PRESERVICE ELEMENTARY TEACHERS' SOLUTION PROCESSES TO PROBLEMATIC ADDITION AND SUBTRACTION WORD PROBLEMS INVOLVING ORDINAL NUMBERS AND THEIR INTERPRETATIONS OF SOLUTIONS

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In this study, we examined 139 prospective elementary teachers' solution processes to additive word problems for which the solution is 1 more or 1 less than the answer produced by the straightforward application of the addition or subtraction of the two given numbers. For each problem, five aspects of their solution processes were examined: (a) the modeling strategy or procedure, (b) execution of procedures, (c) the solution, (d) the type of errors, and (e) the implicit or explicit interpretation of the solution produced by the mathematical model or procedure. The major findings of the study were that a majority of prospective elementary teachers' responses (about 91%) contained incorrect solutions to such problems and that about 93% of the errors were ±1 errors. That is, errors due to preservice elementary teachers' failure to interpret correctly the solution produced by the straightforward addition or subtraction of the two numbers given in each word problem.

PURPOSE OF THE STUDY

Story problems have played and will likely play a prominent role in elementary school mathematics. Verschaffel, Greer, and De Corte (2000) mention, among others, the following reasons for the inclusion of word problems in the mathematics curriculum: (a) word problems provide practice with real life problem situations where students will apply what they learn in school, (b) word problems motivate students to understand the importance of the underlying mathematical concepts because they will use such concepts and abilities to solve problems in the real world, and (c) word problems help students to develop their creative, critical, and problem-solving abilities. However, as argued by several critics (e.g., Gerofsy, 1996; Lave, 1992) word problems, as currently presented in instruction and textbooks, fail to accomplish these goals. The failure is due, in part, to their stereotyped nature (Nesher, 1980) and unrealistic approach needed to solve them: Most, if not all, word problems that students are asked to solve require the application of a straightforward arithmetic operation (Verschaffel, Greer, & De Corte, 2000). As a consequence, when faced with word problems in which the situational contextual plays an important role in the solution process, students fail to connect school mathematics with their real-world knowledge.

Children’s lack of sense making (Silver, Shapiro, & Deutsch, 1993) or suspension of sense-making (Schoenfeld, 1991; Verschaffel, Greer, & De Corte, 2000) has been examined in several studies. However, we do not have empirical data about prospective elementary teachers’ (PETs) solution processes when solving addition and subtraction word problems whose solution is 1 more or 1 less than the one produced by the straightforward addition or subtraction of the two given numbers. In this paper we attempt to bridge this gap. First, we examine prospective elementary teachers' modeling strategies to solve problematic or non-routine word problems involving addition or subtraction of ordinal numbers. Second, we analyze their interpretations of the solutions produced by the mathematical models or procedures. For purposes of this paper, a problematic problem is a problem in which the solution provided by the arithmetic procedure involving the given numbers does not represent the solution to the problem.

THEORETICAL AND EMPIRICAL BACKGROUND

We will refer to the process of representing aspects of reality by mathematical structures as mathematization or modeling. There are several schematic diagrams to represent this process (e.g., Verschaffel, Greer, & De Corte, 2000; Silver, Shapiro, & Deutsch, 1993) but Silver et al.'s (1993) referential-and-semantic-processing model will suffice for our purposes (Figure 1).

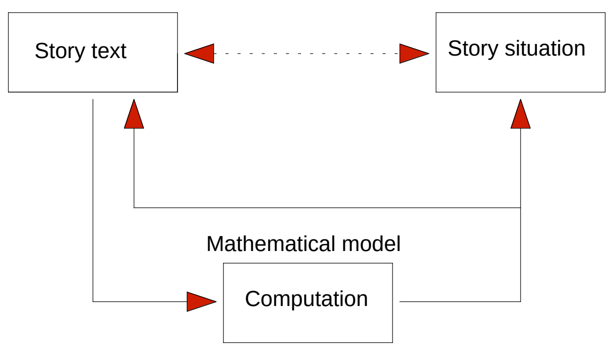

Figure 1: Silver et al.'s (1993) referential-and-semantic-processing model

The process of mathematization or modeling can be described briefly as follows: (a) to understand the structure of the mathematical situation embedded in the story text of the problem, (b) to construct a mathematical model or to choose a mathematical procedure to obtain the solution to the story problem, (c) to execute the computations or procedures called by the mathematical model, and (d) to interpret the outcome or solution produced by the mathematical model in terms of the situational context described in the story text or the constrains of the real-world story situation. Students' realistic solutions to problematic word problems could be enhanced by paying particular attention to the appropriateness of the mathematical model and the appropriate interpretation or meaning of the outcome produced by the mathematical model. Silver, Shapiro, and Deutsch hypothesize that many unsuccessful solutions use the model displayed in Figure 2.

