AFFECTIVE ASPECTS ON MATHEMATICS CONCEPTUALIZATION: FROM DICHOTOMIES TO AN INTEGRATED APPROACH

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The present paper aims to propose a theoretical reflection in order to overcome a strong tradition in psychology concerning the analysis of cognition and affectivity as dichotomic processes explaining human behaviours. A general theory of the human subject is presented to discussion, followed by the proposition of a new unit of analysis for the study of mathematical activity, integrating affectivity and cognition. The choice of such a unit of analysis takes into account the specificity of conceptualization and learning in mathematics, the interest of pre-conceptual competencies-in-action, and the need of studying culturally meaningful situations. This theoretical effort is considered especially relevant for increasing the contribution of psychology of mathematics education in the research context of mathematics education.

INTRODUCTION

How the soul and the body act one against another.

(Descartes, in *The Passions of the Soul,* head-title of the 34th Article)

Theoretical and methodological efforts have been made towards the inclusion of affectivity as a valid explicative variable concerning cognitive abilities and competencies in general (e.g., Ginsburg, 1989), competencies at school (e.g., Frias and cols., 1990) and particularly competence in school mathematics (e.g., McLeod, 1992). This is an important issue for most of those interested in complex psychological processes such as *learning*, *development* and *conceptualization,* since it concerns crucial points in the general (and hopefully less fragmentary) domain of psychology, as well as *urgent* questions in psychology of education in specific domains, such as mathematics (Hazin and Da Rocha Falcão, 2001; Da Rocha Falcão, 2001).

In fact, we seem to have overcome a three-century tradition of opposing affectivity and cognition (see, for example, Descartes, 2003), the first seen as a source of perturbation, a kind of disturbing screen between a rational mind and the real world. In a second moment, we have seen important theoretical efforts in order to emphasize a conjoint effect of affectivity AND cognition (as explicative variables) on specific aspects of complex behaviours (school abilities, achievement and adaptability in work contexts,

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etc.). This approach implies a valorization of affectivity, not only seen as a disturbing aspect but also as a cooperating one (at least until an 'optimum level of excitation', as exemplified by the 1908' formulation of the Yerkes-Dodson Law of the relationship between anxiety and cognitive performance (Yerkes & Dodson, 1908, also referred by Evans & Tsatsaroni, 1996)). Nevertheless, it is important to admit that even in the context of this contemporary approach, the same Cartesian dichotomy is still present. A strong tradition of splitting human phenomena in rational / spiritual aspects (*res cogitans*) and somatic / emotional / animal aspects (*res extensa*) (Descartes, 2003) nourished theoretical systems in psychology stressing one of these poles (to the detriment of the other one), without an integrative approach to show the functional interconnection between affectivity and cognition (Damasio, 1994).

There are central questions to be addressed in order to progress in the theoretical-psychological debate about the topic under discussion. Firstly, it is important to conceptually clarify what is meant in psychological literature by the words *affects*, *affectivity*, *cognition*; discussing this issue implies in clarifying theoretical choices, connected to specific focuses of interest in the domain of psychology of mathematics education. Secondly, it is *fundamental* to take into account that the theoretical choices referred above must be coherently based upon a *theory of the human subject*, which requires the combination of previously mentioned psychological-theoretical approach with a philosophical-epistemic analysis. Once these two aspects have been minimally worked out, it is possible to think about *research methodology* in a more sophisticated context. In the present paper, the intention is to offer some hints for the reflection about the three points mentioned above, towards an *integrative* approach of affectivity and cognition in terms of a third, post-Cartesian approach.

AFFECTS, AFFECTIVITY, COGNITION: WHAT IS THIS ALL ABOUT?

