COLLECTIVE LEARNING STRUCTURES: COMPLEXITY SCIENCE METAPHORS FOR TEACHING

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The purpose of this paper is to discuss the activities of collective learning structures. Drawing from ecological-complexity theory it elaborates on a theoretical framework for observing and acting upon collective learning. To interpret the relation between individual action, social interaction and the collective cognitive domain we adopt the thermodynamic notion of energy-rich matter and the enactivist notion of inter-objectivity. We study the structure, the behavior and the ecology of the collective cognitive domain not only as a catalyst for individual learning, but as a learning body in itself, that emerges from the actions and interactions of individual learning agents.

OBJECTIVES

Through our research we seek to understand the character of collective learning structures and to develop theoretical frameworks for observing and acting upon the learning systems in which students and teachers, as individuals, are nested. To study the relation between the individual and the social learning factors, we draw from ecological-complexity theory. Specifically, we use von Foerster's (1981) notion of energy-rich matter and Maturana's (2000) notion of inter-objectivity to interpret the collective learning that emerged in the context of a grade 7 classroom. We then read those interpretations against our theoretical framework in order to further develop it.

Many mathematics education researchers have recently turned their attention to the social aspects of learning (see *Educational Studies in Mathematics*, 2002, a special issue on discursive approaches). Even though most social cultural theorists engage in the systematic study of social-cultural processes (e.g. power relations, classroom practices and norms), a majority of them, akin to individual constructivists, unquestionably privilege the individual student (singular) as the only learning agent (Burton, 1999). Particular emphasis is placed on processes of interaction; less is said about the activities of the collective structures that emerge from and co-emerge with the interactions.

Some researchers such as Burton (1999), Cobb et al. (1997), Kieren and Simmt (2002), and Sfard (2001) have begun to reconceptualize the notion of "learning agents" to include the collective learning structures. To Sfard (2001) mathematical ability is not only an individual property but can be regarded as a property of the joint actions, "one that does not have an existence beyond these individual interactions" (p.49). Sfard developed analytical tools to explore joint activities of partnered students solving problems. Cobb et al. (1997) developed a construct, collective reflection to distinguish collective mathematics development. Cobb et al. recognize that the relation between the knowing acts of the individuals and those of the class as a whole is not linear, but rather one of recursive elaboration in which individual and collectives of learners are brought forth in immediate action (Kieren & Simmt, 2002). Kieren and Simmt explore collective understanding as a dynamical process arising from the "interactions of interactions" that is "related to the whole collective and its embodied setting rather than referenced to individual knowing per se" (p. 866). They call for further research on how collective learning structures relate to individual acts and social interaction.

THEORETICAL FRAMEWORK

Ecological-complexity draws metaphors from complexity science. In the complexity view such phenomena as thought are construed not as solely individual-psychological events, but rather as part of a more inclusive phenomenon, namely, cognition (von Foerster, 1981). Cognition is described as an emergent property of a level of organization above that of the internal dynamics of a system (Maturana, 2000). In humans, cognition is a property arising from the interplay of brain, body and environment. In a manner compatible to that of distributed cognition theorists who suggest that intelligence is distributed among the social and material dimensions (See *Journal of Interchange on Learning Research*, a special issue on distributed cognition, 2002), complexity science allows us to study both the cognitive domain of collectives and of individuals.

The ecological-complexity view encompasses the neo-Vygotskian view that emphasizes the role of social activity in learning. However, individual learning is recognized as nested in the activities of the class (as a social system that, in turn, is nested in broader systems such as the cultures of a school). At each level of emergent organization new activities appear (See Davis & Simmt, 2002). Hence, the cognitive domain of the organs, of the individual learner, of collectives of students, and of the different cultures need not be subjected to the same possibilities and rules (Davis & Sumara, in press).

Rather than evacuating individual students' experiential and sensory-motor accounts as some social-cultural theorists such as Lerman (2000) suggest, complexity theory explains the behavior of collectivities in a classroom in a different domain. As individual students act and interact, collective structures emerge. The behavior of the collective is a historical consequence of individual and inter-individual actions and that, in a recursive manner, the behavior of the collective occasions individual students' sense making (Kieren & Simmt, 2002). The emphasis on emergent structures offers insights into studying the dynamics that afford learning systems (individual and collective) coherence without collapsing them into one. The meaning of objects, artifacts and concepts are not pre-given (in the culture or, even, in situated practice) but they arise in interactions and actions (Maturana, 2000). The objects that co-emerge with the learners acting and interacting provide the energy-rich matter for collective as well as individual learning. Von Foerster (1981) suggests that it is the thermodynamic concept of energy availability and transformation that may metaphorically explain links between organizational levels: the social to the individual level, the empirical to the abstract level, and so on.

