CRESST REPORT 734

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USING DATA AND BIG IDEAS: TEACHING DISTRIBUTION AS AN INSTANCE OF REPEATED ADDITION

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National Center for Research on Evaluation, Standards, and Student Testing

Graduate School of Education & Information Studies
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Abstract

The inability of students to become proficient in algebra seems to be widespread in American schools. One of the reasons often cited for this inability is that instruction seldom builds on prior knowledge. Research suggests that teacher effectiveness is the most critical controllable variable in improving student achievement. This report details a process of formative assessment and professional development (called PowerSource©), which is designed to improve teacher effectiveness and student achievement. We describe the process we used to develop a model of distribution over addition and subtraction, one of three big ideas developed during the year, and the interactions we had with teachers about teaching distribution in various ways. As a consequence of these interactions, we were able to test whether teaching distribution using the notion of multiplication as repeated addition (a concept which students had learned previously), using array or area models, or teaching it procedurally had the greatest effects on student learning. We found that the repeated addition model was not only less likely to create certain student misconceptions, but also found that students taught using the repeated addition model were more likely to correctly answer questions involving distribution than were their counterparts taught using either of the other methods. Teachers subsequently reported that they preferred teaching distribution as an instance of repeated addition than teaching it using other available methods.

Introduction

Although general mathematics ability among U.S. fourth and eighth graders has seemingly improved over the last 15 years (Mullis, Martin, Gonzalez, & Chrostowski, 2004; Perie, Grigg, & Dion, 2005), the inability of many of these same students to understand and apply their learning in courses like elementary algebra appears to be an unyielding problem for the nation. The problem seems to become most apparent in courses like elementary algebra because it is both a transitional and a gateway course (Gollub, Bertenthal, Labov, & Curtis, 2002). First-year algebra is transitional because it is often the first course where students must abstract the concrete representations of arithmetic; it is considered a gateway course because proficiency in this course is so highly correlated with success in higher education and future economic success (Atanda, 1999; Horn, Nunez, & Bobbitt, 2000).

Increasingly, however, algebra is becoming less of a gate*way* and more of a barrier to students, especially students of color (Ball, 2003; Berkner & Chavez, 1997).

The inability of students to become proficient in algebra seems to be widespread in American schools. In one 4-year study involving more than 50 school districts, the percentage of graduating seniors who had earned a grade of "C" or better in algebra was only about 63% and some individual schools reported pass rates of less than 38% (Business Wire, 2000). Recent media accounts from large urban schools districts report similar disappointing results (see for example, Helfand, 2006). This trend is not confined merely to American high schools. A recent study of the California Community College system, for example, suggested that only about 75% of students who initially enroll in first semester algebra are still enrolled at the deadline to withdraw and, of these students, over half ultimately fail the course. This trend mirrors nation-wide results (Meehan & Huntsman, 2004).

At least one of the reasons cited for the difficulty students often encounter when transitioning from arithmetic to algebra is that the instruction they receive (in both arithmetic and algebra) seldom builds on the ways in which people actually learn (Bransford, Brown, & Cocking, 1999; Kilpatrick, Swafford, & Findell, 2001). In particular, teachers often have difficulty building on students' prior preconceptions, understanding, intuition, or innate problem solving strategies (Saxe, 1999). Because teachers are one of, if not *the* most powerful instruments of change in education (Alliance for Excellent Education, 2003; Carey, 2004; Hanushek, 2002; Mayer, Mullens, & Moore, 2000; Mullis et al., 2000; National Commission on Mathematics and Science Teaching for the 21st Century, 2000; Wright, Horn, & Sanders, 1997), it seems that changing the way algebra and its predecessor courses are taught should produce some of the greatest improvements in the current state of affairs. In fact, the work of Marzano (2006) suggests that improving teacher effectiveness would enhance student learning more than any other controllable variable in education.

Although the suggestions for enhancing teacher effectiveness are too numerous to outline here, many of those recommendations can be categorized into three areas: (a) efforts to improve teachers' content knowledge; (b) efforts to improve teachers' knowledge for teaching; and (c) efforts to improve teachers' ability to assess student understanding and use such assessments to inform instruction. Research suggests that mathematics instruction is a function of what a teacher knows about the domain (content knowledge), what a teacher knows about how to teach that content knowledge (pedagogical content knowledge), and a teacher's beliefs about the importance of each (Millsaps, 2005). Research seems just as clearly to lead to the conclusion that knowing what students actually know and how to use that knowledge to inform instruction is critical to student improvement (Black & William, 1998; Hiebert & Stigler, 2004).

Teacher Content Knowledge

It is generally believed that increasing a teacher's content knowledge will lead to improvements in student achievement; however, the relationship does not seem to be as linear as one might expect. Whereas research does seem to support the conclusion that content knowledge is critical to effective teaching, especially in elementary algebra (Hawk, Coble, & Swanson, 1985), it seems that acquiring content knowledge well beyond the content taught can produce decreasing returns in terms of student achievement (Monk, 1994). We might conclude, therefore, that teachers need a depth of content knowledge about the material taught and knowledge of how that material will be used to build more abstract concepts, but that they may not need advanced degrees to teach courses like introductory algebra effectively. Similarly it seems that an appreciation of how one goes about learning such content is important. For teachers, this suggests that an important trait for effectiveness is a deep understanding of what they are teaching, an appreciation of how others learn the content of interest <u>and</u> knowledge about how one goes about facilitating such learning (pedagogical content knowledge).

