An Inventive Use of the WWW as a Teaching and Learning Tool in Mathematics: Structured Web Materials for Courses Across the Mathematics Canon.

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ABSTRACT

An Inventive Use of the WWW as a Teaching and Learning Tool in Mathematics: Structured Web Materials for Courses Across the Mathematics Canon.

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The author of this paper submits that students have a natural inquisitiveness; hence, students must be **active** in learning. Thus, the student must learn to conjecture and prove or disprove said conjecture. Ergo, the purpose of the paper is to submit the thesis that learning with the use of the World Wide Web (WWW) can enhance the students' experience in mathematics and proposes some innovative uses of the web that the author has constructed for his students.

The paper is organised in the following manner. In the first part of the paper the author submits the model of his web site and how it fits into his pedagogical programme, "the fusion method," of traditional, reform, and Moore methods for teaching. In the second part of the paper the author focuses on the dissemination of material, exercises for the students, assessment of progress and mastery by the students, and programme for the students to submit work for inclusion on the site. In the third part of the paper the author details the e-book and how development of said material has been of assistance to the department and to the mathematics programme so course appropriate materials is available for use by the students in any class (even those not taught by the author). Creation of such materials enables students to bridge the gap that exists between high school and college level work. The final part of the paper discusses the successes and lack thereof of maintaining the site, including both instructor and student created work, cost and time expense, and reaction of colleagues to the materials being available.

Nonetheless, the author's use of the web is not what one might typically think of when one mentions a mathematics web site. It does not include the usual "bells and whistles" one might associate with web sites. It is used primarily as a tool to disseminate materials to students in an efficient manner, encourage students to create mathematical arguments and solutions to problems and then learn the skills necessary to write technical solutions or arguments. The difference in participating and witnessing cannot be understated. The author of this paper submits that students have a natural inquisitiveness; hence, students must be **active** in learning. Thus, the student must learn to conjecture and prove or disprove said conjecture. Ergo, this paper submits that the use of the WWW is within this context - - a programme of mathematical pedagogy such that the students' experience of **doing** a mathematical argument is the reason for the exercise along with a finished product. So, the instructor constantly monitors the progress of individual students and adjust the notes, offers "hints" on the site, and constructs handouts or modules based on the progress of the students in the class (therefore, it is an ongoing use of the web rather than a pre-existing construct).

INTRODUCTION

Mathematics is built on a foundation which insists on sound reasoning, proper justification for a claim, and with abstraction, generalisation, specification, and imagination. One considers examples and abstracts to systems but must reason carefully and prove the claim; one considers a case then can prove a generalisation with an inductive step in mathematical induction; one can consider a particular function with a domain A and codomain B then note that for a specific (a, b) whether or not it is an element of the function; and, through all of these one must imagine in order to create. Applications yield theory; theory might precede application. In all there is an overriding need for students to wonder, conjecture, hypothesise, analyse, argue, critique, prove, disprove, and in some instances memorise. Hence, mathematical education should be centred on encouraging students to think for themselves and not accept that which they read, hear, or believe. They should not accept a maxim which centres on ‘going along with the crowd,’ passively ‘soaking up’ material, witness others actively engaging in an enterprise and believing that they are a part of it, etc. The mathematical community should continue to focus on creating meaningful exercises to assist students in realising their potential, learning new and exciting truths, and critically differentiating between perception and reality.

The author of this paper submits that students have a natural inquisitiveness; hence, for students to learn students must be **active** in learning. Thus, the author of this paper submits the thesis that learning requires **doing**; only through doing is learning achieved; and, hence this paper proposes a philosophy such that process is the important component of mathematical education -- the experience of creating a mathematical argument is the important reason for an exercise and should be advanced above the goal of generating a solution, an answer, a polished proof, or any other such end (in and of itself).

THE WEB-SITE AND THE FUSION METHOD

The ‘use of technology’ has become a scheme (the latest in a long line) that will ‘save’ education and messianically deliver the world a better future. The literature is replete with examples of the use of technology in and out of the mathematics classroom that may or may not be of practical educational utility. Herein is described a modest use of technology that the author has developed over the course of the 1990s that he feels is of practical use to his students, creates a meaningful educational experience, and does not diminish the standards that the student is expected to achieve.

