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DIVISION C — LEARNING AND INSTRUCTION; SECTION 3: MATHEMATICS

SESSION:

Students' Cognition in Mathematics

TITLE

Investigating Image-Based Perception and Reasoning in Geometry

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ABSTRACT

In this study we seek to identify brain and body activities that correlate in valid and reliable manners, and with a high degree of statistical significance, with different aspects of geometrical image-based perception and reasoning. In so doing, we seek a better understanding of cognitive processes associated with geometrical image-based learning, instruction, and assessment in mathematics education, and to contribute to extending multidisciplinary boundaries of educational research.

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WORD COUNTS

ABSTRACT (67 WORDS); PAPER (3208 WORDS); REFERENCES (629 WORDS).

RUNNING HEAD

... Geometry

Introduction

Honoring this year's program theme, this paper outlines two pilot case studies bridging research in mathematics education, cognitive psychology, and cognitive neuroscience; concerned with investigating image-based perception and reasoning in geometry. The aim of our research, as per Division C's invitation, is to gain insight into various modalities of geometrical cognition to better inform our understanding of mathematics learning, instruction, and assessment.

Geometry is required for many students, and is often learned, taught, and assessed more in a heuristic image-based manner, than as a formal axiomatic deductive system (Handscomb, 2005). Students are required to prove general theorems, but diagrams are usually used. It follows that understanding how students engage in perceiving and reasoning about such diagrams can provide educators with greater insights into learning, instruction, and assessment of these matters. Simply put, the end in view of this research program should provide as much insight as possible into two fundamental questions for any given learner considering a geometrical diagram. What aspects of that diagram are being perceived from one moment to the next, and in what manners are those aspects of the diagram being considered. It assumed that such a learner is having their brain waves monitored by an electroencephalograph and that the geometrical diagram is being presented on an eye-tracking monitor, recording their point of gaze.

In accord with our theoretical framework of embodied cognition (Campbell, 2001, 2003; Campbell & Dawson, 1995; Handscomb, 2007; Varela, Thompson, & Rosch, 1991), subjective experience and objective behavior are unified in such a manner that any change in mental states and processes exhibit physiological manifestations in brain and body behavior, and vice versa (Campbell & the ENL Group, 2007). Thus, we attempt to identify physiological manifestations of geometrical image-based perception and reasoning (Campbell & Handscomb, 2007, April) through brain and body activity that we can correlate with observations of overt behavior.

The focus of our first pilot study is on identifying physiological manifestations of perceptual shift when a participant is presented with a multi-stable geometrical figure (i.e., a geometrical gestalt), known as the Necker cube. There are three reasons for using the Necker cube in our first pilot study: First, it is a simple geometrical diagram that affords 2 dominant ways of being perceived, thus increasing the likelihood of our being able to identify the characteristics of a perceptual shift; Secondly, it is geometrical figure that has been extensively studied by cognitive neuroscientists; Thirdly, and our main goal here, is to simply demonstrate how shifts in sensory stimuli manifest as shifts in brain waves recorded by electroencephalography.

In our second pilot study, our participant is presented with a set of slides, where each slide contains 6 geometrical figures, 5 of which are related by a geometrical concept. The participant’s task in this study, for each slide, is to identify the “odd one out.” We are using these slides for three main reasons as well: First, they offer a broad range of opportunities for geometrical reasoning; Secondly, they are being used to establish that various aspects of geometrical reasoning are culturally universal; Thirdly, and our main goal here, is to simply demonstrate how our participant’s brain waves change from initial perception of these geometrical diagrams to subsequent considerations on these very same diagrams, once he has had ample opportunities to reflect upon them.

Thus, our initial aim for these pilot studies is simply to establish that brain states can correlate in valid and reliable manners, and with a high degree of statistical significance, with different aspects of image-based perception and reasoning in geometry. The extent to which such correlations can be established, and the granularity that can be achieved, remains to be determined, and is a long-term goal of a much larger program of research.

Theoretical framework

We take all cognition to be embodied cognition. Cartesian dualism rigorously separates the two worlds of mind (*res cogitans*) and body (*res extensa*). Monist formulations variously claim that all is mind (e.g., idealists) or all is matter (e.g., materialists). Our embodied framework occupies a middle ground between these monist and dualist extremes (Campbell & Dawson, 1995). As material beings we exist in the world, but also we are also mindful of a world, and in that sense the world also exists in us. As embodied beings, then, we are both in the world, and of the world. Varela et al. write,"*[E]mbodiment* has this double sense: it encompasses both the body as a lived, experiential structure and the body as the context or milieu of the mind" (1991, p. xvi, authors' italics). We interpret this double sense of embodiment as two different epistemological perspectives on a single embodied ontology.