Teacher Pedagogical Content Knowledge

In fact, Hill, Rowan, and Ball (2005) report findings suggesting that an increase of one standard deviation in what they termed a teacher's *mathematical knowledge for teaching* (pedagogical content knowledge) was associated with a 0.1 standard deviation increase in student achievement. Similarly, Darling-Hamond (2000) has concluded that knowledge of how to teach content is more predictive of student achievement than a student's demographics, the class size, or the level of teacher education (e.g. number of math courses taken). Nevertheless, it does seem clear that <u>both</u> content knowledge and pedagogical content knowledge are critical to student achievement (Nathan, Koedinger, & Martha, 2001). Taken together with previous research on what teachers need to know, the research of Cohen and Hill (2000) suggests that addressing both deep content understanding and pedagogical content knowledge of the content to be taught is important to improving student achievement.

Teacher Knowledge of What Students Know

Substantial evidence from prior reform efforts, however, also indicates that changes in teachers' course taking, classroom curriculum content, or textbooks makes little difference if teachers do not know how to use these tools well and how to diagnose their students' learning needs (Darling-Hammond, 1997). Whereas using information from formative assessments, for example, has proven effective in convincing teachers to alter their pedagogical approaches and thereby improving student achievement (see for example, Carpenter, Fennema, Levi, Franke, & Empson, 2000; Wiliam, 2007), merely giving teachers formative assessment tools or information collected from assessments that are designed to inform instruction may not actually have the intended effect unless teachers know what to do with that information (Black & William, 1998). Effective formative assessment requires effective professional development focusing on improving engagement and enriching the knowledge of educators. Once these components are in place, then formative assessment can dramatically improve student achievement (Wiliam & Thompson, 2007). Like students, Wiliam (2007) suggests that we have to develop the ability of teachers to react appropriately to situations for which they have not been specifically prepared. Teachers must be able to provide contextually appropriate instruction.

Developing Appropriate Instruction

Even with improved content knowledge, information about pedagogy and access to formative assessments, teachers often develop their pedagogical methods based on their own experiences as a student (Ball, 1994), teach the way they were taught (Walsh & Sattes, 2005) and teach the way they feel comfortable learning (Britzman, 1991). The adage that "teachers teach the way they were taught" has become common in the literature over the last thirty years[1], and seems to appear often when discussing knowledge domains like mathematics. Whether the adage is true or not, a review of teaching practice in the United States suggests that math, in particular, is often taught as a set of rules, procedures, and facts; that these bits of knowledge are often presented in a seemingly random or disorganized manner; and that procedural knowledge is often divorced from what the process or results *mean* (Fuson, Kalchman, & Bransford, 2005).

Because many U.S. teachers were taught and have often been successful in a system that stressed procedure rather than understanding, and because of the dearth of guidance on how teachers might change their teaching (Kieran, 2003), changing practice can prove difficult.

Although the authors' personal experiences suggests teachers can and will change for the better when shown how and why such changes are efficient and effective in improving

[1]While often attributed to Lortie (1975) and his notion of the "apprenticeship of observation" (see for example, p. 61), many have expressed their view that this conclusion results from an erroneous reading of Lortie's actual findings (for example, Mewborn & Tyminski, 2004) and others have suggested that, given the limited scope of the original study, such a conclusion should not be extrapolated to all teachers (see for example, Geer, 1976). Nevertheless, the adage persists.

student performance, there seems to be little statistical evidence that links participation in quality professional development and teacher effectiveness (Mayer et al., 2000, p. ii). The modicum of work that does exist, however, is encouraging. For example, using instructional methods such as direct instruction to build meaningful connections (Miller & Hudson, 2007), guided learning and practice, especially a "concrete to representational to abstract" (CRA) instructional sequence (Polloway, Patton, & Serna, 2005; Swanson & Deshler, 2003; Witzel, Smith, & Brownell, 2001), and student formative assessment and feedback (Marzano, Pickering, & Pollock, 2001; Wiliam, 2007) have all been shown to be effective. In the area of student formative assessment, the research of Black and Wiliam and others suggests that improving the use of assessment for learning has been shown to roughly double the speed of learning (Wiliam, Lee, Harrison & Black, 2004), and is most effective when it is frequent (Bangert-Drowns, Kulik, & Kulik, 1991) and when it tells students not just what to improve, but how to go about making improvements (Black, Harrison, Lee, Marshall, & Wiliam, 2003).

Nevertheless, in the absence of guidance on how to use these methods and to incorporate the results of formative assessment to develop lesson plans, research suggests that teachers, especially novice or pre-service teachers have little idea of how to proceed (Schmidt, 2005). Research further suggests that, absent such guidance, many teachers rely on the course textbook for direction (Brown, 1998; Fan & Kaeley, 1998; Sturino, 2002). For various reasons the textbook is likely to be an insufficient guide (Skowron, 2006).

In fact, a review of common texts and discussions with teachers suggests that procedural expositions are used extensively and almost exclusively in available textbooks. A more empirical review of many of the most popular, commercially available middle-school math textbooks concluded that they are weak in their development of conceptual and sophisticated understanding, weak in their instructional support of teachers and students, and typically do not promote thinking or account for different student ideas (AAAS Program/Committee: Project 2061, 1999). In particular, our review of how rational number equivalence and the distributive property were taught in commonly used texts in California revealed that both topics were frequently explained in an algorithmic manner and were unlikely to be connected with key mathematical principles a student had already learned (e.g. "Use repeated addition, arrays, and counting by multiples to do multiplication" [CA Grade 2 Mathematics Content Standards] or "Understand the special properties of 0 and 1 in multiplication" [CA Grade 3 Mathematics Content Standards]). For example, rational number equivalence is taught by a number of textbooks as "multiply the top (numerator) and bottom (denominator) by the same thing," or is represented as shown in Figure 1.

