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The Building Blocks of Success

Higher-Level Math for All Students

May 2008

INTRODUCTION

Claims about the vital importance of equipping young people with the knowledge and skills they need for success in college, careers, and life have become commonplace. Specific descriptions of the knowledge and skills needed, however, vary and often remain quite vague. Books such as *The World is Flat* by Thomas Friedman, forecasts about the demands of the new knowledge-based economy, and worrisome statistics about the poor preparation young people are receiving in public schools, especially compared with their international peers, justifiably have raised concerns, and many prescriptions have been offered to address the problem. But if there is wide agreement that high schools need to change if they are to prepare young people for new demands, there is far less agreement about what sorts of changes are needed.

Almost any formulation of the knowledge and skills young people will need includes a basic building block for success, mathematics. Policymakers are exploring a range of strategies to improve student success in mathematics, including aligning academic standards to the expectations of the postsecondary and business communities, providing more comprehensive mathematics offerings in high schools, and even requiring all students to complete a college- and career ready sequence of mathematics courses.

The urge to define the knowledge and skills young people need for college and careers is motivated by two realities. First, the world has become a more complex place. To thrive in a global economy, in work contexts that rely on constantly evolving technologies, and in a world challenged by complex social and environmental problems, young people will need significant intellectual skills and inner resources. Second, too many young people in the United States are leaving school woefully unprepared for the demands of postsecondary institutions and the workplace, and it is disadvantaged young people who are often the least well-equipped.

High schools may still be anchored to 20th century expectations, but what are the critical guideposts for a 21st century high school education? There are many specific skills and competencies that young people will need to succeed, but more than particular skills, they will need the cognitive capacity to educate themselves throughout their entire lives. Young people need the ability for complex reasoning and the self-confidence to apply it in new situations. These are precisely the skills that are developed in higher-level mathematics courses, beginning with the foundational Algebra I and extending beyond Algebra II, in which students begin to use abstract reasoning to solve complex problems.

High schools already offer higher-level mathematics, but few recognize that this coursework is a critical building block for success in school, work, and life, for all students. Successful completion of higher-level math courses is associated with subsequent success in college and the workplace for two reasons. Higher-level math courses are practical prerequisites for further study or professional advancement, including both success in postsecondary courses and mastery of complex technical challenges in the workplace. These courses also develop students' intellectual capacity in ways that benefit them in whatever career pathways they may follow, either directly following or long after high school graduation.

Higher-level math is important for all students, not just those who are preparing for a four-year academic degree or a math-related career—and all students can succeed in these courses, with appropriate supports. Researchers who have studied mathematics learning reject the notion that some students can "do math" and others can not. Comparative research on math learning, attitudes, and achievement in different cultures has shown, for example, that, in many cultures, it is expected that everyone can succeed at math if they persevere.[1]

[1] National Research Council, 2005

In the Unites States, by contrast, the focus is often on perceived innate ability—if some students do not make an effort in mathematics this is accepted on the grounds that some students just do not have a gift for it. Since higher-level mathematics is the foundation for success in college and careers, we must change the way we approach mathematics and understand the importance of higher-level mathematics for *all* students.

This paper explores the intellectual and practical benefits to all students of taking higher-level mathematics courses, through and beyond Algebra II, or its equivalent, during high school.

MATH AS THE FOUNDATION FOR HIGHER-ORDER THINKING

Some question whether all students need advanced math to tackle the practical challenges of adulthood, such as managing personal finances, choosing a health care plan, selecting a mortgage, or planning for retirement. They wonder what value Algebra II and other higher-level math courses have in the "real world," especially for students who do not plan to pursue a career in the sciences, technology, engineering, or mathematics, or students who are already struggling in math classes.

To appreciate the importance of higher-level math skills, it is helpful to start with an overview of how mathematical learning develops. Researchers who study learning and cognition describe mathematical learning as a progression in which conceptual understanding builds logically, and expertise is developed gradually.[2] They describe the development of "mathematical proficiency" in terms much broader than simple mastery of increasingly difficult computational skills, facts, and operations.[3]

Developing competency as a mathematical thinker requires individuals to develop both a cumulative body of knowledge and a comprehension of how that knowledge is connected and organized, and to understand the kinds of reasoning expert mathematicians use. The key components of mathematical thinking, which are developed beginning in the elementary grades, are shown in the box below.

