EFFECTIVE TEACHING WITH VIRTUAL MATERIALS: YEARS SIX AND SEVEN CASE STUDIES

Annette R Baturo, Tom J Cooper and Kylie Thompson

Centre for Mathematics and Science Education, Queensland University of Technology

This paper describes an approach to mathematics teaching and learning using teacher-constructed "virtual" materials. In this approach, virtual copies of real materials are manipulated with mouse movements to replicate the traditional physical manipulation of real materials. The paper reports on three case studies where the approach was used to teach different mathematics topics (e.g., equivalent fractions and decimal fraction scales) to a Year 6-7 and a Year 7 class. It describes and analyses the virtual materials in relation to how they relate to prior teaching with physical and pictorial materials. It speculates on the strengths and weaknesses of the approach, and the effect differences between physical and mouse manipulations have on learning.

Over the last 10 years, educational authorities have invested billions of dollars placing computers in classrooms with the expectation that integration of technology and instruction would enhance learning outcomes by changing the nature of teaching and learning in terms of content, delivery, teacher-student interactions, and the roles of both teachers and students. However, many teachers who have not grown up with computer technology have developed high levels of stress (technophobia) when faced with a teaching future that is inexorably leading to the integration of learning technologies (e.g., Morton, 1996). These teachers have difficulty accessing mathematics software that suits their students' needs (Becker, 1994). However, of more concern is the prevalent belief that mathematics cannot be taught effectively with computers (Sarama, Clements & Jacobs-Henry, 1998; Norton, 1999).

To help teachers overcome their technophobia and beliefs about the efficacy of computers in mathematics instruction, the authors have established a Virtual Mathematics Program in which they work collaboratively with teachers in their classrooms helping them construct and implement their own mathematics activities using virtual copies of concrete materials. This paper explores three examples of this use of virtual materials.

Virtual materials and their role in teaching. As argued in Baturo & Cooper (2001), most activity with real or concrete materials in mathematics involves *sliding, joining, separating, grouping, ungrouping, partitioning, turning* and *flipping* actions. All of these actions are available on computer through mouse movements and images of the materials ("virtual materials") using the commonly available generic "office" software (e.g., *MicroSoft* Office, *ClarisWorks*). As described in Baturo and Cooper (2002), virtual activities reflect a variety of options. They can be simple "click and drag, copy and paste" activities (e.g., representing numbers with MAB) through to ones that have capacities for actions and representations not easily available with concrete materials (e.g., modifying the polygon shapes). In particular, shapes can be enlarged by specific amounts, or turned by specific degrees. Some virtual activities have mouse actions that closely imitate the physical actions with real materials (e.g., sorting shapes, turning the hands of a clock). Others have mouse actions that are very different and nowhere near as richly kinaesthetic as the physical actions (e.g., flipping a shape).

From a teaching perspective, virtual materials activities can be "debugged, reconstructed, transformed, separated and combined together" (Healey & Hoyles, 1999, p. 59) and saved for later reuse by the same or other students. As well, virtual activities enable students' manipulations to be saved and stored for later assessment, providing teachers with unique knowledge of all students' proficiency with all components of the manipulations. The strength of a teaching approach that builds virtual materials into its repertoire of activities is that it is multi-representational (providing visuals, language & symbols) and dynamic (showing transformations and changes as well as relations). In this way, as Healy and Hoyles stated, virtual materials use the visual, symbolic and operational power of the technological media and provide another pedagogical and didactical tool for the media. Initial findings from the trials in the Virtual Mathematics Program (Baturo & Cooper, 2001) are indicating that, for technophobic teachers, virtual materials provided a bridge from the acquisition of computer skills to the implementation of classroom activities; the teachers found virtual activities easy to develop, did not require specialist software, and promoted positive learning outcomes. It seems as though, because virtual activities have comforting similarities to concrete activities, teachers are more able to recognise opportunities for translating their traditional teaching activities to computer activities. Furthermore, in every class trialed thus far, the teachers have been impressed by their students' excitement, prolonged engagement, and natural collaboration that have been provoked by the virtual activities.