There are two important theoretical systems in psychology which are good examples of the contemporary approach of affectivity and cognition: Piagetian genetic epistemology and Freudian psychoanalysis. Cognition, from a Piagetian point of view, is related to a biological need of equilibration, where affective aspects are seen as "combustible" for logical structures (the "engine"): "(...) *affectivity is considered as the energetic pole of behaviour*" (Piaget, 1972). Cartesian heritage of dualistic approach of affective and cognitive aspects is clearly present here. Freud, for his turn, will stress unconscious pulsional (libidinal) aspects as central in the theoretical explanation of human behaviour, viewing cognition (or *epistemophilic motivation*) as a derivative of libidinal impulse by sublimation or neurosis (Freud, 1910).

There are certainly other important theoretical contributions addressing this specific aspect of an integrative affect-cognition view of human behaviour (see, for example, Henry Wallon, Donald W. Winnicot and John Bowlby's works on child development); none of them, nevertheless, seem to propose theoretical elaboration concerning specific aspects of cognition (i.e., specific knowledge domains or *conceptual fields* (Vergnaud, 1990)) taking into account affective aspects as constitutive, not merely adjuvant.

A pervasive approach concerning ***affectivity*** proposes that this complex psychological process encompasses *changing states of feeling* (local affect) as well as *more stable,*

longer-term constructs (global affect) (DeBellis and Goldin, 1999, pp.250; italics added). In a similar approach, D.B. McLeod suggests three dimensions of variation in affect: *intensity*, *directions* (positive vs. negative) and *stability* (McLeod, 1992). According to these dimensions, beliefs and attitudes would be seen as "cool" and "stable", while emotional reactions would be classified as "hot" and "unstable" (Schlöglmann, 2003). In a recent effort to systematize theoretical contributions to the present issue, J. Evans and A. Tsatsaroni mention four theoretical models in order to link cognitive and affective domains in educational research: two cognitivist models (Individual-differential and 'Constructivist' models), a Psychoanalitic (traditional-Freudian) model, and a Post-structuralist (Lacanian) model (Evans & Tsatsaroni, 1996).

This diversity of approaches explain the variety of research efforts, in terms of the choice of units of analysis and methodological tools for the study of the dipole affectivity-cognition/mathematical abilities: *emotions* concerning mathematical experience (Breen, 2000; Weyl-Kailey, 1985), psychoanalitic *transfer* and *counter transfer* phenomena in student-teacher relationship (Cabral & Baldino, 2002), *self-esteem* and *self-concept* and performance in school mathematics (Hazin & Da Rocha Falcão, 2001; Ginsburg, 1989), *attitudes* and *beliefs* towards mathematics (De Brito, 1996; Pehkonen, 2001), *personality traits* and *cognitive styles* in mathematical problem-solving (Régnier, 1995; Ginsburg, 1989). In fact, an important aspect to take into account when choosing one or more approaches among the various possibilities briefly mentioned above is: what *theory of the human subject* underlies these various theoretical and methodological propositions? This is the central issue of the next session.

SOME RELEVANT ASPECTS CONCERNING A THEORY OF THE HUMAN SUBJECT

The contemporary contributions of cultural-historical psychology strongly emphasize the need of taking into consideration a developmental perspective, crossed with a psychology of human acts interested in here-and-now phenomena, including classroom scenarios (Valsiner, 2001). According to L.S. Vygotsky, there has been two main philosophical approaches concerning this issue: a dualist/pluralist approach, in which the human subject is segmented in various spheres or aspects, like: biological (endogenous, nature) vs. cultural (exogenous, nurture); brain vs. mind; cognitive, rational vs. affective, passional; individual vs. social, and the like; on the other hand, there is a minor effort towards a *monist* approach in which a unified consideration of human consciousness and/or activity in real contexts is proposed (Vygotsky, 1996). Most theoretical/methodological approaches, as mentioned in the previous section, include the idea of a subject splitted in two or more aspects. In this context, affectivity (in its various instances) is clearly seen as a variable that could be isolated.

