Arizona Mathematics Standard Articulated by Grade Level

GRADE 3

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Strand 1: Number and Operations

Number sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world application. It includes an awareness of the different ways in which numbers are used, such as counting, measuring, labeling, and locating. It includes an awareness of the different types of numbers such as, whole numbers, integers, fractions, and decimals and the relationships between them and when each is most useful. Number sense includes an understanding of the size of numbers, so that students should be able to recognize that the volume of their room is closer to 1,000 than 10,000 cubic feet. Students develop a sense of what numbers are, i.e., to use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to estimate to determine the reasonableness of results.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.

In Grade 3, students build on their previous work with numbers and deepen their understanding of place value in various contexts. They extend their understanding of the base ten number system to larger numbers and apply this understanding by representing numbers in various equivalent forms. Students develop an understanding of the meanings and uses of fractions. They solve problems that involve comparing and ordering fractions and learn to represent fractions in different ways.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Express whole numbers through six digits using and connecting multiple representations. Connections: M03-S1C1-02, M03-S1C1-03, M03-S1C2-01, M03-S1C2-03, M03-S2C1-01, M03S3C2-02, M03-S3C3-01		Use models, pictures, symbols, spoken and written words, and expanded notation. Models may include money, place value charts, or physical objects such as base ten blocks. Continued on next page

The bulleted items within a performance objective indicate the specific content to be taught.

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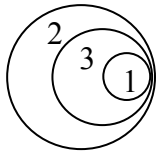
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		Examples: • If the diagram represents the number 231, how would you represent the number 4,521?  • The US Census Bureau estimates that the number of children between the ages of 5 and 13 in Arizona in 2006 was seven hundred ninety-one thousand, nine hundred thirty-one. What is this number written in numeric form?
PO 2. Compare and order whole numbers through six digits by applying the concept of place value. Connections: M03-S1C1-01, M03-S1C1-04, M03-S1C3-01, M03-S2C1-02, M03-S2C4-02, M03-S3C3-01	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Use comparative language and symbols (<, >, =, ≠).
PO 3. Count and represent money using coins and bills to $100.00. Connections: M03-S1C1-01, M03-S1C2-01, M03-S1C2-02, SS03-S5C2-01, SS03-S5C5-01	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	

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Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 4. Sort whole numbers into sets and justify the sort. Connections: M03-S1C1-02, M02-S1C2-04	M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.	Numbers may be sorted into categories such as even and odd, magnitude (number between 1-9, numbers between 10-99, etc.), multiples of 5, digits in the numbers (all of the numbers in the first category have a 3 in the tens place). Sorting numbers by their divisibility can be used to reinforce multiplication and division facts. Examples: • Tarin drew the cards 4, 26, 18, 102, 75, 60, and 55 from a deck of cards labeled with the numbers 1 through 120. He sorted the cards into two groups. Group 1: 4, 26, 18, 60, 102 and Group 2: 75, 55. o What categories might Tarin have used to sort the cards? o Where would you place the card 57 if it were drawn next? • The numbers 1-20 can be sorted into numbers that have a factor of 3 and numbers that have a factor of 4. NOTE: 12 would belong in both sets.
PO 5. Express benchmark fractions as fair sharing, parts of a whole, or parts of a set. Connections: M03-S1C1-06, M03-S1C2-03	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths and tenths. Students are not expected to compute equivalent fractions but they should recognize that fractions can have more than one name. Examples: • Amy has 12 pencils. She is going to share the pencils fairly among 3 people. What fraction of the pencils will each person get? Continued on next page

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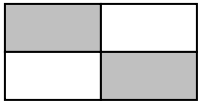
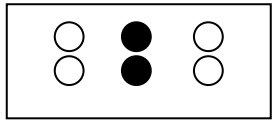
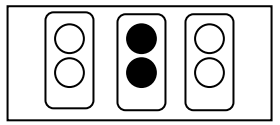
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		• What fraction of the rectangle is shaded? Write the fraction in numerals and words. How might you draw the rectangle in another way but with the same fraction shaded?  Solution: $\frac{2}{4}$ or $\frac{1}{2}$ What fraction of the set is black?  Solution: $\frac{2}{6}$  Solution: $\frac{1}{3}$

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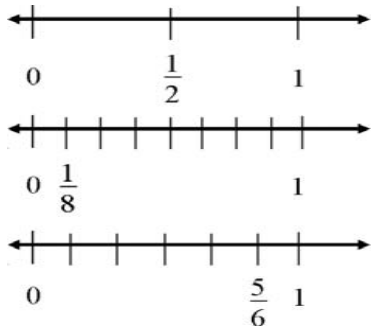
Arizona Mathematics Standard Articulated by Grade Level