Teaching of mathematics. Mathematics consists of things, relations between things, and transformations of things (Scandura, 1971). Within this paradigm of mathematics, importance lies in the relations and transformations not in the things; yet, within our research experience (Baturo & Cooper, 2001; 2002), teachers tend to focus primarily on the "things" and neglect or downplay transformations that often give rise to patterns and therefore relationships. Mathematics learning is about the refinement, abstraction, and integration of concepts and processes, and mathematics teaching is about facilitating this process of refinement, abstraction, and integration. Current pedagogical beliefs emphasise that the abstraction of concepts and processes is best served by a combination of work with appropriate manipulatives and reflection with peers and teacher (English & Halford, 1995).

According to Halford (1993), understanding mathematics involves representing one mathematical structure by another and determining what is preserved and what is lost between the structures. Students translate *external representations* (concrete, virtual, pictorial, diagrammatic, written symbols, spoken words) to *internal representations* (e.g., mental models/perceptions of the external representations). Similarly, they store the kinaesthetic actions (physical and mouse movements) undertaken to represent the relations and transformations in memory. Thus, the use of external representations (real and virtual) should provide a mental image to scaffold the concomitant concept development and abstract symbolism. The current mathematics syllabus and curriculum documents for Queensland schools draw heavily on Payne and Rathmell's (1975) model of concept development that has three main components (representations, language, symbols) and six concomitant interactions, all of which they claim are essential for full concept construction. Whilst all three components can be thought of as different representations of the concept, it is useful to separate them when planning teaching and learning activities. Figure 1 adapts the model to include a continuum of materials with respect to degree of abstraction.

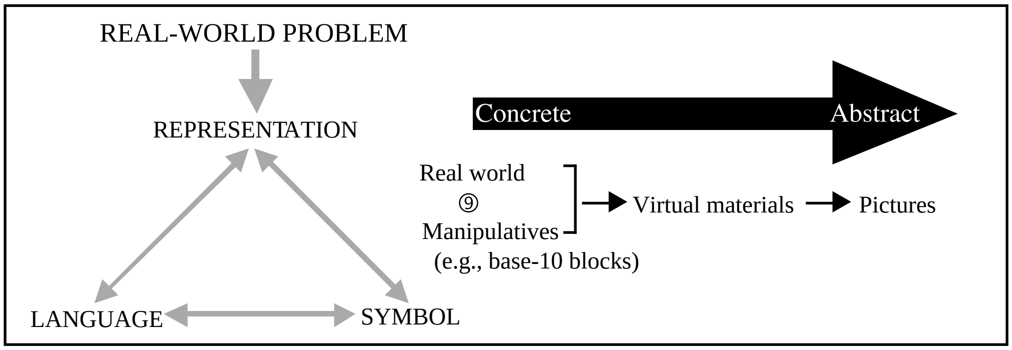

Figure 1. Adaptation of Payne and Rathmell's (1975) model components and interactions required for concept construction (Baturo & Cooper 2002)

Noss, Healy and Hoyles (1997) contended that, although students can mentally replicate (in their schemas) the relations and transformations represented by concrete material and abstract this mental replication to symbols and mental models, *there is a gap between action and expression that is difficult to bridge*. Baturo and Cooper (2002) argued that virtual materials are more abstract than concrete materials but less abstract than pictorial representations and therefore are able to help *bridge the gap from concrete to pictorial representations and, then, to abstraction*. Concrete materials are less abstract than virtual materials because they are multisensory (i.e., they can be seen, smelt, moved, picked up, touched, weighed) whilst real/concrete materials are essentially bisensory (seen and moved). They argued that, although the multisensory nature of concrete materials may develop more detailed memory structures (schema), for example, a tactile memory, the more abstract bisensory virtual materials develop deeper mathematics understandings.

CLASSROOM STUDY

The virtual material activities reported in this study were trialed with a Year 6-7 and a Year 7 class in a state primary school in a regional city in Queensland. The school's students were predominantly from low socio-economic backgrounds. The school's performance in mathematics was low even when compared to schools of similar background.

The trial of the virtual materials was part of a collaborative action research project (Kemmis & McTaggart, 1988) in which University mathematics-education lecturers and teachers worked together to improve the teachers mathematics-teaching practice in terms of enhancing students learning outcomes. The collaboration involved the development of units of mathematics instruction that represented exemplary practice. Therefore, the units began with students exploring physical materials to develop the required concepts, processes and principles and then moved on to virtual materials.

