Quantitative reasoning and problem-solving strategy of children in different ethnic groups*

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Abstract: The purpose of the research is to explore second graders' concept of number development and quantitative reasoning. For this purpose, there were two stages of trials for the children. The first trial was concrete objects. After three months, the children participated in the second trial of half concrete objects. Since understanding the process of solving problems in children is necessary, the researcher observed how children used strategies to discriminate numerosity. After that, the researcher interviewed some students. The research sample came from second graders in Taiwan and Hawaii elementary schools. Twenty students participated in each place. In addition, it was observed whether numerical discrimination strategy in children is the same as in adults. The sample included 20 adults. The result of the research showed that Taiwanese and Hawaiian children reached up to a 90% rate of correct answers in the trial of real objects. However, in the trial of half concrete objects, the rate of correct answers was down to 65%, especially when children compared two quantities of 5:6 ratios. As for strategy, the strategies of the children who gave correct answers were the same as the adults who gave completely correct answers. Because their discrimination was not influenced by distance effect, they could judge the numbers correctly. However, the children who got 90% correct answers were impacted by distance effect, but not influenced by the size of the object. The research suggests that teachers give children real objects to estimate or discriminate the quantities of numbers in their lives.

Key words: quantitative reasoning; problem-solving; child mathematics

1. Introduction

It is a common skill for children to use quantitative reasoning to estimate numerosity in daily life. For example, when children try to figure out how many jellybeans are in two jars, they use quantitative reasoning to estimate the amount. Before understanding a numerical sequence or recognizing numeric symbols, quantitative reasoning is necessary for children. Even if they do simple numeric operations, basically, they need the capacity of quantitative reasoning to complete them successfully. Furthermore, for children to understand numeric calculation and recognize numeric symbols, quantitative reasoning is required. Quantitative reasoning has received the most research attention since the 1990s. The more recent research is concerned about the pivotal role of quantitative reasoning in numeric concept development. Even though the theory and practice relating to the development of numeric concepts in children was influenced by Piaget (1952), he found that cultivating children's informal counting skills is a new insight as an essential aspect of the development of numerical concepts.

1.1 Quantitative comparison without number counting

People who do quantitative comparison need a mental reference unit such as, numerosity estimation (Sowder,

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1992, p. 371). Sophian defined, "Numerosity is the numerical properties of collections" (2007, p. 74). She noted that numerosity of comparison can be processed along with quantities and length, area which can provide cues to discriminate. However, Gallistel and Gelman (2000) pointed out that adults estimating amounts might be influence by a "distance effect" in the trial task, for distance effects are a function of the ratios of two sets of pairs of numbers.

Without any reference to numbers, children could use one to one correspondence to compare all the elements of the two arrays. This means that children apply correspondence relations to compare quantities without numerical counting. However, without counting, some factors may influence the accuracy of the comparison for children. Items' ratio may influence children to perceive relations within and between objects. Children also are impacted by distance effects in numerosity of estimation (Sophian, 2000). However, it is easier for them to discriminate arrays in a 1:2 relationship than arrays in a 2:3 relationship (Huntley-Fenner & Cannon, 2000; Lipton & Spelke, 2005).

Lipton and Spelke (2005) provided evidence that adults discriminate numbers well, at about a 1.15 ratio (4 vs. 6 and 8 vs. 12 elements, but not 4 vs. 5 or 8 vs. 10). The research also showed five-year-old children comprehend tasks of numbers 20-120 by applying a linear relation between number words and non-symbolic numbers. Numerosity of estimation does seem to improve with age (Verschaffel, Greer & Corte, 2007). Quite a few researchers have supported that children with 7 or 8 years of age perform well on problems in numerical ratios but the ratios to be evaluated corresponded to distinct spatial configuration (Dixon & Moore, 1996; Fischbein, 1990; Kieren, 1988; Mack, 1990; Moore, Dixon & Haines, 1991; Nunes, Schliemann & Carraher, 1993; Piaget & Inhelder, 1975; Reyna & Brainerd, 1994; Singer, Cohn & Resnick, 1997; Sophian, 2000).

