Edith Cowan University

An examination of the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students

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Degree of Doctor of Philosophy

By

Jemmy Emmanuel Georges Louange

BEd Honours (Mathematics Education), Sussex University, UK

Principal Supervisor: Dr. Jack Bana

School of Education
Faculty of Education and Arts
Edith Cowan University Perth Western Australia

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Use of Thesis

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Abstract

Independent studies of teaching for number sense and problem solving have revealed that teaching for either of them separately poses a great challenge for the teacher. Yet research focussing on the relationship between number sense and problem solving was virtually non-existent, although the relationship between students' number sense and problem solving ability was becoming more and more evident through various modes and endeavours.

Since both number sense and problem solving were being promoted as two of the major areas of emphasis in mathematics education, there was an urgent need to answer questions such as, "How do they relate to each other in terms of how they are taught, learnt and utilised in solving mathematical problems?" Moreover, teachers were being challenged to ensure delivery of a balanced curriculum, while simultaneously having to develop the number sense and problem solving ability of the students. Hence, it was important to learn how they went about satisfying the latter. This implied that there was also a need to discover whether successful teachers of number sense and problem solving necessarily employed or had a specific teaching style.

This study sought to explore what sort of relationships exist between students' number sense and their problem solving ability, and the contribution of the teacher's teaching style and the students' learning style towards students' performance in these two respective areas. The problem solving ability and number sense proficiency of three classes of Year 7 students, from three metropolitan primary schools, were compared to their learning style, and their mathematics teacher's teaching style. Sixty-eight students, comprised of twenty-six males and forty-two females, and their three Year 7 teachers were involved in this study. Of the three schools, two were private — a boys only and a girls only — and one was a mixed gender state school.

A mixed methods design was employed through which a combination of the ethnographic approach, the case study, framework approach and grounded theory was applied, to investigate the relationships between students' number sense and problem solving abilities, and the teaching and learning style compatibility which promote such abilities. Hence, the method followed was both quantitative — by scoring of test results and quantification of qualitative data — and qualitative — through observations and

tape-recorded interviews. Each teacher and respective students were observed in multiple teaching-learning situations as well as outside the teaching-learning context, and these observations and field notes were documented. Performance data were collected through pre- and post-tests of number sense and problem solving, and also through activities and exercises done in class; the latter were used solely for data validating purposes. A combination of the Think-Aloud and Stimulated Recall Interview (TASRI) protocol was used to gain an insight into how students solved the problems presented and to elicit responses about their thinking at the time the behaviour occurred. A ready-made Index of Learning Style (ILS) inventory was used as a means of ascertaining the learning preference modality of both the students and their teachers. The three teachers were also interviewed both formally and informally and a Teaching Style Inventory (TSI) was used to gather information as to their preferred teaching style and as a means of corroborating the data collected through classroom observations, field notes and interviews. The three principals, two deputy heads and two curriculum coordinators were also interviewed. On-site perusal of various written documents was also carried out.

When triangulated, data obtained from the qualitative observations and interviews, and the quantitative teacher ILS and TSI, suggest that although these three teachers tended to use different teaching approaches, their focus was more on getting students to understand the rationale behind any concept and process under discussion. These teachers taught to the ability of the students first, and in so doing they considered individual learning preferences, although the former was given a lot more prominence than the latter. Classroom observations, student interviews and data gathered through the ILS tended to indicate that although all three teachers expressed a strong preference for receiving information through the verbal learning modality, they taught largely through the visual mode and employed the verbal mode mainly for discussions, with very little teacher exposition. This could be one reason why a large majority of students showed a preference for receiving information through the visual learning modality. This interpretation was supported by the results obtained from the Number Sense (NS) and Problem Solving (PS) tests.

The combined Think Aloud Stimulated Recall Interview (TASRI) protocol revealed that when attempting to solve Number Sense Inherent Problems (NSIP), many of the students with lower number sense seemed to identify and solve a different problem. It is worth noting that such students verbally expressed that they preferred

solving Devoid of Number Sense Problems (DNSP). On the other hand the ability to solve both NSIP and DNSP seemed to increase with level of proficiency in number sense. Both the pre-tests and post-tests results revealed that there is significant correlation between students' number sense and problem solving ability. Performance gain analysis indicated that most students' number sense and problem solving performance improved, and the teaching style of the teacher could be one of the main factors responsible for such an improvement.

The recommendations made pertain mainly to classroom teaching, learning and further research. A striking revelation is that teachers' should find ways and means of combining both contemporary and traditional teaching theories and methods so as to enhance the quality of their students' learning experiences. Although outcomes based education has many advantages, preparation of lessons should not only shift from a focus on content but also pay equal attention to catering for individual learning ability, which is closely tied to learning style. Teachers need to find ways and means of identifying the number sense proficiency level, problem solving ability and also learning preference of their students in order to be able to mathematically empower the latter. Nevertheless, the evidence seemed to point more towards learning preference being dynamic, eclectic and dependent upon many factors, some of which are teaching mode, preference for solving a particular type of problem, the topic under study and the actual mood of the student. Hence, the notion of focusing on learning style was more or less rejected in favour of focusing on students' individual differences which comprised individual affective dispositions, learning strategy preference, problem solving style preference, and academic ability.

There is a need for some form of experimental research to discover the impact of specifically catering for individual learning style upon students' number sense and problem solving proficiency level. Furthermore, this calls for coordinated research to inform the construction of a more specific inventory to gauge the number sense and related problem solving style of Year 7 students.

Declaration

I certify that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution in higher education; and that to the best of my knowledge and belief does not contain any material previously published or written by another person except where due reference is made in the text.

Signed:

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Chapter 1: Introduction

In Chapter 1 of the National Council of Teachers of Mathematics' (2000) *Principles and Standards for School Mathematics*, an idealised vision for school mathematics is presented as such:

> Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by the teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it. (p. 3)

An analysis of this vision reveals a revolving spiral with all its component elements orbiting around teachers helping students become effective problem solvers. This problem-solving theme is made to stand upon four major tenets:

- knowledgeable and reflective teachers;
- high-quality and technologically engaged mathematics teaching-learning;
- mathematically confident and literate students; and
- an integrated curriculum with flexible work settings.

Although not explicitly stated, from these emerge a picture of teachers who value diversity in their students' learning styles and adapt their teaching styles to suit and cater for such diversity. Hence, the envisioned standard for teachers of mathematics is to employ the teaching style(s), which accommodate(s) the differences in students' learning styles, so as to produce good problem solvers. Subsequently an emerging theme, which could be hidden in this vision, is developing students' number sense.

The present study intends to gauge the extent to which such a vision exists in the reality of day-to-day teaching of mathematics in three Year 7 classrooms, within the context of the relationships among four major variables.

1.1 Background to the Study

Reforms in mathematics education have seen many changes in both the content to be taught and learnt, and the teacher's workload. Such reforms have reached a much anticipated stage, which requires that teachers make use of lesson-enriching methods based upon and/or incorporating concepts and activities, such as number sense, problem solving, games, cooperative learning, a laboratory approach, constructivism and other contemporary ideas.

Teachers now have access to such a wide range of research advice and suggested teaching methods which could be used to better, or in some cases replace, traditional teaching methods, that many of them are finding it difficult to keep up with the demand to put into practice all those important ideas gained or developed through research. Such calls have caused others, such as Grasha and Yangerber-Hicks (2000) to express their concerns that with so many choices, it is sometimes difficult to determine which method, approach and/or theories should be used to best enhance the learning experience of students. Nevertheless, one cannot just take a no-more-suggestions approach to the whole situation, as a means of easing the workload of teachers.

Under the sub-title *The Need for Continued Improvement of Mathematics Education*, NCTM (2000) laments the fallen-onto-deaf-ears situation that exists vis-à-vis the vision described above.

> Despite the concerted efforts of many classroom teachers, administrators, teacher-leaders, curriculum developers, teacher educators, mathematicians, and policymakers, the portrayal of mathematics teaching and learning in Principles and Standards is not the reality in the majority of classrooms, schools, and districts. (p. 5)

Yet many primary teachers may believe that they are incorporating number sense, problem solving and many of the proposed recommendations into their teaching. Perhaps the mismatch between what authorities expect and what teachers believe, and are doing could be due in part to teachers seeing these recommendations as being too demanding. Or it could be that both sides are focussing on number sense and problem solving, whereas the influence of two other major contributors, namely teaching style and learning style, have been overlooked altogether. Educational researchers and practitioners are now giving greater attention to teaching style and learning style. According to Borg and Stranahan (2002) learning theory suggests that students will do better in a class when their learning styles are similar to the instructor's teaching style. The NCTM (2000, p. 5) statement, just cited above, acknowledges the fact that there are reflective teachers of mathematics, although many teachers of the subject could be teaching in a state of limbo. In fact, all teachers of mathematics should be, or should be aiming to become, reflective teachers. Such teachers need to upgrade their knowledge and practice of teaching and learning, and in many instances research provides many suggestions for improvement.

At the start of the 1980s the NCTM put forward a much-publicised statement, that "problem solving must be the focus of the mathematics curriculum" (p. 1). The NCTM (1989) reinforced problem solving as the dominant theme of that decade by adding that it should be central to all mathematics programs. As we entered the 1990s there were other calls to incorporate newer teaching methods/approaches and other aspects of mathematics into teachers' repertoires. Most recently there has been increased interest in number sense (Australian Education Council, 1991; National Research Council, 1989a; NCTM, 1989b, 2000), at least from researchers' and mathematics teacher educators' point of view. The literature suggests that it is the belief of most modern educational researchers and policy makers, that the study and development of number sense abilities in students should be an integral part of the mathematics curriculum K-12. For instance, NCTM (2000) emphasises that the mathematics curriculum should not be restricted to number sense only, since other goals for mathematics education are also important. According to NCTM (2000), there is a need for a balanced curriculum which would include number sense as well as practical applications, theoretical development, and problem solving.

Since both number sense and problem solving are being advocated as two of the major areas of emphasis in mathematics education, it was felt that there was an urgent need to answer questions such as "how do they relate to each other in terms of how they are taught, learnt and utilised in resolving mathematical problems". If teachers are feeling constrained by having to ensure delivery of a balanced curriculum, while simultaneously wanting to develop the number sense and problem solving ability of the students, it is important that we learn how they go about satisfying the latter. Therefore, there was also a need to discover whether successful teachers of number sense and problem solving necessarily employ or have a specific teaching style.