The Years 6-7 and 7 classrooms contained nine computers linked to the Internet (three in the Year 6-7 classroom and six in the Year 7 classroom). The teachers of the Years 6-7 and 7 classes, while not technophobic, had limited knowledge in how to use computers to teach mathematics. They, therefore, made little use of the computers for mathematics teaching and, in fact, for any teaching. As the teachers admitted later, the computers in their classroom were used for less than one hour a day.

The two teachers were experienced with good skills in teaching mathematics in a traditional way. They had excellent behaviour management and relationships in the classroom appeared to be based on mutual respect. The Year 6-7 teacher relied more heavily on textbook pages for instruction than the Year 7 teacher. The Year 7 classroom was more used to class discussions than the Year 6-7. For the action research collaboration, he two teachers chose to work as a pair, undertaking the same instructional units, planning together and sharing the development of new materials.

INSTRUCTIONAL UNITS

There were three instructional units in which virtual materials were used.

Unit 1: Equivalent fractions. The two teachers developed a unit on fractions as a prerequisite to a major unit on percent. Part of this fraction unit was the introduction of equivalent fractions. This was done through manipulating real world materials (e.g., chocolate bars, cakes), physical materials (e.g., paper folding showing one-half is the same as two-quarters), pictures (e.g., fraction mats composed of length representatives of halves, thirds, quarters, fifths, and so on aligned so that equivalence can be easily seen vertically) and patterning materials (e.g., fraction sticks – Popsicle sticks with numbers placed regularly as in Figure 2). The virtual materials used were a copy of the fraction sticks. Equivalence was first introduced as a capacity (i.e., it is possible for one-third to be equal to two-sixths) and then as a pattern (i.e., six-ninths is equivalent to two-thirds because both the numerator and denominator are three times larger than for two-thirds).

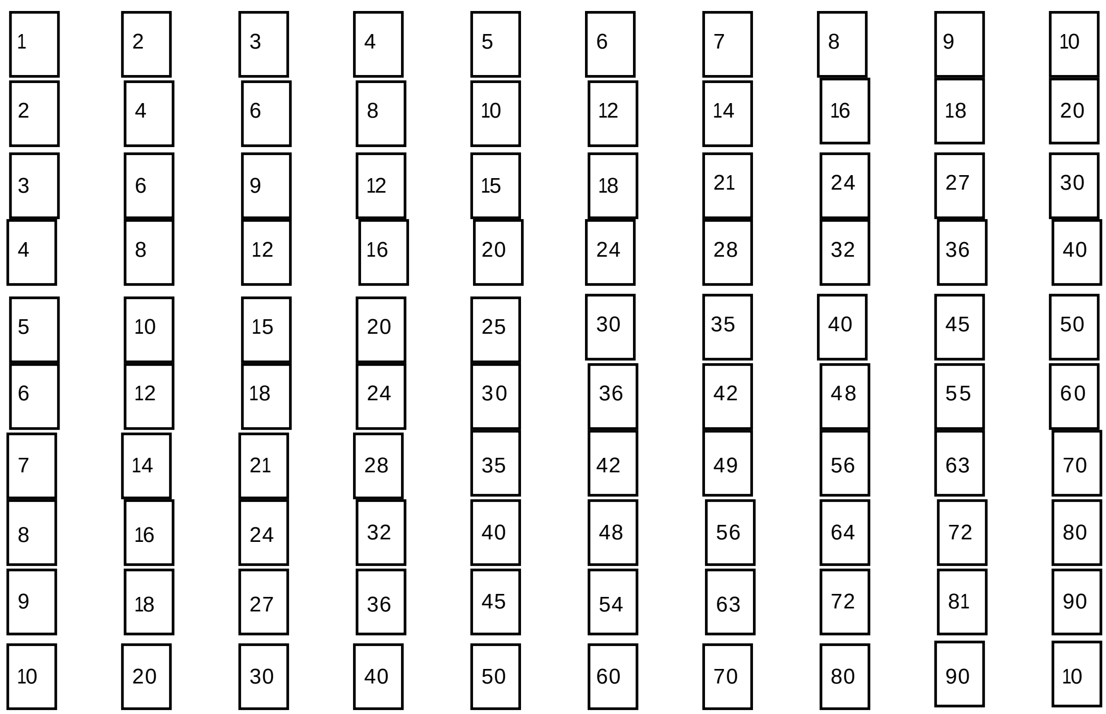

Figure 2. Fraction sticks

The fraction sticks provide the final step in the learning process for equivalent fractions. A representation for, say, two-fifths consists of the 2 stick (naming the stick by its left-most number) placed above the 5 stick. All the fractions equivalent to two-fifths are then displayed for equivalence-relation pattern to be identified, and for the fraction to be compared with, added to or subtracted from another fraction of unlike denominator (as in Figure 3).