1.2 Concrete operational reasoning

When children perform numerical estimation, they need a visual image to remember the amount of objects, as well as knowing how to count with numbers. How does a child estimate numerosity without counting before learning the formal operation? Previous research argued that preschoolers already have numeric sense, so they can estimate quantity, compare number magnitude or do simple numeric operations (Lipton & Spelke, 2005; Sophian, 2000). Five-year-old children have developed well enough in numeric sense (Clements & Sarama, 2007), so that they can discriminate numerosity correctly without counting (Sophian, 2000). Generally, children's quantitative reasoning depends more on their cognitive performances as they age. Regarding Piaget's cognitive theory, 7-8 years old second graders in elementary school may perform better than five-year-old children. It seems confusing that children's quantitative reasoning is reduced as they grow older. Second graders' cognitive development is in the concrete operational stage (7-11 years). When children stay in the operational stage, they are capable of reversible thinking only if they handle physical objects. Piaget (1952) identified that the concrete operational child begins to think logically. In addition, concrete operations allow children to order objects in terms of more than one dimension.

Children at the concrete operational stage can solve conservation tasks. The operational thought is reversible. The concrete operational child can operate an action, and then take the operation back to its original condition. However, the limitation of this stage of cognitive development is that operations are only carried out on concrete objects and are limited to two characteristics at the same time. Hence, if the research design has concrete and half concrete objects for second graders to discriminate numbers with two characteristics, how will the second graders solve problems with strategies of quantitative reasoning? Furthermore, the researcher became interested in children's ability to reason qualitatively about numerosity of discrimination at different levels—whether there were good or expert estimators and poor or novice estimators. Therefore, the experts and novices among the second graders applied strategy to do quantitative reasoning in discriminating numerosity that was explored in the research.

So, selecting the appropriate type of quantitative comparison for a given problem and allowing whatever quantitative features are particularly salient to dominate their reasoning, it was interesting to explore the clear differentiation between the different ethnics of the second graders. Therefore, two major goals stated in this document deal with how second graders discriminate numerosity and how they apply strategies to do quantitative reasoning, which are discussed next.

1.3 Research purpose and question

Hence, the present research will explore how second graders discriminated numerosity and used quantitative reasoning with two experiments in different ethnic groups. The research questions are as follows:

(1) Did second graders of different ethnic groups use different strategy to discriminate numerosity?

(2) How did students perform in order to discriminate numerosity when using different sizes of objects?

(3) How did students discriminate numbers using a different ratio of numerosity of objects?

2. Experiments

In the research, two stages of trials were given to second graders in Taiwan and Hawaii to test their quantitative reasoning ability. The first trial was to estimate numerosity ability with concrete objects. After three months, the children participated in the second trial of half concrete objects to determine their ability to discriminate numbers. In order to understand the process of solving problems by children, observation was done and interviews were conducted to know how children used strategies to discriminate numerosity.

2.1 Participants

The participants came from second graders in Taiwan and Hawaii elementary schools. Twenty students participated from each place. The average age was 7-8 years old. Concerning gender, it was even: twenty girls and twenty boys. In addition, in order to understand whether the quantitative reasoning strategy in children was the same as in adults, the sample included 20 adults who were recruited from different ethnic groups, such as Chinese, American, Hawaiian, Japanese and Korean. Gender was equal in adults as well. Their educational degrees were bachelor, master and doctor. They only participated in experiment B to estimate numerosity.

2.2 Materials

The first trial had real objects to allow children to compare two sets of aggregate amounts to understand which amount was larger as a quantitative representation of an array. Each array of objects was spread on a table with an area of 30 x 21cm. The amount of each array of objects followed 2:3 or 5:6 ratios. The objects serving as stimuli included plastic shells, bells, fish, blue birds, pink birds, apples and bears. There were eight problem sets in experiment A (see Table 1).