As mathematics education researchers, we develop cognitive models about mathematical thinking to help better understand matters concerning learning, instruction, and assessment. To date, we have typically been restricted in our studies to analyzing and interpreting overt behavior and self-reports as the empirical ground for our cognitive models using audiovisual recordings of thought in action (e.g., Campbell & Zazkis, 2002; Zazkis & Campbell, 2006).

A fundamental implication of embodied cognition is that, despite the intrinsic limitations of measurement, changes in subjective experience must necessarily correspond to physiological manifestations in brain and body behavior. This view suggests that beyond traditional data sets such as self-reports, field notes, and audiovisual recordings, we can find further empirical ground to substantiate our understandings of learning, instruction, and assessment by attending more closely to less accessible and more covert behaviors of brain and body activity. To do so, however, requires expanding our methods accordingly. Such is our aim here.

Methods, techniques, and modes of analysis

In accord with our theoretical framework, we consider behavior in a broader sense in such a manner that includes physiological behavior, with special emphasis on recording various aspects of the electrophysiology of brain and body activity, (i.e., electroencephalography (EEG), electrocardiography (EKG), electrooculography (EOG)), as well recording eye-related behaviors such as eye movement, eye gaze, and pupillary response. Extending our methods for research in mathematics education to include these methods requires integrating audiovisual data with physiological data in a time-synchronized manner. We report here on two pilot studies utilizing these methods (also see Campbell & the ENL Group, 2007).

In this paper we focus mainly on our use of EEG, which monitors brain activity. Electroencephalography, using passive sensors placed on the scalp, measures voltage potentials resulting from fluctuations of the electrical component of electromagnetic field generated by the living brain. It is well established that various frequency ranges of electromagnetic energy, generated from various regional sources in the brain, correlate in statistically significant ways with various aspects of cognitive function (e.g., Kahana, 2006).

Analysis techniques

There are two common signal analysis techniques that we are using for analyzing EEG data: Source location and spectral analysis. Source analysis is a form of inverse modeling (Campbell, 2004), which involves determining regional brain source activity that accounts for scalp measurements observed over a fixed period of time. There are various techniques to accomplish this inversion. Here, we have used a method of brain electrical source analysis using a model of multiple discrete equivalent current dipoles or regional brain sources (Scherg 1990). We then decompose the resultant brain source waveforms into contiguous frequency ranges using a technique, which is ubiquitous in signal processing, called the Fast Fourier Transform (FFT). Intuitively, this process is akin to decomposing a beam of light into its composite colors with a prism (see Fig. 1).

Resultant data (e.g., see Fig. 2) from these source location and spectral analysis techniques then evoke more familiar statistical analyses (at least to educational researchers). Statistical techniques enable us to more reliably discern between different brain activities, and thereby, in accord with our aforementioned assumptions regarding embodied cognition, to better substantiate postulated cognitive activities associated with learning, instruction, and assessment.

Modes of inquiry

How are we to tell what parts or aspects of geometrical diagrams a learner is viewing at any given time and in what ways are they thinking about them? Again, these questions constitute the general problematic and program of research that we have embarked upon. The pilot studies reported here are initial forays toward these ends.

Pilot study 1: Image-based perception

Multi-stable perceptual drawings can be perceived in different ways. Geometrical diagrams provide cases in point. A simple geometrical drawing is the Necker cube (e.g., see Figure 3a). This wireframe image allows for the image of a cube to be perceived in two distinct 3-D representational configurations; one has the cube projecting upward to the right (e.g., see Figure 3b) and the other has it projecting downward and to the left (e.g., see Figure 3c).

Once the participant was "wired up" with physiological sensors, his eyes were calibrated to the eye-tracker, and baseline EEG was acquired during a period of rest. He was then presented with a recurring sequence of three screens over 20 trials (Figure 4). The first two screens served as the control; with the first control screen presenting a blank white screen for two seconds

(Figure 4a), and the second presenting a simple wireframe square on a white background for two seconds (Figure 4b). The first two screens control for visual and neural satiation and memory correlates (Gaetz, Weinberg, Rzempoluck, & Jatzen, 1998). The third screen presented the Necker cube (Figure 4c) on a white background. Depending on his initial perception of the orientation of the cube, the task of the participant was to press the "up arrow" key marking the moment his initial perception of the cube shifted from a downward to the upward orientation, or to press the "down arrow" key marking the moment his initial perception of the cube shifted from an upward to the downward orientation. The next trial would immediately begin once an arrow key was pressed. Once the participant had completed the 20 trials, another resting baseline was recorded. We will report our statistical analyses for this pilot study in the presentation. A quick perusal of Figure 6 illustrates that the characteristic of our participant's brain waves were radically different when he was attending to the square and cube than when he was gazing at the blank screen (much higher energy in the alpha frequency range, as indicated by the time frame highlighted in yellow on the right side of the figure).