Hence, this study endeavoured to examine the interplay between the four major emergent issues of number sense, problem solving, teaching style and learning style in the teaching and learning of mathematics.

1.2 Significance of the Study

In subjects such as History (Benson & Eaves, 1985; Booth & Kamal, 1993; McCarthy & Anderson, 2000; McCoy, 2001), Language Learning (Carson & Longhini, 2002; Littlemore, 2001), Science (Ballone & Czerniak, 2001; Felder, 1993) and Design and Technology (Smith, 2001) research has been carried out to identify learning styles that are more suited to the learning of these particular subjects, while in mathematics there seems to be very limited effort in that direction (Parrino, 1997; Sloan, Daane, & Giesen, 2002). Therefore, it was appropriate at this point in time to find out whether there are teaching styles which lend themselves best to enhancing students' number sense and problem solving abilities in mathematics.

Good number sense and problem solving ability are being heralded as the cornerstone of mathematical power (NCTM, 2000). Since teachers are also individually different, it would be interesting to understand how they manage to cater for students' individual differences when it comes to developing the latter's number sense and problem solving ability.

The literature available did not answer this question to this researcher's satisfaction. Hence, this research was motivated by a sense of wanting to know, document and report what actually could be happening in mathematics classrooms with respect to these four factors. Such knowledge will be valuable in providing an applicable framework, which would act as a springboard for further enhancement in the teaching and learning of number sense and problem solving as a means of mathematically empowering the learner, and as a reference point for further research.

There is a limited amount of literature devoted to the connections between number sense, problem solving and teaching and learning styles. The literature review conducted offered weak empirical support for the ability of teaching style to promote number sense and problem solving. Moreover, no research appears to have been done to ascertain whether those with good number sense have better, equivalent or worst problem solving ability than those with poor number sense. If there is such a relationship then there could be certain teacher competencies that are particularly relevant to the combined teaching of number sense and problem solving. The findings that could be generated through such a study can have certain benefits for teaching and learning which would contribute towards enhancing both, with respect to number sense and problem solving ability.

1.2.1 Teaching styles' impact on developing students' number sense

It seems that amongst teachers of mathematics, especially those at primary level, there is a tendency to teach most mathematics topics using the same single approach, in which children are not encouraged to be active thinkers, and skill and drill approaches continue to be the prevalent method of teaching children (Barrett & Allen, 1996). Such a situation could make it even more difficult to teach for the development of number sense, since this calls for adaptation of teaching style or even coming up with new approaches. Combined with this lack of an effective approach to teaching for development of number sense, is the uncertainty of what is meant by the term number sense, which in effect plays a role in shaping the teaching approach the teacher adopts. Hence, a teacher's perception of what such a term means, its relative importance, and ways in which teachers purposefully plan for, promote and support the development of number sense in their students will have great impact upon how they teach the subject.

1.2.2 Teaching to produce good mathematical problem solvers

Research done so far has concentrated mainly upon how children solve problems and some other related aspects (Anderson, 1998; Charles, Lester, & O'Daffer, 1987; Collis & Romberg, 1992; Curtis, 1995; DeBellis & Goldin, 1997; Dole, 1999; Dougherty & Matsumoto, 1995; Gonzales, 1996; Gonzales, 1996; Hacker, Dunlosky, & Graesser, 1998; Johnson & Johnson, 1996; Lesh, 1981; Lowrie, 1999; Nisbet & Putt, 2000; Schoen & Oehmke, 1980; Silver & Cai, 1996; Stacey, Groves, Bourke, & Doig, 1993; Stoyanova, 2000; Wilson, 1993). Once teachers have a notion of what type of teaching promotes or hinders the development of good problem solvers they will be empowered to become more reflective and accommodating in their teaching.

1.2.3 Modelling the relationship between teaching style and learning style, and number sense and problem solving

There is a need for a model which shows the relationship between teaching style and learning style, and their impact upon developing students’ number sense proficiency and problem solving ability. Up-to-date research pertaining to mathematics education lacks a unifying theory that will bring all the important components of research findings together. Although this study does not attempt to do such a mammoth thing, one of its most important contributions will be in providing teachers with necessary insights into how these very important components could be made to work together in a classroom situation.

1.2.4 Students' perceptions of an effective number sense and problem solving mathematics teacher

Research done in this area has mostly concentrated upon the teaching of mathematics in general, teachers' problem solving beliefs (Anderson, 1996), and recently on effective teachers of numeracy (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997). Since both number sense and problem solving are considered two of the most important aspects of mathematics learning (NCTM, 2000), it is necessary that teachers gain access to specific evidence of students' perception of their teaching in these two areas.

1.3 Theoretical Framework

The literature provides very little information about a theoretical framework incorporating the relationship between number sense and problem solving. Furthermore, no literature exist which make specific mention of teaching style and learning style as part of a framework involving number sense and problem solving. Whatever theoretical frameworks have been proposed have most often focussed on one of these four variables, in the case of number sense and problem solving, and in other instances on the link between teaching style and learning style. Hence, the framework being proposed stems from an amalgamation of information which suggest some form of relationship, especially between number sense and problem solving.

Smith (1989) proposed that problem solving be taught first as a separate area of mathematics, and then the skills developed would be incorporated within the whole program. Figure 1.1 is an attempt to illustrate the relationship between problem solving, number sense, teaching and learning using Smith's (1989) stance that:

> ideally the development of problem-solving skills should be fully integrated within the maths program, with students not only learning about problem solving, but also using their problem-solving skills in the development of other areas of mathematics. (p. 209)

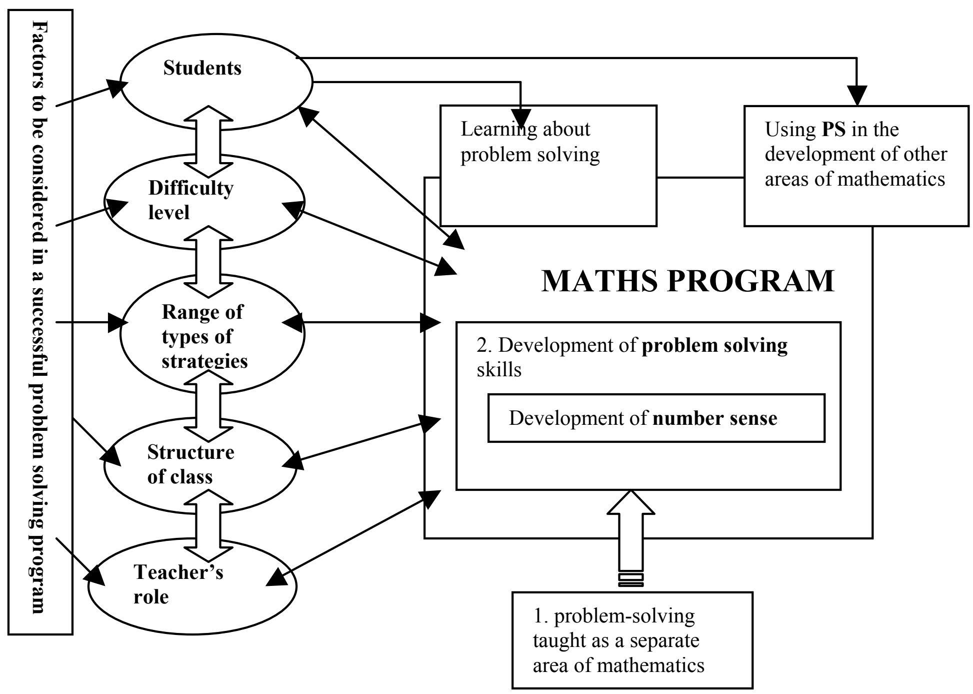


Figure 1.1 Model adapted from Smith's (1989) notion of teaching PS

Since then the notion of teaching problem solving as a separate area has been abandoned for one which advises teachers to embed problems in the mathematics-content curriculum (NCTM, 2000). Both problem solving and number sense have been hailed as being of paramount importance in mathematics education (Lyon, 2001; McIntosh, & Dole, 2000; NCTM, 2000; Nisbet, & Putt, 2000). To Carboni (2001), number sense is a part of children's daily mathematical lives and slowly grows and develops over time. She points out that "in a problem-centred mathematics curriculum, number sense is closely tied to problem solving" (p. 1). Furthermore the literature review has revealed that the role conferred upon both number sense and problem solving in any mathematics program is necessarily a central one. To facilitate and promote such an approach teachers must definitely change or adapt their teaching approaches (Reys, Lindquist, Lambdin, Smith, & Suydam, 2007), which is itself closely related to students' learning styles, to cater for students of various learning styles.

Research results in various fields suggest that teachers with certain preferred learning styles and certain preferred teaching styles will be more likely to be successful in developing their students' number sense and problem solving ability (DeBellis, & Goldin, 1997; Edwards, Bitter, & Morrow, 2005; English, 1998; Glasgow, Ragan, Fields, Reys, & Wasman, 2000; Grouws & Cebulla, 2000; Hatfield, Gill, 2001; Haynes, 1997). This in turn implies that, there is a possibility that students will do better in a class when their learning styles are similar to the teacher's teaching style. Hence, the researcher has proposed a theoretical framework which incorporates these four elements in an interactive and dynamic relationship.

As the teacher's teaching style affects the student's learning style, the impact upon the student's number sense and problem solving ability will set in motion a four-component cycle. Hence, it is being suggested that there are a number of factors acting as the driving force behind teaching styles which are compatible to individual learning styles, and that these also play an important role in producing students with good number sense proficiency and problem solving ability.

The model (Figure 1.2) being proposed is linked to issues generated by the mathematics reform movement, as reflected in the National Council of Teachers of Mathematics (NCTM, 1989) and the reactions to these guidelines (Carnine, 1997; Hofmeister, 1993; Mercer, Jordan, & Millet, 1994; Rivera, 1997). It is now accepted that if number sense can therefore also be seen as helping children to develop procedures for tackling a numerical problem, monitoring and regulating the process of solving the problem, it means that number sense is part of the broader construct of problem solving. The diagrammatical representation of the model is a hypothesised relationship of the four major focus factors of this study based on information extrapolated from literature and research about the envisioned teaching of number sense and problem solving.

