IMPROVING DECIMAL NUMBER CONCEPTION BY TRANSFER FROM FRACTIONS TO DECIMALS

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Seventh and eighth grade students identified as holding an incomplete fraction conception of decimals were tested on related fraction knowledge. Most of these students (78%) had a problem in coordinating the size of the parts and the number of parts in comparing fractions. These students underwent several instructional sessions. Half of them worked on fraction coordination and on mapping it to decimals, while the other half had more instruction in decimals. Treating the source of the problem in fractions was found to be more effective in improving decimal conception. The remaining students (22%), who had a problem in decimals but not in fractions, improved in decimal conception following mapping instruction that promoted transfer of fraction knowledge to decimals.

This study focuses on treating a specific difficulty in decimal number conception (through the article we use the short name 'decimals' to stand for decimal numbers or decimal fractions, e.g. 0.23, 2.072, and the term 'fractions' to stand for rational numbers written as a/b, e.g. 2/3, 7/5). Several researchers in different countries (Sackure-Grisvard & Leonard,1985; Resnick et al., 1989; Nesher & Peled, 1986; Stacey & Steinle, 1999; Stacy et al., 2001) identified children's implicit models of decimals. They observed the different conceptions at a given grade and the changes over the years. The main task used to identify decimal conception was a number comparison task. Given this task, two main implicit number models were observed: treating decimals as if they are natural numbers (this conception leads to two rule variations) and treating decimals as fractions using an incomplete fraction conception (fraction rule). Some other, more technical conceptions, involve the use of rules that do not connect directly to a specific conception (e.g. "it's the opposite of fractions").

The term "fraction rule" was used in the research literature to describe the rule used by children that compared decimals by (only) using their "parts". For example, in comparing 0.2 with 0.34 these children would say that 0.2 is bigger because it has tenths, which are bigger than the hundredths that 0.34 consists of. Similarly, they would say that 0.45 is bigger than 0.457 because hundredths are bigger than thousandths.

According to an international research (Resnick et al., 1989) carried out in the US, France and Israel, about a third of the children show evidence of using the fraction rule in the first year of learning decimals (usually sixth grade). With further instruction some of these children become experts, and yet the ratio of children using this rule does not change much in the next two years, since some of the children holding a more primitive conception (whole number rule) shift to the fraction conception.

In higher grades there is some decline in the ratio of children using this rule. In a series of studies researchers in Australia followed children's conceptions of decimals. In one study Moloney and Stacey (1997) tested children in 4^{th} to 10^{th} grade and found that the fraction misconception persisted in higher grades, and was used by 20% of year 10 students.

Stacey and Steinle (1998) observed that some children behave similarly to children that have the fraction misconception (as defined above) and yet have other reasons for this "overt" behavior. That is, rather than choose 0.3 as bigger than 0.47 because tenths are bigger than hundreds, some of them use, what the researchers term "reciprocal thinking", and choose 0.3 because in fractions 1/3 is bigger than 1/47. Some others use "negative thinking" and choose 0.3 as bigger because they conceive of these numbers as negative numbers, and in negative numbers -3>-47.

In this study we focus on children that hold the fraction conception with the "denominator focused thinking", as Stacey and Steinle (ibid) term the "tenths are bigger than hundredths" explanation. These children have a relatively good decimal conception.

Our research hypotheses were that a large number of students use the fraction rule in 7th and 8th grades, and that most of these children would also have a similar problem with common fractions. We hypothesized that those who have a problem in fractions would benefit from instruction in fractions, and that their new knowledge would transfer to decimals. We also hypothesized that students that use the fraction rule but have no problem in common fractions, would benefit from help in making connections between their fraction knowledge and their decimal number knowledge.

METHOD

Three similar number comparison tests were used in the study. Each of them tested performance and understanding in comparing pairs of fractions and in comparing pairs of decimals. In each item the student was asked to circle the bigger number (or mark that the numbers are equal), and explain her answer.

A pretest was given to 261 seventh and eights grade students, and 59 students were identified as using the fraction rule (FR) in decimals. Out of the 59 students, 46 (also) used a similar rule in common fractions while 13 had no problem in comparing fractions. Following this diagnosis, the students were divided into three groups presented in Table 1: The 13 students, who had no fraction problem, were assigned to the mapping group. The 46 students, who used the fraction rule in both fractions and decimals, were randomly assigned to an experimental group and a control group.