Pilot study 2: Image-based reasoning

Perception entails something we "just see" or something that presents itself within our sensory field. Reasoning, however, involves intellect and reflection, typically comparing one thing with another, seeking relations of difference and similarity. To study spontaneous geometrical image-based reasoning, we used a set of 45 slides, each containing six diagrams, five of which are related by a geometrical concept (Dehaene, Izard, Pica, & Spelke, 2006).

Once the participant was "wired up" with physiological sensors and was calibrated to the eye-tracker, he sequentially proceeded through the 45 slides (Figure 5). The participant's task for each slide was to select the “odd one out” by clicking on the appropriate diagram. The next slide would then appear, and the experiment continued in this way until he had finished the entire set.

The experimenter had the participant go through this set of slides three times. The first episode, i.e., the first time through, the participant was instructed to “just do it.” The second time through, viz., Episode 2, the participant was requested to self-report, i.e., “talk aloud,” about his experiences with each slide in Episode 1. In Episode 3, the experimenter brought the participant’s attention to the unifying concept written in the upper left of each slide, and asked “Had you first attended to this word or phrase when each slide was first presented to you, how might that have changed your experience?” The audiovisual, eye-tracking, and physiological data sets were then integrated and time-synchronized for analysis.

Data sources or evidence

The data sources and evidence that we focus on for this proposal mainly concerns EEG data. We have, however, relied heavily on our audiovisual, eye-tracking, and other data to ensure we were selecting and analyzing the EEG at the appropriate times. Figures 6 and 7 illustrate integrated time-synchronized data sets from Pilot Studies 1 and 2, respectively.

Results and/or conclusions/point of view

We focus in this paper on a findings demonstrating our analyses that get to the heart of our aim to use EEG to discern differences in geometrical image-based perception and reasoning. Most noticeable from our first study, evident on the right hand side in Figure 5, highlighted in yellow, is that progression from each trial is punctuated by focused power in the alpha range (8 – 14 Hz). These power bursts mark the time when our participant was gazing at the blank control screen over a two second period. Strong alpha waves are typically associated with an eyes-closed, relaxed condition. Our eye-tracking and audiovisual data illustrate our participant maintained an eyes-open condition. This observation is consistent with the results of Knyazev, Savostyanov, & Levin (2005), providing an indication, if not a reliable measure, of our participant's level of "alert wakefulness" or "expectant attention" (Nicolelis & Fanselow, 2002) to the upcoming task at hand.

Most noticeable from our second pilot study, comparing brain activity from the first second of exposure to the slides in Episode 1 with the first second of exposure to the slides in Episode 3 are statistically significant variances in different brain regions. In a nutshell, our participant's brain waves exhibited greater power in the higher frequency gamma (30-50 Hz) and beta (14-30 Hz) ranges and less active in the lower frequency theta (4-8 Hz) and delta (0-4 Hz) ranges.

Interestingly, in the alpha range (8-14 Hz), our participant exhibited greater power in the frontal brain regions in Episode 1 and greater power in the posterior brain regions in Episode 3. Our p- values ($p<.05$) are recorded in Table 1. In Episode 1, our participant was exposed to the stimuli for the very first time. Our working hypothesis accounting for these results to this point is that he was then in a more focused and attentive state of mind. In Episode 3, he was now quite familiar with the slides as he encountered them, and thus, potentially, was engaged more with memory at that time. Moreover, his attention in Episode 3 was also more oriented toward linguistic concerns in attending to reading and comprehension in the upper left areas of the slides. Consistent with our observations, both of these cognitive states have been correlated with greater activations in the theta and delta range (e.g., Bastiaansen, et al, 2005).

Educational or scientific importance of the study

Through the Necker cube study researchers may be able to isolate the neural correlate of the subjective experience of perception of a geometric Gestalt. Through the study utilizing the Dehaene paradigm, researchers may gain knowledge of the neural correlates of additional subjective cognitive experiences that are relevant for image-based reasoning in geometry—shifts of attention, for example, or conceptual-verbal identification of a figure. In both cases, there are other indicators of this subjective, cognitive event. For example, the participant may exhibit large-scale behavior that implies subjective cognitive experiences of these types, whether by speech or action. However, the addition of what may be referred to as behavioral data on the small scale, electrophysiological data, for example, significantly expands the scope that educational researchers have for building theories that elucidate processes of geometrical perception and reasoning. In accord with our theoretical framework of embodied cognition, relationships within the electrophysiological data will imply relationships within the flow of subjective, first-person experience when a participant is engaged in geometrical perception and reasoning. Gross behavioral data may have insufficient granularity for these relationships to emerge from its analysis. These relationships between overt behavior and the more covert physiological behaviors, recorded and analyzed using electroencephalography can either form the basis of a theory of image-based perception and reasoning in geometry or they can substantiate or refute an existing theory on geometrical perception and reasoning. There are obvious implications for mathematics educators who wish to base pedagogy on sound empirically grounded theories of mathematical cognition and learning.

