‘SENSING’: SUPPORTING STUDENT UNDERSTANDING OF DECIMAL KNOWLEDGE

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Informed by theory and research in inquiry-based classrooms, this paper examines how classroom practices support students’ understanding of decimals. Data from a six-month teaching experiment, based on the work of Moss and Case’s (1999) use of percentages and metric measure as visible representations for students' emerging understanding of decimals, indicated that understanding was significantly influenced by a classroom climate that supported sense making. Sensing, established by a shared expectation, was used and extended in the form of sociomathematical norms associated with mathematical argument and authority.

INTRODUCTION

Construction of robust decimal concepts is dependent upon students engaging in an active learning process in order to integrate prior whole and fractional number thinking and build multi-levelled and multi-connected decimal concepts (Boufi & Skaftourou, 2002). Within New Zealand, decimal fractions are formally introduced to students aged nine or ten years. For students within this age group, constructing conceptual understanding of decimal fractions is traditionally difficult (Moss & Case, 1999). Such understanding requires reconstruction of prior whole and fractional number concepts and integration of place value concepts using base 10 notation to represent the fractional quantities (Irwin, 1999). The process is lengthy with regular recurring misconceptions and partial understandings occurring as students integrate their prior knowledge with new learning along the path to sense making (Condon & Hilton, 1999).

While many research studies on decimals illuminate the nature of student misconceptions (Stacey & Steinle, 1998), or the role of specific teaching activities (Helme & Stacey, 2000; Hiebert & Wearne, 1986), increasingly classroom studies focus on the role of students' informal understandings (Boufi & Skaftourou, 2002). This study reports on a teaching experiment based initially on the work of Moss and Case (1999) that promoted the use of percentages and metric measure as visible representations for students' emerging understanding of decimals. Fundamental to the success of the teaching experiment was a mathematical learning environment that supported students' sense making—an environment in which student reasoning was foremost, with an expectation that they explain, explore alternative ideas, and validate or reorganise their thinking.

This paper explores the defining features of the classroom climate which supported the extension of the making of explanations from beyond a classroom social norm to a sociomathematical norm and the way in which this promoted students’ development of rich understandings of decimals. Our theoretical framework is derived from Cobb and Yackel's (1996) emergent perspective. From this perspective the construction and reconstruction of decimal concepts is assumed to be both an individual and social activity. In such a learning environment, challenges to thinking patterns occur during discussion and debate as students actively engage in making sense of other's explanations, to elaborate, argue, and justify mathematically their own current thinking (Kazemi & Stipek, 2001; McClain & Cobb, 2001; Wood, 2002). Thus, learning involves reflexivity between individual student activity and participation in classroom practices that support collective activity.

RESEARCH DESIGN

The study involved a 6-month classroom teaching experiment conducted at a large inner city primary school. Students came from predominantly higher socio-economic levels and represented a range of ethic backgrounds.

A collaborative partnership between the researchers and the teacher supported the development of a hypothetical learning trajectory and an instructional sequence, which through on-going discourse and data analysis was revised and modified as required (Cobb, 2000). Following individual interviews, four students were selected as case studies to represent the range of decimal misconceptions common to students within the age group. Data were collected from case study student interviews, 15 lesson observations, and classroom artefacts.

Analysis of data used a grounded approach identifying codes, categories, patterns, and themes. These were used in conjunction with participant dialogue in order to give voice to the students as they participated in classroom practices designed to support the construction of decimal concepts.

The research classroom was first and foremost distinguished by the extent and quality of productive discourse that supported students' reflective reconstruction of decimal knowledge. What characterised this classroom socially and intellectually was a shared expectation that all students actively engage in examination, analysis, and validation of their decimal understandings through reasoned discourse (Wood, 2002). Such expectations, while not unique to this classroom, are fundamentally different from that experienced by many primary aged New Zealand students (Walls, 2002).