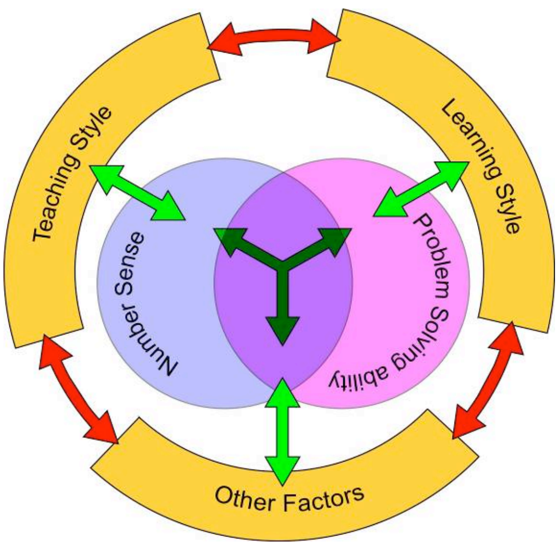


Figure 1.2 The relationship between number sense and problem solving ability

The learner employs his/her preferred learning style to engage in number sense activities and solving number sense and non-number sense problems, which are facilitated and transmitted through a preferred teaching style. Both teaching style and learning style are variable in nature, as they are impacted upon by other factors. These other factors simultaneously affect the problem being solved, and vice versa.

This model can be further interpreted as follows:

- As teaching style (TS) and learning style (LS) interact they are placed adjacent to one another;
- Except for teaching style and learning style, factors which could influence the relationship between students' number sense (NS) and problem solving ability (PSA) are categorised under the umbrella of 'Other Factors';
- Instead of problem solving the focus is directed at problem solving ability (achievement or performance would also fit in the place of ability); and
- NS and PSA intersect for number sense inherent problems, but remain separate when either the number sense item is not a problem for the learner, or the problem is devoid of number sense.
- Many problems are solved without recourse to number sense and not all number sense is applied within a problem solving context.

The ideas presented hitherto were derived mainly from contributions through the literature review; hence they guided the theoretical framework before the main data collection stage.

1.4 Purpose of the Study

The main purpose of this research was to explore the relationships between teaching styles, learning styles, and how they link to the relationship between students' number sense and problem solving abilities. Consideration was also given to the skills and traits demonstrated by teachers in attempting to accommodate number sense and problem solving ability development in their teaching**.** The researcher's experience as a mathematics teacher in secondary schools, and lately as a teacher educator has brought him into contact with children, adults and adolescents who have found it difficult to cope with learning mathematics. To help them improve their performance the researcher has had to delve into the causes behind such failures. Although it was quite difficult to answer the question it seemed that there were many influential factors at play, four of which are teaching style, learning style, ability to understand and use number concepts in solving numerical problems, and ability to solve mathematical problems.

According to D'Ambrosio (2001), in classroom settings students are different and classes are greatly affected by the interaction of students in the class. As the literature review will reveal, a major factor, which could be missing in D'Ambrosio's perspective, is the compatibility of the teacher's teaching style and the students' individual learning styles. This study will therefore be concerned with the relationships between teaching style and learning style, and the number sense and problem solving abilities of students. Hence, this research is an inquiry through observation, recording, interviewing and questionnaires, aimed at providing evidence about the nature and relationships of certain mathematics teaching and resultant learning phenomena. It seeks to clarify the phenomena, illuminate them, explain how they are related to other phenomena, and explain how they may be related to developing number sense and problem solving ability.

1.5 Research Questions

1.5.1 Main question

What is the relationship between teaching and learning styles, and the number sense and problem solving abilities of Year 7 students?

1.5.2 Subsidiary questions

1. What is the relationship between the number sense and problem solving abilities of Year 7 students?
2. How does teaching style impact upon students' number sense and problem solving performance?
3. How does learning style impact upon students' number sense and problem solving performance?
4. How do the teachers' beliefs concerning the link between number sense and problem solving impact on their teaching of number sense?

1.6 Definitions of Terms

Number Sense

McIntosh, Reys, Reys, Bana, and Farrell (1997) observed that a major factor which has contributed to the difficulty encountered in defining the concept of number sense, is that just like it is difficult to convey an explicit meaning of common sense, it is difficult to characterise a concept such as number sense. Consequently, according to

McIntosh et al. (1997), this has "stimulated much discussion among mathematics educators" (p. 3).

Nevertheless, there are certain recurring themes in most definitions of number sense which help in coming up with a generalised definition. For instance, a typical definition would describe number sense as the ability to decompose numbers naturally, use particular numbers as referents or benchmarks, use relationships among arithmetic operations to solve problems, understand number systems, estimate, make sense of numbers, and recognise the relative and absolute magnitude of numbers (Sowder, 1992).

For the purpose of this study a compromise has been reached in reconciling two nearly similar definitions — one from Burton (1993) and the other from Reys (1991) — the result of which allows number sense to be described as a person's general understanding of number and operations along with the ability to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for solving complex problems.

Problem Solving

Although the importance of students demonstrating the skills of problem solving has been determined quite adequately, defining the term itself is quite a complex task (Steen, 1999). In a national report on higher education Jones, Dougherty, Fantaske, and Huffman (1997), and Jones, Hoffman, Moore, Ratcliff, Tibbetts, and Click (1995), provide a general definition of problem solving as a step-by-step process of defining the problem, searching for information, and testing hypotheses with the understanding that there are a limited number of solutions. The goal of problem solving is to find and implement a solution, usually to a well-defined and well-structured problem. Solving a problem requires deliberate searching for appropriate action in order to attain an outcome not immediately obvious to the student: the process of obtaining a solution involves some degree of exploration, analysis and discovery (Booker, Bond, Briggs, & Davey, 1997; Steen, 1999). As cited in Krulik and Reys (1980), Polya captures the essence of problem solving as espoused in most definitions of the term, when he suggests that:

> To solve a problem is to find a way where no way is known off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end, that is not immediately attainable, by appropriate means. (p. 1)

Hence, the stance being taken for this study is one which rests on the premise that “mathematical problem solving is the cognitive process of figuring out how to solve a mathematical problem that one does not already know how to solve” (Mayer, & Hegarty, 1996, p. 31). If “problem solving means engaging in a task for which the solution method is not known in advance” (NCTM, 2000, p. 52), then all problems are novel and non-routine. Thus, throughout this study problem solving will be described simply as “a task for which there is no immediate or obvious solution” (Booker et al., 1997, p. 32), a definition which is in accord with the general use of the word ‘problem’ in mathematics.

Learning Style

Due to the multifarious nature of learning style, no single definition has hitherto sufficed to capture the essence of what is meant by such a term in this study. Felder (1996) purports that learning style denotes the characteristic strengths and preferences in the ways students take in and process information. A person’s style is reflected in his or her behaviour. While others (Boaler, 1997; Keast, 1999; Gill, 2001; Sloan, Daane, & Giesen, 2002) generally state that learning style is the unique collection of individual skills and preferences that affect how a person perceives, gathers, and processes information. Proponents of this second definition proclaim that learning style affects how a person acts in a group, learns, participates in activities, relates to others, solves problems, teaches, and works. (As noted in http://www.sil.org/LinguaLinks/LanguageLearning/OtherResources/YorLrnngStylAndLnggLrnng/WhatIsALearningStyle.htm).

Irvine and York (1995) claim that learning style can be defined as:

> the cognitive, affective, and physiological characteristics that influence how a person learns. Not to be confused with ability, learning style is a measure of preference or habit. It measures not potentials, but propensities. (p. 485)

Keefe and Ferrell (1990) tend to support Irvine’s and York’s definition, when they frame learning style as:

> a complexus of related characteristics in which the whole is greater than its parts. Learning style is a gestalt combining internal and external operations derived from the individual's neurobiology, personality and development, and reflected in learner behaviour. (p. 16)

A general description, extrapolated from the above definition would be that learning style relates to the general tendency towards a particular learning approach displayed by an individual. Hence, it is deemed appropriate that for the purpose of this study, learning style, according to Shaw (1996) refers to:

> the characteristics students bring to situations that influence how they learn. There are several possible dimensions to learning style. For example, students may have perceptual preferences. Auditory learners learn best when they hear instructional material, visual learners prefer material to be presented in a visual format, and tactile-kinesthetic learners learn most effectively with hands-on experiences. (p. 57)

Teaching Style

Teaching style is often confused with teaching approaches and/or strategies, which could be one reason why it is extremely rare to come across a definition for it. In this study teaching style is defined as being distinct from methods of instruction such as lecturing or cooperative learning. Teaching styles are supposed to define the behaviours that teachers exhibit as they interact with learners (Fischer & Fischer, 1979).

1.7 Summary

Through this chapter the scene was set to give an overview of what caused the researcher to investigate the main issues under discussion. This was superseded by a brief historical perspective revolving around a universal vision of a mathematics classroom, and where number sense, problem solving, teaching style and learning style could fit into that vision. Several authorities were evoked as a means of validating the argument presented. In this way, the background to the study was presented, followed by four issues pertaining to its significance, and presentation of the purpose of the study. One main question and four subsidiary ones, stemming from the former, were raised. An outline was given of the major terms of number sense, problem solving, learning style and teaching style.

The next chapter aims to review the literature in the fields of number sense, problem solving, teaching style and learning style. This will start off with making a case for the existence of a relationship between number sense and problem solving, leading into the issue of the need for a teaching style which caters for students' learning styles. This will be followed by discussion of research relevant to the research questions and findings regarding these four elements, followed by an exploration of the studies which are of interest to the present one. Finally these issues from the literature will be brought together to support the modus operandi and *raison d'être* of the research focus.

1.8 Organisation of the Thesis

The previous section has introduced the proposed study, argued for the need to conduct the study and listed the research questions. The purpose of the next chapter is to review the literature in the fields of number sense, problem solving, teaching style and learning style. This will start with an overview of the history of mathematics education relevant to the research topics and research findings regarding these four elements, followed by an exploration of the definition of number sense, problem solving, teaching style and learning style, in their various forms, and how they relate to one another. Throughout this discussion, a case will be made for the existence of a relationship between number sense and problem solving, leading into the issue of the need for a teaching style, which caters for students' learning styles. Studies which are of interest to the present one will be explored, and finally these issues from the literature will be brought together to support the modus operandi and *raison d'être* of the research focus.