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 6. Compare and order benchmark fractions. Connections: M03-S1C1-05, M03-S1C3-01	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection.	Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, and tenths. Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include <, >, =, ≠. Fractions may be compared using $\frac{1}{2}$ as a benchmark.  Possible student thinking: • $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. • $\frac{5}{6} > \frac{1}{2}$ because $\frac{3}{6} = \frac{1}{2}$ and $\frac{5}{6} > \frac{3}{6}$. Continued on next page

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Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection.	Fractions with common denominators may be compared using the numerators as a guide. • $\frac{2}{6} < \frac{3}{6} < \frac{5}{6}$ Fractions with common numerators may be compared and ordered using the denominators as a guide. • $\frac{3}{10} < \frac{3}{8} < \frac{3}{4}$

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Strand 1: Number and Operations
Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

In Grade 3, students build on their previous work with numbers to understand the meanings of multiplication and division. Students apply basic multiplication facts and efficient procedures. They explore the relationship between multiplication and division as they learn related multiplication and division facts.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Add and subtract whole numbers to four digits. Connections: M03-S1C1-01, M03-S1C1-03, M03-S1C2-02, M03-S1C3-01, M03-S2C1-02, M03-S2C4-02, M03-S2C4-03, M03-S3C1-01, M03-S3C1-02, M03-S3C2-01, M03-S3C3-01	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	Problems should include vertical and horizontal forms, including opportunities to apply the commutative and associative properties. Example: • Mary read 1,173 pages over her summer reading challenge. She was only required to read 899 pages. How many extra pages did Mary read over the challenge requirements? Continued on next page

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Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		Students may solve the problem using the traditional algorithm. Here are four other methods students may use to solve the computation in the problem above. • 899 + 1 = 900, 900 + 100 = 1,000, 1000 + 173 = 1,173, therefore 1+ 100 + 173 = 274 pages (Adding Up Strategy) • 900 + 100 is 1,000; 1,000 + 173 is 1,173; 100 + 173 is 273 plus 1 (for 899, not 900) is 274 (Compensating Strategy) • Take away 173 from 1,173 to get to 1,000, take away 100 to get to 900, and take away 1 to get to 899. Then 173 +100 + 1 = 274 (Subtraction Strategy) • 899 + 1 is 900, 900, 1,000 (that's 100). 1,000, 1,100 (that's 200 total). 1,100, 1,110, 1,120, 1,130, 1,140, 1,150, 1,160, 1,170, (that's 70 more), 1,171, 1,172, 1,173 (that's 3 more) so the total is 1+200+70+3 = 274 (Adding by Tens or Hundreds Strategy)
PO 2. Create and solve word problems based on addition, subtraction, multiplication, and division. Connections: M03-S1C1-03, M03-S1C2-01, M03-S1C2-03, M03-S1C2-04, M03-S1C2-05, M03-S1C2-06, M03-S1C2-07, M03-S1C3-01, M03-S2C1-02, M03-S2C3-01, M03-S2C3-02, M03-S2C4-02, M03-S2C4-03, M03-S3C1-01, M03-S3C2-01, M03-S3C3-02, M03-S3C3-03, M03-S4C4-01, M03-S4C4-03, M03-S4C4-04, M03-S4C4-05	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Students use a variety of representations for creating and solving one-step word problems, i.e., numbers, words, pictures, physical objects, or equations. Students explain their thinking, show their work by using at least one of these representations, and verify that their answer is reasonable.

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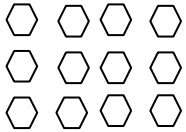
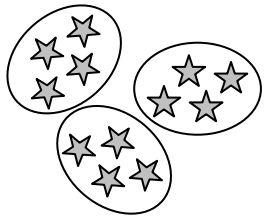
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 3. Demonstrate the concept of multiplication and division using multiple models. Connections: M03-S1C1-01, M03-S1C1-05, M03-S1C2-02, M03-S1C2-04, M03-S1C2-05, M03-S1C2-06, M03-S2C3-01, M03-S2C3-02, M03-S3C3-03, M03-S4C4-04	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection. M03-S5C2-04. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Students are expected to be familiar with multiple representations. The equation 3 x 4 = 12 could be represented in the following ways. • an array:  • equal sets:  • repeated addition or subtraction**:** 4 + 4 + 4 • three equal jumps forward from 0 on the number line to 12:  Continued on next page

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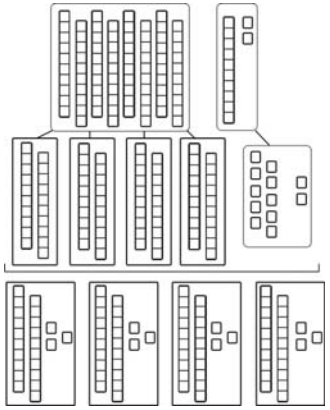
<table>
<tr><th><u>Performance Objectives</u></th><th><u>Process Integration</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>Students are expected to:</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Students should experience problems that involve both sharing and measurement.