MODES OF INQUIRY

Our research is classroom-based; and though some might suggest that it falls under the paradigm of action research, we do not call it such. However, it is similar to action research, for it is grounded in practice and our question is pragmatic. We ask: What can be done to enlarge the sphere of the possible as we engage students in mathematical activity? Our research, however, is also about theory building. We are interested in developing ways of making our observations about learning coherent. To pursue our practice-based and theoretical goals we find ourselves creating explanations based on our observations of the "learning bodies" that emerged in a mathematics classroom. In the study Simmt taught grade 7 mathematics for one year. In a manner consistent with the complexity theory (particularly enactivism) that frames our research, we understood the students as complex systems nested within the class collective—also a complex system. Individual bodies (whether humans or social systems) were observed to learn as a result of their internal dynamics, and coupling with others and with the environment.

Burton (1999) maintained that over privileging the individual, as the only knowing agent, is the basis for individualized syllabi. In today's complex and changing world, teams and networking are prevalent conditions of learning and working. Of particular relevance our analyses are the structure, the behavior and the ecology of collective learning structures. Studying collectives of many agents raises a possibility of construing collectives of students as complex bodies possibly with emergent properties such as mind.

AN INTERPRETIVE POSSIBILITY

We draw illustrative cases from two consecutive lessons on transformational geometry, the first one in which students began by exploring objects with three lines of symmetry.

Part 1

1. "Can you tell me the name of an object that would have 3 lines of symmetry? Some objects" A number of students raised their hands immediately but Edwin in a quiet voice blurted out.
2. "Triangle."
3. Unaware that Edwin had made a response the teacher continued, "Imagine in your heads an object that has 3 lines of symmetry. Agnes."
4. "Triangle."
5. "What kind of triangle?" the teacher prompted.
6. "Equilateral."
7. "Yeah. Do all triangles have 3 lines of symmetry?" the teacher asked.
8. In chorus the students responded. "No."
9. After a number of contributions, there was soon some agreement that an equilateral triangle was the only one that had three lines of symmetry. "Okay, how many lines of symmetry does a square have? Joseph."
10. "Ummm, eight."
11. "Not a cube but a square," the teacher responded as she drew a square on the overhead. A number of students began to call out,
12. "Four."
13. But Joseph was not sure, "Four?"
14. "Four. Ah-ha, that is what people are saying," the teacher nodded.
15. But another student agreed with Joseph's first response. "No. Eight," Stella added loudly.
16. "Let us see …" The teacher began drawing in a vertical bisector. "There is a line here …"
17. "Horizontally and two diagonally," Joseph said guiding the teacher.
18. In a soft voice another student said, "Eight."
19. "Eight?" the teacher acted confused.
20. "Four," another student asserted. "Can you think of an object that would have eight?" the teacher asked.
21. Again in a chorus most of the students shouted, "Octagon."
22. "This is an important question," the teacher began. "Why did I pose it?"
23. John's hand shot up.
24. The teacher called on him to offer an answer. "John."
25. "I think it has more than eight."
26. "Okay. Save it, save it for a second. Somebody said octagon. Let's take a look." The teacher drew an octagon.
27. Esther made an observation seemingly to herself but out loud. "An octagon doesn't have …"

28. *Tim also speaking to himself in an excited tone, "A circle, oh!"*
29. *"Has more than 8," Edwin said, possibly responding to Tim.*
30. *"Y-e-a-h," Tim continued.*
31. *Janelle sitting close to Tim and Edwin said, "A circle has 180."*
32. *"Really?" Tim asked Janelle.*
33. In the meantime, the teacher was still drawing lines in the octagon. It was obvious that she was unaware of Tim, Janelle and Edwin's conversation.
34. As the teacher was drawing in the diagonals she and the students counted, "two three four …"
35. A few students audibly interjected her counting with a discussion of whether the octagon has 8 or 16 lines of symmetry: "That is 16." "Eight."
36. The teacher concluded her drawing by counting together with the students "So if we have 1, 2, 3, 4; 1, 2, 3, 4. I think there are 8. Not 16. Where would the 16 be?"
37. James interrupted at the moment the teacher was waiting to hear from the students who thought it had 16.