Chapter 2 examines respective models of number sense, problem solving, teaching style and learning style. A discussion is presented revolving around the major elements of these models, from which certain key characteristics and features are teased out as a means of developing the broad scope of a model, which would attempt to capture the essence of the relationship, as extracted from the literature, between these four major factors; number sense, problem solving, teaching style and learning style. The resulting model will be employed in the formulation of a theoretical framework, based on the evidence gathered through the research.

In Chapter 3 the research methods used to gather and analyse data, which eventually lead to answering the research questions posed in Chapter 1, are presented. Issues of justification pertaining to the chosen research methods are addressed. In addition, both the context of the study and the background of the participants are explained. Such information will be considered later on in the discussion of the results. The reliability and validity of the research will also be discussed.

Then follows the analysis of the data and respective discussion pertaining to individual research questions. At this point the data is interpreted via the qualitative information and presented in the form of excerpts from observations, interviews and questionnaires, and through analysis of tabulated quantitative data gathered from tests, questionnaires and quantification of qualitative data. All these data will also be triangulated.

In Chapter 5 a summary of the research is presented together with the major findings. In concluding this chapter, the limitations of the study and certain recommendations for further research are discussed.

Chapter 2: Review of the Literature

2.1 An Overview

2.1.1 Shift in research emphasis

The overarching aim of this study is to determine whether relationships exist between teaching and learning styles and students' number sense and problem solving. To this end, the pertinent literature is reviewed in this chapter with respect to *teaching styles* (see Section 2.1), *learning styles* (see Section 2.2), *number sense* with respect to what it is and how students acquire it (see Section 2.3), and *problem solving* (see Section 2.4). The chapter concludes with a theoretical framework that will underpin this study's data gathering and analysis (see Section 2.5).

Progress in research on mathematics education can be described as having grown from the philosophers' mould to the scientists' laboratory, with emphasis shifting from mathematics itself to pedagogical issues affecting it. The following quotation from Kilpatrick (1992) echoes such a sentiment.

> The history of research in mathematics education is part of the history of a field – mathematics education – that has developed over the last two centuries as mathematicians and educators have turned their attention to how and what mathematics is, or might be, taught and learnt in school. From the outset, research in mathematics education has also been shaped by forces within the larger arena of educational research, which abandoned philosophical speculation in favour of a more scientific approach. (p. 3)

Although this shift has its advantages, the literature paints a picture of research in mathematics dealing primarily with problems of teaching and learning as defined by researchers. Despite the desire to understand and improve the learning of mathematics, Kilpatrik (1992) exposes the fact that, "…however, understanding and improvement have not ordinarily meant adopting the participants' views or taking the instructional context as problematic" (p. 4). The present research intends to take Kilpatrick's concern into consideration by scrutinising what is happening in Year 7 mathematics lessons through observation and interviewing of the participants.

From the 1930s onwards there was a growing emphasis placed on learning about factors which could affect the learning of mathematics. The result is that a plethora of suggestions were proposed pertaining to certain more apparent factors such as teaching style (Banks, 1988; Felder, 1988), thinking style (Krutetskii, 1976), thinking structure (Guilford, 1959), thinking strategies (Dienes & Jeeves, 1965), learning style (Claxton &

Murrell, 1987), problem solving processes (Pólya, 1945), assessment (Niss, 1993), and understanding and meaningful learning (Brownell, 1935; Plunkett, 1979); the latter eventually giving rise to the concept of number sense. With the exception of a slow and painful start of research on learning style and teaching style, research in the other mentioned areas proliferated (Clements & Ellerton, 1996). Of particular interest to this study are problem solving, number sense, teaching style and learning style.

2.1.2 From problem solving to learning style: a concise account

Problems and problem solving have a long history in mathematics education (Dewey, 1910; National Council of Teachers of Mathematics [NCTM], 1980; Pólya, 1945; Schoenfeld, 1992; Stanic & Kilpatrick, 1988). The year 1945 is considered a great turning point in the history of research in mathematics education. It was during that year that the translation of Pólya's draft of *How to solve it*, from German to English, took the world by storm (Taylor, 2000). From then on much focus was centred upon problem solving, which has since occupied a central status in mathematics education. Schoenfeld (1987) described the importance of Pólya's contribution, in his article *Pólya, Problem Solving, and Education* thus:

> For mathematics education and the world of problem solving it marked a line of demarcation between two eras, problem solving before and after Pólya. (p. 283)

This focus on problem solving has acted as a catalyst towards an emphasis upon mathematical sense making (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Human, & Olivier, 1997), which eventually gave rise to the number sense concept. The term 'number sense' itself is relatively new, although, as pointed out above, its meaning, which brings to the fore the concepts of understanding and meaningful learning, is abundantly present in the literature of mathematics education (Hiebert, 1984; Brownell, 1935; Burns, 1994; Plunkett, 1979; Skemp, 1982).

Quantitative literacy, a major contributor to the present climate influencing the teaching and learning of mathematics, was popularised during the 1970s, although the change it brought about was not as prominent as the recent emphasis on numeracy projects (Hughes, Deforges, & Mitchell, 2000). The discipline of mathematics as a logical system of axioms, hypotheses, and deductions has a very ancient history, but the expectation that ordinary citizens be quantitatively literate is primarily a phenomenon of the late 20th century (Steen, 2001). Although the term 'number sense' was not popular before the 20th century, the literature suggest that educators were already aware that the factors mentioned previously, and many minor or unidentified others, could have significant bearing upon students' number sense and their problem solving ability (Hiebert, 2003).

Nevertheless, it seems no one had yet asked the question, "What kind of teaching helps/hinders learners' number sense/problem-solving ability?", although by 1970 there was a growing literacy awareness which culminated in the search for an equivalent mathematical literacy, which was eventually termed 'numeracy'. This new focus gave birth to the concept of the numerate citizen, and by the mid 1980's number sense as it is known today was an emergent issue. It was inevitable that both number sense and problem solving would sooner or later meet at a research crossroads, since numeracy entails both number sense and problem solving. Therefore, any discussion about number sense and problem solving would not be complete without an examination of numeracy (Cuban, 2001). The reform emphasis placed on numeracy had to be supported by a change in teaching approaches.

The notion that empirical evidence consistently suggests that the single most important element in a child's success at learning is the quality of the teacher (Grasha, 1996), gave birth to research interest in teaching style and learning style. Such perspectives have met with considerable opposition, which could have slowed down research focussing on teaching style and learning style. In providing advice about roles and responsibilities which will promote better teaching and learning of mathematics, NCTM (2000) cautions that:

> The role of teachers, of course, is central. The choices that mathematics teachers make every day determine the quality and effectiveness of their students' mathematics education. But teachers alone do not make all the decisions—they are part of a complex instructional system. (p. 373)

Such a complex instructional system is necessarily comprised of teaching style and learning style.

Research pertaining to teaching style and learning style has at times focussed solely on one of them only, and at other times on both. Yet it is virtually impossible to alienate one from the other (Banks, Cookson, Gay, & Whawley, 2001). The focus on effective teaching usually results in a sub-focus on how learning is affected. Of the two, teaching style has a greater effect upon learning style, although the latter does influence the former (Dunn & Dunn, 1993).

The 1990s have seen a great surge in research related to teaching and learning style, especially in American educational institutions. In Australia such research has focussed mainly on engineering professors and their students (Holt & Solomon, 1996), the learning style of people coming from particular cultures (Hughes & More, 1997), and using students' perceptions of teaching style and preferred learning style to enhance teaching performance (Woods, 1995). With the exception of a few research endeavours such as Woods' study which focused upon the extent to which students' perceptions of teaching style and preferred learning style can provide meaningful and useful information for teachers wishing to enhance their teaching effectiveness, there is a lack of teaching style versus learning style research concerned with information which might lead towards enhanced mathematics teaching effectiveness.

Teaching style and learning have much to do with personality factors. Hence, a lot is being proposed under the guise of temperament theory, as teachers are warned of the dangers of neglecting the affective components of the students' performance baggage (Krathwohl, Bloom, & Masia, 1964; Gable & Wolf, 1993; Frydenberg, 1997). Various researchers and mathematics educators have attempted to answer this call by producing instruments and materials aimed at helping teachers in catering for their students' various learning styles and hence, their affective performance (Rompelman, 2002). Thus, how conversant or up-to-date the teachers are with such new concepts and instruments needs to be researched.

2.2 The Numeracy Dilemma

2.2.1 Is numeracy greater than or equal to number sense?

Since learning about number is central to the development of numeracy and underpins later success in mathematics (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997), it is appropriate that a link is made to research done on numeracy. Albeit its claim to encompass all mathematics topics, numeracy as proposed by the mathematics community focuses exceedingly upon number problem solving, at the expense of the wider context of mathematical problem solving (Lord & Lester, 1990). This could be confusing to teachers who are endeavouring to upgrade students' number sense and problem solving ability, since numeracy is a very influential factor in recent mathematics educational agendas (Willis, 2000).

One of the most comprehensive and recent studies, pertaining to numeracy, was carried out in England by Askew et al. (1997). Unlike the research being proposed, this study explored the knowledge, beliefs and practices of a sample of effective teachers of numeracy. The present research will make use of such findings by gauging the teachers' perception, and comparing that to what happens in actual practice.

A collaborative two-year research project that began in 1997 called 'Numeracy Across the Curriculum' (NAC), was funded by the Australian Research Council (ARC), the Education Department of Western Australia (EDWA) and Murdoch University. The research had two aims:

- To develop a description of numeracy with examples from across the curriculum;and
- To develop an approach to numeracy based on the practical experience of teachers and the needs of each learning area. (Hogan, 2001, www.redgumconsulting.com.au/numeracy)

As a result of the NAC, a numeracy audit was conducted and a numeracy framework was trialled and perfected. According to the authors the *Numeracy Audit* is a process through which teachers within a school can:

1. extend their understanding of numeracy and
2. collect information about numeracy within the school in order to plan improvement strategies.