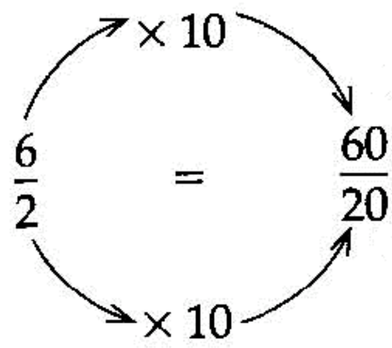

Figure 1. Explaining Rational Equivalence

Similarly, the distributive property is often taught as a procedure that instructs students to multiply each term in the quantity by the multiplier and to separate the resulting products by the addition or subtraction operators present in the original quantity. As shown in Figure 2, this instruction is often accompanied by a graphic which includes arrows to represent the multiplications.

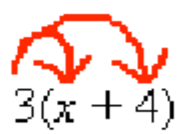

Figure 2. Explaining Distribution

Our experience suggests that such representations are likely to create a number of misconceptions in the minds of students. For example, students often believe that rational numbers are equivalent because "whatever you did to the top, you did to the bottom" or that numbers can be distributed across multiplication and division as well as across addition and subtraction. In this latter case, $2(3 \cdot x)$ becomes $2{\cdot}3 \cdot 2{\cdot}x$ or $6 \cdot 2x$ or $12x$. Moreover, without a complete understanding of distribution, first-year algebra students are likely to incorrectly believe that they can distribute exponents over addition so that $(x + 2)^2$ becomes $x^2 + 2^2$.

Not only does merely memorizing such procedures often lead both students and adults to erroneous conclusions, such memorization leads many to conclude that mathematics is, at its core, about remembering a large number of unrelated facts and recalling them at the appropriate time (Stigler & Hiebert, 2004). Research suggests that this type of belief can have negative effects not only in students' ability to use the tools afforded by mathematics, but in their learning of other subjects as well (Donovan & Bransford, 2005; Dweck, 1999; Kilpatrick et al., 2001).

The PowerSource© Approach

The Center for Research on Evaluation Standards and Student Testing (CRESST) is in the third year of a 5-year study called PowerSource©. This effort is funded by the U.S. Department of Education. The PowerSource© project has the goal of facilitating student and teacher understanding of key big ideas that will form the foundation for proficiency in algebra and connecting concepts being taught in, before, and after middle school. At the heart of PowerSource© is the use of formative assessments (called *Checks for Understanding*) which are designed to provide the teacher vital information about student learning. As was suggested above, however, we know that merely providing teachers with formative assessments, or even assessment data is not likely in and of itself to lead to beneficial changes in practice (Black et al., 2003). Consequently, PowerSource© researchers also developed a program of teacher professional development to provide appropriate content and a set of instructional aids to assist teachers in their teaching of key big ideas underlying proficiency in algebra.

As do others, we see quality professional development as an opportunity to deepen subject content knowledge, improve pedagogical content knowledge, create new knowledge through the interaction of teachers, and to improve the use of formative information to encourage pedagogical changes (Kahle, 1999; National Commission on Mathematics and Science Teaching for the 21st Century, 2000). We are also aware of the need to integrate quality research into professional development sessions, and to encourage each teacher to rigorously evaluate their teaching by studying their own practice (Hiebert, Morris, & Glass, 2003). Furthermore, we acknowledge that educators must be given the time, explicit examples, and an opportunity to explore student response patterns if instruction is to change (Hiebert & Stigler, 2004). At its heart, however, this report addresses the persuasive power of using formative assessment data to motivate and empower knowledgeable teachers to make necessary adjustments to instruction in the course of their normal curriculum.

PowerSource© Professional Development

The design of the professional development (PD) component of PowerSource© is based on findings in the field of cognitive science about how students learn (Donovan & Bransford, 2005), on expert-novice literature that suggests how expertise in a subject like mathematics develops (Chi & Roscoe, 2002; Schraw, 2006), on the role of formative assessment in facilitating this process (Black & Wiliam, 2004), and how these components can be effectively combined to improve teacher practice (Ball, 2003; Carpenter, Franke, & Levi, 2003). Our objective is to provide an intellectually stimulating and supportive environment that builds teacher knowledge and pedagogical content capacity, provides time for reflection, and monitors the effectiveness and the impact of our activities (Lee, 2001).

During the 2006–2007 academic year, we focused on developing sixth-grade teacher capacity around the teaching and understanding of three key organizing "big" ideas that experts have suggested are critical to student proficiency in algebra: (a) the multiplicative identity as applied to rational number equivalence, (b) the distributive property, and (c) the meaning of the equal sign as it applies to solving equations. In each case, it was our goal to ensure that the teachers knew how to use the results of the formative assessments for a particular big idea. In addition, we wanted to minimize known student misconceptions and errors. To accomplish these goals, we presented the content knowledge we felt necessary to effectively teach the big idea and reviewed the instructional guide that suggested how a sample lesson might be taught. While the content knowledge presentation complemented the instructional guide, it did not duplicate it. The teachers were told that, although the content was designed for their growth and edification, they were free to use any of this content knowledge with their students. This process is described in greater detail in the *Methods and Data Sources* section below.