Several researchers (e.g., Cai & Silver, 1995; Greer, 1993, 1997; Reusser & Stebler, 1997; Silver, Shapiro, & Deutsch, 1993; Verschaffel & De Corte, 1997; Verschaffel, De Corte, & Lasure, 1994; Verschaffel, De Corte, & Vierstraete, 1999) have examined children's lack of use of their real-world knowledge to solve problematic word problems. Silver, Shapiro, and Deutsch (1993), examined 195 middle school students' responses to the following augmented-quotient division-with-remainders problem: The Clearview Little League is going to a Pirates game. There are 540 people, including players, coaches, and parents. They will travel by bus, and each bus holds 40 people. How many buses will they need to get to the game? Silver, Shapiro, and Deutsch found that about 22% of the students correctly performed an appropriate procedure but provided an incorrect solution without explicit interpretation. Most of these students interpreted the result of the division (e.g., 13 or 13 with another number) as the number of buses needed to transport the people to the game. In another study, Verschaffel, De Corte, and Vierstraete (1999) investigated 199 fifth and sixth graders' difficulties in modeling and solving problematic additive word problems involving ordinal numbers. The pupils were administered a paper-and-pencil test that included six problems whose solution is 1 more or 1 less than the numerical answer provided by the addition or subtraction of the two given numbers. The researchers found that about 83% of the errors made on these 6 problems were ±1 errors.

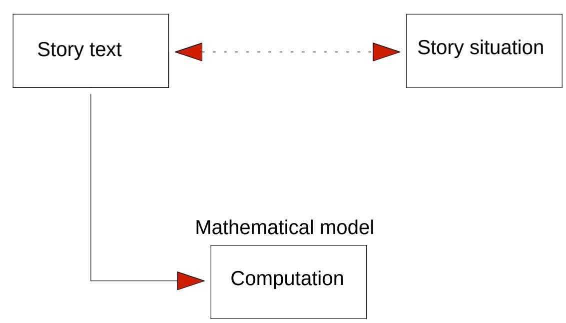

Figure 2: Silver et al.'s (1993) referential-and-semantic-processing model for unsuccessful solutions

In this paper we extend Verschaffel, De Corte, and Vierstraete's (1999) research to PETs. Since teachers are one of the key factors in the instructional environment, it is critical to examine their use (or lack of use) of realistic modeling assumptions in dealing with problematic word problems. Some may argue that there is no reason to believe that PETs will perform better than Verschaffel et al.'s subjects because, when they were children, PETs solved non-problematic word problems. However, PETs are, arguable, a more mature population, and, hence, the possible generalizability of the findings of Verschaffel et al. has to be established *empirically* for PETs.

METHOD AND SOURCES OF EVIDENCE

The sample consisted of 139 pre-service elementary teachers (PETs) from five sessions of their second mathematics content course for elementary teachers in a State University in the USA. The participants were administered a paper-and-pencil test during regular class and told that they would have enough time to complete the test. The directions asked PETs to show all their work to support each of their responses. As in the Verchaffel, De Corte, and Vierstraete's (1999) study, the test contained 9 experimental items and 7 buffer items. The experimental items were adapted from Verchaffel, De Corte, and Vierstraete's (1999) investigation and are displayed in Table 1. The first three of the experimental items can be solved by the straightforward addition or subtraction of the two given numbers while the other six are problematic in the sense described above.

Although written protocols contain some intrinsic limitations when compared to verbal protocols, several researchers have validated the use of written data to uncover cognitive processes (e.g., Hall et al., 1989). In fact, we did not face any difficulty in determining the strategies that PETs used to solve the problems. Written responses are also the most feasible method to collect data on large samples of subjects. However, interviews are being conducted to gain additional insights into the nature of PETs' thinking and reasoning when solving non-routine addition and subtraction problems involving ordinal numbers.