Of course, these investigations are still at the earliest stages. There remains considerable gaps between the experiments described here with any prescriptions for classroom practice. Nevertheless, some illustrative comments may be made with reference to the model for image-based reasoning in geometry that was proposed in Handscomb (2005; summarized and supplemented in 2006). Handscomb's model relates perception of a geometrical figure to conceptual understanding of that figure. Accordingly, the process of geometrical reasoning, with an image, will proceed in a series of stages, with one stage flowing into the next. Each stage in the reasoning process corresponds to one of five "principles of conceptualization." A concatenation of these stages constitutes a geometrical argument. The five principles of conceptualization may be further analyzed into local and global conceptualization and local and global deduction.

Perception of the Gestalt figure of the Necker cube corresponds simply and straightforwardly to a global conceptualization in Handscomb's model (see Figure 8). On the other hand, the reasoning process through the Dehaene paradigm is considerably more complex, even though it does not yet involve the stringing together of a series of deductions. It is possible to isolate local conceptualization as well as global conceptualization, meaning that the participant identifies relationships between components of the figures. The widening and narrowing of focal attention correspond to Handscomb's Principles 4 and 5, respectively, as the participant "steps back," as it were, to review the whole diagram before once again focusing attention on a specific aspect of it. His remaining three principles of conceptualization correspond to relationships between conceptual-verbal understandings developed with respect to geometrical figures. These relationships are present as the participant interacts with the experimental paradigm in the second and third viewings. Clearly, the research discussed in the present paper touches peripherally only on the full model for image-based perception and reasoning. However, there is potential for a more extensive study that would lead to substantiating or refuting Handscomb's model.

Follow-up studies will utilize the further research of Handscomb (2009), in which he develops a model for the brain physiological activity that accompanies mathematical reasoning. Experiments such as those presented here are therefore part of an on-going research program that is still in its infancy. Mathematics educators who are concerned about classroom applications of such a research program should bear this in mind. Nevertheless, it is apparent that these first few steps are leading in a direction that will be of paramount importance for mathematics education through the twenty-first century. Researchers in mathematics education must continue to strive to achieve the multidisciplinary legitimacy of a mature field of academic endeavor.

This paper, then, in accord with the conference theme, illustrates that the boundaries of educational research can continue to be extended to incorporate methods and techniques such as eye-tracking and electroencephalography from other disciplines such as psychophysiology and cognitive neuroscience. The importance of this incipient program of research is to illustrate that qualitatively and quantitatively significant discernments regarding the cognitive processes of learners can be made using these methods. That, in accord with our theoretical framework, is what we have set out to accomplish in this paper. Of course, it is too early to determine the extent and granularity of such discernments. Further research is warranted.

