INVESTIGATING THE MATHEMATICS INCORPORATED IN THE REAL WORLD AS A STARTING POINT FOR MATHEMATICS CLASSROOM ACTIVITIES

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In this report we present preliminary results of a study on the relationship between informal out-of-school and formal in-school mathematics and the ways each can inform the other in the development of new mathematical knowledge, in this case concerning computation in base 12, 24 or 60. This study is based on a teaching experiment involving a sequence of classroom activities in upper elementary school based on the use of cultural artifacts, interactive teaching methods and the introduction of new socio-math norms, in an attempt to create a teaching/learning environment focused on fostering a mindful approach toward realistic mathematical modeling. In this way we wish to create a new tension between school mathematics and everyday-life experience in which cultural artifacts, incorporating mathematics, can play a fundamental role in bringing students' out-of-school reasoning experiences into play.

INTRODUCTION

The critical problem of how to manage the relationship between informal out-of-school and formal in-school mathematics has been the subject of our studies for some years. Although mathematics learning and practice in and out of school differ significantly, (Resnick, 1987; Nunes, 1993) it seems that the conditions which often make extra-school learning more effective can, and must be re-created, at least partially, in classroom activities. While some differences between the two contexts may be inherent, many can be narrowed if classroom learning processes that are closer to those occurring in out-of-school mathematics practice can be created and promoted. This can be implemented in the classroom, for example, by encouraging children to analyze some '*mathematical facts*' (Bonotto, 2001a) embedded in appropriate '*cultural artifacts*' (Saxe, 1991).

The study presented in this report is part of an ongoing research project aimed at showing how the use of suitable cultural artifacts can play a fundamental role in bringing students' out-of-school reasoning experiences into play, by creating a new tension between school mathematics and everyday-life knowledge with its incorporated mathematics. In particular, the artifacts can be used as motivating stepping-stones to launch, at a first stage, new mathematical knowledge.

This quasi-experimental study involves a teaching experiment based on a sequence of classroom activities in upper elementary school aimed at developing new mathematical knowledge concerning computation in base 12, 24 or 60. In this case the artifact used is a weekly TV guide issued as a supplement of a well-known daily paper. These activities are also based on the use of interactive teaching methods and the introduction of new socio-mathematical norms, as outlined by Yackel & Cobb (1996), in an attempt to create a substantially modified teaching/learning environment. This environment is focused on fostering a mindful approach toward realistic mathematical modeling, i.e. both real-world based and quantitatively constrained sense-making (Reusser & Stebler, 1997). In our approach, informal out-of-school and formal in-school mathematics, despite their specific differences, are not seen as two disjunctive and independent entities. Instead, the aim is a process of gradual growth, in which formal mathematics comes to the fore as a natural extension of the student's experiential reality, as in Gravemeijer (1999).

THEORETICAL AND EMPIRICAL BACKGROUND

In common teaching practice the habit of connecting mathematics classroom activities with everyday-life experience is still substantially delegated to word problems. However, besides representing the interplay between in- and out-of-school contexts, word problems are often the only example providing students with a basic sense experience in mathematization, especially mathematical modeling. During the past decades, a growing body of empirical research (e.g. Freudenthal, Schoenfeld, Verschaffel, De Corte) has documented that the practice of word problem solving in school mathematics promotes in students the exclusion of realistic considerations and a "suspension" of sense-making, and rarely reaches the idea of mathematical modeling and mathematization (see Verschaffel, Greer, & De Corte, 2000, for a review of these studies). Furthermore, it has been noted that the use of stereotyped problems and the accompanying classroom climate relate to teachers' beliefs about the goals of mathematics education (Verschaffel, De Corte, & Borghart, 1997; Asman & Markovits, 2001).

If we wish to establish situations of realistic mathematical modeling in problem-solving activities, changes must be made. In particular, the type of activity used to create an interplay between mathematics classroom activities must be replaced with more realistic and less stereotyped problem situations. These should be more closely related to children's experiential world and meaningful. The extensive use of suitable cultural artifacts could be a useful instrument in creating a new link between school mathematics and everyday-life with its incorporated mathematics (Bonotto, 2001b).