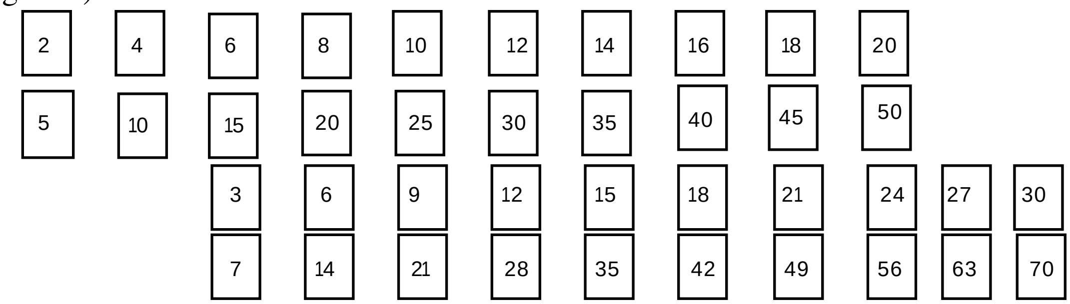

Figure 3. Comparing two-fifths and three-sevenths with fraction sticks

Unit 2: Scales. Both Years 6-7 and 7 students were performing poorly on items that required the reading of scales, especially if there were decimals to be inferred. A unit of instruction was developed that involved the students constructing and interpreting scales. Instruction moved from *complete whole-number scales* with all numbers shown to *incomplete whole-number scales* with partitions not numbered and gradations every fifth or second unit to *incomplete decimal scales*. At the end, virtual materials were used in which positions on incomplete decimal scales could be interpreted and constructed. These attempted to emulate real world instances of scales (e.g., measuring cylinders, odometers, syringes).

Unit 3: Slides, flips and turns. Both Years 6-7 and 7 students had experienced restricted space activities. They had not yet been taught any transformational geometry concepts and processes. A unit of instruction to introduce translations (called *slides* in the Queensland mathematics syllabus), reflections (*flips*) and rotations (*turns*) and to study their properties was developed using tracing paper and Miras as the physical material. This was followed by a series of virtual activities that used mouse movements and the Draw toolbar to practice flips, slides and turns. The virtual activities also used the flips, slides and turns to develop art designs.

RESULTS

The action-research collaboration between lecturer and teacher was based on Lesh and Kelley's (2000) multi-tiered approach where the teacher acts as the major researcher with respect to students' outcomes. The following results with respect to the three units are, therefore, based on accounts given by the teachers on students' responses to the virtual materials. The teachers reached their positions on the virtual materials by observing the students on the computers, talking to the students about their perceptions of the materials after computer use and detecting changes in understanding from tests, homework and textbook activities. The students attempted the virtual materials through a rotation system that enabled students to use computers in turn. They worked on the computers in pairs.

Teaching Unit 1 (Equivalent fractions). Both teachers reported that the virtual use of fraction sticks had been unsuccessful. Their students had found the materials difficult to use and had quickly lost interest in using the computers. Many students said that the activities were "boring". However, both teachers reported that the use of the physical fraction sticks had been very successful and were well liked by the students. They had

therefore focused on the physical materials and not continued with the virtual activities.
A closer analysis of the virtual fraction sticks supports this finding. The virtual sticks and the physical sticks are nearly identical and the mouse movement to pick up and move a stick is similar to the physical movement. However, the virtual movements are more complex. Moving sticks with fingers is a simple task. However, since the sticks are simply lines and numbers, selection of a virtual stick involves quite delicate and precise mouse movements, as the arrow must be placed directly on a line. It is also easy to lose attachment to a virtual stick and once mixed, it is hard to select one of the virtual sticks. The sticks were also unattractive. Taken all together, it is no wonder that students preferred the easy manipulation of physical sticks than the frustratingly delicate manipulation of virtual sticks, particularly when the virtual sticks gave no learning or engagement advantages.