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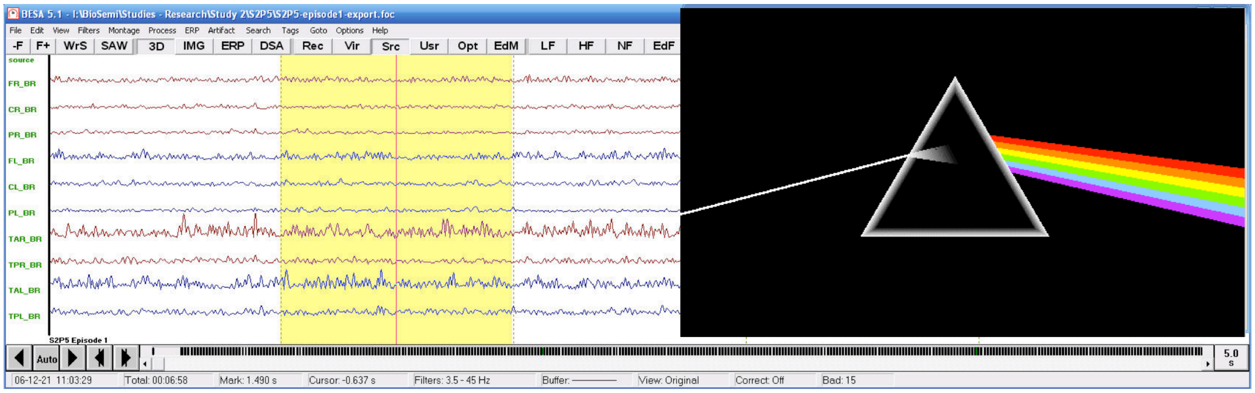
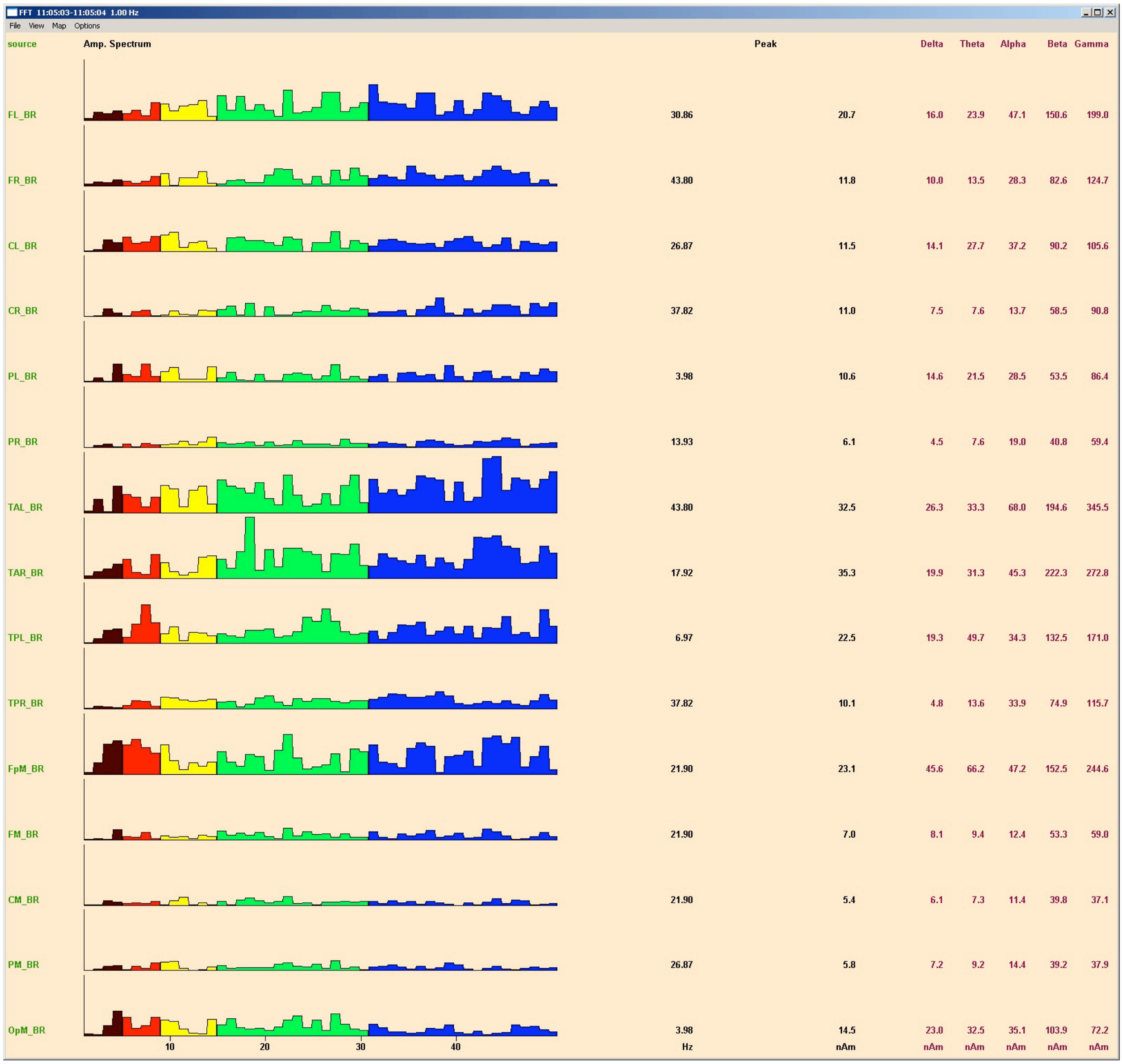


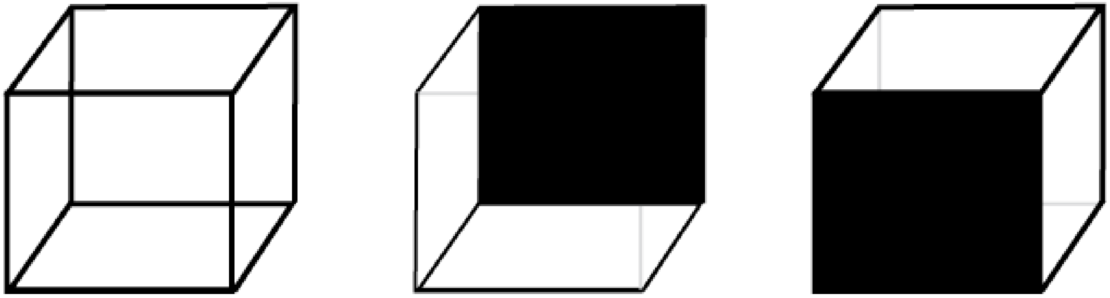
06-12-21 11:03:29 | Total: 00:06:58 | Mark: 1.490 s | Cursor: -0.637 s | Filters: 3.5 - 45 Hz | Buffer: ——— | View: Original | Correct: Off | Bad: 15

