Exploration of the learning expectations related to grades 1-8 algebra in some countries*

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Abstract: This study analyzes the learning expectations related to grades 1-8 algebra across several U.S. states and high performing TIMSS Asian countries and regions, including Singapore, Taiwan and Japan. In order to narrow and focus the investigation, only one topic within the strand is carefully reported. Based on the official curriculum documents, results of this study indicate that the mathematics content, grade placement and cognitive level of learning expectations related to selected topic vary markedly across documents. Thus, these differences in learning expectations result in striking differences in students' opportunity to learn.

Key words: learning expectation; opportunity to learn; TIMSS

1. Introduction

International studies of mathematics and science achievement have consistently reported that students in Asian countries and regions such as Singapore, Taiwan and Japan demonstrate higher levels of mathematics achievement than students in the United States (Mullis, et al., 2004; Wilson & Blank, 1999). Although the reasons are complex, educators generally agree that opportunity to learn (OTL) is a contributing, if not a major factor. Floden (2002) argued, "If OTL is not taken into account, its effect may be mistakenly attributed to some other attributes of the educational system".

The Third International Mathematics and Science Study (TIMSS) used a model called potential educational experience (See Figure 1, Schmidt, et al., 1997) to capture different aspects of how educational opportunities are shaped and how they are potentially related. In this model, national/regional curriculum goals at the system level represent the intended curriculum which contains what students are expected to learn. However, little is known about how the curricula described in the official documents differ from U.S. states' or Asian countries' curriculum frameworks.

2. Research question

This study examines one topic within the algebra strand in the official documents. More specifically, this study addresses the following research question: To what extent and in what ways are learning expectations associated with the algebra strand similar or different in emphasis and grade placement in some Asian countries and regions, and U.S. states as described in their official mathematics curriculum documents?

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This analysis may partially explain differences in performance among students in several countries and states, particularly if the intended curriculum is an important contributor to what students have an opportunity to learn.

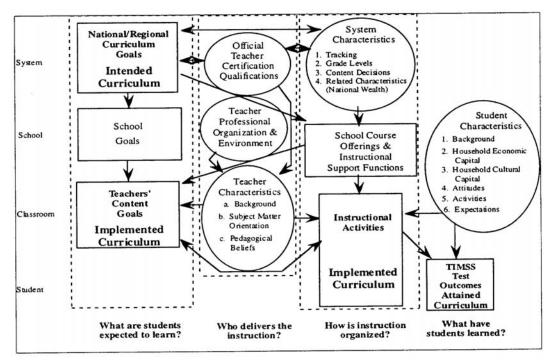


Figure 1 The model of potential educational experiences

3. Source documents

The primary data sources for this study are the state or country official mathematics curriculum frameworks. These include the following documents:

- (1) Singapore (SP): Primary Mathematics Syllabus and Lower Secondary Mathematics Syllabus (implemented from 2001).
 - (2) Taiwan (TW): Mathematics Curriculum Guidelines for Grade 1 to Grade 9 (published in 2003).
- (3) Japan (JP): *Mathematics Program in Japan* (including Elementary, Lower Secondary & Upper Secondary Schools) (published in 2000).
 - (4) Minnesota (MN): Minnesota Academic Standards in Mathematics K-12 (published in 2003).
 - (5) Missouri (MO): Mathematics Grade-Level Expectations (published in 2004).
 - (6) California (CA): The California Mathematics Content Standards (published in 2000).

In addition, k-8 mathematics expectations developed by Achieve were also reviewed because they represent a new proposal for curricular emphases by an independent national organization which was created by governors and corporate leaders in 1996 to help raise states' standards and student performance.

(7) Achieve (AC): *Mathematics Achievement Partnership (MAP) K-8 Mathematics Expectations* (published in 2004).

National documents, such as the *Principles and Standards for School Mathematics* (NCTM, 2000), identify some content strands, such as number, algebra, geometry, measurement, data analysis and probability. Table 1 and

Table 2 illustrate how these strands are reflected in the various documents that were analyzed in this study. Both tables briefly summarize the organization of curriculum frameworks.