The author has created a rather basic web-site for student usage such that class materials are available for students to download from the author created web-site. The web-site is indexed such that for each class the instructor teaches; there is a specific sub-section for the students to navigate to:

Class Links:

- Math 140 ...Finite Mathematics.
- Math 155 ....Basic Statistics
- Math 180 ....Principles of Mathematics
- Math 251 … Analysis I
- Math 252 .... Analysis II
- Math 255 ....Set Theory
- Math 272 ....Linear Algebra
- Math 351 ....Analysis III
- Math 353 ....Advanced Calculus I
- Math 354 ....Advanced Calculus II
- Math 355 ....Probability and Statistics I
- Math 356 ....Probability and Statistics II
- Math 398 ... Directed Reading
- Math 451 ... Real Analysis
- Math 480 .. Topology
- Math 498 Directed Reading and Research

Note: The pages contain links to quizzes, tests, calendars, handouts, etc. If a student has missed class or wishes a copy of any document he may download them (some are in DOC format, some in HTML format, and some in PDF format) It should be noted that to view a DOC the copy of Word must support mathematical symbols. If it does not, reinstall Word with Equation Editor 3.0 or above or download MathType Lite at http://www.mathtype.com.

Other Links:

The Principles of Mathematics (e-book draft 3)

Research Interests

Morehouse College Mathematics Department Home Page

Recommendation Policy

The web-site is constructed in such a manner as not to include what some might consider the usual ‘bells and whistles’ of a web-site. It is rather utilitarian. It contains links to the syllabus and grading policies; a calendar that gives a general approximation as to the pace of the course;

all the handouts that the instructor prepares for the students so that if they miss a class or lose a handout it is there; a running record of homework assigned; an archive of old quizzes; a supposed archive of old tests (the student get a surprise when the try that link); and, student-created solutions to problems, claims, etc. All of the material is readily available for the student to obtain. For example:

Math 255 Set Theory

Math 255 Syllabus & Grading Policies

Math 255 Calendar

Math 255 Handouts

Math 255 Homework

Math 255 Quizzes

Math 255 Tests

Math 255 Students' Presentations

From each the student can proceed to obtain such materials. None of the aforementioned is ground-breaking, but it serves the intended purpose of allowing for students to obtain materials at will. However, two part of this are noteworthy: the handout and student presentation sections.

All of the handouts were created by the author as a reaction to class discussions, in response to requests, through noting certain questions or claims were posed by students in the office, or in rare instances in anticipation of student difficulty grasping the concepts, methods, techniques, or notion of material discuss in class. As such the handouts are a result or consequence of student inquiry and are intended to clarify concepts, provide additional exemplars, be supplements to the text or class discussion, expound upon concepts and expand the discussion beyond the scope of the text, pose questions that are not part of a text, and complement the didactic from the class. The handout section exists such that it is not erased and re-created each semestre; but is enhanced, revised, and edited as ever more questions are posed by students and time progresses.

The student presentation section of the web-site was started in 2000 as a tool to encourage students to learn how to write up solutions, proofs, examples, etc. in a more technical manner than simply through print or script and to allow acknowledgement of the work of the students that was well done. The student presentation section for each class is erased at the end of a semestre so that when a new semestre begins, students are free to write up solutions to problems they presented in class or in the office. It is opined by the author that were this not so, it would become too much like a student solution manual that accompanies many texts, the number and type of problem that students are invited to write up might change over time, and the students might not use the site as intended - - that of a follow-up from class discussion rather than an imposition from the instructor to the student.

The avoidance of imposition of the instructor to the student is an important component of the author's philosophy of teaching, the fusion method. The method is an amalgam of methods that the author experienced as a student. The author has studied under professors who have taught in each of the ways that are fused during his formal educational experience. His professors taught classes or directed the research of the author using the Moore, traditional, or reform methods. Hence, the author developed the fusion method over the years of his college-level teaching experience. It is constantly being analysed, refined, and evaluated so per se it is more dynamic rather than static a system. As such it was created via an action research model.