Key Components of Mathematical Thinking

1. ***Conceptual understanding*****—comprehension of mathematical concepts, operations, and relations;**
2. ***Procedural fluency*****—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;**
3. ***Strategic competence*****—ability to formulate, represent, and solve mathematical problems;**
4. ***Adaptive reasoning*****—capacity for logical thought, reflection, explanation, and justification;**
5. ***Productive dispositions*****—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.**

Source: National Research Council, 2001

[2] National Research Council, 2005
[3] National Research Council, 2001

These key components are necessary but not sufficient to ensure student success. Students with this foundation, which includes secure competence in the foundations on which algebra is based (e.g. fluency with whole numbers, fractions, and aspects of geometry and measurement),[4] are ready to tackle higher-level mathematics, beginning with algebra.

Algebra is generally the first math course students take that has a name other than "math," but few students approach it with much sense of why that is so, or of the doors it can open. Algebra is an ancient branch of knowledge; its name refers to the revolutionary idea that originated with it, that one could use known relationships to answer questions about unknown quantities. Algebra was a way of moving beyond calculating and into abstract reasoning. It has been described as "the language of mathematics, which itself is the language of the information age."[5]

Mathematics educators don't necessarily focus on the distinction between Algebra I and subsequent math courses. What is important to them is the development of thinking skills, or mathematical habits of mind. Once this kind of thinking is established, students can apply it in the context of geometry, trigonometry, calculus, data and statistics, or other advanced courses. It is these higher-level courses that provide the opportunity for students to make use of the skills and capacities they develop in a successful core math curriculum. Taking these higher-level courses is the way students learn and reinforce advanced skills, such as multi-step analysis, that complete the development of mathematical thinking and prepare them intellectually for success in college and the workplace.[6]

Algebra is the "gateway" course not just because it as a prerequisite for many high school and postsecondary math, science, engineering, and technology courses, but because it is an intellectual gateway to abstract reasoning. As all students should take four years of mathematics in high school, at least through Algebra II or its equivalent, to be prepared for college and the workplace, Algebra I provides the foundation necessary for students to complete a comprehensive sequence of rigorous mathematics courses throughout high school and beyond. The intellectual preparation that higher-level high school math courses provide maps closely to the kinds of mathematical thinking that university educators believe are needed for success in college, including:[7]

Understanding mathematics: thinking conceptually, not just procedurally; using logical reasoning and common sense to find mathematical solutions; using experimental thinking; taking risks and accepting failure as part of the learning process; the ability to use formulas and algorithms of computation;

Problem-solving, technology, and communication: using a step-by-step approach to solving problems and draw parallels and connections. Successful students understand the process of modifying, adapting, and combining mathematical tools to find new ways to reach a solution. Fluency in mathematics is a basic skill for science courses; and

Orientation toward learning: Students who do well in mathematics classes are prepared to translate real situations into mathematical representation, and to extract meaning from mathematical expression.

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[4] National Mathematics Advisory Council, 2008

[5] Steen, 1999

[6] The National Center for Education Statistics, for example, highlights the advanced course sequences-including geometry, Algebra II and Algebra II, trigonometry-that yield the greatest gains in mathematical understanding, NCES, 2008

[7] Conley, 2003

WHY IS HIGHER-LEVEL MATH SO IMPORTANT?

There are five fundamental reasons why higher-level math courses are important both for every student's prospects for success and for the nation's future.

Taking higher-level math courses is key to access to postsecondary education, especially for disadvantaged and minority students.

Advanced high school math is a ticket to college access. Algebra II and other advanced courses are prerequisites for many college courses, not only in math and science, but also in social science fields, economics, business, and computer and other technology courses. These courses are also a key component of preparation for college and a signal of college readiness. Unfortunately, there is still a large achievement and opportunity gap in mathematics. Disadvantaged and minority high school students both earn fewer mathematics credits than their socio-economically advantaged peers, and are less likely than those peers to enroll in higher-level math courses, such as trigonometry and calculus.[8]

The gaps in course-taking by population subgroup mirror those seen in test scores and other measures of educational outcomes. Black and Hispanic students are twice as likely as their White and Asian peers to take no math beyond the basic level, for example.[9] Nine and ten percent of Hispanic and Black students, respectively, take advanced Algebra or calculus, while 22 percent of Whites and 43 percent of Asians do so.[10]