Group: Study plan	Experimental n=23	Control n=23	Mapping n=13
Pretest results:	FR in decimals	FR in decimals	FR in decimals
	FR in fractions	FR in fractions	Ok in fractions
Instruction sessions 1 & 2:	Coordination in fractions	More in decimals	More in decimals
Posttest 1	Posttest1 given to	all groups	
Instruction session 3:	Mapping to decimals	More in decimals	Mapping to decimals
Posttest 2	Posttest2 given to	all groups	

Table 1: Group allocation and study plan following pretest results. (FR=Fraction Rule)

Each group had 3 sessions of instruction. During two of these sessions the experimental group worked on coordinating the number of fraction parts with the size of the part in perceiving the fraction magnitude. The control group and the mapping group had "more of the same" instruction on decimals (i.e. continued doing regular activities in decimals). During a third session, the control group continued with decimal instruction, while the experimental group and the mapping group received mapping instruction. This instruction involved a guided discussion mapping fractions and decimals (more details on this session are given in the results section).

Two posttests were given following instruction. The first was given after two sessions, and the other after the third session. The purpose of giving two tests was to differentiate between the effect of coordination instruction and the effect of mapping instruction.

RESULTS

As expected, a large number of students, 23% of the 261 seventh and eighths grade students, used the fraction rule in the pretest when they compared decimals. Most of them, 73% of the 59 students using the fraction rule in decimals, had problems in comparing fractions. It should be noted that in checking students' explanations we did not find students that used the fraction rule with any other explanation besides the "denominator focused" explanation. That is, all the students referred to the parts ("tenths are bigger than hundredths") and no one used the "reciprocal thinking" or "negative thinking" explanations that were observed by Stacey and Steinle (1998).

Table 2 presents an example of comparison items together with representative answers of students that use the fraction rule in decimals, but differ in their fraction knowledge. Duha compared fractions incorrectly, focusing on their parts, fifths and sixths, and disregarding the number of parts. Faddy compared the two fractions correctly by finding a common denominator. As a result, Faddy was assigned to the mapping group, while Duha was placed among the 46 that were split into the experimental and control groups.

Test item	Duha	Faddy
2.8 2.85	2.8 is bigger. 2.8 has tenths while 2.85 has hundredths. Tenths are bigger than hundredths because they are less parts.	2.8 is bigger. Tenths are bigger than hundredths.
5/6 4/5	4/5 is bigger. In 4/5 we divide into fifths, while in 5/6 we divide into sixths, and fifths are bigger than sixths.	5/6 is bigger. 5/6 is the same as 25/30, and 4/5 is 24/30. So now it's easy to compare, 25>24 so 5/6 is bigger.

Table 2: An example of children behaving similarly in decimals and differently in fractions.

Following instruction that focused on parts and number of parts in fractions, the experimental group showed significantly improved (paired t test, $p<.05$) decimal performance. Group performance improved even more following an additional session in which children were encouraged to make connections between fractions and decimals. Similar shifts were observed in the mapping group that participated in decimal sessions and 1 mapping session (Table 3).

	Average (scale	Decimal 0-9)	scores
	experimental	control	mapping
Pretest	2.56	2.73	2.30
Posttest1	4.26	3.82	4.53
Posttest2	7.26	5.39	7.30

Table 3: Average decimal scores in all groups.

The control group, that received more decimal instruction without further fraction instruction, showed some improvement (Table 3). The change following the first two sessions was not significant, and yet following a third session the change (between posttest1 and posttest2 scores) was significant (paired t test, $p<.05$).

In addition to looking at the change within the different groups, a comparison of decimal scores was done between groups. The differences in scores between the groups following the first 2 sessions (tested in posttest1) were not significant. Following the third session (tested in posttest2) a significant difference in decimal scores was found between the experimental group and the control group.

The groups were also compared on their fraction knowledge (Table 4). It was found that the experimental group that had 2 sessions of fraction instruction improved to the extent that it approached the knowledge level of the mapping group (that had no problem in fraction comparison to begin with).

	Average	fraction	scores
	experimental	control	mapping
Pretest	0.91	1.13	4.00
Posttest1	3.69	1.52	4.00
Posttest2	4.00	1.95	4.00

Table 4: Average fraction scores in all groups.

