ATTENTION TO MATHEMATICAL STRUCTURE DURING PARTICIPATION IN A MATHEMATICS CLASSROOM TASK BY LEARNERS OF ENGLISH AS AN ADDITIONAL LANGUAGE (EAL)

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How do students who are learning English as an additional language (EAL) participate in classroom mathematics? To investigate this question, I collected recordings of primary school EAL students as they worked with peers on a mathematics classroom task. Drawing on theoretical and methodological work in discursive psychology (Edwards, 1997), I analysed students' interaction and identified several patterns of attention, including a pattern of attention to mathematical structure. In this paper I report how the students in the study used attention to mathematical structure as part of their joint work on the task. My analysis shows how EAL students are able to participate successfully in mathematics classroom interaction. My analysis also highlights the social nature of this interaction.

INTRODUCTION

The aim of the study reported in this paper was to investigate how students learning English as an additional language (EAL) [1] participate in classroom mathematics in the UK. A key starting point was the observation that such students *are* able to take part in mathematics lessons. Rather than see multilingualism as a problem, therefore, I wanted to explore *how* such students participate in doing mathematics.

There are approximately 500,000 EAL students in the UK, most of whom participate in mainstream mathematics lessons. There has, however, been no sustained research in the UK into the nature of this participation and its relation to the teaching and learning of mathematics and little research elsewhere in the world. The research that has been conducted (e.g. Adler, 1995, 2001; Khisty, 1995; Khisty *et al.*, 1990; Moschkovich, 1996, 2002; Setati, 2002) largely focuses on the teacher's perspective and is based in classrooms in which more than one language is used during mathematics lessons. In the UK, the home languages of multilingual students are *not* normally heard in mathematics lessons. This situation raises the question: How do EAL students participate in their mathematics lessons? This paper reports results from a recent study designed to address this question. As there has been little previous research in this area, I was not attempting to identify features of participation in mathematics classroom interaction that were *specific* to EAL students. The aim was to counter deficit model assumptions about EAL learners (described by Moschkovich, 1996) by exploring the nature of such students' participation.

THEORETICAL FRAMEWORK

The participation of students from a diverse range of cultural and linguistic backgrounds presents a challenge for the analysis of classroom interaction. In particular, it is difficult for me, as a researcher from one cultural and linguistic background, to make assumptions

about what students from different backgrounds mean by what they say. Although we may use the same words, what we mean by those words may be quite different. To address this issue, I have developed a theoretical and methodological approach (Barwell, 2001) drawing on discursive psychology (Edwards, 1997; Edwards & Potter, 1992) and conversation analysis (Sacks, 1992). This approach centres around the notion of attention.

For discursive psychologists, language is conceptualised as primarily "a medium of social action rather than a code for representing thoughts and ideas" (Edwards, 1997, p. 84, orig. emph.). Social action is foregrounded as the primary function of language, which is seen as having evolved through social interaction, and therefore as being structured both by and for social interaction. Thus 'psychological' notions, such as thinking, meaning or attending are examined discursively, to understand, for example, how participants in interaction construct thinking, and what different ways of constructing thinking might achieve. Taking this perspective therefore avoids attempting to say what EAL students mean by their words, in preference for analysing how they use language to construct mathematical thinking or meaning.

In the study reported in this paper, I focused particularly on the notion of attention. By attention I do not mean an internal aspect of focusing the mind on some aspect of the outside world (see, for example, Mason & Davis, 1988). Instead I treat attention discursively, drawing particularly on what conversation analysts call 'participants' attention' (Sacks *et al.*, 1974). This view of attention arises from the social organisation of talk, which includes features such as taking *turns* to speak, so that successive turns form an unfolding sequence of interaction in which each turn builds on what has gone before. For this to be possible, participants must indicate through their words, what of the preceding interaction is relevant to their current contribution. Through their words, therefore, participants display *explicit* attention to some aspect of the immediate past. Since this attention is explicit, however, it is also available to analysts. A first level of analysis therefore consists of identifying *patterns* in students' attention.

Language, of course, is highly flexible. There are many possible ways of explicitly attending to relevant aspects of preceding interaction. A second level of analysis therefore examines *how* attention is brought about, and what particular ways of attending achieve in terms of the on-going socially organised talk. Before outlining some of the results of this form of analysis, I will outline the nature of the data collected.

DATA COLLECTION

The study focused on two Year 5 (9-10 years) classes taught by the same teacher in consecutive years. Over the two years, 10 EAL students participated in the study. Since interaction involving the teacher was heavily cued by the teacher's contributions, I recorded students working together without a teacher present. The primary data therefore consisted of transcripts of pairs of students working together on a mathematics classroom task. Both the task and the pairings selected were based on ethnographic observations of mathematics lessons over several months. The task required students to work together to write arithmetic word problems with some general focus, such as addition or division. Students were paired in a variety of ways including: 2 EAL students sharing a home

language; 2 EAL students from different language backgrounds; 1 EAL student with one monolingual student; 2 monolingual students. Altogether I collected and transcribed 20 recordings of students working on the word problem task. I also collected a variety of other data, including: copies of students' work, information about students' attainment; interviews with teachers; classroom observation notes and video-recordings of mathematics lessons.