In the context of the proposition of an integrative approach, Valsiner & Van Der Veer give priority to the critical revision of the antinomy individual-society. For these authors, many others dichotomies could be also overcome by the critical deconstruction of this central antinomy. On the way to this critical revision, Valsiner & Van Der Veer propose to overcome the approaches of the individual's *fusion* in or *captured* by the society, towards the consideration of an ***inclusive separation***, a co-construction of both the subjectivity and intersubjectivity (Valsiner & Van Der Veer, 2000). J. Valsiner

introduces the crucial theoretical idea of *semiotic mediation of affective processes*. According to this idea, the affective experience starts from the most simple level, called level 0 (zero), concerning the 'inner state of excitation', followed by bodily emotions ("general immediate feeling tone", level 1); these basic levels are followed by a crucial level 2, where specific categories of emotions are labeled by words like "sad-sadness" and "fear", and where it is important to mention a co-construction of a subjective experience semiotically mediated (then culturally embedded) by language; at level 3, generalized categories of feeling are construed, once more through discursive actions like the speech construal "I feel bad"; finally, at the most elaborated and complex level 4, over-generalized feelings are semiotically construed, as denoted by speech construals like "I just feel... can't describe it"; at this level, "(...) the person 'just feels' something – but cannot put that feeling into words" (examples of this experience are aesthetic feelings – 'catharsis experienced during a theatre performance', or in 'interpersonal situations of extreme beauty'). According to J. Valsiner, the experiences above "(...) can be seen as examples showing that human affective field can become undifferentiated as a result of extensive abstractions of the emotions involved, becoming overgeneralized to the person's general feelings about oneself or about the world" (Valsiner, 2001, pp. 164). At this level, the meaning of the affective experience cannot be analyzed in terms of the individual *or* the societal-cultural world; this is the very theoretical contribution of the notion of *inclusive separation*, mentioned above: most psychological phenomena must be analyzed in terms of a dialectical *co-construction*. Under the enlightenment of these considerations, some methodological consequences must be emphasized.

FROM RESEARCH ON AFFECTIVITY AND MATHEMATICAL ACTIVITY TO RESEARCH ON MATHEMATICAL SENSE-MAKING

Two central points must be emphasized here: first, it is always valid reaffirming that methodology is not accepted to pre-establish approaches, choices, limits, targets; theory is the riverbed in which methodology flows. Second, mathematical activity is not more or less complex than any other activity in diverse cultural contexts (Rogoff & Lave, 1984). Nevertheless, mathematical activity has a *specificity* that must be taken into account. The teacher is expected to *consider* the mathematical activity in its complexity, *including* for example the need – of the student – to have approval and love from the teacher; at the same time, a challenge to both teacher and psychologists of mathematics education is: how to be open to these aspects and at the same time not to change the classroom into another cultural place (e.g., familiar or therapeutical place).

The psychological contribution to mathematical activity of sense-making must take into account the systemic complexity of this and any other human activity, keeping in focus the epistemic specificity of mathematics. A psychological approach that loses sight of this last aspect offers a poor contribution to the domain of psychology of mathematics education (Da Rocha Falcão, 2001); on the other hand, "operational splitting" of human activity would ascribe psychological contributions to a tradition of theoretical oversimplification (mostly dictated by epistemological choices concerning psychology as a "valid" science). These considerations lead to a central question concerning the link between theoretical and methodological aspects, in the context of the integrated psychological approach of mathematical activity: which should be the *minimal unit of*