As a result of these shifts in decimal and fraction knowledge, when the three groups were compared on a total score (combining decimals and fractions) the state of the experimental group relative to the mapping group shifted. In the first test the differences in the total score were mainly attributed to the mapping group that was better in fractions than the two other groups. In the second posttest, the differences in scores were attributed to the control group. Although the control group improved to some extent, it still lagged behind the other groups in both decimal knowledge and fraction knowledge. The experimental group and the mapping group improved in decimals in a similar manner, and the experimental group closed the gap in fractions, making the two groups equally good.

As mentioned earlier, the third session in the experimental group and in the mapping group involved the same mapping instruction that was conducted through guided discussion. The purpose of this session was to help the students improve their knowledge in decimals by making connections to their fraction knowledge. As seen in Table 3 and Table 4, the postest1 fraction scores of these two groups were high: 3.69 for the experimental group and 4.0 for the mapping group (on a scale of 0-4). The corresponding decimal scores were 4.26 and 4.53 (on a scale of 0-9).

During mapping instruction the students faced a conflict situation when they got one (incorrect) answer by comparing two decimals, 0.24 and 0.253 and yet a different (correct) answer by comparing the corresponding pair of fractions, 24/100 and 253/1000. In the course of discussion the students immediately realized that the answers should have been the same. They all agreed that the answer in fractions was correct, but then wondered how 0.253 could be bigger than 0.24. One of the students suggested that 0.253 has more parts, and the idea was further elaborated and accepted by others. When one student (S1) wondered about the original rule, his colleague (S2) answered and others (S3, S4) added and summarized:

S1: So the rule about hundredths being bigger than thousandths and tenths bigger than hundredths is incorrect?

S2: I believe that one part out of a hundred is [still] bigger than one part out of a thousand.

S3: But that doesn't mean that the more we [continue to] take parts of [that] hundred it would stay smaller than the number of parts we took of ten.

S4: It's not enough to look at the size of the part, but also at the number of parts that we color.

The mapping session effect can be observed by comparing decimal scores in posttest1 and posttest2 (Table 2 and Table 4). The shift for the experimental group was from an average of 4.26 to 7.26 (on a scale of 0-9), and for the mapping group from 4.53 to 7.30. In both cases the differences were significant (paired-t test, $p<.001$ for the experimental group, $p<.05$ for the mapping group).

DISCUSSION

Following research that identified decimal number conceptions, this study focuses on the source of a specific conception, the fraction rule. Two instructional treatments were used in the study to improve decimal conception of seventh and eighth grade students that were identified as using this rule: coordination of the size of the fractional part and number of parts in comparing common fractions, and mapping fraction knowledge to decimal knowledge. Students who used the fraction rule and had a similar problem in fractions underwent coordination instruction. This instruction improved their understanding and performance in fraction comparison. It also improved their understanding of decimals by transfer of relevant (size of part & number of parts coordination) fraction knowledge.

The second type of instruction, mapping instruction, promoted connections from fractions to decimals, followed by knowledge accommodation. This instruction created conflict for students that got different results in fractions, where they performed well at that point, and decimals, where they still had problems. Group discussion caused pressure to adjust decimal conception accommodating it to allow for transfer of fraction knowledge.

Mapping instruction was used with the experimental group following coordination instruction, achieving further (significant) improvement in decimals. It was also used successfully with students who, to begin with, had no trouble with fractions, and yet needed some help in transferring their fraction knowledge and reorganizing their decimal number knowledge to allow for taking both size of parts and number of parts into account.

The effect of these two types of instruction was compared to the effect of "more of the same", i.e. more decimal instruction for the control group. Following two sessions of this instruction the control group improved, not significantly, in decimals and fractions. An additional session resulted in significant improvement in decimals but not in fractions, with the average decimal score still significantly below the average final score for the experimental and mapping groups. The improvement in decimals apparently resulted from improving knowledge within decimals without making connections to fraction knowledge.

This study used remedial teaching, i.e. treated the assumed source of the problem after learning decimals and after the fraction rule was observed by some of the students. The effect of the treatments implies that working with students on coordinating the number of parts with the size of parts and helping them transfer this knowledge to decimals, can help students construct better decimal knowledge. Instruction of this kind can be used either prior to teaching decimals or after teaching decimals in order to prevent or to remediate fraction rule conception.

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