Examples:
• This is an example of a partitive division or fair sharing problem:
o The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips would each person get?

• The following is an example of a measurement or repeated subtraction problem:
o Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how may days will the bananas last?

<table><tr><td>Starting</td><td>Day 1</td><td>Day 2</td><td>Day 3</td><td>Day 4</td><td>Day 5</td><td>Day 6</td></tr><tr><td>24</td><td>24-4=
20</td><td>20-4=
16</td><td>16-4=
12</td><td>12-4=
8</td><td>8-4=
4</td><td>4-4=
0</td></tr></table>
Solution: The bananas will last for 6 days.</td></tr>
</table>

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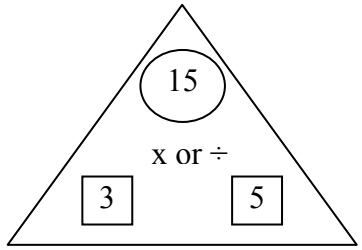
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 4. Demonstrate fluency of multiplication and division facts through 10. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C2-05, M03-S1C2-06, M03-S1C2-07, M03-S2C3-01, M03-S2C3-02, M03-S3C1-01, M03-S3C1-02, M03-S3C2-01, M03-S3C3-03		Students demonstrate fluency with multiplication facts through 10 and the related division facts. Fact fluency includes working with facts flexibly, accurately, and efficiently. This means that students have quick recall using strategies that are efficient. Strategies for learning facts include: • Zeros and Ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • Tens Facts • Five Facts (half of tens) • Skip Counting (counting groups of --) • Square Numbers (Ex: 3 x 3) • Nifty Nines • Turn-around Facts (Commutative Property) • Fact Families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24) • Missing Factors Students may be able to master multiplication facts more easily if they can relate new facts to prior knowledge. When students think about 6 X 8, they might think about the familiar fact of 5 X 8. They know 5 X 8 = 40, so then they add 8 more to 40. They arrive at the answer of 48.

Arizona Mathematics Standard Articulated by Grade Level

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 5. Apply and interpret the concept of multiplication and division as inverse operations to solve problems. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C2-04, M03-S1C2-06, M03-S3C3-03	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Multiplication and division facts are inverse operations and that understanding can be used to solve the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the four possible facts using the same three numbers. Examples: • 3 x 5 = 15 5 x 3 = 15 • 15 ÷ 3 = 5 15 ÷ 5 = 3 
PO 6. Describe the effect of operations (multiplication and division) on the size of whole numbers. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C2-04, M03-S1C2-05, M03-S1C3-01	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	Multiplying whole numbers causes the quantity to increase. Dividing whole numbers causes the quantity to decrease. It is important to note that this is true for whole numbers, but not necessarily for all numbers.

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Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 7. Apply commutative, identity, and zero properties to multiplication and apply the identity property to division. Connections: M03-S1C2-02, M03-S1C2-04		Properties of multiplication can be used to help remember basic facts. • 5 x 3 = 3 x 5 (Commutative Property) • 1 x 5 = 5 or 5 x 1 = 5 (Identity Property) • 12 ÷ 1 = 12 • 0 x 5 = 0 or 5 x 0 = 0 (Zero Property)

Strand 1: Number and Operations
Concept 3: Estimation

Use estimation strategies reasonably and fluently while integrating content from each of the other strands.

In Grade 3, students build upon their previous experience with estimation of numbers and quantities. They use multiple strategies to make estimations. Students compare the reasonableness of their estimate to the actual computation. Multiple and continuous estimation experiences lead to greater understanding of number sense.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Make estimates appropriate to a given situation or computation with whole numbers.	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection. M03-S5C2-04. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	Students estimate using all four operations with whole numbers. Students will also use estimation to compare fractions using benchmark fractions. Estimation strategies for comparing fractions extend from students' work with whole numbers. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Continued on next page

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Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
	M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	Estimation strategies include, but are not limited to: • front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), • clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), • rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), • using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and • using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate). Specific strategies also exist for estimating measures. Students should develop fluency in estimating using standard referents (meters, yard, etc) or created referents (the window would fit about 12 times across the wall).

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Strand 2: Data Analysis, Probability, and Discrete Mathematics
This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs. This prepares students for the study of discrete functions as well as to make valid inferences, decisions, and arguments. Discrete mathematics is a branch of mathematics that is widely used in business and industry. Combinatorics is the mathematics of systematic counting. Vertex-edge graphs are used to model and solve problems involving paths, networks, and relationships among a finite number of objects.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization, and representation to analyze and sort data.