RESULTS AND DISCUSSION

Establishing classroom norms that support students' understanding requires both in-depth knowledge of the mathematics involved and the students' mathematical thinking (Peterson, Fennema, Carpenter, & Loef, 1989), and the ability of the teacher to advance students' mathematical thinking (Fraivillig, Murphy, & Fuson, 1999). The focus of this paper is to articulate the teacher's development and support of students' understanding of decimal through her expectation, use, and extension of 'sensing'□ a norm, which represented not only active listening and making sense of explanations, but also questioning, clarifying meaning, making predictions and justifying conjectures.

Expecting 'sensing'

The establishment and maintenance of this expectation of sensing was achieved though a variety of pedagogical strategies. Clearly evident in the classroom participation structure was an expectation by the teacher that all students would actively engage, not only physically, but also mentally in all mathematical activity. This included an expectation that individual students were responsible for active listening and making sense of explanations in their collaborative groups and in the large group sharing sessions which occurred at the completion of each teaching/learning episode. The teacher regularly reinforced this notion of 'sensing' through her directives to the class:

> Now while you are listening to the explanations I want you to turn your sensing on, ask questions at any time and search for answers. Listen carefully so that you can predict what might be said next.

Moreover, students consistently maintained the expectation that during any mathematical discussion all explanations should make sense. They recognised that it was their personal responsibility to actively question further in order to understand or clarify an explanation. This is illustrated in the following extract when one student has expressed confusion and another student, assuming collective responsibility, instructs the first student:

> Well just listen to him and see if he can make it clearer otherwise think of some other questions we need to ask him to make him explain it better.

The effect of individual students asking questions not only probed all group members' current understanding of decimal concepts, it supported thinking at a deeper conceptual level, fostered through the exchange of ideas. This is illustrated in the following episode in which three students discuss the ordering of 0.9, 0.9015, and 0.90146.

Sara: That's biggest [pointing at .9] and that's smallest [pointing at .90146].

Fay: But why? Why do you think .9 is the biggest?

Sara: I think that is the biggest because that is the tenths, like that's only tenths so like that's a larger number. This [.90146] goes into something like thousandths and they are smaller bits, so it makes it smaller.

Fay: Yeah but this could be .90 and this [.90146] is just a teeny bit more than .9.

Sara: So is the .9 bigger than .90146?

Fay: [Using decimal arrow cards] But if you see that .9 means 9 tenths but this [.90146] has 9 tenths, one thousandth, 4 lots of tenths of thousandth, and 6 lots of hundredths of thousandths more, so it is the biggest.

Jane: Can you explain that again?

Sara: What she is saying is that one is only nine tenths, but that also has ones of thousandths, tenths of thousandths and hundredths of thousandths, so it's the biggest actually, not the smallest.

Through such discussion a model was developed which represented the group's collective strategy and culminated in a problem solution.

Exploration of problems involving decimal operations that were counter intuitive to whole number thinking supported reconstruction of understandings through cognitive conflict as the students argued a path through to group consensus. This is illustrated in the following episode involving three students' joint construction of a notation scheme to represent the subtraction of 0.7 from 2.30. In response to Eric's recording of 1.60, Brenda using 'whole number thinking' says:

Brenda: So it will be 1.23?

Eric: No it will be 1.60.

Brenda: But that's not .70 it's 7 [pointing at the .7].

Eric : It's .70, seven tenths is 70% [pointing at approximately 70% on an enlarged number line on the floor]

Philip: [Extending the explanation] Oh yes, it's .70 because if that was a 7 there would be a zero and it would be .07.

The listening audience used other students' questions and answers to reformulate their own thinking about decimals. This is illustrated in a written reflection recorded following a problem-solving activity involving a prolonged collective discussion about decimal numbers below 0.1 and the role of zero as a placeholder.

> I lurnt [sic] from Jane's mistake. I had ideas about it and when Helen commented on it I thought back over my thinking, sort of recapping and found out I was write [sic].

In doing so the students are demonstrating a shift from participating in making an explanation—to making the explanation itself an object of reflection.