Table 1: The nine experimental items

Type	Item	Required operation(s)*
I-L	1. In January 1985 a youth orchestra was set up in our city. In what year will the orchestra have its twenty fifth anniversary?	S + D
I-D	2. Our youth club was set up in September 15, 1970. I became a member in September 15, 1999. How many years had the club already existed when I became a member?	L - S
I-S	3. In March 2000 it had been 34 years since our school had held its first annual school party. In what year was the school party held for the first time?	L - D
II-L	4. In September 1975 the city's youth orchestra had its first concert. In what year will the orchestra have its fiftieth concert if it holds one concert every year?	(S + D) - 1
II-D	5. Last October (2001) I participated for the first time in the great city running race that is held every year. This race was held for the first time in October 1959. How many times has the race been held?	(L - S) + 1
II-S	6. In November 1994 the twenty fifth annual school party took place. In what year was the school party held for the first time?	(L - D) + 1
III-L	7. There was a summer market in our city every summer from 1950 up through 1969. Since then the summer market was cancelled 30 consecutive times. In what year did the summer market restart?	(S + D) + 1
III-D	8. For a long time the city held a fireworks display every year on the last day of the October festival. In October 1982 we had our last fireworks, and thereafter there was no fireworks display. In October 1999 they restarted the tradition of the annual fireworks display. How many years did we miss the fireworks?	(L - S) - 1
III-S	9. In December 1999 our sports club held its annual election for its officers. Because of a lack of candidates, there had not been elections for the 23 years preceding 1999. Prior to this election, in what year did the last election occur?	(L - D) -1

ANALYSIS AND RESULTS

Preservice elementary teachers' (PETs) written responses were examined with respect to five aspects of their solution processes related to the process of mathematization: (a) procedure or mathematical model (strategies), (b) execution of procedures, (c) numerical solution to the problem, (d) errors, and (e) interpretations of the solutions produced by the mathematical model. The analysis of errors helped us to better understand PETs' interpretations of the solutions produced by the procedure or strategy.

PETs produced a total of 1247 responses out of 1251 possible. We found that 1243 (99%) of the responses to the items contained an appropriate modeling strategy or procedure. A strategy, model, or procedure was judged appropriate if it could potentially lead to the correct solution to the problem. That is, a procedure that, if executed correctly, could yield either the correct solution or 1 more or 1 less than the correct solution. The remaining 4 responses contained an inappropriate procedure. Regarding PETs' types of modeling strategies for the solution processes containing an appropriate procedure, 30 responses contained strategies using counting techniques, one contained the use of a picture, and 25 contained only answers, which suggests that PETs performed the operations mentally. All of the other responses (1187 or 95%) involved strategies that relied on formal methods (i.e., adding or subtracting the two given numbers). We did not find any evidence of other useful heuristic strategies such as solving an analogous simpler problem. With respect to the execution of the procedures, PETs performed 1179 (95%) appropriate procedures correctly.

Regarding the numerical solution to the problems, PETs performed well on the three non-problematic items (Table 2). Only 35 (8%) responses were incorrect. On the other hand, PETs' performance on the problematic items was poor. The percentage of correct responses for each problematic word problem varied from 4% to 14% (Table 2). Overall, only 9% of the solutions to the problematic items was correct.

Problem	Number of correct solutions	Percentage of correct solutions
1	127	91%
2	129	93%
3	126	91%
4	7	5%
5	12	9%
6	5	4%
7	19	14%
8	12	9%
9	18	13%

Table 2. Percentage of correct solutions for each experimental word problem

Given that 781 (95%) of the responses to the problematic word problems contained an appropriate procedure executed correctly, we hypothesized that PETs' lack of success with the problematic word problems was due to the interpretation that the addition or subtraction of the two given numbers produces the correct solution. To verify this hypothesis, we conducted an analysis of errors for the problematic word problems (Table 3). As Table 3 indicates, in all but one case, at least 90% of the errors were ±1 errors. On average, about 93% of the errors were ±1 errors. PETs also made ±1 errors with execution of procedures. These errors are errors due to procedures executed incorrectly for which, in addition, PETs did not adjust their responses. That is, even if PETs had executed the procedures correctly, they would have still made ±1 errors. In this sense, these types of errors are potential ±1 errors. Combining these two types of errors, we conclude that for each problematic word problem, at least 94% of the errors were