The fusion method[1] amends several philosophical positions from the Moore, traditional, and reform methods. The fusion philosophy[2] of education assumes that a person learns best individually but needs a little help, encouragement, or reinforcement from time to time.

Learning takes time and so only through a struggle with concepts, principles, methods, techniques, etc. can a person understand, appreciate, and ultimately master said. Students do not always perform at peak efficiency; so, the fusion method allows that students have a natural

[1] Herein when referring to an instructor who uses the fusion method in a classroom, sometimes the term 'fusion classroom' will be used to refer to the classroom experience and 'fusion instructor' to reference an instructor who adheres to the fusion philosophy.

[2] A more detailed discussion of the fusion philosophy or method is contained in the paper 983-O1-559, "Initiating and Continuing Undergraduate Research in Mathematics: Using the Fusion Method of Traditional, Moore, and Constructivism to Encourage, Enhance, and Establish Undergraduate Research in Mathematics." It is also discussed in paper 983-T1-363, "The Fusion Method of Traditional, Moore, and Constructivism and the Incorporation of Undergraduate Research through the Mathematics Curriculum."

inquisitiveness that ebbs and flows. Books are not banished in a fusion classroom. The student is encouraged to use as many books as he opines is necessary to understand the material.

This philosophy of education does not seek maximal 'coverage' of a set amount of material, but standard competency in a given field with some depth and some breathe of understanding of material under consideration. The fusion method accepts the concept of minimal competency, a criterion that a student should meet so that he can progress, and so student understanding builds from elementary to complex material. Hence, aspects of 'coverage' are included in the fusion classroom. A course description, syllabus, or objectives for a class exist and were agreed to by the faculty of the department so the fusion instructor is duty-bound to include a discussion of material contained within said. But, the fusion method does not attempt to maximise 'coverage' of a syllabus. The goal of education is not, under the fusion methodology, 'vertical' knowledge (knowing one subject extremely well) nor 'horizontal' knowledge (knowing many subjects superfluously), but this philosophy attempts to strike a *balance* between the two.

The fusion method demands no collaboration on material *before* student presentations and no collaboration on any graded assignment and requests minimal collaboration on material after student presentations. After student presentations, if a student does not understand a part of an argument or nuance of said argument, the students are permitted to discuss the argument as well as devise other arguments.

The fusion method includes minimal lectures before student presentations over definitions and terminology, an occasional exemplar argument, as well as subsequent lectures after the students discuss the work(s) presented when the instructor finds there is confusion or misunderstanding about the material amongst the students. The fusion method also requires that students critically read proposed arguments and note whether or not the proposed solution is correct. Thus, the fusion method includes reading of mathematics materials and papers.

The pace of a fusion class is somewhat defined by the students whilst regulated and adjusted by the instructor. The fusion method acknowledges that not all questions can be answered and that each time a question is answered a plethora of new questions arise that may not be not answerable at the moment. Therefore, the fusion method seeks to balance the question of 'how to' with the question of 'why.' It is a philosophy which seeks moderation and strives to be reasonable in expectation and outcome.

The fusion method includes class discussion and allows for the discussion to flow from the students but be directed by the instructor. The fusion method allows for applications (minimal discussion of applications exists in the pure mathematics courses since the emphasis is on the foundations of theoretical mathematics) and modelling (with regard to the fact that students present their arguments before the class and there exist exemplars for the students as well as critical reading exercises). The fusion method also requires that students critically read proposed arguments and note whether or not the proposed solution is correct. Thus, the fusion method includes reading of mathematics materials and papers from sources such as texts, journal articles, commentary, and instructor-created or student-created solutions to problems.

The fusion method includes regularly administered quizzes, tests, and finals. A part of each quiz or test (no less than ten percent or more than thirty percent) is assigned as 'take home.' Upper division course taught with the fusion method include at least one oral quiz where a student contracts a time to meet with the instructor and then be asked to argue whether or not a set of claims is true or not in the office. The fusion method does *not* include group assignments of any kind.