Thus, the *Numeracy Audit* had these main goals:

1. to provide teachers with information about numeracy demands across the curriculum.
2. to develop teachers' skills in recognising numeracy demands in their classroom and their curriculum.
3. to support the school to make judgements about the extent to which numeracy requires action and where that action should be directed.
4. to extend teachers' knowledge of the strategies required to develop their students' numeracy.

Also, conclusions derived from the numeracy framework proposed that being numerate then involves a blend of three knowledges:

1. Mathematical knowledge
2. Contextual knowledge
3. Strategic knowledge

Being (becoming) numerate involves being able (learning) to take on three roles:

- The fluent operator - Being (becoming) a fluent user of mathematics in familiar settings.
- The learner - Having (developing) a capacity for the deliberate use of mathematics to learn.

- The critical mathematician - Having (developing) a capacity to be critical of the mathematics chosen and used. (Hogan, 2000, p. 16)

Evidently the NAC audit is emphasising that teachers need to be reflective in their practice if they are to produce numerate learners as advocated by the resulting framework. This is a fact, which is supported by the following policy statement of the Tasmanian Department of Education, which in itself summarises the central theme of the numeracy policies of all Australian states.

> It is essential that all students become numerate. Numeracy is fundamental to students at all stages of their schooling, from kindergarten to year 12, and in all areas of the curriculum - not just mathematics. For this reason all teachers share the responsibility to help their students become numerate. While many students are highly numerate, evidence suggests that a significant number of students are not sufficiently numerate to function effectively at school or in their future lives. (http://www.discover.tased.edu.au/mathematics/numpol.htm.)

This suggests that numeracy is concerned with teaching that develops students' learning ability to make sense of mathematics and solve problems (Moser, 1999). Yet most people, not familiar with the term, tend to immediately think of it as having to do only with number. Quantitative literacy—or numeracy, as it is known in British English—means different things to different people (Steen, 1999). Although quantitative literacy is often confused with its close relatives, such as basic skills, elementary statistics, logical reasoning, or advanced mathematics, none of these by itself offers a complete or balanced view of numeracy (Steen, 1997). Steen contended that numeracy was rarely mentioned in national standards or state frameworks, although it nourishes the entire school curriculum, including not only the natural, social, and applied sciences, but also language, history, and fine arts. Even though the proponents of numeracy proclaim that it permeates all subject areas, little numeracy is taught anywhere except in mathematics classes, and not as much as one might expect is taught (much less learned) even there (Steen, 1999). This state of affairs has been further aggravated by the looseness of the meaning of numeracy itself (Orrill, 2001). To empower teachers to come up with a strategy and a coordinated set of programs for promoting numeracy, a better term is needed to encapsulate all that this term purports to represent (Sowey, 2003). Otherwise this controversy is making it even more difficult for teachers to help students become sufficiently numerate to function effectively at school or in their future lives (Steen, 1997).

2.2.3 The need for a less ambiguous term

It is very unfortunate that the concept of numeracy has so much ambiguity surrounding it, and that the term itself does not necessarily express its intended meaning. Gal and Schuh (1994) acknowledge that the word 'numeracy' is a neologism — an invented word that has come into use among specialist communities in Britain, Australia, Canada and the United States— used as the quantitative, mathematical counterpart of literacy. Even at the end of the twentieth century, it is not yet a household word (Moser, 1999). Moreover, in some languages, such as French, there is, as yet, no single word that is equivalent to 'numeracy'. However, there is a need for a short word (such as 'numéracie') to express the concept, since otherwise it is difficult to discuss the subject in a concise way or to give the concept a sufficiently wide range of meaning (Ciancone, Tom, & Jay, 1991). In his book *The Values of Precision*, Wise (1995) warns that unless there is a recognised concept such as numeracy (or a close equivalent), it is difficult to come up with a strategy and a coordinated set of programs for promoting numeracy.

Before the word numeracy came into use, there was discussion of terms such as 'mathematical literacy' and 'quantitative literacy', which placed the focus on calculations and the ways in which numbers and mathematical concepts were embedded in texts, but which did not take into account the wider practical uses of numbers and mathematics in the workplace and in personal life on an everyday basis (Murnane & Levy, 1996).

The concept of numeracy has been widely debated internationally without singular agreement . For instance, although a broad definition has been adopted in New Zealand and published in *Curriculum Update 45* as "to be numerate is to have the ability and inclination to use mathematics effectively – at home, at work and in the community" (http://www.tki.org.nz/r/governance/curric_updates/curr_update45_e.php), its suggestions, as proposed in *Mathematics in the New Zealand Curriculum*, about what is expected of students at each stage, relates more to number sense than it does to numeracy, which implies that at all stages students should:

- develop an understanding of numbers, the ways they are represented, and the quantities for which they stand;
- develop accuracy, efficiency and confidence in calculating – mentally, on paper, and with a calculator;
- develop the ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements. (p. 31)

Hence, it is not surprising that current national numeracy testing, in Australia, is placing an emphasis on number sense (McIntosh, & Dole, 2000). Albeit the above-mentioned contradictions and overt emphasis upon number sense; the notion that an understanding of numeracy is needed to solve everyday problems (Hughes, Deforges, & Mitchell, 2000) still permeates most relevant numeracy literature (Verschaffel, DeCorte, Lasure, Van Vaerenbergh, Bogaerts, & Ratinckx, 1999; NCTM, 2000; Pierce and Stacey, 2001). Given such a situation it is a truism to state that the focus on numeracy is being highlighted through its number sense and problem solving components, although there is an imbalance favouring the former at the expense of the latter. In their book *Understanding the Mathematics Teacher*, Desforges and Cockburn (1987) purport that such contradictions cause teachers to focus on one mathematical aspect at the expense of others.

2.2.4 Controversy: Number sense in numeracy without problem solving

Although mathematical power is all about empowering students through developing their mathematical sense-making ability, of which number sense is of paramount importance, most proponents of numeracy fail to emphasise its non-number sense problem solving component. Pierce and Stacey (2001) propose that for arithmetic number sense is important; and for algebra, it is symbol sense. To Pierce and Stacey both concepts apply to the whole problem solving cycle. Hence, such controversy makes it even more important and urgent to gather knowledge and understanding pertaining to how teachers could be dealing with the challenge of boosting the number sense of students through a problem-centred approach (NCTM, 2000).

In addition, Kelly (2002) further elucidates another contradiction which could be hampering the practice of effective teaching and learning of number sense and problem solving. Kelly alleges that the National Numeracy Strategy (NNS), being promulgated in the UK, describes numeracy as a key life skill, and adds that without basic numeracy skills, our children will be disadvantaged throughout life (DFEE, 1999). Kelly (2002) points out that the word ‘skill’ is important because it implies that such an ability is unproblematically transferable from one context — the context of learning — to another in the context of application. According to him, this agrees closely with Cockcroft's (1982) view of numeracy as the ability to use mathematical skill to enable individuals to cope with the practical mathematics of everyday life. In reality, however, numeracy within NNS practices does not attempt to meet such worthwhile aims. Obviously there is a lot of truth in Kelly’s (2002) claims, which are duly supported by the NNS (DFEE, 1999, Section 1) suggestion that:

> Many teachers already find it difficult to instill confidence in most students, and harder still to upgrade their competence, constraints which are not being

> alleviated through the ongoing controversy. For instance, according to NNS numeracy is a proficiency which involves confidence and competence with numbers and measures. It requires an understanding of the number system, a repertoire of computational skills and an inclination and ability to solve number problems in a variety of contexts. (p. 4)

Yet, instead of honouring its proposed vision, the NNS attempts simply to develop children's ability to carry out calculations involving number (a lesser aim than Cockcroft's 'use mathematical skill') in a variety of almost entirely artificial contexts and measured almost entirely through national tests. Rarely are contexts from everyday life or methods of appraising children's performance other than testing included (Kelly, 2002).

Such subliminal bias could cause teachers to neglect teaching from real life contexts, which are necessary ingredients in implementing a problem-oriented curriculum through which students' number sense could be developed (Australian Education Council, 1991; Cockcroff, 1982; Hughes, Deforges & Mitchell, 2000; Markovits & Sowder, 1994; McIntosh, Reys, Reys, Bana, & Farrell, 1997; National Council of Teachers of Mathematics, 1989, 2000; National Research Council, 1989; Thiessen & Trafton, 1999). Number sense is a major component of numeracy, and for the teaching of number sense to be effective the teacher must necessarily engage children in solving problems (Bobis, 2000).

2.3 Problem Solving

2.3.1 Problems in teaching and learning

Problem solving is an unfortunate term in that it is universal to all subjects and difficult to separate from any endeavour which involves thinking. Although the importance of students demonstrating the skills of problem solving has been determined quite adequately, defining the term itself is quite a complex task. In a national report on higher education Jones, Dougherty, Fantaske, and Huffman (1997), and Jones, Hoffman, Moore, Ratcliff, Tibbetts, and Click (1995), provide a general definition of problem solving as a step-by-step process of defining the problem, searching for information, and testing hypotheses with the understanding that there are a limited number of solutions. The goal of problem solving is to find and implement a solution, usually to a well-defined and well-structured problem. Solving a problem requires deliberate searching for appropriate action in order to attain an outcome not immediately obvious to the student. The process of obtaining a solution involves some degree of exploration, analysis and discovery (Booker et al., 1997; Broomes & Petty, 1995; Steen, 1999).

Even before the Cockcroft Report (1982) was published there was growing support for improving the quality and standard of mathematics teaching and learning in school, and one aspect which has slowly been gaining impetus for quite a while is developing the learners' ability to solve problems. Starting with Polya's (1945) *How to Solve it* four problem solving strategies, numerous mathematics educational researchers have expanded on these and come up with new lists of problem solving strategies and suggestions for improvement in the teaching and learning of mathematics. Yet many students who have gone through all these programs have left school without having much understanding of the mathematics they have learnt (Van de Walle, 2001; Stein, Smith, Henningsen, & Silver, 2000). Instead of focussing on sense making there was much emphasis placed on procedural instruction and efficiency in computing (Battista, 1999; Schoen, Fey, Hirsch, & Coxford, 1999; Hogan, 2000). Hazlett (2000) states "mathematics education has long been in need of improved methods of instruction, particularly in the area of problem-solving skills" (p. 3). A major goal of mathematics education is to develop students' ability to use mathematics effectively in their daily lives (NCTM, 2000). Although problem solving offers a unique opportunity for establishing the relevance of mathematics in students' daily lives, it is still difficult to teach, as it requires students to read with understanding and then apply their mathematical knowledge in creative ways (Boud & Feletti, 1997; Savin-Baden, 2000). Since teachers are being encouraged to apply their teaching through a problem solving approach (Smith, 2002), it is necessary that information is obtained as to how they could be dealing with the difficulty expressed by Dickerson.

