 5, Data Analysis, Statistics, and Probability

SPI 0306.5.3- Make predictions based on various representations of data.

SPI 0406.5.4 - List all possible outcomes of a given situation or event.

SPI 0506.5.1 - Depict data using various representations, including decimal and/or fractional data.

SPI 0506.5.3 - Calculate measures of central tendency to analyze data.

SPI 0606.5.1 - Determine the theoretical probability of simple and compound events in familiar contexts.

SPI 0606.5.3 - Determine whether or not a sample is biased.

SPI 0706.5.4 - Use theoretical probability to make predictions.

SPI 0806.5.1 - Calculate probabilities of events for simple experiments with equally probable outcomes.

#### **Georgia Performance Standards**

Grade 3 – M3N5 a, c; M3D1 a

Grade 4 – M4N5 a; M4D1 a, b, c

Grade 5 – M5N2 a; M5N4 h; M5N5; M5D1 a; M5D2

Grade 6 – M6N1 f; M6A1; M6D1 a, b, c, e; M6D2 a, b, c

Grade 7 - M7D1 b, c, f

Grade 8 – M8D3 a, b

#### **Alabama Standards**

Grade 3 -

Determine the likelihood of different outcomes in a simple experiment.

Grade 4 -

Recognize equivalent forms of commonly used fractions and decimals.

Determine if outcomes of simple events are likely, unlikely, certain, equally likely, or impossible.

Represent numerical data using tables and graphs, including bar graphs and line graphs.

Grade 5 -

Use common fractions to represent the probability of events that are neither certain nor impossible.

Grade 6 –

Solve problems involving decimals, percents, fractions, and proportions.

Find the probability of a simple event.

Grade 7 -

Determine measures of central tendency (mean, median, and mode) and the range using a given set of data or graphs, including histograms, frequency tables, and stem-and-leaf plots.

Grade 8 -

Interpret data from populations, using given and collected data.

Determine the theoretical probability of an event.

### **Attribute Eights**

Using a standard deck of cards, identify the following attributes: (a) suit (clubs, diamonds, hearts, spades); and (b) number (1/ace through 10, jack, queen, king). Deal seven cards to each student. Turn over one card next to the deck as a discard pile. The student to the right of the dealer begins play. The student must play a card that shares an attribute with the card next to the deck, or an "8." If the student cannot play a card, the student must draw one card from the deck. Play continues until one student plays all cards in his or her hand. If no student has played all cards before the deck is used, the winner is the student with the fewest cards.

#### **Variations**

• Use factors and multiples of the card on the discard pile.

## **NCTM Standard 1, Number and Operations**

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Grades 3-5

Describe classes of numbers according to characteristics such as the nature of their factors.

Grades 6-8

Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

## NCTM Standard 9, Connections, Grades P-12

Recognize and apply mathematics in contexts outside of mathematics.

## Tennessee Mathematics Standards, Standard 2, Number and Operations

SPI 0306.2.6 - Recall basic multiplication facts through 10 times 10 and the related division facts.

SPI 0306.2.8 - Solve problems that involve the inverse relationship between multiplication and division.

SPI 0406.2.4 - Find factors, common factors, multiples, and common multiples of two numbers.

SPI 0506.2.7 - Recognize equivalent representations for the same number.

## **Georgia Performance Standards**

Grade 3 – M3N3 b, g; M3N4 a, f

Grade 4 - M4N4 a

Grade 5 – M5N1 b

Grade 6 – M6N1 a

# Georgia Performance Standards - Process Standards (throughout activities)

P1 a, c

P2 b

P3 a, b, d

P4 a, b, c

P5 a, b, c