MATERIALS

As previously noted the web-site is constructed in such a manner as not to be flashy or interactive but to be functional and utilitarian. It contains links to the syllabus and grading policies; a calendar; handouts; a record of homework assignments; some old quizzes; a supposed archive of old tests; and, student-created solutions to problems, claims, etc. Some of the handouts are disseminated by the instructor to the students in a class period (especially those centring on definitions, terminology, symbols, etc). Materials from the web-site that are follow-ups to class discussions, that are reaction documents to the dialectic from the class period, or are student created are not disseminated to the students in class. The students are required to download that information themselves; if they so desire. If any material is to be used for homework or for a grade it is provided to the student in class; hence, only those materials that are supplemental are the type of material that the instructor directs students to download themselves.

Over the years students regularly come to the author with problems with the technique of downloading, trouble with working with computers, etc.; the number of incidents of said has not decreased over time (much to the astonishment of the author). There are students who, for whatever reason, are unfamiliar with or uncomfortable with downloading documents from the internet. On amusing anecdote of such was a student who had difficulty with downloading and asked for assistance. In the course of the conversation about how to download a document he mentioned his love of downloading music from the web. It was quite disconcerting to find him comfortable with music downloads but not document downloads (it was, it should be noted a document in PDF form).

The college computer support is minimal. Many students do not own computers so they must download material from a computer in one of the labs. Unfortunately, the mathematics department computer lab does not support printing, so they must use the general college computer lab. Students report problems with the staff of the facility, the condition of the machines, lack of paper, etc. Whilst not harping on negatives - - there are many computer labs at the college; however, they are linked to certain departments such that only students from that department are allowed to use them. Simply put, in many ways the college functions in a feudal manner where each department represents an earldom, dukedom, or vassal state.

The standard exercises that are part of the handouts for a class usually focus on problems that the students encounter with the concepts, with terminology, with methods of proof, with construction of counterexamples, and supplement text exercises. For example, in Math 180, Principles of Mathematics, students study logic, method of proof, etc. One area where students have trouble is with mathematical induction. So, there is an expository discussion paper about mathematical induction that includes exercises such as:

1. <u>Claim</u>: $2^n < 2^{(n+1)} \ \forall \ n \in \mathbb{N}$

2. <u>Claim</u>: $2^n > n \ \forall \ n \in \mathbb{N}$

3. <u>Claim</u>: Let $a \in \mathbb{R} \wedge a > 1$. It is the case that $(1 + a)^n \geq 1 + an \ \forall \ n \in \mathbb{N}$

6. <u>Claim</u>: $\sum_{j=1}^{n} 2^j = 2^{(n+1)} - 2 \quad \forall \ n \in \mathbb{N}$ (Handout / Worksheet 7, Math 180)

The expository discussion is usually of the form such as:

Math 255

Set Theory

Handout 2

Proof Versus Counterexample

There has been some confusion (as it has always been) between a proof and a counterexample. Please refer to a text such as the text, "100% Mathematical Proof" (the old Math 180 text; or another available in the reading room [Dansby Hall 339]) for a detailed discussion of the difference between a proof and counterexample.

First consider the following claim:

Claim 1 Let x and y be integers. If x is even and y is even, then x + y is even.

You must first READ the claim and decide whether or not you think it is true (you may be wrong, but you have to practice this step; it is based on your prior experience and knowledge). It is an inductive step; hence, there is no guarantee that you are right.

The expository handout continues:

Next, after considering claim 1, suppose we think it true. *Thinking it is true is not proving it is true*. Hence, we need to construct a proof. We must announce it is a proof and frame it at the beginning (Proof:) and at the end (Q.E.D.).