±1errors. Overall, about 97% of the errors made by PETs on the problematic word problems were ±1errors or potential ±1 errors. About 3% of the remaining errors were due to other factors such as adjusting both numbers, using the inverse operation (subtraction instead of addition), executing procedures incorrectly, etc. We conclude that ±1 errors and potential ±1 errors were influenced by PETs' interpretation that the result of the addition or subtraction of the two given numbers yields the correct solution to the problematic word problems. As one PET said: "this is just a subtraction problem. We get the correct answer by subtracting the numbers."

Table 3: Type errors for each of the six problematic word problems

Type of problem	Required operation(s)	Type of ±1 error	± 1 errors (%)	± 1 errors and execution of procedures (%)	Execution of procedures (%)	Other (%)
II-L	(S + D) - 1	+ 1 error	83%	14%	2%	1%
II-D	(L - S) + 1	- 1 error	94%	2%	2%	1%
II-S	(L - D) + 1	-1 error	90%	8%	0%	2%
III-L	(S + D) + 1	-1 error	98%	0%	1%	2%
III-D	(L - S) -1	+ 1 error	98%	0%	1%	1%
III-S	(L - D) - 1	+ 1 error	93%	1%	1%	5%

DISCUSSION AND CONCLUSION

While we expected, based on previous research with school children, that some prospective elementary teachers (PETs) would not provide the correct solution to the problematic word problems, it was alarming to find out that a high percentage of the total responses to the problematic items contained unrealistic solutions. In fact, only 9% of the solutions to the problematic items were correct. An analysis of errors revealed that about 97% of the errors were either ±1 errors or potential ±1 errors. We offer several explanations to understand this finding. One explanation might be that PETs are used to approach word problems in a superficial or mindless way because the word problems posed in the traditional instructional environment can be solved by the straightforward application of arithmetical operations. In this sense, PETs probably expected that all the test problems were of that kind. As stated by one PET “I didn’t even know that this type of problems existed.” Another explanation may be PETs’ insufficient repertoire of useful heuristic strategies (such as thinking of an easier analogous problem or making a diagram). This was evident during class instruction where PETs were asked to solve problematic subtraction and addition word problems using an analogous simpler problem or making a diagram. Some PETs stated that these heuristic strategies helped them to understand and solve the problems. On the other hand, we also may explain some of the PETs’ unsuccessful solutions as due to PETs’ lack of understanding of heuristic strategies. This was also evident during classroom instruction. A few PETs stated that they did not know how to pose an analogous simpler problem. Others said that using a picture was confusing. Another explanation for PETs’ unsuccessful solutions may be an insufficient understanding of the enumeration process to solve subtraction and addition word problems involving ordinal numbers. This lack of understanding might have prevented PETs from interpreting that we need to adjust by one the solution produced by the addition or subtraction of the two given numbers. This explanation is similar to the one offered by Silver, Shapiro, and Deutsch (1993) to account for some of their students’ responses involving 13 or a combination of 13 with another number. The researchers advanced the explanation that these responses reflected a lack of interpretation of the remainder of the division of 540 and 40. That is, these students' faulty solutions were influenced more by a lack of understanding of the meaning of the remainder than by an understanding that the solution to the word problem can be represented by the division of the two given numbers. In this case we know that if the remainder is greater than zero, then an extra bus is needed. In the present study, however, the solution to the problematic word problems is not represented by the addition and subtraction of the two given numbers. Successful solutions involve understanding something else: the connection between the nature of the enumeration process needed to find the solution and the answer produced by the addition or subtraction of the two given numbers. There is not an algorithmic hint we can give students to obtain the correct solution to this type of problems. Understanding the nature of the enumeration process of addition and subtraction word problems involving ordinal numbers is cognitively more complex than understanding the meaning of the remainder of division word problems. Evidence to support this claim comes from observing, during classroom instruction and on tests, that some PETs chose to use counting techniques because they did not know what numbers to use or how to adjust the solution produced by the addition or subtraction of the two given numbers. The use of a picture to understand the enumeration process was too confusing for some of these PETs.