Table 1 Experiment A—plastic objects array

1. 20 fish (2.5×2×0.5) cm vs. 30 blue birds (4×3×1) cm
2. 20 frogs (2×1.5×0.8) cm vs. 30 apples (diameter 8cm×2) cm
3. 20 pink birds (2×1.8×0.5) cm vs. 24 bears (2×1.8×2.5) cm
4. 20 shells (1.3×0.8×0.8) cm vs. 24 bells (diameter 5cm×1.8) cm
5. 20 bear (2×1.8×2.5) cm vs. 30 pink birds (2×1.8×0 .5) cm
6. 20 bells (diameter 5cm×1.8) cm vs. 30 shells (1.3×0.8×0.8) cm
7. 20 blue birds (4×3×1) cm vs. 24 fish (2.5×2×0.5) cm
8. 20 apples (diameter 8 cm×2) cm vs. 30 frogs (2×1.5×0 .8) cm

In experiment B, each array had two groups of different stickers of different sizes, with ratios of 5:6 or 2:3, and different pictures of animal stickers which included turtles, apples, bears and frogs. In addition, even the same animal had different pictures.

The experiment B materials arrangement was a paired amount of stickers which was composed of two types of objects, and students were asked to judge whether each type of item could be greater. The items were presented in a spatially separated oval or rectangular area on A4 size papers. The amount in each group followed the 2:3 or

5:6 ratios in arranging the objects on display for the children. Those objects comprised different sizes and pictures of animal stickers. There were twelve problem sets which were combined into a file, so that it was convenient to test the children for numerosity comparison (see Table 2).

Table 2 Experiment B—animal stickers array

1. O 20 fish (1.8×1) cm vs. R 24 frogs (2.5×2) cm
2. O 24 fish (1.4×1) cm vs. R 20 turtles (1.8×1.2) cm
3. R 20 apples (1.6×1.2) cm vs. O 24 turtles(1.8×1.2) cm
4. O 30 fish (1.5×2) cm vs. R 20 apples(2×1.5) cm
5. O 30 bears (2×1.5) cm vs. R 20 fish (1.8×1) cm
6. R 20 frogs (2.5×2) cm vs. O 30 turtles(1.8×1.2) cm
7. R 24 fish (1.5×1) cm vs. O 20 frogs (2.5×2) cm
8. O 20 bears (2×1.5) cm vs. R 24 turtles (1.8×1.2) cm
9. R 24 apples (2×1.5) cm vs. O 20 fish (1.5×1.3) cm
10. O 20 turtles (1.5×2.2) cm vs. R 30 fish (1.5×1 .8) cm
11. O 20 fish (1.5×1) cm vs. R 30 apples(2×1.5) cm
12. R 30 frogs (2×1.3) cm vs. O 20 bears (2×1.5) cm

Notes: O—Oval; R—Rectangular.

2.3 Procedure

Participants were tested individually by a female experimenter in a single session with each child that took five minutes for experiment A and 3 minutes for experiment B. During the process of the experiment, the children were asked not to count but to use eye tracking to reason quantitatively and solve problems.

The problems were intermixed and presented in an order that was randomized independently for each individual. Participants were simply asked which of the two pair amounts was just like the bigger one. No feedback was given about the correctness of responses, but children were reminded that either the left or the right alternative could be the bigger one and that they needed to look carefully at both alternatives to identify the correct one. Furthermore, in two experiments, the second graders and adult participants were asked to compare the different ratios' aggregate amounts in the collections, including between 20 and 30, or 20 and 24 objects.

3. Results and discussion

3.1 Results

When children were comparing two sets of objects with relational correspondences, they were asked to compare two sets of objects without counting. In the latter experiment, children should adjust their cognitive structure and adapt new strategies to solve their problems. From the two experiments of comparing quantities, it was learned that one real object array showed no ethnic differences in numerosity of discrimination, but in the experiment of the two dimensions of half abstract objects, an ethnic difference was indicated. Moreover, the data showed that there were different performances in comparison among the children.