Part 2

38. "I know one that has infi—nite!"
39. "You know one that has an infinite?" the teacher asked. "Don't say it," she said playfully.
40. "There is a shape with lots," another student added.
41. By now a number of hands were up. "You know one that has … lots," the teacher pointed at students one by one as they shot their hands up to show that they knew.
42. "Me too," a student uttered almost inaudibly.
43. "You know one that has what?"
44. "Lots," he replied.
45. "Me too."
46. "Infinite," another student said.
47. "Infinite," the teacher repeated. "Infinite," another student said.
48. *"There are 16," another student said to another in the midst of the new "game." She was likely referring back to the octagon that was still being projected on the screen.*
49. "I think eventually … It will run around," another student commented.
50. *Esther and Janelle had a side conversation, "There is 16…." "Why did she say [there is 8]?" Esther asked.*
51. "Okay, at the count of 3," the teacher instructed. "An object with an infinite number of lines of symmetry. 1, 2, 3."
52. "Circle," the students called out.
53. Edwin was a lone voice, "Nothing," he said.
54. Although the teacher did not take up his suggestion. (It is not clear whether she heard it.) *On the video record Janelle, John and Tim can be observed to discuss the question, of whether it would be possible to draw lines of symmetry for nothing.*
55. *It was in that conversation that Tim turned to his colleagues saying, "I was thinking a sphere with the same diameter as a circle; a sphere will have more lines of symmetry than the circle."*
56. At the end of the class the researcher-observer asked Tim about his conjecture. He responded, "A sphere might have 360 times more lines than the circle.

Figure 2. Individual and collective knowing systems

Energy-rich materials

In most lessons in the class, the moment-to-moment actions of individuals unfolded into what can be observed as the behavior of a collective learning structure. It was not the teacher's explicit intention, to explore with seventh graders the symmetrical properties of a circle. But by the time the teacher posed a question about the lines of symmetry a square has, it appears, the teacher and the students were drifting into naming objects with 4, 8, 16, … lines of symmetry. While the teacher was drawing a square to assist students in determining whether it had 4 or 8 lines (lines 16-27 [16-26]), in the collective domain the project at hand arose to find an object with more lines. As the class was exploring objects with 8 lines of symmetry, Tim and Janelle began examining the circle [28-32]. James interrupted the discussion [38] saying that he knew an object with infinite lines. In the collective a project had emerged to find an object with most lines of symmetry.

The groups' interactions and conversation (including the teacher's drawings) appeared to have potentially availed energy rich materials to individual students. For instance, the numbers 180 and 360 that Janelle [31] and Tim [56] used in their conjectures, even at first glance, were not arbitrary. Why was it convenient for the students to use 180 and 360 but not, for instance, 32 and 64? To Bussi (2000), who explored the emergence of enriched use of mathematical tools, this is not arbitrary. Upon reflection we certainly could see where the numbers might have come from. However in the moment those interactions were simply part of the immediate collective intelligence of the class.

In the lesson that followed the significance of a protractor became apparent. When the teacher returned to the idea of an object with most lines of symmetry, other students conjectured, "a circle has 360 lines of symmetry; a circle has 1800 lines of symmetry". One student referred to taking a semi circle to deduce the properties of a circle. As the students empirically tested Tim's conjectures the teacher picked up a protractor. The protractor was a significant object that even in its physical absence (in the earlier lesson) had been a readily available visualization—the numbers 180 and 360. Yet the fact that it was a significant object did not only require the teacher to initiate the students to its cultural use. More was involved; the students together with the teacher, as they recursively coordinated their actions (when measuring angles), brought forth a protractor as energy-rich materials that could coordinate further actions even in transformational geometry. It is probable that in another grade 7 class with different experiences such *conceptual blending* of a measuring tool with lines of symmetry would not have been possible. Here we adopt Maturana's concept of inter-objectivity to elaborate on Vygotsky's notion of symbolic mediation. Whereas Vygotsky's notion emphasizes initiation and internalization, Maturana's (2000) notion of inter-objectivity focuses on the bringing forth of the objects in actions and interactions; objects emerge "as co-ordinations of doings that coordinate doings" (p. 462). Our retrospective analysis revealed how the protractor was frequently used and became a valued tool in the previous weeks. For example, it was used in measurement and in the introduction of the transformational geometry unit. Throughout these lessons the teacher showed special interest in the need for every student to have a protractor. As students together with the teacher consensually coordinated actions, the protractor arose as an object that was energy-rich matter not only for individuals but also in the collective domain.

IMPORTANCE OF THE STUDY

Educators are likely to benefit from conceptual tools for observing the nested learning structures. Our analysis is in its early stages. We, nonetheless, speculate that the shifting of teacher's attention to include focusing on the class as a collective learning body has the potential to generate insights as it transcends the duality between the social and individual, the emergent and formal, and the mental and physical aspects of learning.

The complexity view of cognition makes nested learning structures visible. This illuminates the relation between the individual acts, social interaction and collective learning. For instance Tim's brilliant conjectures in this (and other) lessons can be explained as more than an individual attribute. Given the emergent classroom project and the availability of common experiences with the protractor, he, as a structure-determined system, was able to select "elements from rich sources on offer" in the collective and transformed them for his own use (Kieren & Simmt, 2002, p. 871). To a larger extent Tim's conjectures about the sphere were occasioned by the collective project. He was, perhaps, seeking to refute an earlier stated conjecture that the circle had most lines of symmetry. His conjecture offered to a group of his classmates made sense to them and to him because it arose in a community in immediate action, in the collective.