Problem solving is a central issue in current reform initiatives in mathematics education (Fairhurst and Fairhurst, 1995; NCTM, 2000). By the early nineteen-nineties curriculum developers had designed problem-solving oriented curricula to help move reforms into K-12 mathematics classrooms. Although such an endeavour was seen as a bold and positive one, researchers like Ball (1996) felt that there was still the problem of the mathematics community not knowing much about how teachers actually use problem-solving oriented mathematics curricula to teach. Hitherto the literature still suggests a lack of information with regard to the teaching of problem solving, albeit some research has focussed on this aspect of mathematics teaching and learning (Sztajn, 2003). In addition there is the problem of lack of sense making in both the teaching and learning of mathematics (McIntosh, Reys, Reys, Bana, & Farrell, 1997; Buzeika, 1999), which could be a substantial hurdle in the development of students' problem solving ability (Reusser & Stebler, 1997; Schoenfeld, 1989, 1992, 1994, 2001; Verschaffel, Greer, & De Corte, 2000).

2.3.2 The Australian situation

It was during the 1980s that making students better problem solvers was formally established as a goal of the mathematics curriculum in every state and territory in Australia (Stacey & Groves, 1990). Unfortunately there has been a change in the main interpretation of the goal of problem solving in school mathematics in Australia (Keeves & Stacey, 1999). The many ambitious attempts to assess problem solving and to encourage the teaching of problem solving, through both experience and reflection, have been abandoned or reduced, largely in response to external pressures (Stacey, 2000). Nevertheless, there are encouraging signs that the use of a problem solving approach to teaching is becoming much more sophisticated (Williams, 2000). Nisbet and Putt (2000) observe that the second aspect of the problem solving goal, which is to teach mathematics through problem solving approaches, has been given impetus in recent years by the popularity of constructivist theories to guide teachers as facilitators of learning. In-depth research is providing guidance on how to replace traditional teaching with approaches that engage students more fully in mathematical thinking (Nisbet & Putt, 2000).

Recently most emphasis has been placed on monitoring achievement on basic skills, but against all expectations this has not excluded any emphasis on problem solving (Owens & Mousley, 2000). This in itself tends to suggest that emphasis on number sense, of which basic skills is a component, does not necessarily imply exclusion of problem solving. Although not receiving the prominence that it had several years ago, problem solving has continued to be regarded at least as an essential competency in mathematics. Nisbet and Putt (2000) attribute this to two causes, one of which was the identification of using mathematical ideas and techniques as one of a small number of key competencies for national vocational education.

Most problem solving research being undertaken in Australia is focusing on the way that students solve problems in different content areas of mathematics (Nisbet & Putt, 2000). Research into students' solving of number sense problems has so far failed to deal with the problem solving aspect per se (Keeves & Stacey, 1999).

Wagener (2002) reiterates the fact that number sense and problem solving are related. Yet, albeit the rising popularity of problem solving and number sense as research foci, there is still a seemingly unconscious reluctance to link the two, apart from their expressed relationship in definitions of number sense. Extensive reviews of Australian research on problem solving for four-year intervals provide no indication that much attention will be paid to studies of the relationship between number sense and problem solving (Nisbet & Putt, 2000; Nisbet, Putt, & Taplin, 1996). Even Keeves' and Stacey's (1999) reporting of other aspects of problem solving, in the context of broad developments in research in mathematics education in Australia since 1965, do not indicate any former or recent attempt to go in this direction. Yet problem solving and number sense are two intertwined concepts, central to the call for reform in the teaching of mathematics (Shaw & Blake, 1998).

2.4 Number Sense

2.4.1 Lack of congruence between school mathematics and real mathematics

The *National Statement on Mathematics for Australian Schools* supports an increased emphasis on developing number sense through mental computation in recognition of the role of the calculator (Groves & Cheeseman, 1992). Research done in the area of number sense has focussed on mental computation, calculator use, estimation and sense making (Swan, 2002; Taylor, Simms, Kim, & Reys, 2001), while research efforts to investigate the importance of problem solving in relation to number sense is yet to materialise. This could be due to the lack of congruence between reality and practice, especially when it comes to promoting problem solving and number sense through the use of technology (Owens & Mousley, 2000).

Willis and Kissane (1989) have expressed their concerns at the lack of congruence between school mathematics and real mathematics. They observe that the emergence of calculators and computers serves to highlight this lack of congruence. Recently in Australia, powerful attempts have been made to change this situation, where "paper-and-pencil methods still receive the most emphasis in schools, with none or very little emphasis being placed upon mental and calculator computations" (p. 58). This has resulted in the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990) endorsing the 1987 national policy on calculator use, recommending that all students use calculators at all year levels (K-12) and that calculators be used both as instructional aids and as learning tools. In line with world-wide trends, the national statement has increased emphasis on developing number sense through mental computation, partly in recognition of the role of the calculator. For instance, 1990 saw the commencement of the *Calculators in Primary Mathematics* project, which is a long-term investigation into the effects of the introduction of calculators on the learning and teaching of primary mathematics. The revolution proposed through the reform was expected to bring about better teaching and learning of mathematics. Unfortunately many students, and school leavers, suffer from lack of sense making in their application of mathematics (Carpenter, Fennema, & Franke, 1996).

2.4.2 The need for number sense

This lack of sense making caused certain concerned researchers to lament that "the notion that mathematics instruction and learning should be based on reflective inquiry and sense-making has long been overshadowed by a quest for high levels of efficiency in computing" (McIntosh, et al, 1997, p. 5). Hence, there was a call to raise the standard of teaching and learning mathematics which was echoed by the "Number and Operations Standard" in *Principles and Standards for School Mathematics* (NCTM, 2000) when it stated that, "Central to this Standard is the development of number sense" (p. 32). By then, number sense was being seen as having as much importance as problem solving. Many research reports and documents have stressed the need to develop number sense and the importance of teaching to develop learners' number sense in school mathematics (Australian Education Council, 1991; Cockcroft, 1982; Markovits & Sowder, 1994; McIntosh, et al, 1997; NCTM, 1989, 2000; National Research Council, 1989a). Without lessening the degree of emphasis upon problem solving, researchers and education policy makers were pushing for the development of number sense to be a priority on the mathematics teacher's teaching agenda (Ball 1991; NCTM, 1989; Russell, 1996).

Unfortunately, one possible reason for lack of a combined number sense and problem solving research could be due to the difficulty of divorcing one from the other. Both the 'number sense' and 'problem solving' terms have suffered many diverse definitions. The controversy stems mainly from the fact that number sense is akin to common sense (McIntosh, et al, 1997), which is a necessary tool in solving any problem. Compared to problem solving, number sense seems to have proved the most difficult to define. McIntosh et al. (1997) state:

> Like common sense, number sense is a valued but difficult notion to characterize and has stimulated much discussion among mathematics educators (including classroom teachers, curriculum writers, and researchers) and cognitive psychologists. (p. 3)

Although there is a great deal of truth in the above statement there is one major unifying concept involved in all definitions pertaining to number sense. Trafton and Thiessen (1999) encapsulate this central theme, number, when they proclaim that number sense “describes a cluster of ideas, such as the meaning of a number, ways of representing numbers, relationships among numbers, the relative magnitude of numbers, and skill in working with them” (p. 8).

In many industrialised countries the need for students to develop number sense has been emphasised through various documents calling for reforms of school mathematics (Australian Education Council, 1991; Cockroft, 1982; Emanuelson & Johansson, 1996; Japanese Ministry of Education, 1989; NCTM, 1989).

Despite the existence of variations as to what is meant by this relatively new concept, in summary number sense refers to a person’s general understanding of numbers and operations and the ability to handle daily life situations that include numbers. As such it entails the development of useful, flexible, and efficient strategies, involving mental computation or estimation, for handling numerical problems (Burton, 1993; Howden, 1989; McIntosh, Reys, & Reys, 1992; Reys, 1994; Sowder, 1992; Treffers, 1991). The interrelatedness of number sense and problem solving is glaringly evident, since all of the various definitions of number sense make mention of problem solving. Surprisingly, enough most, if not all, research efforts pertaining to number sense have so far managed to avoid the issue of including problem solving as one of the variables.

The difficulty to characterise number sense has instigated various enquiries, which have stimulated some vital discussions amongst researchers, classroom teachers, curriculum writers, including mathematics educators. These discussions, according to Reys et al. (1997) have included:

> A listing of essential components of number sense (Resnick, 1989; Sowder & Schappelle, 1989; Willis, 1990; Sowder, 1992; McIntosh, Reys & Reys, 1992), descriptions of students displaying number sense or the lack thereof (Howden, 1989; Reys et al., 1991), a theoretical analysis of number sense from a psychological perspective (Greeno, 1991), and discussions of instructional strategies which promote the development of number sense (Brownell, 1945; Kamii, 1989; Reys et al., 1991, Burton, 1993; Burns, 1994). (p. 4)

This study is partially concerned with the last category of discussions, as it seeks to add a new dimension of compatibility of teaching style and learning style to these discussions. According to Menon (2002) research on number sense has focused mainly on:

"average" children's number sense (e.g. Reys & Reys et al., 1999; Turner, 1996; Yang, 2002), while some studies have focused on the number sense of children with learning disabilities (e.g. Gersten & Chard, 1999; Griffin, Case, and Siegler, 1994). (p. 1)

However, this study seeks to look at the relationship between number sense and problem solving through the interaction of the teachers' teaching style and the students' learning style.