Our general hypotheses were that: (a) PowerSource© would not only minimize known student errors, but also would not make other misconceptions more likely to occur than would normally occur in students of teachers involved in the "normal" district professional development program. In other words, we wanted to ensure that PowerSource© wasn't mitigating certain misconceptions and replacing these with one or more other misconceptions; and (b) students of PowerSource© teachers would be *more* likely to correctly apply the concepts of rational number equivalence, the distributive property and to solve one-step equations for an unknown than would students of teachers involved in the "normal" district professional development program.

This report describes a unique opportunity we had to test these hypotheses using the second of the three big ideas—the distributive property—with 22 teachers working in a medium-size, suburban Southern California school district.

METHODS AND DATA SOURCES

After an introductory professional development session of 4 hours designed to elaborate the foundation of the project and to provide information on teaching Rational Number Equivalence, PowerSource© treatment teachers and researchers met for three more professional development sessions of approximately 90 minutes each. During the first 45 minutes of each of these 90-minute sessions, teachers and researchers discussed student work on prior *Checks for Understanding*, possible misconceptions identified by those assessments and instructional interventions to correct those misconceptions. The last 45 minutes of each session focused on another single "big idea", how that big idea would be developed from its nascent form into abstract concepts in algebra, and how the big idea could be appropriately taught and applied to sixth grade subject matter. To aid teachers with their instruction, teachers were given an instructional handbook on each of the big ideas during the second half of each 90-minute session. The professional development integrated this instructional handbook (pedagogical content) with the conceptual development of each of the big ideas (content knowledge).

The teachers then returned to their classrooms to develop their actual instructional plan and to instruct their students on the applicable big idea for two class periods of approximately 40 minutes each. Although researchers asked that teachers instruct each big idea in a way similar to that presented in the instructional handbook, the teachers were not required to follow the handbook exactly, nor did researchers actually monitor the match between actual instruction and the pedagogy found in the handbook. Researchers, however, did collect teacher self-reports of the similarity of their instruction to the handbook following their classroom presentation of each big idea. After the initial presentation of a big idea to their students, teachers were encouraged to continue to use each big idea in other instructional units they developed during the year.

At the end of the first day of instruction, each teacher assessed student understanding using a researcher developed *Check for Understanding*. Because of time constraints on the curriculum, the instructional unit and *Check for Understanding* were designed to last approximately 55 minutes total. On the second day of instruction, each teacher used a researcher-developed worksheet to allow students to practice a big idea and to allow teachers an opportunity to correct student misunderstandings identified by the first *Check for Understanding*. Researchers encouraged each teacher to use the worksheet in a way that best responded to the needs of their students as suggested by student work on the first *Check for Understanding* (e.g. whole class, small group, individual, etc.). At the start of the third day, teachers collected worksheets (if they had not already done so) and then administered a second, 15-minute *Check for Understanding*.

Subsequent to each complete instructional unit, teachers and researchers met in another 90-minute professional development session. Here again, the first 45 minutes were devoted to reviewing the composite results of all students in the teachers' school district, and discussing apparent student misconceptions and the pedagogical implications of the results from the previous unit. The second 45 minutes developed the next big idea in the sequence. In total, researchers spent 90 minutes with the teachers on each of the three big ideas and 45 minutes on an end of the year session called "Review and Applications" designed to tie the other three big ideas together. Consequently, the total face-to-face professional development time teachers and researchers shared during the year was approximately 9.25 hours.

A randomized group of control teachers in each district only administered the researcher-developed assessments. These teachers did not participate in the professional development aspects of PowerSource© and were not given the instructional handbook. These teachers were, however, encouraged to continue participating in their ordinary district professional development program in lieu of PowerSource© professional development. Although the content of and teacher participation in that professional development were not directly monitored by researchers, these teachers were asked to self-report on their pedagogical methods.

Our experience with 22 teachers working in a medium-size suburban Southern California school district is illustrative of the importance of the synergistic contribution practitioners and researchers can make to professional development and improving pedagogy. This experience also offered an opportunity to investigate the hypotheses outlined above in the context of teaching the distributive property.

To develop the big idea of distribution, the instructional handbook showed PowerSource© teachers how to instruct students on the distributive property using array and area models. The professional development session added the idea that the distributive property could be built on the notion that multiplication means to repeatedly add the same quantity a given number of times. For example, 3 times 4 can be interpreted as 3 + 3 + 3 + 3 or, because multiplication is commutative, the product could be represented as 4 + 4 + 4. This same conceptualization of multiplication could also be applied to multiplication of fractions and integers (e.g. ¾ times 5) to produce ¾ + ¾ + ¾ + ¾ + ¾ and multiplication involving quantities [e.g. 5 times the quantity (x + 2) to produce (x+2) + (x + 2) + (x+2) + (x + 2) + (x+2)]. In turn, each product could be simplified. In the case of the first multiplication, the sum would be $15/4$ and, in the case of the second multiplication the sum would be $5x + 5 \cdot 2$ or $5x + 10$. Teachers in the professional development group were encouraged, if they felt it appropriate, to review the meaning of multiplication with students and to expand this idea from integer representations to representations involving an integer multiplier and a quantity to be multiplied, in addition to using representations such as the array and area models.