In the real object experiment, the data showed that Taiwanese students' quantitative reasoning was not different from the Hawaiian students'. Taiwanese and Hawaiian children reached up to a 90% rate of correct answers in experiment A. Taiwanese students got 93.13% and Hawaiians 92.50% correct answers.

However, in experiment B, the performance of the students in the two schools was significantly different in numerosity of discrimination. Hawaiian students had a 76.50% correct rate; Taiwanese students got up to a 90% correct rate. Furthermore, the correct rate of some problems was down to 65%, especially in 5:6 ratio aggregate amount because it was difficult for the students to compare one set with a big size but a small amount of objects versus another set of a small size but a big number of objects. Under the conditions, Hawaiian students declined greatly in numerosity of estimation of the 5:6 ratios. In the first problem of the array, Taiwanese students got 80%

correct, but Hawaiian students were down to 25% correct. As for the rate of the second problem, Taiwanese students had an 85% correct rate, but Hawaiian students had only a 30% correct rate. The reason why Hawaiian students got such a low number correct was because a distance effect influenced their performance. Without doubt, the visual factors impacted the Hawaiian students' performance in numerical estimation of half concrete objects. So, in the trial of quantitative reasoning, visual influence will be considered in judging numerical discrimination for the concrete operational stage child.

3.2 Discussion

3.2.1 Strategy of discrimination

In the discussion of the findings, good estimators are the students who solved problems, with a 90% correct rate, and novices are those who fell below the 90% correct rate. Then it was discovered that children with different competence levels use different strategies to discriminate between numbers.

The first discussion focuses on the time consumed for the participants to solve each problem. In this respect, it was found that the good estimators took fifteen minutes to solve each problem on average; on the contrary, the novice spent less than fifteen minutes solving problems. More than that, when the first type of novices solved problems, they looked at each problem with only a quick glance, then solved the problem. The second type of novices spent longer—up to 25 minutes longer—solving the problems because they just focused on one set and neglected the other set of the comparison. So, these children were likely to center on only one dimension of a set and ignore other important characteristics.

As for how the amount of objects impacted the discrimination, it was found that students could not perform the task as with the changes made in experiment B. In experiment A, however the arrays were displayed, showing different sizes and different amounts of the real objects, even increasing or decreasing the amounts gradually, children could complete well all the trial tasks and reach high correct rates. In experiment B, children failed the trial when they coped with the problems of the descending amount array. Hawaiian children's correct rate fell to 75%, Taiwanese children's down to 85%. Under the situations, children responded with incorrect answers, based on the arrays which were designed with the objects referring to the small amount first then bigger amount of objects later.

According to Bright's (1976) view, good estimators could apply mental reference to detect the area or denseness of the aggregate amount of objects on the arrays. The visual factors of the object of size were further considered for children in numerosity of judgment. In experiment A, obviously, when children detected the array of the small size but big amount of objects verses the big size but small amount of real objects, they gained high incorrect rates. The data presented the two schools' children correct answer rates down to 75%. Moreover, in experiment B, in the sixth array—30 turtles verses 20 frogs—due to the frog's big size but small amount, Hawaiian students had a 70% correct rate, and Taiwanese students got 80% correct. The ability of numerosity of discrimination was affected by the children's perceiving of the objects' size. Hence, numerosity of estimation needs children's intuitive ability of reference for quantities associated with understanding the estimation of size.

3.2.2 High level of discrimination

Speaking of the particular strategies used, the good estimators used very different ones than the novices used in numerosity of discrimination. The good estimators or experts judged the numbers with density and area, which means they used high-level strategies. The experts used these strategies to discriminate numerosity, but novices did not.

