KEY TRANSITIONS IN COUNTING DEVELOPMENT FOR YOUNG CHILDREN WHO EXPERIENCE DIFFICULTY

Ann Gervasoni
Australian Catholic University

This paper explores the Counting development of Australian children participating in the Early Numeracy Research Project who were identified as low-attaining using an individually administered assessment interview and a research informed framework of growth-points. The progress of Grade 1 and Grade 2 children who participated in an intervention program was compared to children who did not. Results suggest that the intervention was more effective for Grade 1 children, but that the effectiveness of the intervention seemed to depend on the growth point transitions children needed to make.

BACKGROUND

Counting is not only an everyday 'survival skill', but provides a basis for the development of number and arithmetic concepts and skills (Baroody & Wilkins, 1999). Although children need to develop more powerful strategies, being able to count a collection of about 20 items enables young children to solve many of the numerical problems they encounter. Learning to count collections is therefore an important development in mathematical learning. However, there is a group of young children who have difficulty developing this knowledge. These children are in danger of being "left behind" and of not benefiting from the curriculum provided in the regular classroom.

Teachers argue that it is often difficult to help children who have been *left behind* in the classroom. Most teachers do not have adequate time to single out children for significant periods of individual instruction. However, the children in danger of being *left behind* need opportunities to accelerate their learning; regular instruction that targets their individual needs. This is the purpose of intervention programs.

As part of the Early Numeracy Research Project (ENRP, Clarke, McDonough & Sullivan, 2002), a large scale project conducted in Australia from 1999-2001, an intervention program entitled *Extending Mathematical Understanding* was developed for Grade 1 (six year old) and Grade 2 (seven year old) children who were being *left behind* in their number learning. This paper explores the effects of the intervention program on Counting development, and insights gained about difficult progressions in Counting knowledge.

KEY GROWTH-POINTS IN LEARNING TO COUNT

As part of the ENRP, a research-based framework of six growth-points (see Figure 1) was created to describe the key developments, during the first three years of schooling, of children's counting knowledge. Similar to the work of Wright (1998), the ENRP Growth Points are concerned with children's production of number name sequences. However, the ENRP Growth Points focus also on children making the count-to-cardinal transition in word meaning described by Fuson (1992a) so that they are able to think about the number sequence to solve problems. The growth points do not describe children's use of counting strategies in addition, subtraction situations. These strategies are described in ENRP growth points pertaining to the addition and subtraction domain.

1. Rote counting: *Rote counts the number sequence to at least 20.*

2. Counting collections: *Confidently counts a collection of around 20 objects.*
3. *Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.*
4. Counting from 0 by 2s, 5s, and 10s: *Can count from 0 by 2s, 5s, and 10s to a given target.*
5. Counting from x (where x>0) by 2s, 5s, and 10s: Can count from x by 2s, 5s, and 10s to a given target.
6. Extending and Applying: *Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks*

Figure 1. ENRP Counting Growth-points

For some young children, the progression to *counting collections* (2) and *counting forwards and backwards from various starting points* (3) is prolonged or difficult. These growth-points relate to two of the counting levels described by Fuson (1992b), the *Unbreakable List Level*, and the *Breakable Chain Level*. These levels describe the development that occurs in order for children to *count collections*, or *count forwards and backwards* by ones. The *Unbreakable List Level* involves the number name sequence being broken into individual words, which are used in counting by relating each number word to a perceptual item to be counted (Steffe, von Glasersfeld, Richards, & Cobb, 1983). Children begin to relate the last word counted to cardinal meanings for the group of counted objects (the cardinality principle). They can then use *count-all* strategies to add two numbers.

The *Breakable Chain Level* involves children being able to start saying the number word sequence from any number word. They eventually use this ability in combination with an embedded cardinal-to-count transition in word meaning to add by a more efficient *counting-on* method, in which counting to determine the final sum begins with the first addend number word, instead of beginning the count from one.

These two levels, as they relate to *counting collections* and *counting forwards and backwards*, are not only important for children's counting development, but are also important for the development of numerical problem-solving strategies. It is the progression to these growth-points that is difficult for young children *left behind* in Counting.

IDENTIFYING AND ASSISTING CHILDREN *LEFT BEHIND* IN COUNTING

As part of the ENRP, all children took part in assessment interviews conducted by their teacher at the beginning and end of each year (March/November). The interviews were coded to determine the growth points each child reached in nine areas of mathematics, including Counting. The processes for ensuring the reliability of scoring and coding are outlined in Rowley and Horne (2000).

Table 1 shows the percentage of Grade 1 and Grade 2 children in ENRP trial schools who reached each of the Counting Growth Points in March 2000. These data enable the children *left behind* in Counting to be identified.