In the initial 4-hour session in this district, three researchers met with 13 treatment teachers to discuss the rationale for teaching to the big ideas and then, in another 45-minute session, researchers and teachers discussed the distributive property, as described above. In this context, teachers were shown how to develop the meaning of multiplication as repeated addition of the same thing and how to apply this concept to integer multiplication of other integers, fractions, variables and quantities such as $(3 + 1)$ or $(x + 2)$. Specifically, teachers were shown how to connect concepts with this definition rather than to develop new rules for the students to memorize (such as "combine like terms" or "multiply the outside number with the first and last numbers in the quantity and then drop the operator"). As such, our professional development activities addressed the growth of big ideas from nascent forms of understanding to abstract algebraic concepts with the teachers.

During the follow-up session (the 45-minute session dedicated to reviewing student work), three of the treatment teachers voiced their reluctance to continue to use the repeated addition model of distribution developed in our professional development. Based on their review of their students' work, these teachers expressed their belief that the use of the repeated addition approach was more likely to cause a common misconception than were the array and area models. Specifically, these teachers felt that representing a distribution such as $2(3 + 1)$ as $(3 + 1) + (3 + 1)$ and then as $2 \cdot 3 + 2 \cdot 1$ was far more likely to lead to the student misconception that a distribution like $3(2 + 1)$ should be expanded to $(2 + 1) + (2 + 1)$ or $3 \cdot 2 + 3 \cdot 2$. It should be noted that, while all three teachers were new to PowerSource© (as were all the other treatment teachers), one of the three had taught mathematics for 4 years, the second for 14 years, and the third teacher had taught mathematics for more than 35 years.

To explain their view, these three teachers each suggested that they had followed the professional development presentation much more closely in developing their instructional plan than other PowerSource© teachers and they felt that such an adherence to teaching the repeated addition model was the source of perceived misconceptions in their students. For example, the teacher with the most experience suggested that, "I think my students were confused by the idea of multiplication as repeated addition and transferred the idea when they answered (the third question on the last day)." Another PowerSource© teacher (not one of the three) noted that, "I did not teach or mention repeated addition as multiplication in the lesson because it was not in the [instructional] handbook."

Based on this feedback and the ensuing discussion, researchers agreed to investigate the hypothesis that the students of these teachers (hereafter referred to as "the teachers of interest") were more likely to answer distribution questions on our *Checks for Understanding* in a way indicative of this particular misconception than other PowerSource© teachers, as well as the hypothesis that the students of these teachers were more likely to answer in a way indicative of the misconception than the students in the control group.

Because the *Checks for Understanding* addressed the big idea of distribution in a number of ways, we focused our analysis on 3 of the 11 assessment questions we felt likely to identity this misconception. One question came from the *Check for Understanding* given after the first day of instruction (Check 1) and two other questions came from the *Check for Understanding* given after the second day of instruction (Check 2). Respectively, these questions are:

Check 1 (item 3):

Fill in the missing number

$3\,(15 + 5) = 3 \bullet \_\_ + 3 \bullet 5$

Check 2 (item 2):
Fill in the missing number
$6\,(3 + 1) = 6 \bullet \_\_ + 6 \bullet 1$

Check 2 (item 4, Step 2):
A student simplified the expression 2 (7 + 4) like this. Can you fill in steps 2, 3, and 4?

Simplifying Step	
1	2 (7 + 4)
2	(2 • 7) + (2 • __)
3	14 +
4	

To test the contention that using a repeated addition model of multiplication to develop the distributive property was more likely to cause the unwanted misconception in these three items than was an area or array model of instruction, we developed the following null hypotheses:

- H_{01}: Students of the teachers of interest were no more likely than students of the other treatment teachers to write “5” in the blank of Check 1 (item 3).
- H_{02}: Students of the teachers of interest were no more likely than students of the other treatment teachers to write “1” in the blank of Check 2 (item 2).
- H_{03}: Students of the teachers of interest were no more likely than students of the other treatment teachers to write “7” in the blank of Check 2 (item 4, step 2).

In addition, researchers also wanted to investigate whether the repeated addition conceptualization of multiplication as a basis to teach distribution was more likely to produce the response indicative of the misconception than was the traditional method. To investigate this relationship, researchers developed the following three null hypotheses:

- H_{04}: Students of the teachers of interest were no more likely than students of the control teachers to write “5” in the blank of Check 1 (item 3).
- H_{05}: Students of the teachers of interest were no more likely than students of the control teachers to write “1” in the blank of Check 2 (item 2).
- H_{06}: Students of the teachers of interest were no more likely than students of the control teachers to write “7” in the blank of Check 2 (item 4, step 2).

Given that the ultimate goal of PowerSource© is to aid in the improvement of student learning, we also wanted to know if the students of the teachers of interest were more likely to answer each of the problems correctly than their counterparts in the control group. Therefore, researchers developed the following three null hypotheses:

- H_{07}: Students of the teachers of interest were no more likely than students of the control teachers to answer Check 1 (item 3) correctly.
- H_{08}: Students of the teachers of interest were no more likely than students of the control teachers to answer Check 2 (item 2) correctly.
- H_{09}: Students of the teachers of interest were no more likely than students of the control teachers to answer Check 2 (item 4, step 2) correctly.

Finally, we wanted to know if the students of the teachers of interest were more likely to answer each of the problems correctly than were the students of the other PowerSource© teachers. In particular, we were interested in knowing if teaching distribution from a context of repeated addition rather than just using diagrammatic representations like area or array models made a significant difference in student outcomes. To test this comparison, researchers developed and tested the final three null hypotheses:

- H_{010}: Students of the teachers of interest were no more likely than students of the other treatment teachers to answer Check 1 (item 3) correctly.