Proof:

1. Let x be an integer	1. Premise
2. There exists an integer, m, such that x = 2m.	2. Definition of even integer.
3. Let y be an integer	3. Premise
4. There exists an integer, k, such that y = 2k.	4. Definition of even integer.
5. Consider x + y	5. Hypothesis.
6. x + y = (2m) + (2k)	6. Substitution
7. = 2m + 2k	7. Associative axiom of multiplication
8. = 2(m + k)	8. Distributive axiom of multiplication over addition.
9. Hence, x + y = 2(m + k)	9. Transitivity of "="
10. But, m + k is an integer, say n.	10. Closure of integers under addition.
11. So, x + y = 2n, such that n is an integer.	11. Substitution
12. Thus, x + y is even.	12. Definition of even integer.

Q. E. D.

Comment: note in line 4 we had to express y as 2 times an integer; but, we can not use the same variable as m (for x) since we do not know [we do not have a premise, hypothesis, or prior information] hence can not opine that y = x.

Claim 2 Let x and y be integers. If x is odd and y is even, then x + y is even.

You must first READ the claim and decide whether or not you think it is true (you may be wrong, but you have to practice this step; it is based on your prior experience and knowledge). It is an inductive step; hence, there is no guarantee that you are right. Next, after considering claim 2, suppose we think it false. *Thinking it is false is not proving it is false*. Hence, we need to construct a counterexample. We must announce it is a counterexample, present the counterexample, and demonstrate that indeed the premises are true but the consequent is false. A counterexample is concrete - - it is not writing a paragraph or two explaining why one opines the claim false - - it is an example!!!!! Also, note it is not framed at the beginning (Proof:) and at the end (Q.E.D.: Quod Erat Demonstratum) as with a proof; we only need announce at the beginning and compleat the counterexample.

Counterexample:

Consider x = 3 and y = -8.
Note that x is odd since x = 2(1) + 1 and 1 is an integer.
Note that y is even since y = 2(-4) and -4 is an integer.
Now, x + y = 3 + (-8) = -5 = 2(-2) + 1. Since -2 is an integer, -5 is an odd integer (by the definition of odd integer).
Therefore, x + y is not even.

E. E. F.
Finally, as with all the discussions, examples, proofs, counterexamples, claims, etc. that we encounter; it is my opinion that few can do well in this class through just attending and watching others do the work. I opine that only through doing can we understand and KNOW. Hence, my advice is: "practice, practice, practice." Notice that this too is framed - - by the announcement of a counterexample and by the end (E. E. F.: Exemplum Est Factum).

Back to handouts page

At the point of the course from whence this handout came; the students are required to use a ‘vertical’ form for a proof. That requirement ends when they begin discussing sets in detail and they are free to use the more typical ‘horizontal’ form such that justifications become a part of sentences.

The students are not tested directly over any material on the web-site. It is strictly used as assistance for them.

In each class because the author uses the fusion method of instruction, there exist claims that students are to prove or disprove which naturally percolate from the students themselves. In addition, there are the aforementioned exercises that are part of handouts from the web-site. Further, there are exercises from the text used for each course. Since students do presentations in each class the author teaches, students present their work on the board and if it seems to be something that should be included in the notes for students to access, the instructor invites the presenter to write up the solution (usually in Microsoft Word with Mathtype). The author usually has to show the presenter how to use Mathtype and direct the student where to download it (it comes with a 30 day trial then deteriorates into ‘Mathtype Lite’ [sic] that has all the symbols a student needs). Typically, it takes about a week or so for the student to create the document because of time constraints, difficulty acclimating to using such a programme, nesting the symbols and equations into a document, etc. For time-to-time, a student will return to the office for a refresher on how to use it; but it has been the case that they acclimate to it fairly rapidly once they 'get the hang of it.' They bring a copy of the work to the author on floppy disc or CR-RW and he reviews it. If any corrections need to be made; he does so but uploads the original from the student and the modified file. It is then ready to be downloaded by a student from the instructor's web-site.