Figure 1: Regional brain source decomposition (left side) combined with a prism simile for spectral decomposition of brain sources into discrete frequency ranges using a Fast Fourier Transform (right side)



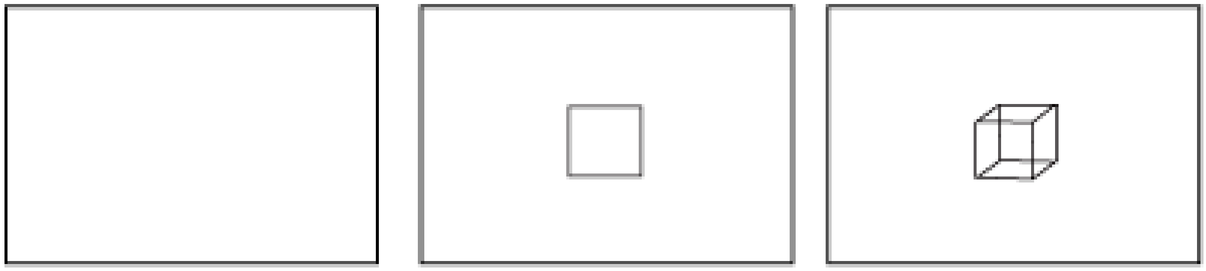
source	Peak		Delta	Theta	Alpha	Beta	Gamma
FL_BR	30.86	20.7	16.0	23.9	47.1	150.6	199.0
FR_BR	43.80	11.8	10.0	13.5	28.3	82.6	124.7
CL_BR	26.87	11.5	14.1	27.7	37.2	90.2	105.6
CR_BR	37.82	11.0	7.5	7.6	13.7	58.5	90.8
PL_BR	3.98	10.6	14.6	21.5	28.5	53.5	86.4
PR_BR	13.93	6.1	4.5	7.6	19.0	40.8	59.4
TAL_BR	43.80	32.5	26.3	33.3	68.0	194.6	345.5
TAR_BR	17.92	35.3	19.9	31.3	45.3	222.3	272.8
TPL_BR	6.97	22.5	19.3	49.7	34.3	132.5	171.0
TPR_BR	37.82	10.1	4.8	13.6	33.9	74.9	115.7
FpM_BR	21.90	23.1	45.6	66.2	47.2	152.5	244.6
FM_BR	21.90	7.0	8.1	9.4	12.4	53.3	59.0
CM_BR	21.90	5.4	6.1	7.3	11.4	39.8	37.1
PM_BR	26.87	5.8	7.2	9.2	14.4	39.2	37.9
OpM_BR	3.98	14.5	23.0	32.5	35.1	103.9	72.2
	Hz	nAm	nAm	nAm	nAm	nAm	nAm

Figure 2: Actual spectral decomposition (FFT) of brain source location waveforms with data from different frequency ranges in columns on the right hand side (cf. Fig. 8)



(a) (b) (c)

Figure 3: The Necker cube (a); perceived with blackened face in foreground (b) & (c)



(a) (b) (c)

Figure 4: Pilot Study 1 stimuli: 20 trials of (a) for 2s; (b) for 2s; (c) upon choice

