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RESEARCH BRIEF

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A Road Map for Mathematics Achievement for All Students

FINDINGS FROM THE NATIONAL MATHEMATICS PANEL

By Carolyn Brown, Ph.D., Research Director for The Center

What can we learn from the National Mathematics Panel Report that can help to improve mathematics achievement among American students?

The Study

Foundations for Success: The Final Report of the National Mathematics Advisory Panel (2008). Washington, DC. U.S.: Department of Education. http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

Methodology

In 2007, the federal government appointed a group of education professionals, researchers, and stakeholders to study and advise on ways to "foster greater knowledge of an improved performance in mathematics among American students…with respect to the conduct, evaluation and effective use of the results of research relating to proven-effective and evidence-based mathematics instruction…based on the best available scientific evidence" (National Mathematics Advisory Panel, 2006). The charge was aimed at preparing students to be successful in algebra at the high school level.

This executive order defined a set of topics for the National Mathematics Panel to examine:

- **CURRICULAR CONTENT AND INSTRUCTIONAL MATERIALS.** What is the essential content of school algebra and what do children need to know before starting to study it? How should published materials present the curricular content?
- **LEARNING PROCESSES.** What is known from research about how children learn mathematics?
- **INSTRUCTION.** What is known about the effectiveness of instructional practices?
- **TEACHERS AND TEACHER EDUCATION.** How can we best prepare, recruit, retain, and provide ongoing development for effective teachers of mathematics?
- **ASSESSMENTS.** How can we make assessments of mathematical knowledge more accurate and more useful?

The National Panel reviewed more than 16,000 research publications and policy reports and received public testimony from 110 people, including parents, teachers, school administrators, board of education members, educational researchers, textbook publishers, and others interested in improving mathematics education. In addition, the panel reviewed written commentary from 160 organizations and individuals and analyzed survey results from 743 active teachers of algebra.

In Brief

The National Panel outlined six overarching recommendations as a comprehensive approach to mathematics education. This research brief will focus on the first three topics and the recommendations for PK–12 education.

Panel Findings

CURRICULAR CONTENT AND INSTRUCTIONAL MATERIALS. The K–8 mathematics curriculum should be streamlined to emphasize the most critical topics in the early grades.

Recommendations:

- State algebra standards should include the Major Topics of School Algebra as defined by The National Panel, along with a thorough outline of mathematical connections among these topics. [See the text box, "Major Topics of School Algebra," for more information.]
- The curriculum at the elementary and middle school levels must require fluency in the Critical Foundations of Algebra as defined by The National Panel. [See the text box, "Benchmarks for Critical Foundations of Algebra," for more information.]
- Benchmarks for the Critical Foundations of Algebra should drive curricula, instruction, and assessment, and be interpreted with flexibility to meet the needs of all students.
- Mathematics instruction must be presented in a focused, coherent sequence that builds on proficiency in key topics from year to year. Repetitive, spiraled curriculum should be avoided.
- School districts should ensure that students have access to an algebra course, preferably in Grade 8.
- States and districts must have curricula and materials that are sequenced and articulated across grade levels.

LEARNING PROCESSES. Rigorous research on how children learn should drive mathematics instruction by recognizing the advantages of a strong start for young children; integrating conceptual understanding, procedural fluency, and automatic recall of facts; and emphasizing that effort, not just inherent talent, leads to achievement in mathematics.

Recommendations:

- Instruction in computational estimation and concepts of rounding and estimating must be emphasized.
- Fractions, decimals, and percents—both conceptual and procedural knowledge—must be taught to mastery.
- Teachers should include explicit instruction to teach spatial visualization skills in elementary schools.
- Teachers and educational leaders must emphasize the vital role of effort (as opposed to natural talent) in mathematics achievement.
- Schools need to emphasize task engagement and self-efficacy in mathematics—factors that have been recognized in research as particularly effective in improving achievement for African-American and Hispanic students.

Major Topics of School Algebra

SYMBOLS AND EXPRESSIONS
- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

LINEAR EQUATIONS
- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

QUADRATIC EQUATIONS
- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square of quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

FUNCTIONS
- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions, absolute value, rational functions, step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

ALGEBRA OF POLYNOMIALS
- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's Triangle)
- Mathematical induction and the binomial theorem

COMBINATORIES AND FINITE PROBABILITY
- Combinations and permutations as applications of the binomial theorem and Pascal's Triangle

Source: National Mathematics Advisory Panel, 2008

NOTE: These topics were derived from a review of state standards for Algebra I and II, mathematics textbooks, National Assessment of Educational Progress Algebra Objectives 2005, American Diploma Project's benchmarks for a high school exit test, and the algebra standards in Singapore.