These course-taking patterns have a profound impact on college access. These students are the least well-prepared for college, and the least well-represented among college freshmen: just 11 percent of freshmen are Black and seven percent are Hispanic, although 14 and 17 percent of the total population of 18 year olds are Black and Hispanic, respectively.[11] Mathematics course-taking is important for access to and success in two-year institutions as well, but many minority students and girls, of all races and ethnicities, lack access to advanced math classes or are discouraged form enrolling in them.[12] Under these circumstances, higher-level math courses function not as the intellectual and practical boost they should be, but as a filter that screens students out of the pathway to success.[13]

Some people fear that pushing all students to take challenging math courses will further discourage some of those who have traditionally not signed up for them. But requiring students to take challenging math does not increase high school dropout rates, as long as students are given the support and high-quality instruction they need to succeed.[14] Such supports might include afterschool or summer programs with a math focus, small-group or one-on-one tutoring sessions, and professional development and team teaching initiatives that help teachers target the full range of students' needs. Researchers who have looked at the effects of exposing students at all ability levels to an accelerated curriculum have found that the benefits are universally shared. For example, in a study that controlled for other factors that influence performance, researchers found that students who enter ninth grade with differing levels of preparation all show improvement on mathematics

[8] Bozick, et al., 2008
[9] Rose and Betts, 2001
[10] NB: Rose and Betts have somewhat different figures from the NCES data cited in "Using Research to make the Case." Both data sets support the main point.
[11] Greene and Forster, 2003.
[12] National Action Council for Minorities in Engineering, 1999
[13] National Research Council, 1989
[14] National Mathematics Advisory Council, 2008; Lee and Bukam, 2003

assessments if they have the opportunity to take algebra and other higher-level math courses.[15] Other studies have shown similar results, particularly for so-called "average" students who take higher-level classes; research does not show benefits to any students of being placed in lower-level classes.[16]

Math is critical for college success and degree completion.

Entering college without having taken the necessary foundational and advanced mathematics courses in high school is not just a practical obstacle for students. Poor preparation in mathematics can be a significant roadblock to degree completion. Students who study math at least though Algebra II in high school are more than twice as likely as those who do not to earn a four-year degree, and the level of math a student reaches is the most accurate predictor of whether that student will earn a Bachelor's degree.[17]

Additionally, students who need remedial courses in math once they enter two-and four-year institutions have much lower completion rates than other students. Of students at two- and four-year colleges, just 45 percent of those who require remediation in math only will complete a degree, while 56 percent of those who require no remediation will earn a degree.[18] This is a large and growing number of students: by one estimate 35 percent of those enrolled in two-year institutions and 22 percent of those in four-year institutions require remedial math.[19] Community and technical colleges report the greatest increase in need for remediation.[20] It is clear that high school course-taking affects the need for remediation, and that the rigor and content of the courses matter more than the title. These statistics make it clear that many high school math courses are simply not rigorous enough to prepare students adequately for college-level work.[21]

In a study that examined math course-taking and later education outcomes, Rose and Betts similarly found that not only does taking math courses matter, but that the nature of the courses makes a difference as well. After controlling for demographic factors that also affect college entry (such as ethnicity, parental education and income, school size and demographics, and teachers' education level), 73 percent of students who took calculus during high school later earned a bachelor's degree, while just three percent of those who took "vocational" math (e.g. courses labeled vocational, general, basic, or consumer math) did so. Among those who took "advanced" algebra in high school (that is, a course beyond Algebra II but not calculus, such as statistics or pre-calculus) 56 percent earned a BA. Thirty-four percent of those who took "intermediate" algebra (comparable to Algebra II), and just 13 percent of those who only took some algebra and/or geometry earned BAs.[22]

Why might math preparation be so strongly correlated to completing college? A close look at the content of higher-level math courses suggests an important reason. Studying higher-level math equips students with the ability to use logical reasoning, rigorous argument, and proof as tools in any field. Beginning with Algebra II, students learn to apply strategies in solving multi-part problems, establish connections between multiple pieces of information, and use reasoning to determine which tools are applicable and how to use them. Math educators describe the mathematical thinking that fosters success in math courses (and beyond) as habits of mind, or ingrained logical thinking (e.g. problem-solving, communicating, reasoning, making connections,

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[15] Gamoran and Hannigan, 2000
[16] Epstein and MacIver, 1992; Hoffer, 1992; Sanders, 2000
[17] Adelman, 1999 and 2006
[18] Venezia, Kirst, and Antonio, 2003
[19] National Center for Education Statistics, 2004
[20] Bettinger and Long, 2006
[21] Bettinger and Long, 2006
[22] Rose and Betts, 2001
[23] Cuoco, Goledenberg, and Mark, 1997

informed guessing and detecting patterns).[23] Students may not recognize how they draw on these intellectual experiences in other contexts, but educators recognize that they are key to college success.[24]