The strategies of the children who completed correct answers were the same as the adults who also completed all correct answers. The clue in tracing the array was that these children could not focus on the object occupying the space to estimate the number, but they quickly detected each row which composed the number of items. Because their discrimination was not influenced by distance effect, they could judge the numbers correctly. However, the children who had a 90% correct rate were impacted by distance effect but not influenced by the size of the object. The children judged the aggregate amount with the objects' density and space but not size. In the aspect of the higher wrong rate judgment, those students had never developed their strategies for numerosity of discrimination.

In reference to ratio, only in the 5:6 ratio aggregate amount of objects did the children fail to judge amounts accurately. After computing the correct rate for two schools' children, it was discovered the rates were low. Taiwanese children correct level reached 66.7%, but Hawaiian children only had a 33.3% correct rate. Obviously, the ability of numerical estimation was impacted by distance effect, especially for Hawaiian students. On the contrary, when displaying the array of 2:3 ratio aggregate amount, all students did well in comparison amounts, all correct rates were up to 100%. With this in mind, it is noted that when children judged aggregate amount objects, all of them had poorer performances on the 5:6 ratios than on the 2:3 ratios. Therefore, the skilled estimators integrate various strategies to judge amounts, such as by density, space and area to trace the number of items.

3.2.3 Ethnic group's quantitative reasoning

The discovery concerning concrete perception is cross-culturally valid. There were not different performances of numerosity of discrimination between the second graders of Taiwan and Hawaii, for the result of experiment A showed that both schools' students' abilities of numerosity of discrimination reached up to a 90% average of correct answer rates in real objects. Regardless of ethnic group, the children's cognition development fitted the common model in concrete operation stage for conservation. That is, they understood quantity or number of items as related to the arrangement or appearance of the object or items.

However, contrasting experiment B, there were different performances between Taiwanese students' comparisons and that of the Hawaiian students. Hawaiian students were influenced by the limitation of perceptual concentration more than Taiwanese students. For they were likely to center on only one dimension of the aggregate amount of half concrete objects and ignore other important details, such as ratio. Their correct rates in trials were lower than in experiment A because their performance depended upon how they perceived the information, such as the oval or rectangular area of the organized objects. In the first and second questions in experiment B, the objects were arranged on oval and rectangular areas. Hawaiian students failed to discriminate amounts on these questions, they got correct rates of only 25% and 30%. The trials displayed 24 frogs on the rectangular area and 20 fishes on oval area. It was difficult for Hawaiian students to distinguish whether the amount of the frogs were larger than the fish because the two ratios were so close.

In comparing the motivation of solving problems, Hawaiian students were more proactive than Taiwanese students. During the process of the trials, Hawaiian students were motivated to solve the problems. When they were tested, they were curious about the objects in the trial and touched the objects, even giving names for objects, such as "Oh! This is Nimo (fish)". In this respect, the children gave names to objects, thereby labeling objects to classify them with characteristics. It is important for organized cues to be designed to allow for children to explore the cues as a strategy for distinguishing amounts. So all arrays of the displays should promote them to solve problems. However, Taiwanese students did not classify objects with names. They answered question positively and took testing for granted because they thought the test was just like another mathematics task. So they lacked enthusiasm to explore new questions during the trial, and lacked motivation to take a risk or accept new challenges.

4. Conclusions

According to these two experiments, Taiwanese and Hawaiian children made accurate quantity judgments of concrete objects. However, their judgments were less accurate when the response alternatives differed in spatial configuration, oval or rectangular areas in close ratios (2:3 or 5:6) of objects. Experiment A showed that students from both schools had comparable performances and no ethnic-related differences in the quantitative reasoning for concrete objects. Experiment B found that ethnic-related differences that existed were due to the close ratio amounts between 20 versus 24 items or size of items in half concrete objects.

The findings suggest that the concept of ratio may have important foundations in the way second graders perceive relations within and between objects. Furthermore, student and adult participants' estimation performance was influenced by physical features of stimuli only for very close ratios of the displayed amounts. It is natural that the mind is able through the eye to successfully estimate any large number of objects without counting them. These experiments gave further understanding of how participants accomplish numerosity of estimation tasks and the effects of aging in this domain.