In clarifying its goal of the *Illinois Learning Standards* the Illinois State Board of Education (1997) proposes that all people must develop number sense and operations and be able to use the skill to solve problems using mental computation, paper-and-pencil algorithms, calculators and computers. Clearly this implies that there is an existing link between number sense and problem solving.

2.5 Linking Number Sense and Problem Solving

2.5.1 The reciprocal relationship of number sense and problem solving

The content analysis confirms the findings of the literature review that there is a lack of consensus about how to define problem solving and about the distinction between problem solving and number sense. The myriad of definitions of both problem solving and number sense have so many overlapping concepts, which in essence is tending towards making it increasingly difficult to distinguish between the two.

To Sowder (1992) number sense is synonymous to the development of 'quantitative intuition' or a 'feel for number'. Since the thought of teaching, 'intuition' is difficult to conceive, Sowder (1992) attempts to 'add weight to the mass of number sense', by reference to the contributions of Resnick (1987, 1988). Resnick used the term 'higher order thinking' in preference to the term 'number sense'. In the following extract Sowder substituted 'number sense' for Resnick's original 'higher order thinking'.

> [Number sense] resists the precise forms we have come to associate with setting of specified objectives for schooling. Nevertheless, it is relatively easy to list some key features of [number sense] when it occurs. Consider the following:
>
> [Number sense] is *nonalgorithmic*. That is, the path of action is not fully specified in advance.
>
> [Number sense] tends to be *complex*. The total path is not 'visible' (mentally speaking) from any single vantage point.
>
> [Number sense] often yields *multiple solutions*, each with costs and benefits, rather than unique solutions.
>
> [Number sense] involves *nuanced judgement* and interpretation.

> [Number sense] involves the application of *multiple criteria*, which sometimes conflict with one another.
>
> [Number sense] often involves *uncertainty*. Not everything that bears on the task is known.
>
> [Number sense] involves *self-regulation* of the thinking process. We do not recognise it in an individual when someone else 'calls the plays' at every stop.
>
> [Number sense] involves *imposing meaning*, finding structure in apparent disorder.
>
> [Number sense] thinking is *effortful*. There is considerable mental work involved in the kinds of elaborations and judgments required. (p. 381)

Without hesitation, one can also comfortably fit the term 'problem solving' in the brackets above. Hence, the overlap between number sense and problem solving is so great that it would be a truism to state that they are closely related. How teachers cope with this apparent ambiguity is important to this study since it is in the classroom that theory is put into practice. In the context of this present research any problem which necessarily requires knowledge and skill in number, to arrive at an acceptable resolution, will involve number sense and any question for which the solver has no immediate and apparent way of solving will constitute a problem (Thiessen & Trafton, 1999). Yet the teacher, and the students, might not see it this way.

Given such constant and consistent linking of number sense and problem solving in definitions of number sense, it could be expected that a student with good number sense would be able to transfer such knowledge into solving given problems involving related concepts. To take a specific topic as an example, some number sense research (Markovits & Sowder, 1994; Reys & Yang, 1998; Sowder & Wheeler, 1989) have shown that, when given two fractions, students have far more difficulty figuring out which one is closer to a third fraction than they do simply comparing them to each other. For example, deciding whether $^3/_8$ or $^7/_{13}$ is closer to $^1/_2$ is more difficult than comparing $^3/_8$ and $^1/_2$. Students need to consider the use of a benchmark, understand the size of the fractions, and consider the use of a multi-step strategy when solving a problem. All three of these abilities are characteristics of number sense. Hence, inherent in this observation is that those who are weak in fraction number sense would find it very difficult to solve problems involving fractions, as compared to those who have good fraction number sense. This final statement also applies to other number sense ability in other topics such as decimals, integers and so on.

A close analysis of the literature reveals that both number sense and problem solving rely upon nearly the same environment and teacher/learner qualities. Given such a situation it would be expected that number sense would have a significant effect upon students' problem solving. Yet it seems that no research has been done to ascertain such a subliminal assumption; although one researcher, English (1998), has compared students in terms of their number sense and problem-posing performance level, using the results of students' performance on a number sense test and a novel problem solving test.

Historically, number has been the cornerstone of the entire mathematics curriculum internationally (Reys & Nohda, 1994). All the mathematics proposed for pre-kindergarten through Year 12 is strongly grounded in number. The principles that govern equation solving in algebra are the same as the structural properties of systems of numbers. In geometry and measurement, attributes are described with numbers. The entire area of data analysis involves making sense of numbers. Through problem solving, students can explore and solidify their understandings of number (NCTM, 2000).

In the NCTM (2000) *Standards*, "understanding number and operations, developing number sense, and gaining fluency in arithmetic computation form the core of mathematics education for the elementary grades" (p. 32). It is envisaged that as they progress from pre-kindergarten through Year 12, students should attain a rich understanding of numbers. Hence, the acknowledgement of the existence of a reciprocal relationship between number sense and problem solving is not a far-fetched idea.

Despite their evident connection, problem solving and number sense are not necessarily synonymous because while number sense is inherent in problem solving, many problems are solved without recourse to number sense (Hiebert et al., 1997). An overview of what is meant by problem solving reveals that it is the vehicle through which number sense is transported (Bolster & Reys, 2002). The differences between the two stems from the fact that number sense deals more specifically with number whilst problem solving permeates all aspects of mathematics, and even other subjects. Problem solving means that students can engage in tasks for which the solution is not known in advance (Mason & Davis, 1991). Problem solving is both a goal and a means of learning mathematics (Ronis, 2001). Problem solving helps students learn to deal with unfamiliar situations and develop habits of persistence (Collis & Romberg, 1992). It requires that students explore, make conjectures and question one another (Clarke, 1997). Nevertheless the literature does not necessarily attempt to distinguish between the two, although a few definitions of number sense fail to make mention of its problem solving aspect.

2.5.2 Definitions of number sense and the embracement of problem solving

The connection between number sense and problem solving permeates most definitions of the former. For instance, a typical definition would describe number sense as the ability to decompose numbers naturally, use particular numbers as referents or benchmarks, use relationships among arithmetic operations to solve problems, understand number systems, estimate, make sense of numbers, and recognise the relative and absolute magnitude of numbers (Sowder, 1992). Most definitions of number sense incorporate a sense of problem solving (Denvir & Bibby, 2002; Ritchhart, 1994), which serves to show that it is virtually impossible to separate the two. Although in these definitions the intention weighs more towards number sense inherent problems, it could be that number sense ability is intricately linked to mathematics problems, which are devoid of number sense (Lyon, 2001; Anghileri, 2000).

Fennell and Landis (1994) define number sense as "the foundation from which all other mathematical concepts and ideas arise" (p. 187). To them, number sense "is good intuition about numbers and their relationships" (p. 188). Students with number sense can automatically tackle a variety of problems. They can break down the problem and use the numbers as references. In other words, they can make connections between their knowledge and newly learned mathematical concepts and skills. In general, they know how to make sense of numbers, how to apply them, and are confident that their problem solving processes will enable them to arrive at solutions. This necessitates a more open-ended, innovative and problem-oriented teaching approach (Anderson, 1996, 1997; Thiessen & Trafton, 1999), as required by the calls for reform (NCTM, 2000; Whitehead, 1997; Haynes, 1997).

2.5.3 Teaching for and learning to develop number sense

Oppositions to the current reform emphasis, although not numerous, have provided food for thought for everyone concerned with the teaching and learning of mathematics (Prais, & Luxton, 1998). For instance, Gill (2001) laments the focus of debate which centres on forms and approaches to teaching: direct instruction versus inquiry, teacher-centered versus student centered, traditional versus reform. She warns that:

> These labels make rhetorical distinctions that often miss the point regarding the quality of instruction. Our review of the research makes plain that the effectiveness of mathematics teaching and learning does not rest in simple labels. Rather, the quality of instruction is a function of teachers' knowledge and use of mathematical content, teachers' attention to and handling of students, and students' engagement in and use of mathematical tasks. Moreover, effective teaching — teaching that fosters the development of mathematical proficiency over time takes a variety of forms. (http://www.aft.org/thnkmath/addingitup.htm)

It is the investigation of such forms that this study is concerned with: the type(s) of teaching styles being used. With the ongoing emphasis upon reforming the teaching and learning of mathematics, comparative studies have examined the effectiveness of these contrasting approaches to teaching (Cuevas & Driscoll, 1993a). The result seems to favour those approaches advocated by the proponents of reform, since they contend that such approaches are more appropriate for reaching all students with mathematics (Cuevas & Driscoll, 1993b).

At the heart of such reforms is the distinction between *inquiry mathematics* and *school mathematics* (Cobb, Wood, Yackel, & McNeal, 1992). In contrast with traditional classroom activities that emphasize repetition, practice, and routinised means to some focused endpoint, inquiry mathematics instruction emphasizes student engagement in problem-solving and theory-building about important mathematical situations and concepts. Although teaching which favours the traditional approach is (painstakingly) being replaced by the inquiry approach (Silver, Kilpatrick, & Schlesinger, 1990; Adams & Hamm, 1998), more research is needed to discover the types of teaching approaches, successful or unsuccessful, being used in the teaching of number sense per se.

Number sense and problem solving research have in most cases always concentrated on either the teacher or the students (Greeno, l989; Howden, l989; McIntosh, Reys, & Reys, 1992; Markowitz & Sowder, 1994; Menon, 2004; Resnick, 1988; Sowder, l992; Reys, l99l;). Rarely has there been any attempt to study both parties' contributions. Yet research findings and discussions pertaining to number sense and problem solving provide suggestions about the teaching and learning of these two important elements (Willis & Checkley, 1996; Gersten & Chard, 1999).

Comments such as "although different students may initially use different ways of thinking to solve problems, teachers should help students recognise that solving one kind of problem is related to solving another kind" (NCTM, 2000, p. 83), are teacher-advisory in nature and scope, they acknowledge the fact that teachers are partially responsible for helping children develop their problem solving ability. Suggestions abound as to how teachers could help children develop a problem- solving attitude, but very little exists on how teachers are actually doing this (Silver, 1985), let alone how they are teaching number sense .