INSTRUCTION. High-quality instruction uses both student-centered and teacher-centered strategies.

Recommendations:

- High-quality research supports a mix of student-centered and teacher-centered instruction.
- Regular formative assessment (weekly or biweekly) should be used, especially in the elementary grades.
- Districts and schools need to provide teachers with training on how to use formative assessment to differentiate instruction.
- Students with learning disabilities need regular, explicit, systematic instruction in areas such as computational fluency, translation of word problems, and mastery of foundational concepts.

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Benchmarks for Critical Foundations of Algebra

FLUENCY WITH WHOLE NUMBERS

- By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
- By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

FLUENCY WITH FRACTIONS

- By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
- By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percents and with the addition and subtraction of fractions and decimals.
- By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.
- By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
- By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
- By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.

GEOMETRY AND MEASUREMENT

- By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
- By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface areas and volumes.
- By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

Source: National Mathematics Panel Report, 2008
NOTE: These were based on review of skills and concepts in Grades 1 through 8 curricula of the highest performing countries on the Trends in International Mathematics and Science Study, the National Council of Teachers of Mathematics *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence,* K–8 mathematics curriculum frameworks for the six highest rated state curricula, a 2007 American College Testing (ACT) survey, and a panel-sponsored survey of 743 teachers of introductory algebra across the country.

- Computer-assisted instruction for drill and practice should be considered a useful tool for developing fluency in mathematical skills and in teaching specific content to special populations.
- Computer programming should be considered as an effective tool for developing specific mathematics concepts, applications, and problem solving, particularly in elementary grades.
- Mathematically gifted students should be allowed to progress through the curriculum at an accelerated rate.

Suggestions for School District Improvement

Many of the findings and recommendations from The National Panel will ring true for classroom teachers, school administrators, and education researchers. This careful and comprehensive analysis of the most rigorous

research and professional expertise provides a guideline for what schools and districts can do to improve mathematics achievement for all students:

- Make sure the district curriculum is carefully sequenced across all grade levels to include the Critical Foundations of Algebra as defined by The National Panel. District mathematics curriculum coordinators should focus on assuring that all important skills and concepts are taught to mastery and that the curriculum is streamlined to reduce repetition.

- Teachers should emphasize the importance of student effort, task engagement, and self-efficacy in mathematics, especially for African-American and Hispanic students.

- Fractions, decimals, percents, spatial visualization, and computational estimation and rounding should be introduced in the appropriate sequence and taught to mastery in the elementary years. [See the text box, "Benchmarks for Critical Foundations of Algebra" for more information.]

- Algebra should be provided for all eighth graders.

- Regular, formative benchmark assessment should be administered at all grade levels.

- Students with learning needs can be supported through explicit and computer-assisted instruction.

- Gifted students should be allowed to progress at their own rate.

Challenges

The National Panel found that teachers who were knowledgeable in mathematics, particularly in the varied and specific teaching methods in mathematics, were the most effective teachers. More development is needed in the area of professional development and preservice preparation to produce a cadre of highly qualified mathematics teachers. Also, very little research was found that addressed the teaching of fundamental mathematics in early childhood education.

A shift needs to occur in the way teachers, students, and parents think about learning mathematics. Ideas about mathematics achievement being a matter of natural ability must be cast off, along with schools' practices of reserving algebra for the high achievers. All students must receive preparation, beginning in prekindergarten, for algebra and other advanced mathematics topics.

Bottom Line

To produce a generation of students who can compete globally will require schools to prioritize the effective teaching of mathematics, including articulating curriculum, streamlining textbooks, producing challenging examinations, and training teachers in the skills needed to instruct students for high achievement. By focusing on clear steps and procedures that will prepare students to master algebra, this report points the way to a future of mathematics achievement for American students that will help America maintain its position as a center for cutting edge science, technology, engineering, and mathematics research and development.

Other Resources

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1100 17th Street NW, Suite 500
Washington, DC 20036

P: 877-277-2744 > **W**: www.centerforcsri.org
P: 202-223-6690 > **F**: 202-223-8939