Before children conduct numerical comparison or develop their quantitative reasoning, teachers should help students use correspondence relationships to compare numerical quantities without counting for lower graders, so that the students can develop well in numerical comparison advance number estimation. The research suggests that teachers give children real objects to estimate or discriminate the quantities of objects in their lives. The implications for mathematics teaching: to help children make judgments about aggregate amounts, students need to practice estimating amounts under the influence of distance effect.

References:

Antell, S. E. & Keating, D. P. (1983). Perception of numerical invariance in notated. *Child Development, 54*, 695-701.

Bright, G. W. (1976). Estimating as pat of learning to measure. In: D. Nelson & R. E. Reys. (Eds.). *Measurement in school mathematics*: *1976 yearbook*. Reston, VA: NCTM, 87-104.

Clements, D. H. & Sarama, J. (2007). Early childhood mathematics learning. In: F. K. Leater. (Ed.). *Second handbook of research on mathematics teaching and learning*. Reston, VA: National Council of Teachers of Mathematics.

Dixon, J. A. & Moore, C. F. (1996). The developmental role of intuitive principles in choosing mathematical strategies. *Development Psychology, 32*, 241-253.

Fishchbein, E. (1990). Intuition and information processing in mathematical activity. *International Journal of Educational Research, 14*, 31-50.

Gallistel C. R. & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences, 4*(2), 59-65.

Huntley- Fenner, G. & Cannon, E. (2000). Preschoolers' magnitude comparisons are mediated by a preverbal analog mechanism. *Psychological Science, 11*(2), 147-152.

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In: J. Hiebert & M. Behr. (Eds.). *Number concepts and operations in the middle grades*. Hillsdale, NJ: Lawrence Erlbaum, 162-181.

Lipton, J. S. & Spelke, E. S. (2005). Preschool children's mapping of number words to non-symbolic numerosities. *Child Development, 76*(5), 978-988.

Mack, N. (1990). Learning fractions with understanding: Building on informal knowledge. *Journal for Research in Mathematics Education, 21*, 16-32.

Moore, C. F., Dixon, J. A. & Haines, B. A. (1991). Components of understanding in proportional reasoning: A fuzzy set representation of developmental progressions. *Children Development, 62*, 441-459.

Nunre, T. Schliemann, A. D. & Carraher, D. W. (1993). *Street mathematics and school mathematics*. Cambridge: Cambridge University Press.

Piaget, J. P. (1952). *The origins of intelligence in children*. New York: International Universities Press.

Piaget, J. & Inhelder, B. (1975). *The origin of the idea of chance in children*. New York: Norton.

Reyna, V. F. & Brainerd, C. J. (1994). The origins of probability judgment: A review of data and theories. In: G. Wright & P. Ayton. (Eds.). *Subjective probability*. New York: Wiley, 239-272.

Singer, J. A., Kohn, A. S. & Resnick, L. B. (1997). Knowing about proportions in different contexts. In: T. Nunes, & P. Bryant. (Eds.). *Learning and teaching mathematics*. East Sussex, UK: Psychology Press, 115-132.

Sophian, C. (2000). From objects to quantities: Developments in preschool children's judgments of aggregate amount. *Developmental Psychology, 36*, 724-730.

Sophian, C. (2007). Rethinking the starting point for mathematics learning. In: O. N. Saracho & B. Spodek. (Eds.). *Contemporary perspectives in early childhood education: Mathematics, science and technology in early childhood education*. Information Age Publishing, 21-44.

Sowder J. (1992). Estimation and number sense. In: Douglas A. Grouws. (Ed.). *Handbook of research on mathematics teaching and learning*. NY: MacMillan Publishing Company, 371-389.

Verschaffel, L., Greer, B. & Corte, E. D. (2007). Whole number concepts and operations. In: Frank K. Lester, Jr. (Ed.). *Second handbook of research on mathematics teaching and learning.* Reston, VA: Nation Council of Teachers of Mathematics, 557-628.

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