THE E-BOOK

The first course beyond the Calculus sequence in the mathematics major programme at Morehouse College is Mathematics 180, The Principles of Mathematics. It is the first of a two course sequence that is designed to transition students to upper division courses. Principles and the subsequent course Math 255, Set Theory, are designed to be taken by the students concurrent with Calculus I and II or Calculus II and III. The courses are intended to prepare students for upper division courses with special emphasis on the Real Analysis and Abstract Algebra sequences which are normally started after Set Theory.[3] The first course centres on logic and the second course centres on naive set theory.[4] In each course students are required to understand definitions and axioms and create proofs, examples, and counter-examples. In the Principles course two texts were used, Garnier & Taylor's *100% Mathematical Proof* (2nd Ed., Wiley) and Tan's, *Finite Mathematics* (6th Ed., Wiley). But, there was much consternation on the part of a significant number of faculty in the department as to the costs associated with purchasing two texts for the course. Additionally, the Garnier & Taylor text became difficult for the bookstore to obtain (it went out-of-print, it seems). No text was available that discussed the material the course was designed to include and the decision was made that the material should be defined by the faculty rather than a text define the material discussed in a course. Hence, the author of this paper volunteered to begin work of construction of an e-book for the course. Now, the course uses an e-text (http://facstaff.morehouse.edu/~pmclough/Math180text.html) and Tan; whereas, eventually the course will just use the e-book (when it is completed). Handouts, worksheets, ancillary materials, etc. are available at the instructor's web site.

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[3] For a compleat overview of the mathematics programme see the mathematics web-site, http://math.morehouse.edu/, and for the revised mathematics programme see, http://facstaff.morehouse.edu/~pmclough/majorproposal.html. The revised curriculum has been approved by the department and by the Division of Science and Mathematics and is currently being reviewed by the College Educational Policy and Curriculum Committee.

The Principles course introduces reasoning, first order logic, predicate calculus, syllogistic arguments, existentials, universals, elementary set theory, counting methods, matrix algebra and its axioms, probability and its axioms, and statistics. The statistics section of the course is optional. The students are familiarised with mathematical notation used in upper division course work, direct proof (three different methods), *reducto ad absurdum*, cases, mathematical induction, proof by example for existentials, generalised point method of proof for sets, as well as counterexample for universals and counter-arguments for existentials. Throughout the course they are presented with claims and proposed proofs and counterexamples which they critique.

The first meeting day of Principles the students are given a syllabus, given a grading policy, told of the expectation of student responsibility, told of the e-book, etc. Definitions are presented and they are sent home with some basic drill exercises. The second meeting day student presentations begin. Rather than calling on students like the Moore method [2], volunteers are requested like Cohen's modified Moore method [1]. Throughout the first few weeks the class proceeds in this fashion with short talks about new definitions, methods to prove or disprove claims, and introduction to new terminology, notation, etc. By the end of the first third of the semestre, the amount of time the instructor talks decreases from perhaps half of the class period (at the end of the class session) to perhaps a fourth or not at all. In this manner, the students are encouraged to take more responsibility for their education and regard the instructor less as a teacher and more as a conductor. Nonetheless, it must be noted that some days there are no student presentations; so, the instructor must be prepared to lead a class in a discussion over some aspects of the material or be prepared to ask a series of questions that motivates the students to conjecture, hypothesise, and outline arguments that can later be rendered rigorous.

The second half of the course is a hodgepodge of introductory exposition to some material the students will study in upper division courses beginning with sets that will be studied in the second course of the sequence. This part of the course uses the Tan text supplemented by handouts from the web-site. For each subject, definitions, terminology, and notation are established and a series of facile claims are presented for the students to prove or disprove. In the Principles course, the objective is to introduce the student to the subjects and to allow them to

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[4] The course are numbered and titled: Math 180, The Principles of Mathematics (designed for second semester freshmen) and Math 255, An Introduction to Set Theory (designed for first semester sophomores). Syllabi, grading policies, etc. can be downloaded from http://facstaff.morehouse.edu/~pmclough.

work on elementary claims in the areas so they may connect the logic and methods of proof earlier discussed with the particular areas of math under discussion at the time. For example, in the combinatorics section the students are introduced to the Peano axioms and then given parochial claims on factorals, permutations, combinations, etc. Usually the first course ends during discussion of probability, but in rare instances during a long semestre and with a mature group the course may end with discussion of statistics or additional material.