The artifacts we introduced into classroom activities (e.g. supermarket bills, bottle and can labels, rulers, the cover of a ring binder, see Bonotto, 2001a; Bonotto & Basso, 2001; Bonotto 2003) are concrete materials that are meaningful to children as they are part of their real life experience, offering significant references to concrete situations. This enables children to keep their reasoning processes meaningful and to monitor their inferences. As a consequence, they can off-load their cognitive space and free cognitive resources to develop more knowledge (Arcavi, 1994). We can thus make use of children's familiarity with the chosen artifacts and allow them to express their intuitions and produce their own anticipations, in the sense of "*prospective learning*", Freudenthal (1991). These anticipations precede, and may be functional to, any systematic learning process. Furthermore, the double nature of the artifacts, that is belonging to the world of everyday life and to the world of symbols, to use Freudenthal's expression, makes it possible to move from the situations in which it is usually utilized to the underlying mathematical structure and vice versa, in agreement with '*horizontal mathematization*' (Treffers, 1987).

THE STUDY

In this study we decided to use the TV guide from a well-known weekly magazine in order i) to extend students' capacity to calculate from base 10 to b ase 12, 24 or 60, ii) to develop the conceptof equivalence between time intervals expressed in different ways (days, hours, minutes), iii) to introduce informally the concept of fractions.

The children in the classes involved did not know how to carry out computation with hours and minutes, however they all knew how to add and subtract in base 10, and remembered from the previous scholastic year that an hour is made up of 60 minutes.

To check students' familiarity with TV program guides, the experience was preceded by a phase in which children were asked to bring to class magazines and daily papers they usually use to choose TV programs. Magazines read and used only by parents, that led however to discussion within the family, were also accepted. It was found that the timetable of television programs, directly or indirectly, is part of the experiential reality of the children involved in the experience. All said that they knew the starting time and duration of their preferred programs, and that they were able to regulate TV viewing with their daily activities.

Participants

The study was carried out in two third-grade classes (children 8-9 years of age) in a suburb of the city of Padua by the official logic-mathematics teacher, in the presence of a research-teacher. The first class consisted of 20 pupils (10 girls and 10 boys), the second class of 21 (10 girls and 11 boys). In each class there were three children with learning difficulties, and two in the first class and one in the second who displayed demotivated behavior towards school activities.

As a control, two third-grade classes (children 8-9 years old) were chosen from another area of Padua, in keeping with the following criteria: i) congruence of socio-cultural background, ii) homogeneous level of performance with the two classes involved in the teaching experiment, (as confirmed by the outcome of the pretest), iii) use by teachers of a traditional teaching method.

Materials

After time to collect, read and comment on the various TV guides gathered by the children, it was decided that all children should work on the same TV guide in order to be able to manage and organize the classes better. This guide also has a section, on the two following pages, dedicated to a review of the films to be televised, where the starting time, duration, but not the finishing time, can be found. Among the details presented is the date of production from which it is possible to calculate the age of the film.

Procedure

The teaching experiment took place from February to April 2001, for a total of 12 hours a week of class activities, divided into 10 sessions. The first 5 sessions were dedicated to familiarization with the artifact, classification of the various programs according to typology (news, cartoons, films, etc) and to discovering the mathematical facts included, selecting from the many found. The remaining 5 sessions concerned 2 experiences involving two different opportunities offered by the artifact chosen.

In the first experience, using the table of television programs, the children were asked to organize their day, and then the week, keeping in mind their activities and commitments, and not exceeding an hour and a half of television a day.

The second experience, which took place in 2 two hour sessions, was aimed at reading and interpreting the numerical data in the artifact used - this time the reviews of the two films. The aim also included calculating the duration of the two films in minutes and converting them to hours, and finally establishing a strategy to find the finishing time of the film (see fig.1 for the requirements of the second experience). The children were then left free to discover other spontaneous scientific dilemmas, for example the age of the film.

Each session of these two experiences was divided into three phases. In the first, each pupil was given an assignment to carry out individually. The children were asked to answer all the questions in writing, individually. In the second phase, the results obtained through personal reflection and elaboration were discussed collectively, sometimes corrected, and then systematized and re-elaborated. The third was aimed at the elaboration of a collective written text comprising the clearer and more convincing explanations emerging from the whole-class discussion.