Table 1 Summary of strand-organization in Singapore's document

| Grade Strand | G1 | G2 | G3 | G4 | G5 | | G6 | | G7 | | G8 | |
|-----------------|----|----|----|----|---------|-----|---------|-----|-----|----|-----|----|
| | | | | | EM1/EM2 | EM3 | EM1/EM2 | EM3 | E/S | NR | E/S | NR |
| Whole number | X | X | X | X | X | X | | | | | | |
| Measurement | X | X | X | X | X | X | X | X | X | X | X | X |
| Statistics | X | X | X | X | X | X | X | X | X | X | X | X |
| Geometry | X | X | X | X | X | X | X | X | X | X | X | X |
| Fraction | | X | X | X | X | X | | | | | | |
| Decimals | | | | X | X | X | | | | | | |
| Aver./Ra/Sp | | | | | X | | X | X | | | | |
| Ratio/Prop. | | | | | X | | X | X | | | | |
| Percentage | | | | | X | | X | X | | | | |
| Algebra | | | | | | | X | | X | X | X | X |
| Arithmetic | | | | | | | | | X | X | X | X |
| Graph | | | | | | | | | | | X | X |
| Problem Sol. | | | | | | | | | X | X | X | X |
| Trigonometry | | | | | | | | | | | X | |

Notes: (1) EM3 stream intended for pupils less able to cope with language and mathematics. A few differences of strands exist in Primary 6 (EM1/EM2) and Primary 6 (EM3), for example: Primary 6 (EM3) has narrow strands-average/rate and direct proportion; (2) Even though Primary 5 & 6 (EM1/EM2) and Primary 5 & 6 (EM3) have the same strand, they don't cover exactly the same topics or materials; (3) Primary 7 (express/special) and Primary 7 (Normal) have the same strands, but their coverage is not exactly the same, for example: Simple linear equations and simple financial transactions are in Primary 7 (express/special) but they are in Primary 8 (normal/academic); (4) Quadratic equations, motion geometry (reflection, rotation, translation, and enlargement), Pythagoras' theorem, and trigonometrical ratios (sine, cosine, tangent) are all in Primary 8 (express/special), but they are not in Primary 8 (normal/academic); (5) Likewise, Normal (Academic) and Normal (Technical) have the same strands, but the content materials are not exactly the same, for example: real number, sphere, cone, arc length, sector area, similar and congruent figures, scale drawing are all in the Normal (Academic) stream, but those are not in the Normal (Technical) stream.

Table 2 Summary of strand-organization in curriculum standards in this study

| Country/State | Year | Grades | Strands | | | | |
|---------------|------|-----------------------------|--|--|--|--|--|
| | | Grade 1, 3, 4, 5, 6 | Number & quantity; geometry; algebra; Statistics & probability | | | | |
| Taiwan | 2003 | Grade 2, 8 | ade 2, 8 Number & quantity; geometry; algebra | | | | |
| | | Grade 7 | Number & quantity; algebra | | | | |
| Ionon | 2000 | Grades 1-6 | Number and calculations; quantities and measurement; geometrical Figures; math relation | | | | |
| Japan | 2000 | Lower secondary 1 & 2 | Numbers & algebraic expressions; geometrical figures; math relations | | | | |
| Minnesota | 2003 | Grades 1-8 | Mathematical reasoning; number sense & computation and operations; patterns & functions, and algebra; data analysis & statistics, and probability; spatial sense & geometry, and measurement | | | | |
| Missouri | 2004 | Grades 1-8 | Number and operations; algebraic relation; measurement; data & prob | | | | |
| California | 2000 | Grades 1-7, Grades 8-12* | Number sense; algebra and functions; measurement and geometry; statistics & data analysis and probability; math reasoning | | | | |
| Achieve, Inc. | 2004 | Grades K-8 | Algebra; data & measurement; geometry; number & operations | | | | |

Note: A geometry course is also provided in grades 8-12.