In Grade 3, students construct and analyze frequency tables, single bar graphs, and single line graphs in addition to pictographs and tally charts from previous grades and use them to solve problems. Students' understanding of number and operations are reinforced as they interpret information from the displays of data.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Collect, record, organize, and display data using frequency tables, single bar graphs, or single line graphs. Connections: M03-S1C1-01, M03-S2C1-02, SC03-S1C2-04, SC03-S1C2-05, SC03-S1C3-01, SS03-S4C1-05, SS03-S4C6-02	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Single bar graphs should be created horizontally as well as vertically. Determining appropriate scale and units should be emphasized and provides an opportunity to reinforce multiplication and division skills. The construction and interpretation of data displays can be reinforced during social studies and science.
PO 2. Formulate and answer questions by interpreting and analyzing displays of data, including frequency tables, single bar graphs, or single line graphs. Connections: M03-S1C1-02, M03-S1C2-01, M03-S1C2-02, M03-S1C3-01, M03-S2C1-01, SC03-S1C1-02, SC03-S1C3-02, SC03-S1C3-03, SS03-S4C1-02	M03-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	

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Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 2: Probability

Understand and apply the basic concepts of probability.

In Grade 3, there are no performance objectives in this concept. Performance objectives begin in Grade 4.

Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 3: Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

In Grade 3, students use lists and charts to systematically organize information and determine the outcomes of a given situation.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Represent all possibilities for a variety of counting problems using arrays, charts, and systematic lists; draw conclusions from these representations. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C2-04, M03-S2C3-02, SC03-S1C2-05	M03-S5C2-05. Represent a problem situation using words, numbers, pictures, physical objects, or symbols.	After students solve many of these types of counting problems, they should begin to organize their initial random enumeration of possibilities into a systematic way of counting possibilities, particularly through the organization of information in a chart (array) or systematic list. Ultimately, students should begin to make connections to the multiplication principle of counting. See the examples below. Continued on next page

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<tr><td>Students are expected to:</td><td></td><td></td></tr>
<tr><td></td><td></td><td>Examples:
Jan is hungry for a snack. A snack consists of one drink and one fruit. List all possible snacks that Jan could eat?
<u>Drink</u> <u>Fruit</u>
Milk Apple
Juice Banana
o A Systematic List
Milk-Apple (MA), Milk-Banana (MB), Juice-Apple (JA), Juice-Banana (JB)
o A Chart (Array)<table><tr><th>Drink
Fruit</th><th>Milk</th><th>Juice</th></tr><tr><td>Apple</td><td>MA</td><td>JA</td></tr><tr><td>Banana</td><td>MB</td><td>JB</td></tr></table>
List all the different two-topping pizzas that a customer can order from a pizza shop that only offers four toppings: pepperoni, sausage, mushrooms, and onion.
o A Systematic List
Mushroom-Onion
Mushroom-Pepperoni
Mushroom-Sausage
Onion-Pepperoni
Onion-Sausage
Pepperoni-Sausage
o A Chart (Array)<table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>Pepperoni</td><td>x</td><td></td><td></td><td>x</td><td>x</td><td></td><td></td><td></td></tr><tr><td>Sausage</td><td>x</td><td>x</td><td></td><td></td><td></td><td>x</td><td></td><td></td></tr><tr><td>Mushroom</td><td></td><td>x</td><td>x</td><td>x</td><td></td><td></td><td></td><td></td></tr><tr><td>Onion</td><td></td><td></td><td>x</td><td></td><td>x</td><td>x</td><td></td><td></td></tr></table></td></tr>
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Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Solve a variety of problems based on the multiplication principle of counting. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C2-04, M03-S1C3-01, M03-S2C3-01	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	Students should be able to solve problems based on everyday situations using models or manipulatives. Example: • How many outfits can be created using four different shirts and three different pants?

Strand 2: Data Analysis, Probability, and Discrete Mathematics
Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

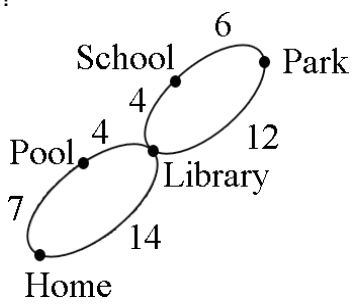
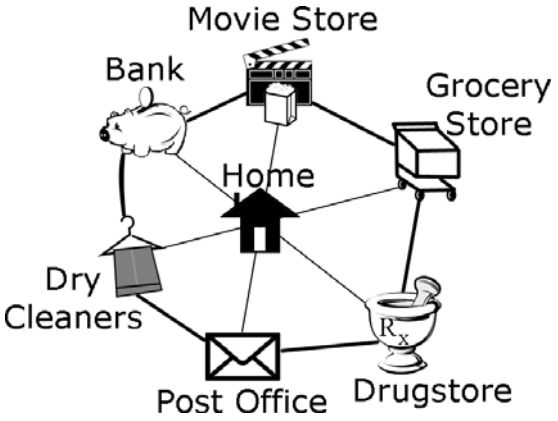
In Grade 3, students expand upon their previous experience with coloring pictures and maps in second grade to include more complex maps. Students should be able to justify how they know they used the least number of colors. Students learn that a street map can be represented by a vertex-edge graph and that routes can be represented by paths in graphs.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Color complex maps using the least number of colors and justify the coloring. Connections: SS03-S4C1-01	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection.	Students should be given many opportunities to explore and color different types of maps and make conjectures about patterns they notice. Continued on next page