Questions asked: Make an evaluation of the information presented in the film review, in particular the time the film ends. Write down the procedure you used.	**Film** ***Courage*** *** Rete 4 / time: 16.00 Producer... With... Review… Comedy Italy 1956 Duration 95' Ó
Questions asked: Make an evaluation of the information presented in the film review, in particular the time the film ends. Write down the procedure you used.	**Film** *The Secret of the Old Forest* ** Channel 5 / time: 0.30 Producer... With... Review… Fairytale Italy 1993 Duration 134' Ó

As far as the control classes were concerned, the class teachers dedicated, within the same time period, exactly 12 hours to class activities regarding reading and calculation of time duration measured in hours and minutes, according to the modality and techniques normally used in elementary school.

DATA

The research method was both qualitative and quantitative. The qualitative data consisted of students' written work, audio recordings and fields notes of classroom observations and audio recordings of mini-interviews with students. The quantitative date was collected by means of pre- and post-tests, administered to the two experimental classes as well as the other two control classes. The two tests were constructed by the official class teachers, not the research-teacher, by taking some items normally used in the bimonthly tests utilized by the same teachers.

Both the pre- and post-tests were organized in such a way as to evaluate the effects of learning on time duration and fractions. The structure of items remained basically the same in the pre- and post-test, although post-test items included more difficult data or figures.

Research questions and hypotheses

In terms of learning processes, it was decided to continue gathering information on the way a particular artifact could be utilized as a motivating stepping-stone to launch new concepts or algorithms.

The first general hypothesis was that the children in the teaching experiment class, thanks to the opportunity they had to refer to a concrete reality (the cultural artifact), explore their strategies and compare them with those of their schoolmates, were able to grasp the calculation of hours and minutes more effectively, and the equivalence between time intervals expressed in different forms (days, hours, minutes) compared with the control class, who received a more traditional teaching method.

It was also hypothesized that using the clock face, which is divided into half and quarter hours, would allow participants to work out the concepts related to fractions according to "*prospective learning*" (hypothesis II).

Furthermore, we hypothesized (hypothesis III) that, contrary to the practice of word-problem solving documented in the literature, children in this teaching experiment would not ignore the relevant, plausible and familiar aspects of reality, nor would they exclude real-world knowledge from their observations and reasoning. Finally children would also exhibit flexibility in their reasoning, by exploring different strategies, often sensitive to the context and quantities involved, in a way that was meaningful and consistent with a sense-making disposition and closer to the procedures emerging from out-of-school mathematics practice.

SOME RESULTS

Some early results from the second experience are reported. From the first film review, all the children except one, were able to elaborate in their own words the information regarding starting time, channel, year of production etc. Of 41 children, 28 noted and commented on the judgment of the review (for example "*as it has 3 stars, it means it is good*"). Some noted that the film had a green symbol (children's viewing) and 16 children calculated the age of the film, even if they were not explicitly asked to do so, therefore activating a problem-posing procedure. 26 children worked out the equivalence "*95 minutes = 1 hour and 35 minutes*" and 29 mentioned the time the film finished. We note the case of a child, who we will call Emanuele, a repeating student with serious scholastic demotivation and learning difficulties. At the end of the first phase, the written report, he handed in a blank sheet. It was therefore decided to test his knowledge and thought processes by individual interview. We discovered that he knew how to read the data in the artifact and how to correctly work out the equivalence by referring to his preferred interest, football. In fact he knew that the duration of a football match is 90 minutes, and that it corresponds to an hour and a half because he always watches sports programs with his father, the most well-known of which is called "*novantesimo minuto*" ("*ninetieth minute*"). Therefore, 95 minutes for him was equivalent to an hour and half plus 5 minutes. The case of Emanuele therefore confirmed our third hypothesis.

Regarding the task for the second review, we found that 15 children calculated the age of the film, 20 correctly interpreted the time 0.30 as "*half past midnight*", 27 children worked out the equivalence between minutes and hours correctly, and 21 worked out the time the film would have finished. During the whole-class discussion the entire class participated with great interest in the problem raised by a child that 0.30 and 24.30 may not be the same times.

Some significant extracts are presented from written work regarding the finishing time of the film. These show the activation of strategies sensitive to the context and quantities involved and also the emergence of problem posing activities.

Claudia: *"I pretended that the film started at exactly 0. I put the 30 minutes to one side. I added 2 hours and that makes 2. I added the 30 minutes and so I got 2 and 30. I added the 14 minutes and so arrived at 2 hours and 44 minutes."*

Claudia tried to simplify the data as much as possible to be able to calculate with greater surety. The explanation was extremely clear, expressed in the language and terminology normally used by children, and for these reasons during the class discussion it led to curiosity, attention, understanding and participation by classmates who were unable to find the finishing time.