Success in higher-level math also gives students self-confidence they will need to persevere in college-level study. According to a national survey of recent high school graduates, students who take at least Algebra II feel more prepared for college work, and more prepared to handle math-related challenges they may face at work than their peers who did not reach that level.[25] Specifically, 60 percent of college students who took Algebra II or higher-level math courses in high school report that they feel prepared for the math they are expected to do in college, while just 26 percent of those without that preparation do. Similarly, 68 percent of those who took Algebra II or higher feel prepared for the math they are expected to do at work, while just 46 percent of other high school graduates do. These students seem to be correct—recent research has shown a strong correlation between taking challenging math and performing well in college. For example, students who have taken rigorous high school math are found to earn higher grades in college science courses.[26]

In short, higher-level math course-taking influences college access and success in two ways. The credits themselves make students more desirable candidates, and increase the likelihood that students will seek out more postsecondary education. Many public and private four-year institutions require students to take at least Algebra II, if not a course beyond it in high school, just to be eligible for admissions. The substance of the courses has an effect as well. Students who take higher-level math courses benefit both by learning specific skills they can use in college classes and in the workplace, and they also develop reasoning skills that enable them to perform better generally in both of these settings.[27]

The association between the study of mathematics and economic success is so strong that many observers have identified access to advanced math courses as a key dividing line that separates disadvantaged and advantaged youth.[28] Or, put another way, higher-level math courses are a critical tool for reducing the gaps in postsecondary achievement and completion between these groups.

Students who take higher-level math courses are better prepared for the workplace and earn higher salaries.

The impact of differences in mathematics course-taking on young people's lives is evident in their salary gaps once they enter the workforce. Taking math courses throughout high school is beneficial, but these effects are much stronger for advanced math courses than for more basic math. For example, the annual earnings of students who had taken calculus in high school were about 65 percent higher than the earnings of students who had only completed basic math.[29] This analysis is consistent with other analyses of the links between educational attainment and earnings. The link between advanced math course-taking and attainment of a college degree reinforces this connection, since Associates degree-holders can expect to earn 25 percent more in the course of their careers—and BA holders 73 percent more—than workers with only a high school diploma.[30]

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[24] Conley, 2003
[25] Achieve, 2005
[26] Harvard University, 2007
[27] Rose and Betts, 2001
[28] Carnevale, no date
[29] Rose and Betts, 2001
[30] Carnevale and Desrochers, 2003

Workers who lack advanced skills will not only earn less, they will struggle to find satisfying employment.[31] The profile of jobs open to young people starting out has changed dramatically. High school graduates in 1950 could seek work among the 60 percent of all jobs that required no particular skills. In 2005, only 14 percent of jobs were classified in that way. The Bureau of Labor Statistics has estimated that 80 percent of the 50 fastest-growing job categories will require postsecondary education or equivalent training.

Employers are not interested in hiring high school graduates who lack the skills and abilities needed to advance beyond entry-level jobs, and they are increasingly dissatisfied with the skills they see in applicants.[32] Higher-level math courses signal to employers that candidates have the flexibility and intellectual agility to succeed in the workplace. These courses also equip individuals with the critical thinking and analytic skills they will need to navigate multiple job and career changes, and training in skills that may not yet have been conceived, which they are likely to face throughout their working lives.

Higher-level math skills are used in many kinds of work.

The intellectual skills that are cultivated in the study of mathematics are valuable both directly and indirectly to many work settings. The skills employers say they are looking for are the same ones postsecondary faculty members look for in their students. Through the State Career Clusters Initiative secondary, postsecondary, business, industry and government leaders developed Plans of Study for the 81 Career Clusters Pathways. Each Pathway Plan of Study—ranging from Transportation Operations and Financial and Investment Planning to Agribusiness Systems and Telecommunications—identifies a recommended set of academic and career-oriented courses for students to take at both the secondary and postsecondary levels to pursue a career track in the relevant field. All 81 Plans of Study recommend that students take Algebra II and at least one other higher-level math course, such as College Algebra, Statistics, or Trigonometry, either explicitly or as an option among equally rigorous math courses.

A glance at the knowledge and skills needed in several key job categories—areas in which promising, well-paying jobs are growing—further demonstrates the point. These descriptions represent just a few of the many fields in which a strong foundation in mathematics plays an important role in on-the-job success.