The distribution of children's counting ability across the growth points demonstrates a wide range in understanding, and highlights the challenge for teachers to cater for the range of abilities in classrooms. Further, the results suggest that a number of children being *left behind.* Eleven percent of Grade 1 children were not yet able to count a collection of 20 items, even after one year at school, and three percent of Grade 2

children were yet to develop this knowledge. A further 22 percent of Grade 2 children who could not yet count forwards and backwards by ones beyond 100 were also in danger of being *left behind* their peers and faced with a curriculum with which they could not adequately engage in order to learn successfully.

Counting Growth Points (March 2000)	Grade 1 *(n=1505)*	Grade 2 *(n=1544)*
0. Number names	5	1
1. Rote counting	6	2
2. Counting collections	56	22
3. Counting forward/backward by ones	15	14
4. Skip counting by 2, 5, 10 from 0	16	47
5. Skip counting by 2, 5, 10 from *x*	2	13
6. Extending and applying	0	1

Table 1: Percentage of Trial School Grade 1 and Grade 2 Children in 2000 Who Reached Each of the Counting Growth Points.

In order to assist the Grade 1 and Grade 2 children, who were being *left behind*, ENRP trial schools could elect to implement an intervention program. Twenty-one of the thirty-five schools elected to do so in 2000. The intervention program, *Extending Mathematical Understanding* (EMU) comprised daily 30-minute sessions for between 10 and 20 weeks, depending on the progress of students. Specialist teachers worked with groups of three or four students or with individual students. The program was not remedial in nature, but was built upon constructivist learning principles (see, e.g., von Glasersfeld, 1989). Children were engaged in experiences that required 'hard' thinking, and were required to reflect upon their activity and articulate what they had learnt and how they had learnt. The specialist teachers were trained to provide intensive instruction and feedback that was directed to the particular learning needs of each child.

Typically, each EMU session was structured to include 10 minutes of counting and place value activities, 15 minutes of rich problem solving activities (often with an addition, subtraction, multiplication or division focus), and 5 minutes reflection on the key ideas explored. Counting activities included: estimating the numerical value of large collections and then counting these collections; grouping items to emphasise the tens structure and meaning of number names using materials such as *ten frames;* using number charts and vertical number lines to emphasise patterns in the number sequence; and prediction games using the constant function on calculators, with justified argument required for the predictions.

COUNTING PROGRESS OF THE CHILDREN *LEFT BEHIND*

To determine the effect of the intervention program on the development of children's counting knowledge, the Counting growth of children in ENRP trial schools who participated in an EMU Program (the EMU Group) was compared to children in ENRP trial schools who had reached the same Counting Growth Point in March, but who did not participate in an EMU Program (the Comparison Group). Of particular interest is whether

the EMU Program was more effective than the regular classroom program in assisting children to count collections, and whether children were able to advance further to counting forwards and backwards by ones from any number. These are important developments in Counting knowledge for those *left behind.*

Table 2 describes the growth for Grade 1 children who were not yet able to count a collection of 20 items at the beginning of Grade 1 (March).

Low-attaining Students	Counting Growth Points (November)				
	1	2	3	4	5
	Rote count	Count Collections	Forwards & backwards	Skip-counting	Skip-counting from X
EMU students (*n*=18)	0	50	6	33	11
Comparison Group (*n*=120)	10	58	13	18	1

Table 2. November 2000 Counting Results for Grade 1 Low-attaining Students who in March were not yet able to count collections (expressed in percentages)

The results indicate that Grade 1 children in the EMU group made better progress in Counting than the comparison group. There are 4 points to note. First, all children in the EMU group were able to count collections of 20 items by the end of the year. Second, half of the EMU group were at least able to count forwards and backwards by the end of the year, compared with about one-third of the comparison group. Third, children in the EMU group were more likely to progress further than *counting forwards and backwards* and be able to *skip-count*, or skip count from various starting points. The final point to note is that at least half of the children in both groups did not progress beyond *counting collections*. It appears that progressing beyond *counting collections* to Growth Point 3 is a prolonged transition for many children, even when children participate in a daily intervention program.

Low-attaining Students	Counting Growth Points (November)					
	0	1	2	3	4	5
	Not apparent	Rote count	Count Collections	Forwards / backwards	Skip-count from 0	Skip-count from X
EMU Group (*n*=9)	0	11	33	22	22	11
Comparison Group (*n*=19)	10	5	42	16	21	5

Table 3. November 2000 Counting Results for Grade 2 Low-attaining Students who in March were not yet able to count collections of 20 items (expressed in percentages)

Table 3 below shows the results for the 28 Grade 2 children who were not yet able to count collections of 20 items in March. Considering that there were more than 1500 Grade 2 children in the cohort, it is clear that these 28 children were being *left behind*.

A surprising result was that not all Grade 2 children in the EMU group learnt to count collections, whereas all Grade 1 children in the EMU group did. All Grade 2 children in the EMU group learnt to rote count, and a higher proportion of the EMU group reached each of the subsequent growth-points. As with the Grade 1 children, a large proportion of each group did not progress beyond *counting collections*. Overall, the children in the EMU group made better progress than the comparison group, but this was not

pronounced. It is possible that the experiences provided by the EMU program were more effective for Grade 1 children who were not able to *count collections* than for Grade 2 children.