ATTENTION TO MATHEMATICAL STRUCTURE

My discursive analysis of EAL students' interaction revealed four patterns of attention [2]. In this paper I focus on one: students' attention to the mathematical structure of their emerging word problem. For a definition of the structure of a word problem I drew on Verschaffel, *et al.* (2000):

the nature of the given and unknown quantities involved in the problem, as well as the kind of mathematical operations(s) by which the unknown quantities can be derived from the givens...[and] the way in which an interpretation of the text points to particular mathematical relationships (Verschaffel, Greer & de Corte, 2000, p. x).

This definition does not concern any particular operation which may be used (or expected to be used) to solve a word problem, but is concerned with the mathematical relationship between the various quantities which occur in such problems. In examining EAL students' attention to the mathematical structure of their word problems, I am not attempting to categorise the structure of their problems for myself. My task is to analyse how the students themselves attend to aspects of the problem structure, as *they* see it.

I will illustrate the pattern of attention to mathematical structure, before setting out some of the ways in which this attention was used by students. In the following extract, Tahira (EAL) and Verity (non-EAL) are starting to work on writing a new word problem. I have asked them to write problems 'about' division (for transcription notation, see [3]).

Tahira and Verity

36	T	I can think of times one
37	V	mm
38	T	if/[if there were um
39	V	[y-
40	T	if you had um/ twelve sweets
41	V	twelve/ sweets/
42	T	and you had um/
43	V	you had/ how many people?/ six people
44	T	six people
45	V	yeah 'cause half of twelve is six/ if you/ had/
46	T	six people
47	V	six people/ how many sweets/ how many sweets would they get each/ ac-
48		no/ actually/

- 49 T twelve divide six
- 50 V yeah twelve divided by six/
- 51 T do times
- 52 V no we're not allowed to do times/

In this extract, the two participants attend to mathematical structure on several occasions. Tahira offers to 'think of times one' (line 36) as the opening move in writing the problem. I cannot say whether she actually has a problem in mind; only that she explicitly attends to the structure of the future problem. The two students then propose and negotiate various generic aspects of the problem (see Gerofsky, 1996), including a scenario about a quantity of sweets. Having considered 'six people' (lines 43-44), Verity attends to mathematical structure again, this time in the form of an arithmetic relationship between 12 and 6, ''cause half of twelve is six' (line 45). Following further discussion of the wording of the problem, Tahira attends to the same relationship using different words, 'twelve divide six' (line 49). This attention to structure is maintained by Verity, who reformulates Tahira's statement (line 50). Tahira then continues the attention to structure, saying 'do times' (line 51), a request which Verity rejects. Thus at several points in the above sequence the two students attend to mathematical structure, attention which is seen as co-constructed by both participants. Throughout the data collected in this study, EAL students and their peers attended to mathematical structure as part of the process of creating word problems, as in the above extract.

SOME USES OF ATTENTION TO MATHEMATICAL STRUCTURE

Analysis of *how* the students who took part in the study used attention to mathematical structure as they created word problems together revealed a number of uses, including:

- justifying generic details,
- managing the social relationship between the participants,
- critiquing the emerging word problem,
- modifying word problem structure.

The first three of these uses are illustrated by the sequence involving Tahira and Verity. For the fourth, I will use another extract from the data. I will address each use in turn.

Justifying generic details

Tahira and Verity discuss a number of generic details for their problem, including the number of sweets and the number of people. One of the features of the word problem genre is that such details are essentially arbitrary (Gerofsky, 1996). In generic terms, it makes little difference how many sweets or people there are. In the above sequence, mathematical structure is used as a way of *justifying* the choice of particular numbers:

- 40 T if you had um/ twelve sweets
- 41 V twelve/ sweets/
- 42 T and you had um/
- 43 V you had/ how many people?/ six people
- 44 T six people

45 V yeah 'cause half of twelve is six/ if you/ had/

Verity uses attention to structure, 'cause half of twelve is six' (line 45), to justify her suggestion of 'six people' (line 43). Indeed it is difficult to see "half of twelve is six" acting in any other way. Consider some alternative ways this exchange could have been. Verity could have said 'you had/ six people/ half of twelve is six.' The recourse to structure seems to act as a justification, even without the causal language of "cause". Even if the structure is introduced first, it still appears to act as a justification. Verity could have said, for example 'half of twelve is six/ so let's do if you had six people.'