Teaching Unit 2 (Decimal scales). Both teachers were delighted with the virtual scales and reported that the students had both enjoyed the activities and learnt from using them. Unlike the teaching of equivalent fractions, where there were many physical materials available, the teaching of scale had been a slow concise development of knowledge across more and more complex number lines. Students read the values off the number lines or marked the values onto the number lines.
Thus, the move to computers was much more of an attraction in the scale lessons than it was in the fraction lessons. As well, the virtual scales had authenticity (they were moving drawings of real measuring instruments) and movement (e.g., the syringe reading was changed by using the mouse to move the plunger, while the odometer reading was changed by using the mouse to rotate a pointer). They were an extension of what was being done in class not a copy and they gave an *engagement* advantage.

Teaching Unit 3 (Slides, flips and turns). Again both teachers were delighted with this virtual activity and reported that the students both enjoyed the activities and learnt from using them. This is interesting because, with flips and turns, virtual materials move into an area where the mouse movement are very different to physical movements. A tile may be turned with a rotation of a hand or flipped with a movement of the hand upside down, but a virtual tile requires an icon on the Draw toolbar to be activated and a round handle to be dragged in a circle for a turn and the Draw menu to be activated and either “flip vertical” or “flip horizontal” to be selected for a flip. In particular, flips are very different; they do not require a movement but rather a selection from a menu and they can only be flipped in two directions. Any other direction for a virtual flip requires turning as well as flipping.

However, the virtual materials are very different to the physical materials for sliding, flipping and turning. Using tracing paper to trace, move and retrace is detailed work and is often inexact, as hand movements cannot replicate the perfection of abstract mathematical movements. It also lacks colour and is slow, making designs difficult. On the other hand, virtual slides, flips and turns, although different, are exact, can involve colourful materials, are quick and easily lead to attractive and complex designs. Furthermore, in virtual materials, the sliding, flipping and turning actions have to remain separate and be done in a measured way; hands often slide, flip and turn physical materials at the same time thus confusing the actions. Thus virtual slides, flips and turns are not only extensions of physical slides, flips and turns, they provide greater exactitude, allow for detailed experience of each action in isolation and enable easy preparation of attractive complex designs; they have both *learning and engagement* advantages.

DISCUSSION AND CONCLUSIONS

As the tool of this generation's time, computers should be utilised whenever and wherever possible in the educational arena (Baturo & Cooper, 2002). However, like physical materials, the case studies in this paper have shown that virtual materials are not effective simply by their presence. They have to add something in terms of engagement or learning to the instruction, and this from the students' perspective. Virtual materials, along with physical materials, have to be understood in terms of what the students perceive from the images and what they do with them (Baturo & Cooper, 2002).

Student manipulation of virtual materials is a very different use of computers in mathematics education than that commonly seen in Queensland schools. It has all the virtues of digital material; programs can be saved for later reuse by the same or other students and there is the opportunity for all students' manipulations to be saved and stored for later assessment (Baturo & Cooper, 2002). It has the strength of being multi-representational (providing visuals, language & symbols) and dynamic (showing transformations and changes as well as relations). In this way, it uses the visual, symbolic and operational power of the technological media and provides another pedagogical and didactical tool for the media (Healy & Hoyles, 1999). Its abstract nature gives it capacities for actions, activities and representations not easily available with physical materials; for example, shapes can be enlarged by specific amounts, or turned by specific degrees (Baturo & Cooper, 2002). It is less time consuming in terms of preparation, particularly with respect to space activities, requiring only one template that can be downloaded for individual student's use. It is a colourful, vibrant and dynamic way to teach. It extends and amplifies existing technologies, modifying, reshaping, and blending the ways in which humankind works mathematically and enhances the teaching and learning value of physical materials, particularly when integrated with manipulation of physical materials (Kaput & Rochelle, 1997). On top of this, the closeness of virtual to physical materials makes it more comforting for mathematics teachers (and more familiar to students) (Baturo & Cooper, 2001).

However, as we have seen with the fraction sticks, closeness with the physical materials with which it is integrated may be a factor in ineffectiveness as well as effectiveness. In these cases, some added ability in terms of authenticity, speed or attractiveness may be needed to make the use of virtual materials worthwhile. Of course, difference is inherent between virtual and physical materials in terms of the form of manipulation –mouse and hand. This difference can be positive, as Unit 3 shows; sliding, flipping and turning with virtual materials requires each of the three actions to be separately carried out and enables the consequences of each action to be differentiated from the others.

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