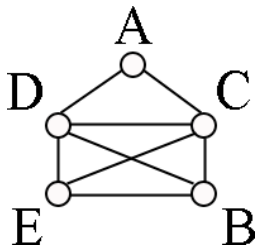
The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
	M03-S5C2-04. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem. M03-S5C2-08. Make and test conjectures based on data (or information) collected from explorations and experiments.	Examples: • County map of Arizona • Map of United States
PO 2. Investigate properties of vertex-edge graphs • circuits in a graph, • weights on edges, and • shortest path between two vertices. Connections: M03-S1C1-02, M03-S1C2-01, M03-S1C2-02, SS3-S4C1-03	M03-S5C2-02. Identify relevant, missing, and extraneous information related to the solution to a problem. M03-S5C2-04. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	It is very important to give students multiple opportunities to find paths and circuits in graphs, before adding weights to the graphs. Once weights are added to the graphs, students can reinforce their addition skills. In social studies, students construct maps of familiar places. These maps can easily be connected to vertex-edge graphs. Definitions of properties of vertex-edge graphs include: • path – connected sequence of edges that starts at a vertex and ends at a vertex • circuit in a graph – path that starts and ends at the same vertex • weight on an edge – value (or some number of objects) placed along an edge in a vertex-edge graph to represent some quantity such as distance, time, cost, or number of traffic lights Continued on next page

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		Example: • What is the shortest path (in minutes) from home to school?  The weights (values) on the graph represent time in minutes. Example: • If Liz leaves her home and visits all the locations on the graph only once and then returns home, she has traveled a circuit. List all the possible circuits. 

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 3. Solve problems using vertex-edge graphs. Connections: M03-S1C2-01, M03-S1C2-02, M03-S1C3-01, SS3-S4C1-03	M03-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	Example: • How many different paths can be traveled from point A to point B based on the graph below?  This problem can be made more complex by adding weights to the edges and directing the students to find the shortest path. Addition skills can be reinforced with this type of activity.

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 3: Patterns, Algebra, and Functions
Patterns occur everywhere in nature. Algebraic methods are used to explore, model and describe patterns, relationships, and functions involving numbers, shapes, iteration, recursion, and graphs within a variety of real-world problem solving situations. Iteration and recursion are used to model sequential, step-by-step change. Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically while integrating content from each of the other strands.

In Grade 3, students understand that logical patterns exist and are a regular occurrence in mathematics. Students recognize, extend, and generalize numerical sequences with both words and symbols.

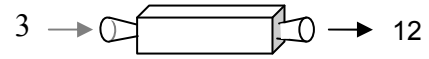
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Recognize, describe, extend, create, and find missing terms in a numerical sequence. Connections: M03-S1C2-01, M03-S1C2-02, M03-S1C2-04, M03-S3C1-02, M03-S3C2-01, M03-S4C1-01, SC03-S1C1-02	M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.	Working with missing terms in sequences provides an opportunity to reinforce addition, subtraction, multiplication, and division facts. Examples: • 3, ___, 9, 12, 15, … • 80, 72, 64, __, __, __, … Possible descriptions for the second pattern include: • Each number is 8 less than the previous number. • The first term is 8 x 10. The second is 8 x 9. The 3rd term is 8 x 8. So, the next term must be…
PO 2. Explain the rule for a given numerical sequence and verify that the rule works. Connections: M03-S1C2-01, M03-S1C2-04, M03-S1C3-01, M03-S3C1-01, M03-S3C2-01, M03-S4C1-01	M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.	Example: • What is the rule for the pattern? 2, 4, 6, 8, 10, … o rule: *add 2 to the previous term* o verification: 2 + 2 = 4, 4 + 2 = 6, 6 + 2 = 8

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 3: Patterns, Algebra, and Functions
Concept 2: Functions and Relationships

Describe and model functions and their relationships.