Aerospace: To progress beyond entry-level maintenance type jobs in the fast-growing aerospace industry, new hires will need a strong math and science background.[33] According the FAA, for example, aircraft maintenance technicians need to be comfortable working with complex measurement, proportions and ratios, solving equations with multiple variables, calculating volumes, and modeling with linear equations. Checking for wear and tear in an aircraft's fuselage and repairing any damage is a life and death responsibility—workers may need to calculate the strength of various materials to select the right one for a repair, or use sophisticated geometric and algebraic operations to determine the shape of the replacement material and other factors to perform this task. To determine the effect of the repair on a plane's center of gravity, workers use extremely precise measurements of length and weight and calculations of balance relationships.

Construction: Construction is a field that is open to people with a range of backgrounds and preparation, but success as a builder and an entrepreneur in this field depends on mathematical skill.[34] Architects

[31] Department of Labor, 2000
[32] Achieve, 2005; Achieve, 2004
[33] Achieve, 2008
[34] Achieve, 2008a

develop blueprints for building projects, but contractors must be able to read and understand them, and devise strategies for following them. They rely on number sense, trigonometry, and geometry as they prepare foundations, build customized irregularly shaped designs, test load-bearing capacity, work with structural engineers to make adjustments, and make the calculations that allow them to correctly install everything from tiles to complex systems such as plumbing and electrical work. Contractors also use complex math skills, including algebra and forecasting, in running their businesses. Developing accurate estimates that factor in inflation, costs that vary with supply and demand, and fluctuating interests rates for loans to calculate labor needs and profit margins—these are some of the many mathematical tasks that contractors carry out every day.

Health: Jobs in the health care sector are growing rapidly, many of which require advanced-level math skills.[35] These jobs vary in the educational and industry certification they require, but all of these workers have serious responsibilities, and much depends on their accuracy and precision. To use one example, a radiologic technologist, the individual who takes and processes X-rays, cannot simply memorize the settings for X-ray equipment. He or she must take a variety of factors into account and use complex calculations to determine the correct amount and time of exposure for every image. Technologists who use CT scanning equipment use geometry and spatial relations to isolate the target area for screening and obtain accurate images.

Information Technology: IT is another field that is growing fast and provides a wealth of opportunities for high school graduates who may have some postsecondary training but not a four-year degree. The skills needed for success are those taught in high school and postsecondary courses such as Algebra II, Linear Algebra, Plane Geometry, Statistics, Discrete Mathematics and Calculus.[36] Network technicians are likely to be in high demand for the foreseeable future. In this job a worker may need to, for example, calculate the electrical demands of a network, together with other electrical demands on the space and anticipated expansion, in order to route the many miles of cable that may be needed. He or she may use algorithms and Boolean algebra and logic to determine network usage and plan for maximum efficiency. Financial applications are important as well—a network technician may use sophisticated spreadsheets to budget for upgrades and equipment costs and other tasks critical to maintaining smooth network operation.

Manufacturing: Another field that traditionally has not had stringent entrance requirements, manufacturing, more than ever before, now demands mathematical skills, such as reasoning and problem solving skills.[37] Employers seek new hires who have taken Algebra II, Trigonometry, Statistics, Computer Applications, and applied manufacturing technology. Why? A worker in this fast-paced industry may need to use skills gained in these courses to design or refine the operation of complex electronic equipment. For example, to design a lifting arm on an automotive assembly line, workers might need to calculate proportions; model and solve linear equations; read and present data; and calculate using integers, fractions, and decimals.

These job categories—and many others—require the kinds of skills and knowledge students learn in higher-level math courses. Since many high school students are not sure exactly what career pathway they will choose to follow, the range of occupations in which these skills are needed underscores the importance of ensuring that all students have this preparation.

[35] Achieve, 2008b
[36] Achieve, 2008c
[37] Achieve, 2008d

Continued U.S. economic growth will depend on a highly skilled workforce.