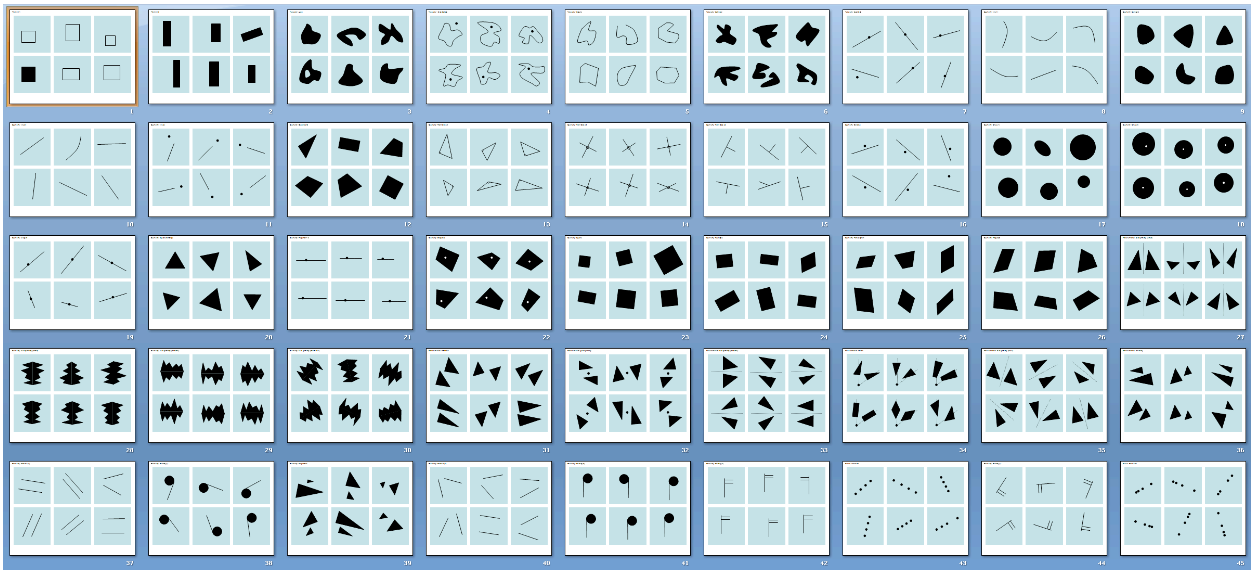
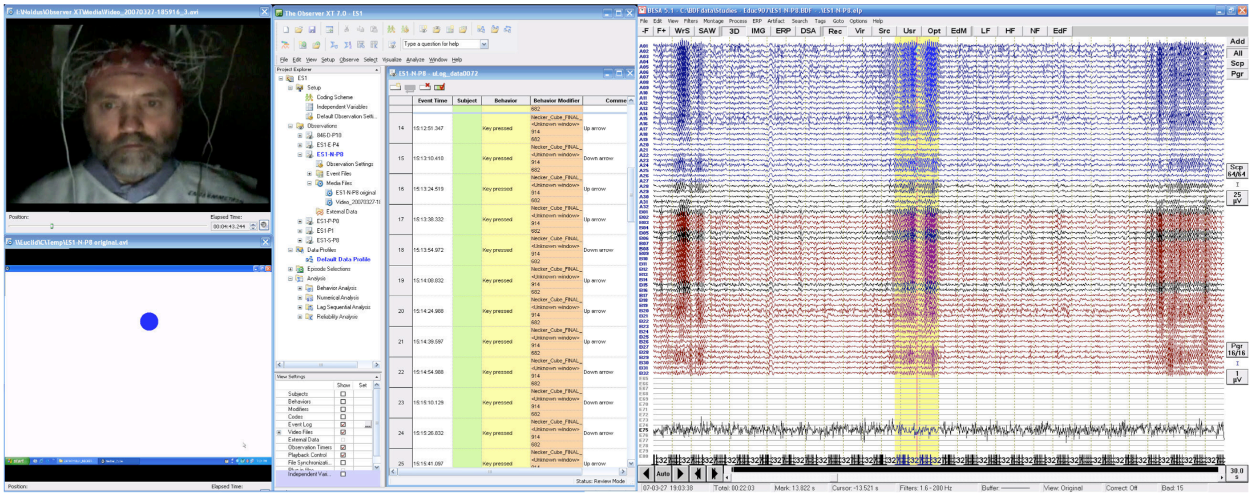
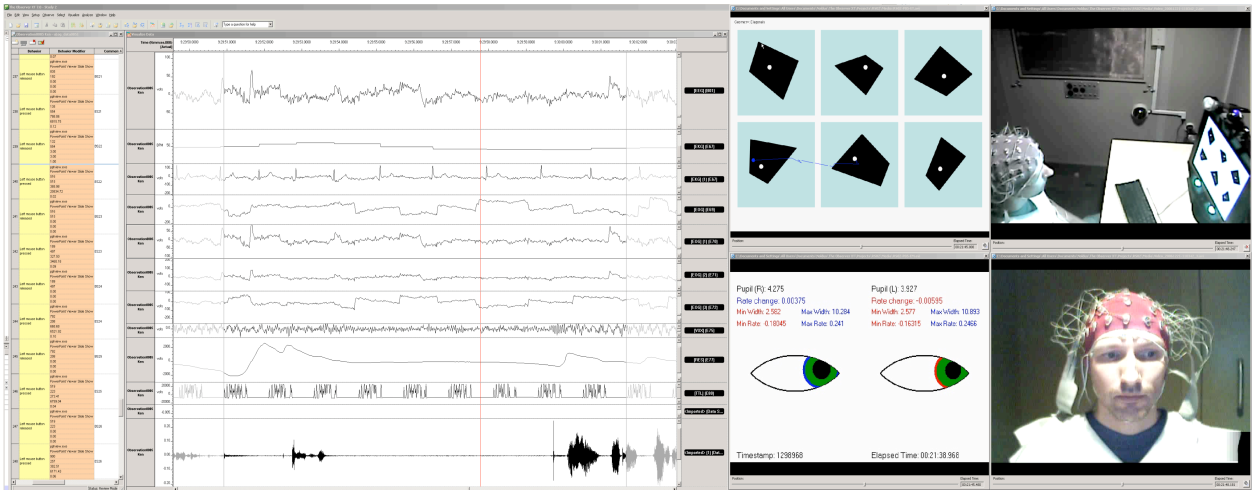