Managing the social relationship between the participants

What, socially, is achieved by using attention to structure in the way described above? Other forms of justification are possible. Verity could have said 'trust me', or 'I know best', or 'obviously', all persuasive devices rather than accounts for her ideas. She could have invoked previous experience of word problems, remembering a particular problem or citing the teacher, for example. All these possibilities, however, rely on Verity and therefore have implications for the social nature of the discussion. In particular such approaches require Tahira to accept either Verity's judgement or her memory. Attending to mathematical structure, by contrast, distances Verity from her account, so making it both more authoritative and less personal (Edwards and Potter, 1992, p. 162). By attending to mathematical structure, Verity draws on the objective rhetoric of mathematical reasoning. In mathematical discourses, the arithmetic relationship between numbers is not seen as a personal matter. By drawing on the objective authority of mathematical discourse, Verity avoids forcing Tahira to submit to Verity's personal authority. Thus attention to mathematical structure is also used to manage the *social relationship* between the two participants.

Critiquing the emerging word problem

At the start of the above sequence, Tahira offers to 'think of times one' (line 36). Towards the end of the negotiations, she brings attention to the operation required to solve the emerging word problem. She formulates this attention in terms of division:

- V six people/ how many sweets/ how many sweets would they get each/ ac no/ actually/
 twelve divide six
- 50 V yeah twelve divided by six/
- 51 T do times

Tahira is characterising the problem as one of division, a proposal with which Verity agrees. Tahira then argues for a different kind of problem, 'do times' (line 51) [4]. The form of Tahira's initial attention to structure (line 49) sets up 'do times' as a contrast, so that her words act as a *critique* of the problem which has been created. This critique ties in with Tahira's opening offer to 'think of times one', a task which has not been completed. A frequent use of attention to mathematical structure in the data collected in this study is to critique word problems in this way.

Modifying word problem structure

The above uses of attention to mathematical structure can be seen in the sequence shown below, which also illustrates an additional use: modifying word problem structure. The sequence features Cynthia (EAL) and Helena (non-EAL) and is taken from the last few lines of the transcript of the students' work on their first problem. They have been asked to write problems 'about' addition. Prior to this sequence, Cynthia and Helena have agreed on and Helena has written down: "Daniel has a job he gets pay £415 in a month". They are now negotiating the question which will conclude the problem.

Cynthia and Helena

100	C	how many in a week/ no oh yeah/ how many in a week
101	Н	() okay then/ how many/ how much money does he get/ in
102		a year /
103	C	in a week
104	Н	a week?
105	C	no that's ()
106	Н	no cause/ you said in a month /
107	C	yeah/ no/ I said/ [no/ I said/ Daniel has a job he gets paid four &
108	Н	[how many
109	C	& hundred and fifteen pound in a month/ how many in a week
110	Н	how much he gets
111	C	yeah/ how-how much he get/ on one week
112	Н	that's dividing innit
113	C	oh yes that's divide/
114	Н	that's sort of like dividing cause there's four/ four weeks in a month so
115		that's four divided by (three) I mean four hundred and fifteen
116	C	I'll just do/ how many in a year//

In this sequence, the two students disagree over whether to conclude with a question about 'how many in a year' or 'how many in a week'. In generic terms, either would be acceptable. Their negotiations consist of each student restating their preferred question until Helena shifts attention to mathematical structure, characterising the use of week as 'that's dividing innit' (line 112). Cynthia agrees and Helena then elaborates through continued attention to the structure of the problem. This attention to structure then provides a basis for Cynthia to accept Helena's preference for the questions 'how many in a year'. Thus for Cynthia, a division problem is transformed into what is potentially an additive problem. Attention to mathematical structure is therefore implicated in *modifying* the word problem structure.

CONCLUSION

The analysis reported in this paper shows how the EAL students in this study are able to participate in the use of attention to mathematical structure to conduct a number of social actions, including: justifying generic details, managing the social relationship between participants, critiquing emerging word problems, and modifying word problem structure.

These findings highlight the close inter-relation between social and mathematical concerns. Attention to mathematical structure is an intrinsic part of the mathematical nature of the word problem task. At the same time, this attention is used to conduct social actions and to manage the on-going relationship between the students, as for example, when the objective nature of mathematics is used to render a point of view more authoritative and less personal. Finally, these findings show some of the ways in which EAL students *can* participate in mathematics classroom interaction, at least in the context of the word problem task.

NOTES

- 1. English additional language (EAL) refers to any learner in an English medium environment for whom English is not the first language and for whom English is not developed to native speaker level. Native English speakers are described simply as monolingual.
- 2. The 4 patterns of attention are: attention to the word problem genre, attention to narrative experience, attention to mathematical structure and attention to written form.
- 3. Transcription conventions: Bold indicates emphasis. / is a pause < 2 secs. // is a pause > 2 secs. (...) indicates untranscribable. ? is for question intonation. () for where transcription is uncertain. [for concurrent speech. & for utterances which continue on a later line.
- 4. The interpretation 'do times' is supported by Tahira's intonation.

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