The educational objective of Principles (and of Set Theory) is to transition the student from an elementary view of mathematics to a more multifarious view of mathematics. Since these courses are taken commiserate with Calculus courses, it assists the student to be able to move beyond the concept of mathematics strictly as a tool or an applied subject to a more theoretical subject. Indeed, even in the second part of the Principles course which consists of discussion of counting methods, matrix algebra and its axioms, probability and its axioms, and statistics the aim is to note the theoretical underpinnings of the subjects whilst reviewing (one assumes it is a review; but for some students it is not) the applied component of the material. Also, the subjects discussed in this part of the course are precursors to courses that the student might take later in his programme.

The sequence of courses establishes a foundation for preparation intended for advanced course work by including the Peano axioms, the axioms of the reals, and the axioms of sets. In particular, the discussion of the axioms of the reals is revisited in the first Real Analysis course, so that the claims considered and the discussion of the axioms of the reals in the Set Theory course is very much so a basic axiomatic introduction to the real line. The discussion of mathematical induction is presented in the Principle of Mathematics course via the Peano axioms so that students understand the basic logic of mathematical induction and how the principle is grounded in the axioms, so as to acclimate them to how there is a beginning point in a discussion, argument, proof, etc. The discussion of mathematical induction follows the first order logic, predicate calculus, syllogistic arguments, existentials, universals, and elementary set theory. This placement allows the student to see the connection between that they learnt about logic and how it is actualised in mathematical systems.

The creation of the e-book and its use is experimental; to my knowledge our department has never undertaken such an enterprise before. The e-book is intended to be a flexible text for the

Principles class such that if at a future date a subject may be added or deleted; it can be expanded or contracted; and is such that it is intended to be of use and be able to be altered by any person in the department. A couple of chapters (yet to be written) are not a part of the current syllabus; but were a part of the syllabus in the 1970s and are to be included as optional sections so that the department will have the ability to define a main core for the course, the logic, and then allow some variation with the subsequent material. Indeed an individual instructor may take a part of the e-book, rework it, and then use it with his or her students. So, discussion continues in the department as to what material should be in the Principles course and what in the Set Theory course, so it may be that not soon after this paper is delivered, the course objectives might increase or decrease.

SUCCESSES AND THE LACK THEREOF

Student reaction to the inclusion of the material has been overall positive, especially with follow-up exposition, exemplars, or discussion. However, the reaction is not so positive with the added exercises (but, then when are students ever enthusiastic about extra work?). Faculty reaction to the materials has been somewhat negligible such that faculty whose interests include computer programming or usage have reacted positively and faculty who do not programme or use computers much have reacted with benign indifference. The department chairmen have been supportive of the endeavour, especially in the construction of the e-book.

The time it takes to create, revise, maintain, etc. the site is rather much. Since the author is not a typist and is a rather poor programmer (HTML, SAS, SPSS, BMDP, DOS, BASIC, Maple, etc. are his only skills) the time it takes to do such maintenance and revision of the site is rather substantial. Furthermore, the author is not fond of writing and so the task of creating, maintaining, and revising the site is rarely one that is enthusiastically pursued by the author. This is ameliorated by his desire to assist the students; but it is easier to want to help others than to actually help others.

The e-book gets special mention in this regard because the author has over the years been fond of criticising the state of commercially available mathematical texts (especially Calculus texts; but it spans the undergraduate curriculum). He has found that criticising others’ work is facile compared with attempting to do the work himself. It has been an extremely difficult task to get the first three of the eight or so chapters of the e-book written. Each chapter has gone through at least four drafts that were presented to colleagues for review. There were many more that were works in progress between each draft. The difficulty lay primarily with the author's problem in addressing fundamental questions of, 'how to say X,' 'is this too much or not enough exposition,' and 'are these exercises too repetitive or is there not enough to adequately tie material together,' etc. The author has attempted to create the e-book so that it is somewhat conversational in tone, addresses material so that it is not 'dumbed-down,' and is (of course) correct. Juggling these seemingly contrary objectives was easy in theory but much more difficult in practice.