Gregorio's protocol included the following:

"1) I found 2 hours and 14 minutes in this way: 60+60=120+14=134. 2) To arrive at 2.44 it was 0.30+2 ore=2.30+14=2.44. 3) To get 8 years we worked out 1993 to arrive at 2001 makes 8 years. We can see 51 minutes of the film "The executors".

It can be seen that in the end that Gregorio faced a spontaneous dilemma with a film whose review was next to the one assigned and whose viewing time partially overlapped. He posed the question *"Once the film "The Old Forest" is finished, how much of the film "The Executors" can I watch?"*. This shows how the use of an artifact may evoke situations that are in fact experienced, activating the ability to pose and resolve problems.

On the basis of the qualitative results we can say that this experience has reinforced knowledge of the hours in the day and led to calculation in base 60 by means of an informal, non conventional, procedure on the basis of intuition linked to the context or the quantities involved. Among the children's protocols, attempts at formalizing calculation in rows and columns also appeared.

As far as the outcomes of the pre- and post-tests are concerned, the errors in the experimental group diminished by 46%, while those of the control group remained more or less stationary. The two parts of each test are outlined, that is the first part testing reading ability, calculation of hours and the equivalence between time intervals expressed in different forms, and the second part regarding knowledge of the concept of fractions. It emerged that the greater improvement in the experimental group's performance is relative to the abilities tested in the second part of the test, where the concept of fractions was evaluated. There was in fact a 63% reduction in errors in the case of the experimental group, while errors increased for the control group. This is also documented by the statistical analysis that we are elaborating.

CONCLUSION AND OPEN PROBLEMS

From the results it appears that the teaching experiment had a significant positive effect on achieving learning goals, in particular enhancing and understanding the calculation of hours and minutes and the equivalences between time intervals expressed in different forms, and enhancing a first approach to the concept of fractions in a way that is meaningful and consistent with a sense-making disposition. This was not the case in the control group where an increase in errors was found in the second part of the test. It could be supposed that the control group, who received a more traditional type of teaching, may have acquired general algorithmic procedures and formal rules, but these were not well mastered and therefore did not improve performance.[1] The first two research hypotheses were therefore confirmed.

It was also confirmed by the qualitative results that using the TV guide did not activate rigid and general algorithmic procedures but rather specific heuristics, that have an inner consistency and value. The strategies were flexible, local and sensitive to number sizes (hypothesis III), and were such that children often made reference to parts of the hour (half and quarter hours) to be able to manage calculations better. This aspect made them more sensitive to the concept of fractions according to *prospective learning* and therefore led to the distinct improvement (63%) by the experimental classes in the second part of the post-test.

Furthermore by asking children i) to select other cultural artifacts from their everyday life, ii) to identify the embedded mathematical facts, iii) to look for analogies and differences, iv) to generate problems (e.g. discover relationships between quantities), the children were encouraged to recognize a great variety of situations as mathematical situations, or more precisely "mathematizable" situations.[2] In this way, children are offered numerous opportunities to become acquainted with mathematics and to change their attitude towards mathematics.

For the real possibility to implement this kind of classroom activity, there needs to be a radical change on the part of teachers as well. They must try to i) modify their attitude to mathematics; ii) revise their beliefs about the role of everyday knowledge in mathematical problem solving; iii) see mathematics incorporated in the real world as a starting point for mathematical activities in the classroom, thus revising their current classroom practice. Only in this way can a different classroom culture be attained. On the basis of the experience of this and our other studies, we entirely agree with Freudenthal (1991), that the main problem regarding rich contexts is implementation requiring a fundamental change in teaching attitudes.[3] As in other studies (Verschaffel et al., 1999), the effective establishment of a learning environment like the one described here makes very high demands on the teacher, and therefore requires revision and change in teacher training, both initially and through in-service programs.

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[1] Many studies have pointed out that local strategies developed in practice are more effective than arithmetic algorithms which are usually taught in school to give students powerful general procedures, but which are, in fact, often useless in out-of-school contexts (Schliemann, 1995).

[2] We think that the development of a problem-posing program integrating the promising outcomes of recent studies (English, 1998; Silver et al., 1996) should also permeate the entire curriculum.

[3] Cultural artifacts used, and the way they have been used, can be considered as " *contexts*" or " *rich materials*" in the meaning given by Freudenthal, who underlines their qualities through a comparison with structured material.

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