Extending 'sensing'

Making explanations of mathematical thinking is common practice in many classrooms. However, a fundamental focus of reform-based mathematical classrooms is the way in which explanations extend beyond procedural description to mathematical reasoning based on justification of specific problem-solving strategies. The teacher regularly reinforced the need for explanations to be extended as she instructed the students:

> Don't forget to work together so that you can explain how and why the strategy you used worked for you all. Don't look for a quick solution; prove it this way and then that way then check back to your benchmarks that you use. If you are having problems go back to what you know so that you are all able to go forward again.

During small group problem solving episodes, student questioning and argumentation extended explanations to include justification. The students recognised the power of supporting verbal mathematical explanations or arguments with a demonstration, usually in the form of an invented notational scheme, graphical representation, or physical re-enactment. This is illustrated as the students collaboratively constructed a notational scheme in order to add 34.07 and .005

Jane: So what did you do? Can you show me again?

Sara: We added them together 34.07 and .005 and that gave a score of 34.075 because…

Jane: Why?

Eric: [Pointing at the notation] Those are five thousandth [records 5/1000 on the sheet] and those are seven hundredths [records 7/100 on the sheet].

Fay: But I thought those were a thousandth. I thought the seven was a thousandth.

Eric: [Points at his earlier recording] No those are the five thousandth and those are the seven hundredths, see, and you need ten of those to make one of those but you haven't got ten of those so it's not one of those.

Sara: Yeah so you just leave it like that 34.075

Jane: I still don't get how you got that .075 because plus .07 and .005 it just doesn't make sense.

Eric: Yeah it does because look… [Eric illustrated his point with a concrete model].

Multiple strategies were often recorded in small groups with the most 'sophisticated' strategy presented to the larger class group. Selection of the most sophisticated strategy, while contingent on understanding by all group members, also included 'taken as shared' knowledge of the most efficient or 'tidy' steps. The students took for granted their right to question or challenge explanations, and in response, receive an explanation based on mathematical reasoning. This is illustrated in the following example in which three students solve a problem involving the subtraction of 0.37 from 3:

Stefan: I think it is .63 because if he has got 3 metres and he only wants 2 metres and 37 centimetres then if you add .6 onto that and .03 more it makes 3 metres.

Sara: Yeah but does that work and how are we going to show it?

Stefan: Well on a number line so start at 2.37 and add .6 oh no put the .03 first.

Georgia: But why?

Stefan: I am making a tidy number, like sort of rounding, so everyone can see, and then you just have to put 6 tenths.

In addition, the students understood that collectively and individually they needed to be able to give clear, logical explanations of a group strategy to the larger sharing group. This influenced their behaviour in collaborative groups where they discussed ways to explain and validate their mathematical thinking. The expectation that other students would also be expected to 'make sense' of one's explanation led to extended discussion and translation across representations as the students sought explanations that they deemed would be understood and acceptable to their peers:

Brenda: What's like maybe an easier way of explaining it?

Eric: Oh yeah because remember that Miss Smith said that we all have to understand how to do it so can you guys do that if you use my way? You could do a number line what about that?

Brenda: Yeah cos I go with number lines. I find them easier to understand and so does everybody I reckon.

Eric: Well you do know that you just write down the same strategies but it looks a bit easier to understand.

As students engaged in these mathematical conversations about decimals, the prolonged discourse, which was a feature of many such interactions, frequently led to the reconstruction of students' erroneous thinking patterns.

Using sensing

The intellectual climate established in the classroom not only held all students responsible to participate in 'mathematical conversations'—it maintained an expectation that all members of the listening audience would consider seriously other student ideas proposed during discussion. During group presentations the teacher used the notion of 'wait time' to allow other students the opportunity to make sense of the mathematical concepts being explained and also to support engagement in reflective analysis of similarities and differences of solution strategies with their own group strategies. This is illustrated in the following episode in which the student making an explanation and recording each step numerically is paused by the teacher and asked by another student to justify his thinking:

Adam: Can you explain where you got the .033 from? Oh but I see…

Eric: We're trying to make a tidy number. See three thousandths and seven thousandths is another hundredth then three hundredths and seven hundredth is another tenth. Nine tenth plus one tenth is another whole so that is seven.