analysis to the study of this specific activity? This is a complex question, for which we propose four central points to be considered: 1. A theory of reference concerning learning and conceptualization: according to G. Vergnaud, learning and conceptualization always refer to specific domains (Vergnaud, 1997), and learning of mathematics has certainly its specificity. As a consequence, any research proposal on learning cannot avoid a previous epistemic analysis of the conceptual field explored. 2. The consideration of pre-conceptual competencies-in-action: we refer here to human competencies that have two major characteristics: firstly, they are *effective,* in the sense that they help people dealing with daily, culturally situated situations; secondly, these competencies are very hard to *express* by any symbolic means (natural language, graphic representations, mathematical models, and so on). Examples of these competencies are those shown by handicraft workers, but also some competencies of very highly school-educated intellectual workers (researchers, engineers, specialized technicians, and so on – see, on this subject, Samurçay & Vergnaud, 2000). Taking these competencies into account in mathematical conceptualization implies in connecting school activity to other socio-cultural contexts. 3. The integrative approach of cognitive-affective aspects: delimitation of affective and cognitive poles reflects a philosophical perspective on human nature that cannot be considered as theoretical *a priori*. Most researchers on the domain of affectivity and mathematical activity, as discussed on section 2 above, have stressed that affective states can vary from very "hot", emotional, "irrational" states to very "cool", attitudinal, cognitive-like states (Schlöglmann, 2003). On the other hand, there are references to "aesthetic feelings and motivation" in mathematical activity (Gadanidis, Hoogland & Hill, 2002), "mathematical intimacy and integrity" (De Bellis & Goldin, 1999), and considerations issued from neuroscience on "memory about emotions as a cognitive memory" (Schlöglmann, 2002). Even though we can always refer distinctly to "affective" and "cognitive" systems (in the context of neuropsychology, for example), it seems that it would be highly productive to overcome this dichotomy in the context of the building of a new unit of analysis in psychology of mathematics education. 4. The proposition of situations to be analyzed in a diachronical process: finally, this new unit of analysis should result of an effort to "bring complexity of psychological phenomena into the analytic focus of psychology" (Valsiner, 2001). This effort implies in considering that culture is a part of the systemic organization of human psychological functions, but at the same time, human beings can distance themselves from any current setting through such cultural (semiotic) means, and yet they remain parts of the setting, as suggested by J. Valsiner (Valsiner, 2000, pg. 59). Psychological unit of analysis, hence, must consider meaningful cultural situations, without evacuating individual *subjectivity*. The consideration of these cultural situations, on the other hand, cannot be reached by the only analysis of discursive production; we assume that the development of conceptualization is based simultaneously on the extraction of regularities in empirical world, on the construction of predicates and inferred, non-directly observable objects, and finally on the establishment of relations between linguistic invariants and operational invariants (Vergnaud, 1997). In other words, if it is not possible to reduce thinking to empirical world, it is not possible to reduce it to specific structures of language either. Finally, an important tool to be extensively used to explore such a complex unit of analysis is *interpretation*, through comparison and generalization. Interpretation, however, must be circumscribed to the specific research context (i.e., psychology of

mathematics education) in which it is exercised, without problematic amalgam with other contexts of use (e.g., the clinical-psychotherapeutic context).

FINAL REMARKS AND CONCLUSION

The main target of the present paper was to offer hints in order to contribute to the proposition of a new unit of analysis concerning mathematical activity, overcoming the traditional-Cartesian dichotomy between affective and cognitive aspects, among other dichotomies. This presumably more productive unit of analysis targets cultural situations, in the context of which a mathematical activity takes place, involving a set of identifiable epistemic contents (a conceptual field). Individuals can act in the context of these situations in various ways, with a common goal; psychological analysis should be able to show both diversity and generality of these phenomena, through the interpretation of situated actions. This interpretation covers discursive acts as semiotic productions necessarily framed by socio-cultural contexts (e.g., mathematical didactic contract in the classroom of mathematics, workplace culture). Furthermore, individual emotional states and conceptual metaphors can only be adequately reached by including the analysis of individual bodily gestures (as developped by Lakoff and Núñez, 2000).

Let us emphasize that all these aspects should be considered in a dialectical, conjoint approach, according to the theoretical concept of *inclusive separation*, discussed above.

Affect and cognition are in fact ways of looking at the same phenomena: human activity. The specific contribution of psychology, in the context of the community of mathematics education, is the proposition of an integrative approach of the human subject as a mathematics learner *possessor of a* subjectivity that is always embedded in culture, but never subsumed by this same culture. Considering this same discussion under a methodological point of view implies in focusing on smaller and *yet* complex enough situations, in order to be able to tell a better *narrative* about people doing mathematics. This is a valuable research target for the coming years.

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