The role of the teacher is key to the nature of the class' collective learning structure. Due to limitations in space, it might suffice to observe the teacher is part of the collective who shares in the control of the collective: indeed she is nowhere to be seen in the immediate interactions around the lines of symmetry of "nothing" [53] or around the comparison of the circle to the sphere [56]. The collective is able to persist in spite the lack of immediate participation by some of its agents. On the other hand it is clear that the teacher's questions and comments were part of the collective project. When James interrupted, "I know one that has infinite, the teacher playfully said, "… don't say it" [39]. This culminated into other students reflecting and offering what they knew. James' comment together with the teacher's response appears to capture a pattern in the joint interaction, a dynamic of mutual learning among members of the community.

Some students just like Esther, perhaps attending to the segments as the amount of symmetry, grappled from far behind the great leaps that the emergent project afforded to the majority of students. Even students, whose voices are not present in the collective, still could never be considered not to support or be supported by the reflections of the collective body. For instance, Joseph's (mis)-understanding that a square had 8 lines of symmetry was central to occasioning the joint project toward examining the symmetrical properties of octagonal shapes [13-36]. It is not our intention to present the teaching in the lesson above as a model. Rather, we emphasize that, on a moment-to-moment basis, the actions and interactions of the students and the teacher in any mathematics class are central to the mathematical behavior of the collective and of individual students. Even conversations of sub-collectives such as Janelle, John and Tim [54] were in feedback loops with the collective and individual students' cognitive domain.

CONSEQUENCES FOR THEORY

We have identified three characteristics of knowing observed in the nested systems: reflections, insights and conceptual blends which co-emerge within the collective space; individual knowing is nested in collective knowing; and the collective reflections, conceptual blends, symbolic objects and the individual knowing recursively become the energy-rich materials to be taken up at either the individual or the collective level.

Collective structures emerge

The class that Simmt taught was a collection of unique students. Despite the diversity, it was apparent that, this group of seventh graders became a coherent collective, with patterns of behaviors. In addition to the learning behaviors that the teacher explicitly encouraged (see Davis & Simmt, 2002), students talked and played lots of the time, with mathematics at the core of their talk and play; they spontaneously broke out into small groups to offer conjectures to each other; and the labor of the mathematical tasks was distributed among the students (not in some organized way but in a locally emergent way) as they took advantage of each others' expertise and interests.

Burton (1999) noted the difficulty of establishing communities of learning in secondary classes that, unlike elementary classes, are taught by several teachers. This was an obstacle to encouraging a community of active mathematics learners. (Simmt had the opportunity of observing the same class with another subject teacher. She observed that the same community brought forth a totally different collectivity with that other teacher, one in which order, silence and independent seatwork was valued). Davis and Sumara (in press) raise the question of studying the differences between collective characters in, for instance, mathematics and language arts classrooms. Simmt's experience supports this.

In this mathematics class the collective that emerged was a student community engaging in mathematics. Moreover, the teacher worked towards avoiding a hierarchical community in which the teacher or textbook was the sole author of knowledge. The students recurrently interacted under mutual acceptance and, as such, the class was a social collective, which maintained a central, although always evolving, character that was certainly distinct from the language arts class that Simmt observed. Using Maturana's (1988, 2000) work, this collective could be distinguished at three levels:

- The structural level of agents at which recognizably unique individuals with diverse abilities and motivations contribute to the collective experiences. (At this level we might observe the internal dynamics, the processes of interaction.)
- The behavioral level—the collective whole that emerges as individual students act and interact has its own dynamics. It interacts with its medium that includes the teaching, the materials, the setting and other collectives. The changes that appear as observable as the collective body structurally changes to compensate for perturbations from its environment, such as collective understanding and knowledge, are the collective structure's behaviors (Kieren & Simmt, 2002; Maturana, 1988).
- The ecological unity level—the collective is nested in a larger unity, say, of the wider school mathematics community. In ecological contexts, the collective learning structure is defined by the whole to which it is a part.

We have discussed the activities of a collective learning structure in order elaborate on previous research on collective understanding reported at the last PME-NA meeting. To interpret how activities of the collective cognitive domain relate to individual acts and social interaction, we have observed that individuals' initial actions are important to both the individual and the collective cognitive domain. At anytime any student's structure is also an expression of the network of the collectives in which he/she co-exists. The behaviors of the collective are connected to its emergence from individual interactions, and they recursively become the conditions of possibility for individual learning. We have identified energy-rich materials, inter-objectivity and symbolic interactions as one aspect of the relationship between joint and individual knowing. Our analyses raise a question: What are the conditions of possibility for more intelligent learning collectives?

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