In Grade 3, students build on the ideas of functions from second grade. Students focus on the relationship between two quantities and how different representations are related.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Recognize and describe a relationship between two quantities, given by a chart, table or graph, in which the quantities change proportionally, using words, pictures, or expressions. Connections: M03-S1C1-01, M03-S1C2-01, M03-S1C2-02, M03-S1C2-04, M03-S1C3-01, M03-S3C1-01, M03-S3C1-02, M03-S3C2-02, M03-S4C1-01	M03-S5C2-02. Identify relevant, missing, and extraneous information related to the solution to a problem. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.	The relationship can be given by a table, model, or input/output (function) machine. Examples: • What rule is shown by the input/output machine?  In Out 1 4 2 8 3 12
PO 2. Translate between the different representations of whole number relationships, including symbolic, numerical, verbal, or pictorial. Connections: M03-S3C2-01, M03-S3C3-02, M03-S4C1-01, SC03-S1C2-05, SC03-S1C3-02, SS03-S4C1-05	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.	Students can represent whole number functions using pictures, numbers, symbols, and words. • Pictures  Continued on next page

The bulleted items within a performance objective indicate the specific content to be taught.

<table>
<tr><th><u>Performance Objectives</u></th><th><u>Process Integration</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>Students are expected to:</td><td></td><td></td></tr>
<tr><td></td><td></td><td>

Symbols
The number of points equals 5 x n (if n = the number of stars)
Words
Each star has 5 points. In order to figure out the total number of points, you multiply the number of stars by 5.
Table

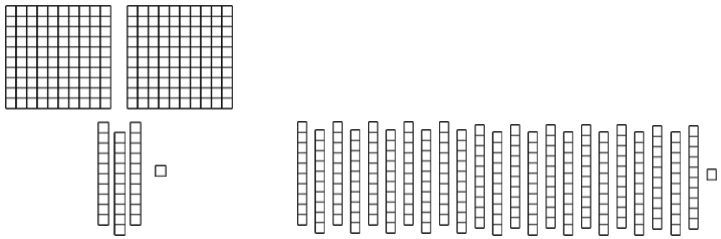
<table>
<tr><th>Stars</th><th>Number of Points</th></tr>
<tr><td>1</td><td>5</td></tr>
<tr><td>2</td><td>10</td></tr>
<tr><td>3</td><td>15</td></tr>
<tr><td>4</td><td>20</td></tr>
</table>
</td></tr>
</table>

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 3: Patterns, Algebra, and Functions
Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

In Grade 3, students use a variety of representations to illustrate mathematical situations and relationships. These representations help students conceptualize ideas and solve problems.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Record equivalent forms of whole numbers to six digits by constructing models and using numbers. Connections: M03-S1C1-01, M03-S1C1-02, M03-S1C2-01	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Students may use manipulatives, pictures, or symbols to model whole numbers and their equivalent forms. Examples: • 142,350 = 100,000 + 40,000 + 2,000 + 300 + 50 • 3 x 8 = 6 x 4 • 3 x 8 = 15 + 9 • 20 = 10 + 5 + 5; 10 x 2; 10 + 10, 5 x 4; 10 + 10, etc. • Base Ten Model: 231 2 – 100's; 3 -10's +1 or 23 – 10's + 1 

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Use a symbol to represent an unknown quantity in a given context. Connections: M03-S1C2-02, M03-S3C2-02, M03-S3C3-03	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Example: • Chen baked 25 crackers. His friend ate some of the crackers. Chen now has 9 crackers. $25 - \Delta = 9$
PO 3. Create and solve simple one-step equations that can be solved using addition and multiplication facts. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C2-04, M03-S1C2-05, M03-S3C3-02	M03-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.	Students may create story problems or equations. When crafting story problems, students should carefully consider the question(s) to be asked and answered. Examples: • Solve the equations below: $6 \times \Delta = 24$ $a \times 2 \times 2 = 24$ $78 + \Delta = 92$ • Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? $3 \times 4 = m$

Strand 3: Patterns, Algebra, and Functions
Concept 4: Analysis of Change

Analyze how changing the values of one quantity corresponds to change in the values of another quantity.

In Grade 3, there are no performance objectives in this concept. Performance objectives begin in Grade 4.

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 4: Geometry and Measurement

Geometry is a natural place for the development of students' reasoning, higher thinking, and justification skills culminating in work with proofs. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Students use geometric methods, properties and relationships, transformations, and coordinate geometry as a means to recognize, draw, describe, connect, analyze, and measure shapes and representations in the physical world. Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. A major emphasis in this strand is becoming familiar with the units and processes that are used in measuring attributes.

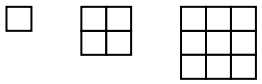
Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.

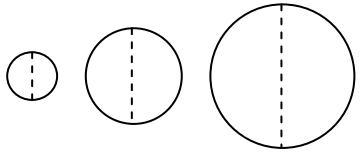
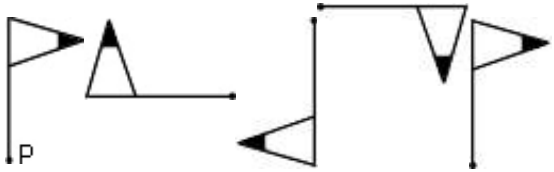
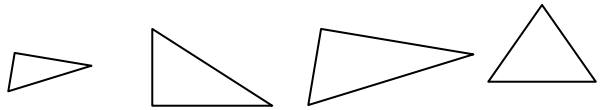
In Grade 3, students describe, analyze, compare, and classify two-and three-dimensional shapes.