In addition to differing by strand organization, the "grain size" of level of specificity of the statements of learning expectations also differ. This can be illustrated, in part, by examining the number of learning expectations related to algebra at each grade in the documents. The following Table 3 summarizes the number of learning expectations related to algebra, by grade, in each of the documents analyzed.

Based on the collected documents, the following Table 3 provides the number of learning expectations within the algebra strand by grade.

| | | 0 | | | 8 | | | | |
|------------|---|----|---|----|----|----|----|----|-------|
| Grade | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
| Singapore | 3 | 10 | 5 | 15 | 21 | 10 | 39 | 25 | 128 |
| Taiwan | 3 | 4 | 2 | 4 | 5 | 5 | 18 | 17 | 58 |
| Japan | 3 | 3 | 4 | 2 | 1 | 0 | 5 | 7 | 25 |
| Minnesota | 2 | 6 | 3 | 3 | 2 | 2 | 5 | 7 | 30 |
| Missouri | 5 | 7 | 6 | 6 | 6 | 7 | 7 | 7 | 51 |
| California | 3 | 3 | 7 | 7 | 5 | 9 | 13 | 0 | 47 |
| Achieve | 7 | 5 | 3 | 4 | 5 | 13 | 26 | 15 | 78 |

Table 3 Number of learning expectations within the algebra strand of each document by grade

Furthermore, comparison can be made about number and percent distribution of learning expectations in algebra strand versus other strands, see Table 4.

| Attribute County or State | Total learning expectations, grades 1-8 | LEs in algebra strand | | LEs in o | other strands |
|---------------------------|---|-----------------------|---------|----------|---------------|
| Singapore | 287 | 128 | (43.8%) | 159 | (56.2%) |
| Taiwan | 188 | 58 | (30.9%) | 130 | (69.1%) |
| Japan | 97 | 25 | (25.8%) | 72 | (74.2%) |
| Minnesota | 147 | 30 | (20.4%) | 117 | (79.6%) |
| Missouri | 159 | 51 | (32.1%) | 108 | (67.9%) |
| California | 178 | 47 | (26.4%) | 131 | (73.6%) |
| Achieve | 330 | 78 | (23.6%) | 252 | (76.4%) |

Table 4 Number and percent distribution of learning expressions by strand

4. Methodology

The selection of countries for this study was based on the performance on the TIMSS assessment. The selection of U.S. states was based on student performance on the NAEP-2000 (Kloosterman & Lester, 2004) assessment and on the evaluation of official state curriculum documents by the Fordham Foundation. Basically, algebra was only one strand of recent studies. Other strands included number and quantity, geometry, measurement, and statistics/probability. In order to present the whole procedures of analyses, only one topic within the strand was reported here.

A coding system was developed which consisted of the general categories: object, action, tools, and cognitive domain. For each Learning Expectation (LE) in the selected topic of the curriculum documents, the following information was coded:

(1) Object-the main noun(s) in the learning expectation.

- (2) Action-the main verb(s) in the learning expectation.
- (3) Tools-equipment specified for use within the learning expectation.
- (4) Cognitive domain-identification of cognitive level of learning expectation based on the *Survey of Enacted Curriculum* protocol (CCSSO, 1999), see Table 5.