The low percentage of correct solutions to the problematic word problems is alarming. Since a high percentage (about 97%) of incorrect responses contained ±1errors or potential ±1 errors, it seems that PETs need some type of instructional intervention to learn how to model whole number addition and subtraction word problems in which the solution is 1 more or 1 less than the sum or difference of the two given numbers. We will assess the effects of both minimal (such as giving students a hint that some of the given problems are problematic or "tricky" or creating a cognitive conflict by asking them to solve similar simpler problems) and more intensive interventions on PETs' modeling strategies to solve problematic addition and subtraction word problems involving ordinal numbers in a sequel to this study (Authors, in preparation). The cognitive conflict technique was used during classroom instruction and during interviews. Some PETs realized that the addition and subtraction of the two given numbers does not always produce the correct solution to word problems involving ordinal numbers.

This study contributed to our understanding of PETs' modeling strategies and errors when solving subtraction and addition word problems involving ordinal numbers. Other studies have examined children's solutions to other types of problematic word problems (e.g., what will be the temperature of water in a container if you pour 1 l of water at 80° and 1 l of water of 40° into it? (Nesher, 1980), John's best time to run 100 m is 17 sec. How long will it take to run 1 km? (Greer, 1993)). To better understand PETs' thinking when solving problematic word problems, we also need to examine the modeling strategies that they use to solve such problems. This is certainly a fertile area for further research. Other problematic word problems for which we lack information about PETs' thinking and modeling strategies are problems similar to the ones investigated in this study but involving multiplication and division (e.g., A farmer wants to fence the front of a square field whose side measures 1000 feet. How many posts does he need, if he wants to place a post every 20 feet?).

References

Authors. (In preparation). The effects of instruction on preservice elementary teachers' modeling strategies and errors when solving addition and subtraction word problems involving ordinal numbers.

Cai, J., & Silver, E. A. (1995). Solution processes and interpretations of solutions in solving a division-with-remainder story problem: Do Chinese and U.S students have similar difficulties? *Journal for Research in Mathematics Education*, 26(5), 491-496.

Gerofsky, (1996). A linguistic and narrative view of word problems in mathematics education. *For the Learning of Mathematics,* 16(2), 36-45.

Greer, B. (1993). The modeling perspective of wor(l)d problems. *Journal of Mathematical Behavior*, 12, 239-250.

Greer, B. (1997). Modelling reality in the mathematics classroom: The case of word problems. *Learning and Instruction*, 7, 293-307.

Hall, R., Kibler, D., Wenger, E., & Truxaw, C. (1989). Exploring the episodic structure of algebra story problem solving. *Cognition and Instruction*, 6, 185-221.

Lave, J. (1992). Word problems: A microcosm of theories of learning. In P. Light & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing* (pp. 74-92). New York: Harvester Wheatsheaf.

Nesher, P. (1980). The stereotyped nature of school word problems. *For the Learning of Mathematics*, 1(1), 41-48.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution-The social rationality of mathematical modeling in schools. *Learning and Instruction*, 7(4), 309-327.

Schoenfeld, A. H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J. F. Voss, D. N. Perkins, & J. W. Segal (Eds.), *Informal reasoning and education* (pp. 311-343). Hillsdale, NJ: Erlbaum.

Silver, E. A., Shapiro, L. J., & Deutsch, A. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education*, 24(2), 117-135.

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modeling in the elementary school: a teaching experiment with fifth graders. *Journal for Research in Mathematics Education*, 28(5), 577-601.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modeling of school arithmetic word problems. *Learning and Instruction*, 4, 273-294.

Verschaffel, L., De Corte, E., & Vierstraete, H. (1999). Upper Elementary School Pupils' difficulties in modeling and solving nonstandard additive word problems involving ordinal numbers. *Journal for Research in Mathematics Education*, 30(3), 265-285.

Verschaffel, L., Greer B., & De Corte, E. (2000). *Making sense of word problems*. Swets & Zeitlinger: The Netherlands.