<table>
<tr><th><u>Performance Objectives</u></th><th><u>Process Integration</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>Students are expected to:</td><td></td><td></td></tr>
<tr><td>PO 1. Describe sequences of 2-dimensional figures created by increasing the number of sides, changing size, or changing orientation.

Connections: M03-S3C1-01, M03-S3C1-02, M03-S3C2-01, M03-S3C2-02, M03-S4C1-02, M03-S4C2-01, M03-S4C4-04, M03-S4C4-05</td><td>M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.

M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.</td><td>Examples:
• Describe how the length and area of the figures shown below are changing.
 
<table><tr><th>Side Length</th><th>Area</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>9</td></tr></table>
Continued on next page</td></tr>
</table>

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
		Example of description: As the side length of the square increases, the area increases. • Describe the pattern shown in the figures.  • Describe the pattern shown in the figures. 
PO 2. Recognize similar figures. Connections: M03-S4C1-01	M04-S5C2-04. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem. M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.	At this level students can only determine if a figure appears to be similar by observing the attributes. They need multiple opportunities to evaluate figures in different orientations. Example: • Which of the figures shown below are similar? How do you know? 

The bulleted items within a performance objective indicate the specific content to be taught.

<table>
<tr><th>Performance Objectives</th><th>Process Integration</th><th>Explanations and Examples</th></tr>
<tr><td>Students are expected to:</td><td></td><td></td></tr>
<tr><td>PO 3. Identify and describe 3-dimensional figures including their relationship to real world objects: sphere, cube, cone, cylinder, pyramids, and rectangular prisms.

Connections: M03-S4C1-04</td><td>M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.</td><td></td></tr>
<tr><td>PO 4. Describe and compare attributes of two- and three-dimensional figures.

Connections: M03-S4C1-03</td><td>M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions.</td><td>Attributes include:

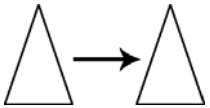
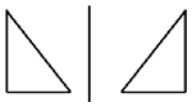
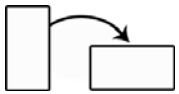
<table><tr><th>2-D Figures</th><th>3-D Figures</th></tr><tr><td>vertices</td><td>vertices</td></tr><tr><td>sides</td><td>edges</td></tr><tr><td>lines of symmetry</td><td>faces</td></tr><tr><td></td><td>base</td></tr><tr><td></td><td>surfaces</td></tr></table>
Students should understand that 2-D figures have perimeter and area and 3-D figures have surface area and volume. At this grade level, area can be described using an array model (M03-S4C4-04). The concepts of surface area and volume should be discussed but not computed.</td></tr>
</table>

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 4: Geometry and Measurement
Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

In Grade 3, students begin to apply their understanding of spatial reasoning and recognize how the positions of 2-dimensional figures change in terms of translations, reflections, and rotations.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Identify a translation, reflection, or rotation and model its effect on a 2-dimensional figure. Connections: M03-S4C1-01, M03-S4C2-02	M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions. M03-S5C2-08. Make and test conjectures based on data (or information) collected from explorations and experiments.	Students recognize that the shape remains the same when translated, reflected, or rotated. Translation (Slide)  Reflection (Flip)  Rotation (Turn)-Shape moves about a point 

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Identify, with justification, all lines of symmetry in a 2-dimensional figure. Connections: M03-S4C2-01	M03-S5C2-06. Summarize mathematical information, explain reasoning, and draw conclusions. M03-S5C2-08. Make and test conjectures based on data (or information) collected from explorations and experiments.	Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.

Strand 4: Geometry and Measurement
Concept 3: Coordinate Geometry

Specify and describe spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.

In Grade 3, there are no performance objectives in this concept. Performance objectives begin in Grade 4.

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 4: Geometry and Measurement
Concept 4: Measurement

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

In Grade 3, students form an understanding of perimeter and area. They select appropriate units, strategies, and tools to solve problems involving perimeter and area. In upper grades, they will calculate area and perimeters of more complex figures.

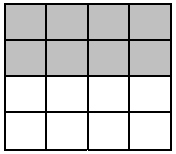
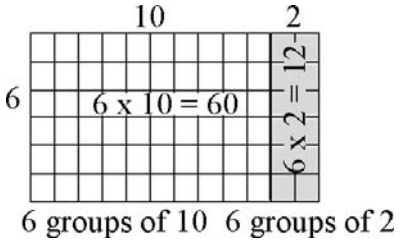
Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 1. Determine elapsed time • across months using a calendar • by hours and half hours using a clock. Connections: M03-S1C2-02, M03-S1C3-01	M03-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	

The bulleted items within a performance objective indicate the specific content to be taught.