Adam: Yeah that's an efficient step.

In this way, the sharing of notational schemes increased students' awareness of more conceptually advanced mathematical thinking:

> Next time I will do William's way because it was easy to do and easy to track and cut down some of our steps. But then again doing it our way I knew exactly what we were doing and as well I was sort of using tidy numbers to make the steps quicker. [Student journal entry]

Verification of mathematical thinking was also an integral part of the process as the students focused their attention on similarities and differences in the mathematical concepts and not merely procedures. In order to maintain focus on the mathematics in explanations, there was an accepted norm that small group solutions were only shared with the large group if the group could verify that their explanation differed mathematically from those already shared. This is illustrated in the following explanation of a strategy to add 1.13 and 1.28.

Brenda: Start from 1.13 and add on .07. Do you know why?

Jane: So it gets you to .2, a tidy number.

Brenda: Then plus 1 metre and 20% of another metre, so 2.4, add .01or 1% so it's 2.41. That's a different way because I was using some percentages as well as some tidy maths and this way is quicker. The last group used fractions and they took lots more steps in that bit…

During large group discussions, listening students also reflected on their own solutions— monitoring the differences and identifying possible errors in their group solution:

Jane: That's different from ours.

Fay: Yeah that is where we are wrong…oh yes see where we added the wrong tenth and hundredth to the wrong ones.

The acceptance of errors as a basis for mathematical inquiry provided ongoing opportunities for all students to engage in mathematical analysis and re-conceptualisation of their developing understandings. Often the teacher would place two group's record of their solutions alongside each other, reinforcing the right of each group to warrant their own explanation:

> These children are checking their thinking, let's give them some thinking time, and then they can explain what they did differently and justify their reasons.

Such 'teachable moments' provided a potential source of cognitive conflict—an essential element in the construction and reconstruction of robust decimal fraction concepts.

In this climate the growth of intellectual autonomy was evident; students demonstrated confidence in their own ability and that of their peers to make mathematical decisions and warrant their own solutions. Rather than appeal to the teacher for help or authority students regularly explored alternative strategies and validated their solutions to decimal fraction problems through alternative representations of percentages, metrics, fractions and decimals.

CONCLUSIONS

The teaching experiment was designed to build on students' informal understandings and strategies. However, descriptions of the learning environment presented in this paper clearly recognise that the teacher needed more than an awareness of the array of possible student misconceptions and current conceptions about decimals; the teacher needed to provide a learning environment that challenged the traditional classroom participation structure for all of the participants. In the classroom involved in this study, a participation structure of collaborative interaction and discourse was central to students' mathematical development, both in its role as supporting individual construction and transformation of decimal concepts and as a social act within the mathematical community.

The participating teacher spontaneously developed the metaphor of 'sensing' to negotiate with her students the shared expectation of sense making as central to the learning process. This strategy overtly positioned the teacher as her own author of an inquiry-based pedagogy. Through her establishment and ongoing support of 'sensing', the teacher provided a classroom environment where all members interactively constituted the social and sociomathematical norms. Their experiences at examining, discussing, and reflecting on their mathematical constructions assisted their on-going development of decimal understandings.

Overall, the classroom environment portrayed a vision of mathematics learning—neither wholly individual nor wholly social—which enabled connections to be made between the person, the cultural and the social. Students appeared to 'learn' from their participation in the cultural milieu of their classroom rather than from other students or the teacher per se (Yackel & Cobb, 1996). The 'sensing' metaphor encapsulated the learning agenda advocated by Kirshner (2002) in which specific mathematical dispositions are targeted for instruction through the development of the classroom microculture. Sensing, as understood by the participants in this study, appeared to go some way to mediate the conceptual and dispositional goals for student learning.

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