Figure 5: Pilot Study 2 stimuli, slides 1 to 45 (after Dehaene, Izard, Pica, & Spelke, 2006)



	Event Time	Subject	Behavior	Behavior Modifier	Comme
				682	
14	15:12:51.347		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Up arrow
15	15:13:10.410		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Down arrow
16	15:13:24.519		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Up arrow
17	15:13:38.332		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Up arrow
18	15:13:54.972		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Down arrow
19	15:14:08.832		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Up arrow
20	15:14:24.988		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Up arrow
21	15:14:39.597		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Up arrow
22	15:14:54.988		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Down arrow
23	15:15:10.129		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Down arrow
24	15:15:26.832		Key pressed	Necker_Cube_FINAL_ <Unknown window> 914 682	Down arrow
25	15:15:41.097		Key pressed	Necker_Cube_FINAL_ <Unknown window>	Up arrow

Figure 6: An integrated time-synchronized data set from Pilot Study 1



Pupil (R): 4.275
Rate change: 0.00375
Min Width: 2.582 Max Width: 10.284
Min Rate: -0.18045 Max Rate: 0.241

Pupil (L): 3.927
Rate change: -0.00595
Min Width: 2.577 Max Width: 10.893
Min Rate: -0.16315 Max Rate: 0.2466

Timestamp: 1298968
Elapsed Time: 00:21:38.968

Figure 7: An integrated time-synchronized data set from Pilot Study 2

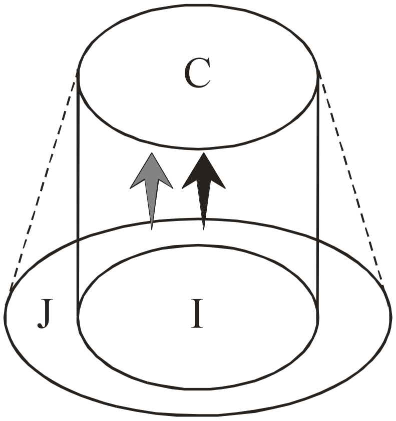


Figure 8: Principle 4 of Conceptualization (Handscomb, 2005), indicating the widening of attention from region I to region J of the geometrical figure, where C represents a conceptual-verbal understanding of the figure.

p values in the ANOVA analyses										
	E1B>E3B					*E3B>E1B*				
Brain Regions[1]	*Delta*	*Theta*	*Alpha*	*Beta*	*Gamma*	*Delta*	*Theta*	*Alpha*	*Beta*	*Gamma*
FL			0.014	0.000	0.000	0.009				
FR	0.000	0.031	0.000	0.000	0.000					
CL		0.008		0.011	0.000					
CR			0.000	0.000	0.000					
PL					0.054		0.003	0.001	0.001	
PR				0.047	0.000		0.000	0.000		
TAL			0.000	0.000	0.000					
TAR			0.001	0.000	0.000					
TPL					0.001	0.002	0.054	0.009		
TPR					0.000	0.000	0.000	0.000		
FpM			0.018	0.000	0.000		0.001			
FM			0.046	0.000	0.000	0.000	0.000			
CM		0.001						0.014		
PM				0.000	0.000	0.006				
OpM					0.002	0.000				
Frequency	1	3	7	10	14	6	6	5	1	0

Table 1: Frequency of statistically detectable differences for E1B>E3B and E3B>E1B. This table displays the *p* values in a series of univariate ANOVA analyses that examined whether the difference over each brain region and frequency range between those regions on which the

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[1] Brain region codes follow the BESA naming scheme of standard sources <www.besa.de>

magnitude of each individual frequency band at E1B is bigger than E3B, and the magnitude of those five frequency ranges at E3B is bigger than E1B. In the univariate ANOVA models, the brain region-frequency range pairs are dependent variables, e.g., FL_D (the mean of Delta frequency range over the Frontal Left region. For instance, the table shows that in the case that the magnitude of Delta frequency range over the Frontal Right brain region at E1B is not only larger than that at E3B, but that difference is statistically detectable at *p* value=.000. It can be concluded from the above table that although the magnitude of Delta frequency range on the Frontal Left brain region for E1B is not only larger than that for E3B, but that difference is not statistically detectable. For the FL region, for instance, the magnitude of Delta on the FL region at E3B is not only larger than that at E1B, but also the difference between those two moments is statistically detectable (*p*=.009).