Table 5 Cognitive level of learning expectation

| Level | Main goals | | | | | |
|---------|--|--|--|--|--|--|
| | Memorize facts/definitions/formulas: | | | | | |
| Level 1 | (1) Recite basic mathematics | | | | | |
| Level 1 | (2) Recall mathematics terms and definitions | | | | | |
| | (3) Recall formulas and procedures | | | | | |
| | Perform procedures: | | | | | |
| | (1) Use numbers to count/order/denote | | | | | |
| | (2) Do computational procedures or algorithms | | | | | |
| Level 2 | (3) Follow procedures/instructions | | | | | |
| Level 2 | (4) Solve equations/formulas/routine word problems | | | | | |
| | (5) Organize or display data | | | | | |
| | (6) Read or produce graphs and tables | | | | | |
| | (7) Execute geometric constructions | | | | | |
| | Demonstrate understanding of mathematical ideas: | | | | | |
| | (1) Communicate mathematical ideas | | | | | |
| Level 3 | (2) Use representations to model mathematical ideas | | | | | |
| Level 3 | (3) Explain findings and results from data analysis strategies | | | | | |
| | (4) Develop or explain relationships between concepts | | | | | |
| | (5) Show or explain relationships between models, diagrams and other representations | | | | | |
| | Conjecture/generalize/prove: | | | | | |
| | (1) Determine the truth of a mathematical pattern or proposition | | | | | |
| | (2) Write formal or informal proofs | | | | | |
| Level 4 | (3) Recognize/generate or create patterns | | | | | |
| Level 4 | (4) Find a mathematical rule to generate a pattern or number sequence | | | | | |
| | (5) Find and investigate mathematical conjectures | | | | | |
| | (6) Identify faulty arguments or misrepresentations of data | | | | | |
| | (7) Reason inductively or deductively | | | | | |
| | Solve problems/make connections: | | | | | |
| | (1) Apply and adapt a variety of appropriate strategies to solve problems | | | | | |
| Level 5 | (2) Apply mathematics in contexts outside of mathematics | | | | | |
| | (3) Analyze data/recognize patterns | | | | | |
| | (4) Synthesize content and ideas from several sources | | | | | |

A sample of how learning expectations were coded is provided in Table 6.

Table 6 Sample of coded learning expectations

| Learning Expectation (LE) | Grade | Action | Object | Cognitive demand | Tools |
|---|-------|----------------|-------------------------|------------------|--------------------------|
| Pupils can understand the meaning that two triangles are congruent through construction with straightedge and compass (Taiwan) | 8 | Understand | Congruent triangles | Level 3 | Straightedge/ compass |
| Student will know and use the decimal notation and the dollar and cent symbols for money (CA) | 2 | Know & use | Notation and symbol | Level 1 | |
| Solve problems involving surface areas and/or volume of a rectangular or triangular prism, or cylinder (MO) | 8 | Solve problems | Volume and surface area | Levels 1, 4 | |
| Find the volumes and surface areas of cubes, cuboids, prisms and cylinders (Singapore) | 7 | Find | Volume and surface area | Level 2 | |

5. Analysis of the learning expectations related to the selected topic

The general strategy for analysis was based on the "topic tracing" method developed by TIMSS researchers. That is, for each topic, all LEs related to that topic within each curriculum document (Singapore-SP, Taiwan-TW, Japan-JP, Minnesota-MN, Missouri-MO, California-CA, Achieve-AC) were identified and the following information was compiled:

- (1) A description of the focus of the topic by grade level and document;
- (2) The grade where the topic is intended to be first introduced to students;
- (3) The range of grades during which instruction was intended to take place on the topic;
- (4) Any grade for which the topic was to be a special emphasis.

5.1 Summary of learning expectations related to the commutative, associative, and distributive law

The concept of the commutative, associative, and distributive law was one of topics analyzed within the algebra strand. Based on the analysis, a summary of the content emphasis and grade placement for this topic was provided.

In all 40 learning expectations related to the commutative, associative and distributive law were identified across the seven documents. The earliest LE appears in grade 1 of the Taiwan and Achieve document and states:

Pupils can recognize the commutative law and associative law to addition in the concrete situation (Taiwan, 1). Use the fact that a+b=b+a to simplify addition problems (Achieve, 1).

Other early grades about the commutative, associative, and distributive law LEs include:

Pupils should be able to build up the multiplication tables of 2, 3, 4, 5 and 10 and commit to memory. Include activities to help pupils see that multiplication is commutative (Singapore, 2).

To know about the simple properties of addition and subtraction and to use them for the purpose of devising algorithms or checking their results (Japan, 2).