- H_{011}: Students of the teachers of interest were no more likely than students of the other treatment teachers to answer Check 2 (item 2) correctly.
- H_{012}: Students of the teachers of interest were no more likely than students of the other treatment teachers to answer Check 2 (item 4, step 2) correctly.

FINDINGS

We tested each of the null hypotheses above using the Pearson Chi-Square statistic at a significance level of $\alpha = .01$. We chose this rigorous level of significance because we wanted the teachers to understand that the inferences we made about differences between groups were very unlikely to have occurred by random chance. From a statistical viewpoint, this level of significance is also warranted because we will be considering numerous null hypotheses (Cohen, 1992). Consequently, when differences between groups are present, they are very likely correlated with differences between pedagogical methods used by members in the various groups. The first three null hypotheses were constructed to compare differences within PowerSource©. Specifically, we wanted to determine the difference in the likelihood of certain student response patterns for teachers teaching the content as presented in professional development and those teachers teaching the content as presented in the instructional materials. It should be noted, however, that all the teachers had been exposed to all the same content. In the second set of hypotheses, we looked for similar differences in response patterns between students who had been taught by the teachers of interest using the instructional methods presented in PowerSource© professional development and students of more traditional methods of instruction.

The results from these analyses of the data are provided below. Observed values are reported on the first line of each cell and expected values are given in parentheses on the second line of each cell.

In the first comparison, we hypothesized that students of the teachers of interest are no more likely than students of the other treatment teachers to answer each of the questions with an answer suggesting that they had misapplied the notion of "repeated addition of the same thing" to the three distribution problems. Each of the null hypotheses tested is given in the respective table below.

Table 1

Null Hypothesis 1[a]: Incorrect Answer of 5 on Item 3 (TI vs. Other Treatment)

Answer responses	Students of teachers of interest	Students of other treatment teachers
All other answers	167 (162)[b]	407 (412)[b]
Answer of "5"	2 (7)[b]	21 (16)[b]

[a]H_{01}: Students of the teachers of interest were no more likely than students of the other treatment teachers to write "5" in the blank of Check 1 (item 3).
[b]Expected values.
Note. TI = Teachers of interest.

Table 2

Null Hypothesis 2[a]: Incorrect Answer of 1 on Item 2 (TI vs. Other Treatment)

Answer responses	Students of teachers of interest	Students of other treatment teachers
All other answers	170 (166)[b]	420 (424)[b]
Answer of "1"	0 (4)[b]	14 (10)[b]

[a]H_{02}: Students of the teachers of interest were no more likely than students of the other treatment teachers to write "1" in the blank of Check 2 (item 2).
[b]Expected values.
Note. TI = Teachers of interest.

Table 3

Null Hypothesis 3[a]: Incorrect Answer of 7 on Item 4 (TI vs. Other Treatment)

Answer responses	Students of teachers of interest	Students of other treatment teachers
All other answers	169 (164)[b]	415 (420)[b]
Answer of "7"	1 (6)[b]	19 (14)[b]

[a]H_{03}: Students of the teachers of interest were no more likely than students of the other treatment teachers to write "7" in the blank of Check 2 (item 4, step 2).
[b]Expected values.
Note. TI = Teachers of interest.

Each of these null hypotheses was accepted. In each of the three cases, the observed values did not differ enough from the expected values to conclude the students in the two groups were significantly different in their response patterns (at a level of $\alpha = .01$). For H_{01}: $\chi^2 = 4.53$, $p = .033$, for H_{02}: $\chi^2 = 5.61$, $p = .018$, and for H_{03}: $\chi^2 = 5.48$, $p = .019$. Consequently, contrary to the original belief of some teachers, we found that students of the teachers who indicated that they had taught the principle of distribution using a repeated addition model (the teachers of interest) were no more likely to exhibit a response pattern indicative of the anticipated misconception than were students of PowerSource© teachers who indicated they had not used that model.

We then analyzed the data to determine if students of the teachers of interest were more likely to respond in a way indicative of the misconception we were investigating than were students of the control teachers. In this comparison, we were concerned that the PowerSource© professional development not prove more likely to contribute to particular student misconceptions than traditionally employed instructional methods. Each of the null hypotheses tested is given in the respective table below.

Table 4:

Null Hypothesis 4[a]: Incorrect Answer of 5 on Item 3 (TI vs. Control)

Answer responses	Students of teachers of interest	Students of control teachers
All other answers	167 (158)[b]	442 (451)[b]
Answer of “5”	2 (11)[b]	39 (30)[b]

[a]H_{04}: Students of the teachers of interest were no more likely than students of the control teachers to write “5” in the blank of Check 1 (item 3).
[b]Expected values.
Note. TI = Teachers of interest.

Table 5:

Null Hypothesis 5[a]: Incorrect Answer of 1 on Item 2 (TI vs. Control)

Answer responses	Students of teachers of interest	Students of control teachers
All other answers	170 (165)[b]	454 (459)[b]
Answer of “1”	0 (5)[b]	20 (15)[b]

[a]H_{05}: Students of the teachers of interest were no more likely than students of the control teachers to write “1” in the blank of Check 2 (item 2).
[b]Expected values.
Note. TI = Teachers of interest.

Table 6:

Null Hypothesis 6[a]: Incorrect Answer of 7 on Item 4 (TI vs. Control)

Answer responses	Students of teachers of interest	Students of control teachers
All other answers	169 (155)[b]	420 (434)[b]
Answer of "7"	1 (15)[b]	54 (40)[b]

[a]H_{06}: Students of the teachers of interest were no more likely than students of the control teachers to write "7" in the blank of Check 2 (item 4, step 2).
[b]Expected values.
Note. TI = Teachers of interest.