Arizona Mathematics Standard Articulated by Grade Level

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 2. Apply measurement skills to measure length, weight, and capacity using US Customary units. Connections: M03-S1C3-01, M03-S4C4-03, M03-S4C4-05, SC03-S1C2-04	M03-S5C2-03. Select and use one or more strategies to efficiently solve the problem and justify the selection. M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-08. Make and test conjectures based on data (or information) collected from explorations and experiments	Measurement skills include: • selecting appropriate unit of measure • selecting the appropriate tool, and • estimating, measuring, and comparing estimate to actual measure.
PO 3. Convert units of length, weight, and capacity • inches or feet to yards, • ounces to pounds, and • cups to pints, pints to quarts, quarts to gallons. Connections: M03-S1C2-02, M03-S1C3-01, M03-S4C4-02	M03-S5C2-02. Identify relevant, missing, and extraneous information related to the solution to a problem. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 4. Determine the area of a rectangular figure using an array model. Connections: M03-S1C2-02, M03-S1C2-03, M03-S1C3-01, M03-S4C1-01, M03-S4C4-05	M03-S5C2-05. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols. M03-S5C2-07. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.	Students should be able to determine the possible rectangles with a given area. Examples: Array models can assist students with understanding square numbers. Students should recognize that a square is also composed of two rectangles. • 4 x 4 = (2 x 4) + (2 x 4)  • A rectangle with an area of 24 could be arrays of 1 x 24, 2 x 12, 3 x 8, 4 x 6, 2 x 12, 3 x 8, 4 x 6. • 6 X 12 = (6 X 10) + (6 X 2) 

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:		
PO 5. Measure and calculate perimeter of 2-dimensional figures. Connections: M03-S1C2-02, M03-S1C3-01, M03-S4C1-01, M03-S4C4-02, M03-S4C4-04	M03-S5C2-04. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.	Students may use objects to represent length, such as string. A shape can be outlined with string and stretched into a straight line. The length can be measured with a ruler. This reinforces the concept that perimeter is a linear measure.

Strand 5: Structure and Logic

This strand emphasizes the core processes of problem solving. Students draw from the content of the other four strands to devise algorithms and analyze algorithmic thinking. Strand One and Strand Three provide the conceptual and computational basis for these algorithms. Logical reasoning and proof draws its substance from the study of geometry, patterns, and analysis to connect remaining strands. Students use algorithms, algorithmic thinking, and logical reasoning (both inductive and deductive) as they make conjectures and test the validity of arguments and proofs. Concept two develops the core processes as students evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems.

In Grade 3, there are no performance objectives in this concept. Performance objectives begin in Grade 4.

The bulleted items within a performance objective indicate the specific content to be taught.

Strand 5: Structure and Logic
Concept 2: Logic, Reasoning, Problem Solving, and Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

In Grade 3, students describe, explain, and justify their solution processes which may include numbers, words (including mathematical language), pictures, physical objects, or equations. Students use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one representation and verify that their answer is reasonable.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 1. Analyze a problem situation to determine the question(s) to be answered.		
PO 2. Identify relevant, missing, and extraneous information related to the solution to a problem.		Any time students approach a problem, they should consider what information is most important and decipher how the information is related to the question to be answered.
PO 3. Select and use one or more strategies to efficiently solve the problem and justify the selection.		Students should be exposed to multiple problem-solving strategies and be able to choose which ones to use.
PO 4. Determine whether a problem to be solved is similar to previously solved problems, and identify possible strategies for solving the problem.		This problem-solving process should be continuously reinforced throughout instruction. This will help students connect to prior learning and consider which problem-solving strategy might be more efficient in a particular case.

The bulleted items within a performance objective indicate the specific content to be taught.

Performance Objectives	***Process Integration***	***Explanations and Examples***
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 5. Represent a problem situation using any combination of words, numbers, pictures, physical objects, or symbols.		
PO 6. Summarize mathematical information, explain reasoning, and draw conclusions.		Summarizing information, explaining your thinking, and drawing logical conclusions are all interconnected and difficult tasks for students to accomplish. These process skills form the foundation of "doing" mathematics and should be encouraged from a very young age.
PO 7. Analyze and evaluate whether a solution is reasonable, is mathematically correct, and answers the question.		Students often do not check their solutions or evaluate whether their answers make sense. These processes should become common practice for efficient problem-solvers.
PO 8. Make and test conjectures based on data (or information) collected from explorations and experiments.		Making and testing conjectures closely connects to M03-S5C2-06 and these are all critical processes to help students create meaning.

The bulleted items within a performance objective indicate the specific content to be taught.