Student will understand that grouping numbers in multiple addend problems, in any order, results in the same sum (Minnesota, 2).

Students can investigate commutative principle with whole numbers (Missouri, 2).

A sample of grades 7 and 8 LEs related to the commutative, associative, and distributive law includes:

Students can simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used (CA, 7).

Students can use the distributive law to derive each of these formulas:

$$(a+b)^2 = a^2 + 2ab + b^2$$

 $(a-b)^2 = a^2 - 2ab + b^2$
 $(a+b)(a-b) = a^2 - b^2$
(Achieve, 8)

Three common learning goals were noted within the set of LEs (see Table 7). For example, in seven of the documents, students are expected to identify the commutative law, including the commutative law to addition and multiplication and this expectation occurs at grade 1 or grade 2 across these seven documents:

Table 7 Common learning expectations related to the commutative, associative and distributive topic

| Common learning expectation | SP | TW | JP | MN | MO | CA | AC |
|---|----|----|----|----|----|----|----|
| The commutative law of addition in integers | - | G2 | G2 | G2 | G3 | G2 | G1 |
| The commutative law of multiplication in integers | G2 | G2 | G5 | - | G4 | G3 | G2 |
| The associative law of multiplication in integers | - | G4 | G5 | - | G5 | G3 | G4 |
| The distributive law in integers | G7 | G5 | G5 | G8 | G5 | G5 | G6 |

Notes: (1) G1 means the learning expectation is provided for grade 1; (2) "-" indicates no specific statement found in the LEs.

Among the 40 LEs some were noted only within one or two documents. For example, the Taiwan and Achieve documents include LEs not found in other documents. These include:

Pupils can comprehend the associative law in the concrete environment and understand the following results: $a \times b \div c = a \div c \times b$ and $a \div b \div c = a \div (b \times c)$ (Taiwan 4).

Pupils can understand the distribution law of the multiplication to addition and figure out the meaning in the concrete situation (Taiwan, 5).

Pupils can use the multiplication formula of the quadratic formula. Ex: $(a+b)^2$, $(a-b)^2$, $(a+b)\times(a-b)$, $(a+b)\times(c-d)$

Show that subtraction and division is not commutative (Achieve, 3).

Since a negative number "-a" is defined by the equation "-a+a=0", the distributive law forces the product of two negative numbers to be positive (Achieve, 6).

These include: $(a+b)^2=a^2+2ab+b^2$ $(a-b)^2=a^2-2ab+b^2$ $(a+b)(a-b)=a^2-b^2$ (Achieve, 8).

Use the distributive law to derive each of these formulas (Achieve, 8).

Based on the analysis of the collected documents, Table 8 summarizes the grade at which the topic of the commutative, associative, and distributive law receives special emphasis. "Special emphasis" indicates that common learning expectations of this topic are addressed and that a substantial amount of time (in proportion to other topics from algebra) is devoted to the commutative, associative, and distributive law. In general, attention of this topic is concentrated in every grade.

Table 8 Grades for special emphasis on the commutative law, associative law, and distributive law topic

| | Grade 1 | Grade 2 | Grade 3 | Grade 4 | Grade 5 | Grade 6 | Grade 7 | Grade 8 |
|------------|-----------|-----------|-----------|-----------|-----------|--------------|--------------|--------------|
| Singapore | | | | | | | | \checkmark |
| Taiwan | $\sqrt{}$ | √ | | $\sqrt{}$ | $\sqrt{}$ | | | \checkmark |
| Japan | | | | | $\sqrt{}$ | | \checkmark | \checkmark |
| Minnesota | | $\sqrt{}$ | | | | | | \checkmark |
| Missouri | | $\sqrt{}$ | $\sqrt{}$ | $\sqrt{}$ | $\sqrt{}$ | $\sqrt{}$ | \checkmark | \checkmark |
| California | | √ | √ | √ | | √ | √ | |
| Achieve | | | | $\sqrt{}$ | | \checkmark | | $\sqrt{}$ |

5.2 Summary of emphasis on the commutative, associative and distributive law by country/state

As noted in Figure 2, the main emphasis for this topic in Singapore is in grade 7 and grade 8. In grade 2, focus is on commutative property of multiplication and relationship between multiplication and addition. In grade

7 and grade 8, students should understand and apply these properties such as the commutative, associative, and distributive law.