In this case, each of the three null hypotheses above was rejected at the $\alpha = .01$ level. On Check 1 (item 3) students in the control group were significantly more likely than students taught by the teachers of interest to complete the item using a number (5) that might indicate that they were blindly following the pattern suggested by repeated addition ($\chi^2 = 10.15$, $p = .001$). Similarly, both null hypotheses concerning the two items of interest on Check 2 were rejected. In both cases, students in the control group were significantly more likely to respond with answers that indicated they might be following a pattern suggestive that they were misapplying repeated addition than were students in the classrooms of the teachers of interest. Consequently, both H_{05} ($\chi^2 = 7.40$, $p = .007$) and H_{06} ($\chi^2 = 18.70$, $p < .000$) were rejected. Contrary to the original hypotheses expressed by the teachers of interest, our findings suggest that students of the teachers of interest are significantly *less* likely to make the anticipated error than are students in classrooms taught by teachers in the control group and no more likely to make the error than other students in PowerSource© classrooms.

Ultimately, the goal of PowerSource© (both the professional development and instructional aids) is to help teachers improve the achievement of their students. Consequently, we also wanted to know if the students of the teachers of interest were more likely to answer each of the problems correctly than their counterparts in the control group. Both teachers and researchers wanted to know if teaching distribution from a context of repeated addition resulted in a significant improvement in student outcomes over traditional instructional methods. The following tables compare the ability of students of the teachers of interest to correctly answer each of the researcher developed distribution questions compared to the students of control group teachers, and the students of other PowerSource© teachers, respectively. The null hypothesis tested for each question and each group is given below.

Table 7

Null Hypothesis 7[a]: Correct Answer Item 3 (TI vs. Control)

Answer responses	Students of teachers of interest	Students of control teachers
Correct answer	157 (114)[b]	280 (323)[b]
All other answers	12 (55)[b]	201 (158)[b]

[a]H_{07}: Students of the teachers of interest were no more likely than students of the control teachers to answer Check 1 (item 3) correctly.
[b]Expected values.
Note. TI = Teachers of interest.

Table 8:

Null Hypothesis 8[a]: Correct Answer Item 2 (TI vs. Control)

Answer responses	Students of teachers of interest	Students of control teachers
Correct answer	167 (143)[b]	373 (397)[b]
All other answers	3 (27)[b]	101 (77)[b]

[a]H_{08}: Students of the teachers of interest were no more likely than students of the control teachers to answer Check 2 (item 2) correctly.
[b]Expected values.
Note. TI = Teachers of interest.

Table 9:

Null Hypothesis 9[a]: Correct Answer Item 4 (TI vs. Control)

Answer responses	Students of teachers of interest	Students of control teachers
Correct answer	167 (139)[b]	360 (388)[b]
All other answers	3 (31)[b]	114 (86)[b]

[a]H_{09}: Students of the teachers of interest were no more likely than students of the control teachers to answer Check 2 (item 4, step 2) correctly.
[b]Expected values.
Note. TI = Teachers of interest.

As was seen in the previous comparisons of the students of the teachers of interest and the students of the control teachers, these two groups were significantly different in their likelihood of providing certain answers on each of the questions of interest. Here again, for all three distribution questions, the students of the teachers of interest were significantly more likely to provide a correct answer than were students of the teachers in the control group. (H_{07}: $\chi^2 = 68.30$, $p < .000$; H_{08}: $\chi^2 = 35.29$, $p < .000$; H_{09}: $\chi^2 = 41.80$, $p < .000$).

Somewhat surprisingly to both researchers and PowerSource© teachers however, was the discovery that the ability of students of the teachers of interest to provide a correct answer to each of the three distribution questions also differed significantly from their counterparts in the treatment group. The null hypothesis tested for each question and each group is given below.

Table 10:

Null Hypothesis 10[a]: Correct Answer Item 3 (TI vs. Other Treatment)

Answer responses	Students of teachers of interest	Students of other treatment teachers
Correct answer	157 (141)[b]	341 (357)[b]
All other answers	12 (28)[b]	87 (71)[b]

[a]H_{010}: Students of the teachers of interest were no more likely than students of the other treatment teachers to answer Check 1 (item 3) correctly.
[b]Expected values.
Note. TI = Teachers of interest.

Table 11:

Null Hypothesis 11[a]: Correct Answer Item 2 (TI vs. Other Treatment)

Answer responses	Students of teachers of interest	Students of treatment teachers
Correct answer	167 (158)[b]	393 (402)[b]
All other answers	3 (12)[b]	41 (32)[b]

[a]H_{011}: Students of the teachers of interest were no more likely than students of the other treatment teachers to answer Check 2 (item 2) correctly.
[b]Expected values.
Note. TI = Teachers of interest.

Table 12:

Null Hypothesis 12[a]: Correct Answer Item 4 (TI vs. Other Treatment)

Answer responses	Students of teachers of interest	Students of other treatment teachers
Correct answer	167 (158)[b]	396 (405)[b]
All other answers	3 (12)[b]	38 (29)[b]

[a]H_{012}: Students of the teachers of interest were no more likely than students of the other treatment teachers to answer Check 2 (item 4, step 2) correctly.
[b]Expected values.
Note. TI = Teachers of interest.