U.S. students are not learning the math they need.[38] Employers in manufacturing, high tech, health care, and other fields are struggling to find employees with the skills necessary to function well and meet expectations, and evidence regarding mathematics course-taking suggests a significant reason. U.S. high school students generally demonstrate a "solid proficiency in lower-level skills such as whole numbers, fractions, and decimals."[39] During high school students also generally develop "intermediate" skills, such as basic algebraic relationships and logic, but they do not master advanced math skills. Just four percent are proficient in these skills by the time they graduate. Not surprisingly, it is the students who enroll in higher-level courses (pre-calculus, calculus, AP and IB mathematics) who score highest on measures of higher-order mathematics skills.[40]

Labor experts agree that workers need math skills to succeed in a global economy in which computers and other complex technology are used everywhere from factories and auto repair shops to stores and dentists' offices. Workers without the skills to use these existing and emerging technologies are likely to be trapped in the lowest-paying jobs and to be ill-equipped to adapt to economic changes.[41] Science- and engineering-related fields are growing faster than other economic sectors, and producing new jobs three times faster than the economy overall.[42] These new jobs range from doctorate-level research positions to openings for technicians with some post-high school education or training, as well as many positions available for engineers, machinists, and managers. Workers lacking mathematical skills limit their own prospects. In addition, if there are not enough workers in the U.S. with the necessary skill sets, the United States will lose economic development opportunities to other countries whose workforces do have them.

The lack of an adequately skilled workforce has long-term implications for the U.S. economy.[43] The nation's economic growth has depended on technological innovation, but our capacity to compete with other countries is impeded by a workforce whose qualifications are not keeping pace. U.S. students are lagging behind their international peers in science and mathematics literacy: U.S. 15 year-olds scored lower than the average for the OECD nations that participated in a recent comparative assessment.[44] U.S. employers estimate that 39 percent of recent high school graduates are not prepared for the entry-level jobs that are available.[45] Recent turbulence in the U.S. economy underscores the seriousness and urgency of the challenges young people in the educational pipeline will face. The workers of the future will need not just specific mathematical and technical knowledge and skills, but also the flexibility and intellectual resilience to keep learning and adjusting to a changing economic environment throughout their careers.

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[38] National Mathematics Advisory Panel, 2008
[39] Bozick, et al., 2008
[40] Ibid
[41] Schoenfeld, 2002
[42] National Mathematics Advisory Panel, 2008
[43] National Research Council, 2007
[44] Bozick, et al., 2008; NCES, 2008; PISA results at nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2008016.
[45] Achieve, 2005

CONCLUSION

The importance of higher-level mathematics for all students is clear in regards to college access and success, the demands of the 21st century workplace, and the nature of mathematical thinking. There is little doubt that the challenge of providing high quality instruction in higher-level math courses to all students is a formidable one. Many states and districts have already focused attention on aligning their high school mathematics standards with college and career expectations, providing better opportunities for mathematics course-taking, and requiring all students to complete a rigorous academic curriculum that prepares them for both college and the workplace.

These broad measures are very important, but more is needed. Even when aligned standards, graduation requirements, and other measures push students to take higher-level math courses, too many students are still left behind. Many students give up on math and believe it is not for them. All too often even parents believe that higher-level math courses are too hard and are out of reach for their own children. Too many students still find ways to meet requirements with lower-level courses.

What strategies can states and districts use to challenge the stigma that math is not for everyone? One is math preparation that begins in preschool to develop the mathematical thinking, skills, and conceptual knowledge students will need to tackle higher-level courses. Another is guidance and support to ensure that struggling students break through their fear of math and catch up on the essential skills they need to succeed. Many states and districts have come up with promising interventions for these students, including afterschool and summer programs. Also, all students need math courses that are genuinely rigorous, not just courses with rigorous sounding titles. Each year of math instruction needs to do its part to build students' conceptual development and the skills they will need to continue progressing. And, finally, to provide that kind of consistent instruction, every student needs teachers who not only know math but also know how to teach math positively and effectively.

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ABOUT ACHIEVE

Created by the nation's governors and business leaders, Achieve, Inc., is a bipartisan, non-profit organization that helps states raise academic standards, improve assessments and strengthen accountability to prepare all young people for postsecondary education, careers and citizenship. Achieve has helped more than half the states benchmark their academic standards, tests and accountability systems against the best examples in the United States and around the world. Achieve also serves as a significant national voice for quality in standards-based education reform and regularly convenes governors, CEOs and other influential leaders at National Education Summits to sustain support for higher standards and achievement for all of America's schoolchildren.

In 2005, Achieve co-sponsored the National Education Summit on High Schools. Forty-five governors attended the Summit along with corporate CEOs and K–12 and postsecondary leaders. The Summit was successful in making the case to the governors and business and education leaders that our schools are not adequately preparing students for college and 21st-century jobs and that aggressive action will be needed to address the preparation gap. As a result of the Summit, 33 states joined with Achieve to form the American Diploma Project Network—a coalition of states committed to aligning high school standards, assessments, graduation requirements and accountability systems with the demands of college and the workplace.

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