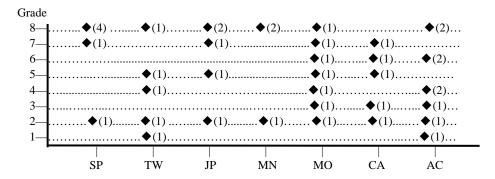


Figure 2 Number and grade placement of learning expectations related to the "commutative, associative and distributive law" within algebra strand

Note: The number inside parentheses indicates the number of learning expectations.

Japan has fewer LEs related to this topic. Students are expected to solve algebra problems using the commutative law, associative law, and distributive law.

In Taiwan, students in grade 1 recognize the commutative law and associative law about addition. In grade 2 and grade 4, attention should be paid to the properties of the commutative law and associative law about multiplication. In grade 5 and grade 8, the distribution law is learned. In addition, students are expected to learn formulas of the quadratic equation using the distribution law.

Only three learning expectations on this topic were identified in the Minnesota document. One at grade 2 and others at grade 8. In grade 2, students should understand the commutative property about addition. In grade 8, students are expected to simplify and evaluate algebraic expressions using the associative property.

In the Missouri document, LEs related to the commutative, associative, and distributive law appear equally in grade 2 to grade 8. From grade 2 to grade 4, students are expected to apply the commutative property of addition and multiplication to whole numbers. Students in grade 5 will learn and apply the distributive and associative properties to whole numbers. In grade 6 to grade 8, emphasis is placed on the distributive and associative laws to simplify the algebraic expression.

The California document expects that students learn the commutative and associative properties of addition and multiplication in grade 2 and grade 3. Basically, students will learn to use the distributive law from grade 5 to grade 7.

The Achieve document includes the largest number of LEs related to the commutative, associative and distributive law (9 in all). Students in grade 1 to grade 3 are expected to understand and use the facts that addition and multiplication are commutative and associative. In grade 4 and grade 6, students should be able to use the commutative, associative, and distributive law associated with the integer and other number system. Finally, students in grade 8 will use the distributive law to derive some basic formulas such as $(a+b)^2 = a^2 + 2ab + b^2$,

$$(a-b)^2 = a^2 - 2ab + b^2$$
, and $(a+b)(a-b) = a^2 - b^2$

5.3 Weight of topic within the algebra strand

In order to gauge the relative emphasis (weight) of the commutative, associative and distributive laws within the algebra strand, Table 9 provides a summary of the number of learning expectation associated with the commutative, associative, and distributive law, and the percent with respect to the total number of LEs within the algebra strand.

Table 9 The weight of topic—the commutative law, associative law and distributive law

| | SP | TW | JP | MN | МО | CA | AC |
|------------------------------|------|------|-----|-----|-------|-----|-------|
| Number of LEs | 6 | 5 | 5 | 3 | 7 | 5 | 9 |
| Percent of total Algebra LEs | 4.8% | 8.6% | 20% | 10% | 13.7% | 30% | 10.7% |

This table indicates that California has the highest percentage related to this topic within the algebra strand. By contrast, Singapore has relatively low weight about this topic.

5.4 Cognitive level of learning expectations related to the commutative, associative and distributive law

Recall that the cognitive level for each learning expectation was coded using the Survey of Enacted Curriculum (SEC) protocol (CCSSO, 1999). Table 10 provides a summary of the distribution of levels in cognitive demand. Note that each LE may be coded by double levels.