The results in Table 2 and Table 3 suggest that progression from *counting collections* to *counting forwards and backwards*, is prolonged for a large proportion of children, even when they participate in an intervention program that includes a focus on counting development. To explore this issue further, the growth of Grade 1 children who began the year being able to count collections was determined (see Table 4). This is the median growth point for Grade 1 children in March (*n*=1505).

Low-attaining Students	Counting Growth Points (November)				
	0/1	2	3	4	5
	Rote count or below	Count Collections	Forwards/ backwards	Skip-count from 0	Skip-count from X
EMU Group (*n*=21)	0	33	38	19	10
Trial School Group (*n*=756)	1	38	17	38	6

Table 4. November 2000 Counting Results for Grade 1 Low-attaining Students who in March were able to count collections of 20 items (expressed in percentages)

The results suggest that about one-third of these Grade 1 children did not progress to the next growth point by the end of the year. The progress of the two groups was similar, suggesting that there was little advantage for the children who participated in the intervention program. This highlights the difficulty of this progression for some children.

Low-attaining Students	Counting Growth Points (November)					
	0/1	2	3	4	5	6
	Rote count or below	Count Collections	Forwards / backwards	Skip Count from 0	Skip Count from X	Extending / Applying
EMU Group (*n*=37)	0	46	11	29	14	0
Trial School Group (*n*=276)	2	25	12	45	15	1

Table 5: November 2000 Counting Results for Grade 2 Low-attaining Students who in March were able to count collections of 20 items (expressed in percentages)

Table 5 shows the progress of Grade 2 children who were able to count collections at the beginning of the year. The median growth point for Grade two children in March was skip-counting (Growth Point 4).

The results show that at least one-quarter of Grade 2 children did not progress from *counting collections* to *counting forwards and backwards* by the end of the year. This indicates again the difficulty of this progression for some children. Further, the results suggest that Grade 2 children who began on Growth Point 2 were disadvantaged by participation in the EMU program with respect to Counting. Indeed, children in the comparison group were more likely to progress to at least Growth Point 3. It may be that

Grade 2 children who can count collections, but who are not yet able to count forwards and backwards from varying starting points, need the broader type of learning experiences provided within the regular classroom, rather than experiences geared precisely to their next growth point transition. The regular classroom program for Grade 2 children is more likely to emphasise *skip counting*, including skip counting from different starting points. It may be that *skip counting* learning experiences help children to construct knowledge about patterns in the number sequence that also assists the progression to counting forwards and backwards by ones. Children are less likely to have these counting experiences within the EMU program if they have not yet reached Growth Point 3. This suggests that children at a particular point in their counting development may be disadvantaged if the learning experiences in which they engage are too narrow.

Grade 2 children's progress in counting may also be influenced by their learning in other mathematical domains. For example, it could be that children who participated in the EMU program were being *left behind* in other mathematical domains. To explore this issue further, the percentage of Grade 2 children who had reached Growth Point 2 in March and who were below the median growth points for Grade 2 in Place Value, Addition and Subtraction and Multiplication and Division were calculated (see Table 6).

Low-attaining students	Place Value	Addition & Subtraction	Multiplication & Division
EMU Group (n=37)	90	72	70
Comparison Group (n=276)	65	42	40

Table 6. Percentage of Gr 2 children who had reached Counting Growth Point 2 in March and were behind in Place Value, Addition & Subtraction and Multiplication & Division.

Clearly, a greater proportion of children in the EMU group were behind in the other domains. This may explain why the children made less progress in Counting than the comparison group: their difficulties were broader in scope, and their learning may have been concentrated in other areas. Further research is necessary to investigate the interaction between these domains in the mathematical learning of children who are being *left behind* their peers.

CONCLUSION

The results reported in this paper suggest that the EMU Program was effective for increasing the counting knowledge of children in Grade 1 and Grade 2 who could not yet count collections of 20 items, although it appears that intervention in Grade 1was more effective than intervention in Grade 2. The extent of children's learning seemed dependent on the growth-point transitions children needed to make. This suggests that children may need different types of experiences, depending upon their age and level of understanding. It seems that Grade 1 and Grade 2 children who have reached the same growth point in Counting do not gain equivalent benefit from equivalent experiences.

The results also suggested that the progression to *counting forwards and backwards by ones* was prolonged for a sizeable proportion of the Grade 1 and Grade 2 children. More research is needed to explore the nature of this progression and how teachers can assist children to make this transition. The type of experiences offered by the EMU program

did not seem to advantage children in this case. It appears that children learning to count forwards and backwards by ones beyond 100 benefited more from the broader type of experiences and interactions offered by the regular classroom program.

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