Another gnawing concern is with the site overall: the inclusion of the handouts, old quizzes, e-book, student presentations, etc. - - is it too much? Is there a point at which the students are left with information overload? Further, are they using it properly or not? The author is concerned that the better students seem to be using it as intended, but some of the students who struggle or have problematic backgrounds (poor high school preparation) seem to use it as they use a text – improperly. They believe that all claims are approached in the same manner and that all must work out in a satisfactorily parsimonious way; hence, especially with expository material they use it not as a guide for finding a way to solve a problem, construct a proof, or construct an example but as *the* way to do said. The study habit which seeks to copy and memorise that which should not be copied and memorised is a disturbing behaviour which is not new, but exists and is not diminished by the materials on the web-site.

Another failure on the part of the author is the site is not interactive where interaction may be of use. This is due to the author's lack of enthusiasm for programming as previously mentioned, busy schedule, etc. which any faculty member in any department in the country could probably understand and appreciate. There are web-sites that have been constructed by faculty at other colleges or universities (the Interactive Real Analysis site at Seton Hall University comes to mind as an example) that are far and away better that the author's site. However, the author opined that instead of merely sitting back and moaning about how much better others are incorporating web material in their classes he should try to do it; so, he did.

Additionally, the institutional technology (IT) office of the college where the author teaches is of little to no use. Much of the supporting software had to be purchased by the author himself since the IT budgets in a manner such that individual faculty are not encouraged to request software. A part of the costs have been offset by small grants the author has been awarded over the past three years. The IT office is under-staffed and under-budgeted and of the staff that exists two of the ten staff members seem to be knowledgeable about technology. It probably is the case at other small liberal arts college, and the author is aware that there are a plethora of colleges where the IT support is less than at the college where he teaches or non-existent; so one should not overly critique the problems that exist. It is fair to say that our department has done 'more with less' for the past fifteen years that I have been a member of the faculty; my colleagues are resourceful, professional, and well-intended.

Another disappointment is with the student presentations that are not included on the site. One may think the previous sentence was a typographical error, but it is not. Perhaps as few as two in ten presentations that are made in class and are of note such that the author invites the students to present their work conclude with existence on the site. Once again, time seems to be the greatest enemy. The students report that they just do not have the time, forgot the invitation, or other reasons such that their work is not submitted for inclusion. A colleague has suggested that the author makes the inclusion a requirement; but that author is recalcitrant to do such because of his educational philosophy.

CONCLUSION

Nonetheless, it should not be construed that the exercise is a failure or that I am remorseful about entering into the enterprise. It has been (as should be the case) a learning experience and there have been vicissitudes but overall the experience has been enriching. It is part of the manner in which I seek to live my life: that which I learnt the best was that which I did myself, rather than be told about, lectured to, or even read about. I must do in order to understand. Hence, by engaging in the creation of material for students rather than using materials created by others, I have gained a better appreciation of the time, effort, and difficulty others must endure to write texts or create meaningful educational materials that are appropriate for students. I can honestly say that I am moderately successful at creating a use of the web that connects directly to the classes that I teach and the research programmes that I engage in with undergraduates in so far as it is an extension of the didactic from the classroom.

It seems advisable to promote the concept that each academician should decide for himself or herself whether or not such inclusion of materials (especially the student-created work) is of interest of him or her and fits within the philosophical framework of education that he or she has created or adopted. I submit that the use of the web as outlined in this paper seeks to assist students to discover the material in a guided (rather than haphazard) fashion; attempts not to 'talk down' to students; and, does not compromise the integrity of an undergraduate mathematical education. It has the property that it may assist the student to progress from an elementary understanding to a more refined understanding of mathematics. Ergo, this paper submits that the use of the WWW in this manner supports a programme of mathematical pedagogy such that the students' experience of doing a mathematical argument is the reason for the exercise along with a finished product. So, the instructor constantly monitors the progress of individual students and adjust the notes, offers "hints" on the site, and constructs handouts or modules based on the progress of the students in the class (therefore, it is an ongoing use of the web rather than a pre-existing construct).

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