Unlike the findings reported for differences between the likelihood of students of the teachers of interest to provide a particular wrong answer more often than students in classes of the other treatment teachers (insignificant at the $\alpha = .01$ level), differences between these groups of students when providing a correct answer were significant. For each of the three different distribution questions, students of the teachers of interest were significantly more likely to provide the correct answer to each of the three distribution problems than students in the classrooms of the other treatment teachers at the $\alpha = .01$ level (H_{010}: $\chi^2 = 15.32$, $p < .000$; H_{011}: $\chi^2 = 10.67$, $p = .001$; H_{012}: $\chi^2 = 9.44$, $p = .002$).

CONCLUSIONS

Teachers often find new pedagogical methods difficult to implement and often rely on experience to suggest instructional methods. Although such a process may produce satisfactory results for these teachers, the currently available statistics suggest that such methods are not producing similar results for the majority of students attempting to master introductory algebra. As a gateway course, algebra proficiency is critical for the economic and academic well being of all students. To help improve proficiency of students in introductory algebra, researchers at CRESST have developed a program of formative assessments and professional development targeted on key big ideas necessary for the mastery of algebra.

In the present report, we discuss the findings of one small part of that study. In this case, the teachers who used a repeated addition model of multiplication to develop the concept of distribution (the teachers of interest) felt their students were more likely than other treatment or control teachers to respond in a way indicative of a misconception engendered by a repeated addition model. Contrary to the original beliefs of their teachers, however, the students of the teachers of interest were less likely to answer in a way that suggested teaching distribution as an example of repeated addition of the same thing was likely to result in a response pattern that suggested a certain misconception in these students. Specifically students of the teachers of concern were less likely to respond to questions like 6 (3 + 1), with the answer of 6 • 3 + 6 • 3. In fact, students of teachers in the control group were significantly more likely to respond in this way than students of any of the teachers who received the PowerSource© professional development and the instructional materials. Moreover, the students of the teachers of interest were not significantly more likely to make this error than were the students of the other PowerSource© teachers. In fact, the students of teachers who indicated that they taught distribution by connecting this concept to their students' understanding that multiplication is repeated addition of the same thing were *less* likely to produce a pattern seemingly indicative of the misconception than were students of the other treatment teachers. The difference, however, was not significant at the $\alpha = .01$ level. This suggests that PowerSource© materials (both professional development and the instructional handbook) were correlated with a decrease in the likelihood students would respond in manner indicative of a particular misconception.

It should be noted here that we are not inferring that students responding in a manner suggestive of repeated addition actually hold that misconception or are answering because they are applying a repeated addition schema to this problem. Rather we are testing whether students of teachers who instructed using a repeated addition model were more likely to respond in a particular manner more often than other groups of students and whether teaching from the standpoint of repeated addition was likely to increase the likelihood of a certain wrong answer pattern than either of the other instructional methods (array–area or more traditional). It was not. It should also be noted that because the instruction of the teachers in the control group was not monitored, researchers had no way of knowing how distribution was *actually* taught in those classrooms. The study design does, however, allow us to conclude how the students of teachers of interest responded to assessment questions about distribution relative to students of teachers who had only been exposed to traditional professional development in this particular school district. In each case, the analysis of the data leads us to believe that teaching distribution using a repeated addition model was significantly less likely to result in a response pattern teachers and researchers felt indicative of a particular misconception. Moreover, neither of the suggested PowerSource© instructional treatments (area–array model or repeated addition) seemed significantly different in their likelihood to engender such a response pattern from students. Rather, such a response pattern was associated with students in non-PowerSource© classrooms significantly more often than with students in the classrooms of teachers receiving PowerSource© professional development.

In addition to analyzing response patterns indicative of a particular misconception, we were also interested in the overall performance of students in different groups. Aside from the significant differences in a specific incorrect response pattern, there were also differences between the students of the teachers of interest and the students of the control group in the likelihood that students answered each of the questions correctly. Students of the teachers of interest were significantly more likely to answer each of the three distribution questions correctly than were students of teachers in the control group. Moreover, these same students were also significantly more likely to answer the distribution questions correctly than were the students of the other PowerSource© teachers. This latter result is interesting as it controls for time in professional development in addition to the teacher and student effects controlled for by randomization between the treatment and control groups. Whereas researchers were not able to control for the amount or type of professional development the control group received, all the teachers in the PowerSource© group received the same amount of professional development and the teachers then decided to teach the unit on distribution in the way they thought would best illustrate the big idea. Although the teachers of interest were a "self selected group" in the sense that they decided to teach the concept of distribution as repeated addition, the results are suggestive that teaching distribution in this way can have positive effects on student achievement without engendering a specific unwanted misconception. Nevertheless, this same result might not be replicated if teachers were constrained to teach distribution using either an array–area model or repeated addition. We would recommend a future experimental study to see if teaching distribution using array and area models versus teaching this concept as repeated addition using randomized groups would generate the significant differences in student performance as suggested in this report.

One final conclusion suggested by this research is that focusing professional development on developing conceptual understanding of a single "big idea" with teachers, over a relatively small number of hours is correlated with significant positive differences in student performance. We plan to explore these differences further by comparing student results on state standardized assessments and pre- to post-test changes on a researcher developed transfer measure to see if the trends reported for the students and teachers involved in the present research are replicated in larger groups of students and teachers and across multiple districts and on non-researcher developed assessments.

In the end, all PowerSource© teachers in this district responded well to the findings reported here and their responses suggested that they would continue or begin to teach distribution as an instance of repeated addition. In fact, our interactions with them 1 year later suggest this repeated addition model has been adopted by nearly every PowerSource© teacher in the district.

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