Table 10 Number and distribution of level in cognitive domain for LEs related to the commutative law, associative law, and distributive law

| SEC Country/State | N of LEs | Memorize fact, def. & formula | Perform procedures | Demonstrate understanding | Conjecture, generalize & prove | Solve problems, connect |
|----------------------|----------|----------------------------------|--------------------|---------------------------|-----------------------------------|-------------------------|
| Singapore | 6 | - | 100% | - | - | - |
| Taiwan | 5 | - | 20% | 40% | 40% | - |
| Japan | 5 | - | 60% | 60% | - | 20% |
| Minnesota | 3 | - | 33% | 33% | - | 33% |
| Missouri | 7 | - | - | - | 57% | 43% |
| California | 5 | - | 40% | 20% | 40% | 20% |
| Ac Achieve | 9 | 22% | 44% | - | 33% | - |

Table 10 has provided evidence that most LEs under the topic are categorized into the levels 2-5. Particularly enough, all countries/states except Achieve have no learning expectations categorized into the level of "memorize Ffact, def. and formula" in the SEC cognitive domain. Also, all LEs in the Singapore document related to this topic are categorized into the level of "perform procedures".

6. Coder reliability

Coder reliability is an important issue. Inter-coder (or inter-rater) reliability is a particularly important consideration for research in content analysis. It represents the extent to which independent coders evaluate a characteristic of a statement and reach the same conclusion. Kolbe and Burnett (1991) argued that inter-coder reliability was often perceived as the standard measure of research quality. "High levels of disagreement among judges suggest weaknesses in research methods, including the possibility of poor operational definitions, categories, and judge training" (Kolbe & Burnett, 1991, p. 248).

Although there are many different "agreement indices" available, this study uses two measures of reliability: (1) percent agreement between coders, and (2) Cohen's kappa (Yaffee, 2005). Note that a second index is considered because most often it can account for agreement expected by chance. Some methodological experts

contend that percent agreement overestimates true inter-coder agreement, however it is widely used because of its simple calculation. Bakeman (2000) argued that Cohen's kappa should be the measure of choice and this index is generally used in research that involves the coding of behavior. Indeed, Cohen's kappa is the only index included in the Statistical Package for the Social Sciences (SPSS) software.

The basic procedures utilized in this study to check reliability of coding include:

- (1) Prepare the coding guidelines, including the description of each of the categories listed in the coding schemes;
- (2) Invite four doctoral students including the researcher as two-team coders. Each coder independently applies the given guidelines to the same samples (randomly chosen) of the data;
 - (3) Calculate the pilot test of reliability, including percent agreement and Cohen's kappa;
 - (4) Whenever disagreement of coding items occurs, coders are discussed and reach final consensus;
- (5) Based on discussions, the original coding guidelines and/or coding schemes are revised to clarify as needed;
 - (6) Code new test samples and measure reliability;
- (7) Loop continued until coding description was clarified and an acceptable level of coder reliability was reached.

7. Conclusion

Mathematics curriculum frameworks typically contain statements that specify the subject content for particular grades. These statements can be used to describe the nature and judge the quality of a mathematics curriculum. That is, these statements are intended to be a set of expectations for mathematics curriculum development and assessment. They indicate the scope of content and highlight the specific topics at all levels for students to learn.

In this study, several approaches and lenses have been used to analyze the learning expectations in each document. Based on the analyses of the set of LEs associated with this topic, we learn that some content similarities and differences are evident across the different documents. More specifically, this examination reveals that the California document has exceptional high weights in the selected topic than Singapore and Taiwan documents. Meanwhile, most LEs related to the topic are categorized into the "perform procedures" level. It is also clear from Table 4 that the Singapore document emphasizes the algebra strand within school mathematics.

In general, each document might have its strength and weakness depending on the topic chosen. Hopefully, the results of this study will provide insight into learning expectations as specified in the official curriculum documents analyzed. Understanding the attention focused on the topic in the intended curriculum may help clarify the context for differences in students' opportunity to learn. Future research may pursue the impact of curriculum documents on teachers' practice.

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