2.6 Teaching Style and Learning Style

Griggs (1991) report that a comprehensive definition of learning style was adopted by a national task force, comprised of leading theorists in the field and sponsored by the National Association of Secondary School Principals (NASSP). This group defined ‘learning styles’ as the composite of characteristic cognitive, affective, and physiological factors that serve as relatively stable indicators of how a learner perceives, interacts with, and responds to the learning environment (Keefe, 1991). Included in this comprehensive definition are ‘cognitive styles’, which are intrinsic information-processing patterns that represent a person's typical mode of perceiving, thinking, remembering, and problem-solving (Griggs, 1991).

The concept of learning style as a construct that affects individual students' learning preferences is not new. Whether defined in terms of self-views, needs, personalities, individual attitudes, differences, processes, temperaments, autonomies, modalities, aptitudes, values, ideal environments, personal touches, motivations, behaviour sets, characteristics, preferences, patterns, or nature and make-up; learning style is accepted by many scholars and educators as a determining factor for individual learners' respective successes and failures in schooling situations (Marion, 2002).

2.6.1 The role of teaching style and learning style

McIntosh et al. (1997) are concerned that “while agreement exists that the development of number sense is an important goal for all children, many important questions remain unanswered about the routes to achieve this goal” (p. 5). This study aims to help in answering their call for providing better information to guide curriculum and instruction efforts in this area. Evidence from recent research suggests that lack of catering for students’ various learning styles is becoming increasingly apparent as one of the major obstacles in teaching and learning (Wagener, 2002)

Proponents of learning styles have based their arguments upon the fact that we all have our own ways of doing things. The preferences, tendencies, and strategies that individuals exhibit while learning constitute what have come to be called "learning styles" (Thompson & Mascazine, 1997). Over the past 30 years formal study of learning styles has developed from various conceptual orientations. One of the most popular learning style instrument was first produced by Rita Dunn in 1967, which has since then been called the ‘Dunn and Dunn Learning Style Inventory”. According to Curry's (1987) review of 21 different learning or cognitive style models through psychometric analyses, the Dunn and Dunn model had one of the highest reliability and validity ratings. Hence, it is claimed that Dunn and Dunn (1978a, 1978b) have produced one of the most comprehensive theories of learning styles, both in scope and practice, for teachers (Schugurensky, 2003). Many of the modern learning style theories and instruments incorporate aspects of the Dunn and Dunn Model, and such theories have provided the foundation for teachers to teach according to the learning styles of their students (De Bello, 1990).

The need to provide students with more problem-oriented mathematics activities is receiving increased attention as a necessary ingredient to advancing the mathematics literacy of Australia’s youth (Lyon, 2001). Propelling this increased attention are repeated reports of students’ lack of understanding of basic mathematical concepts and applications, inadequate mathematical sense making and inability to solve novel mathematical problems. Explicit in the literature is the call for teachers to adapt their teaching styles to meet the demands of the new national standards (NCTM, 2000), which have placed a great deal of emphasis upon effective teaching of number sense and problem solving.

The nature of number sense and its embedded relationship to problem solving requires teaching which focuses on individual differences in the students. Teachers could be finding it difficult to respond to students’ learning styles because influential institutions and publications often fail to use the terms teaching style and/or learning style. For instance, NCTM’s (2000) vision of equity stresses that “mathematics can and must be learned by all students” (p. 13). Yet its description of all students include those who live in poverty, who are non-native speakers of English, who are disabled, females and many non-white students, a list which fails to include explicit mention of differences of learning styles.

Number sense is highlighted in current mathematics education reform documents because it typifies the theme of learning mathematics as a sense-making activity (NCTM, 2000), and teachers are expected to help children develop their sense making ability (Burns 1994), in order for the latter to be better problem solvers (Woods, 1995), but the principle of “equity requires accommodating differences to help everyone learn mathematics (p. 13). Although it is not explicitly stated as such, it seems that the call from sources such as the Australian Education Council (1991), Hughes, Deforges and Mitchell (2000) and Felder (1993, 1996) is stating that unless there is a conscientious effort to raise teachers' awareness of the importance of catering for differences in their students' learning styles, it would be very difficult to encourage students to make sense of the concepts they learn. Since sense making is now a very important component of mathematics teaching and learning, it seems that failure to heed such a call might result in the teaching and learning of number sense being kept as a traditional affair in most cases, instead of taking the centre stage as aspired by NCTM (2000).

The development of number sense is important in mathematics education. The National Council of Teachers of Mathematics (NCTM, 2000), in their *Principles and Standards for School Mathematics*, note that number sense is one of the foundational ideas in mathematics in that students:

> (1) Understand number, ways of representing numbers, relationships among numbers, and number systems; (2) Understand meanings of operations and how they relate to one another; (3) Compute fluently and make reasonable estimates. (p. 32)

To develop such a spectrum of understanding so as to inculcate an appreciation of number sense in their students, teachers need to be very conversant with a range of strategies (Cockcroft, 1982), the application of which depends on the teacher's teaching style (Griggs, 1991).

Moreover, "number sense develops over time. The development is best if the focus is consistent, day by day, and occurs frequently within each mathematics lesson" (Thornton & Tucker, 1989, p. 21). Reys et al. (1991) advocated that the best way to help children develop number sense is through process-oriented activities in a comfortable classroom atmosphere. To develop and/or enhance both the learner's number sense and problem solving ability, many researchers insist that the environment is important: a creative learning environment encourages children to think, probe, communicate, and discuss (Adams & Hamm, 1998; Anderson, 1996; Jones & Jones, 1998; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997). Such reflections also bring into play the notion of teaching style and learning style compatibility.

Treating every child in the classroom the same way is not responding to their styles. Teachers and parents want more for students than the old hit-and-miss strategies of the past. Minimally, both want to help the underachiever and enhance the potential of successful students (De Bello, 1990). Thus, the need to investigate present practices in mathematics classrooms is growing ever stronger (Black, & Atkin, 1996; Fullan, & Stiegelbauer, 1991; McLeod, 1992).

Current research affirms that learning styles-based programs statistically increase student achievement (Dunn, Beaudry, & Klavas, 1989; Montgomery, 1995; Noble & Bradford, 2000). Though controversial in some quarters, research continues to build a strong case for the impact of learning style upon acquiring and mastering knowledge. The underlying thesis is that one learns more effectively when information is presented in a manner congruent with one's favoured method of acquiring and processing information (Montgomery, 1995).

Considerable research has dealt with the mathematical performance of elementary school students, but far less research has dealt with what their teachers understand. The few studies that have investigated the mathematical understanding of elementary teachers and preservice elementary teachers indicate that many exhibit weakness in mathematics, may misapply mathematical rules, do not understand true meanings of mathematical concepts, and that they are, generally, not prepared to teach the mathematical subject matter entrusted to them (Cuff, 1993; Hungerford, 1994).

Understanding how teachers implement a problem solving approach to teaching and learning is vital (Lester, & Garofalo, 1982; Takahashi, 2000) to answering the questions posed through this study. Furthermore, very little is known about teaching and learning which aims to develop the number sense ability of students (McIntosh et al., 1997).

2.6.2 The case for investigating teaching style and learning style

Studies in teaching and learning style in general or in subjects other than mathematics can equally help us enrich both teacher and student perception of the importance of knowing their particular teaching and learning style, respectively. Though addressed for the most part to a scholarly audience, such studies afford insights into the congruence of style structures and content that can be adapted to classroom use (Morphord, & Willing, 1991a, 199b). Furthermore, the notion of an affective domain in mathematical problem solving has strengthened the latter's link with both teaching style and learning style (DeBellis & Goldin, 1987, 1997). In a more specific sense, Goldin (2002) observes that "when individuals are doing mathematics, the affective system is not merely auxiliary to cognition – it is central" (p.60).

In the literature, interest in teachers' teaching style and the students' learning styles has grown during the last decade or so. This has led to a firm conviction of their influence on instructional practice (Woods, 1995). For instance, Noble and Bradford (2000) have written a book which looks at reasons for boys' under-achievement, ways of adapting teaching styles to maximise learning gains for boys, and girls, guidance on how to plan successful pyramid, whole school and classroom approaches, practical strategies for subject leaders and teachers, examples of successful case studies, and how to introduce ideas through professional development. Though there is still some sceptical fear of accepting that teachers' teaching styles' impact upon students' learning styles play a great role in the latter's' learning, research in this area is gathering momentum, especially at post secondary level (Felder, 1993; Harp & *Orsak,* 1990; Park, 2001).

Montgomery (1995) presented a scheme which classifies learning styles of college students into four domains with each domain subdivided into two styles, as shown in Table 2.1. She then goes on to explain how these domains and categories play out in the typical classroom. According to her, current research, though sketchy and preliminary, strongly suggests that college students are generally active, sensing, visual, sequential learners; as opposed to reflective, intuitive, verbal and global learners (Table 2.1). Roughly translated, most college students receive instruction by the traditional lecture method, while their learning styles are incompatible with that delivery mode. In short, there is a disconnection between teaching style and learning style. It is like teaching the blind with pictures and teaching the deaf with the spoken word (Montgomery, 1995).

Table 2.1 Information Handling Domain vs Learning Styles

Information Handling Domains			
Processing	Perception	Input	Understanding
Active 67%	Sensing 57%	Visual: 69%	Sequential 71%
learn best by doing something physical with the information	prefer data and facts.	prefer charts, diagrams and pictures.	easily make linear connections between individual steps
Reflective 32%	Intuitive 42%	Verbal 30%	Global 28%
do the processing in their heads	prefer theories & interpretations of factual information	prefer the spoken or written word.	must get "big picture" before individual pieces fall into place

Montgomery’s advice that the college-age students in schools grew up with television, movies, video, and video-games exacerbates the situation, adds a new dimension in the discussion about teaching style versus learning style. Moreover, NCTM’s (2000) ‘Technology Principle’ states that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning” (p. 24). The call here is for teachers to be in tandem with the culture of the present time. In his timely thesis, which is appropriately entitled *Learning style and teaching style interaction and the effect on psychological reactance,* McMillin (1999) warns that the more time wasted in not accepting that there is a great need to further research the relationship between students’ learning styles and their teachers’ teaching styles, the greater the chasm being created between teaching and learning.

The point being made is that the present generation of students are living in a completely different world from those of the pre-twentieth century; they live in a technological world (Means & Olson, 1994; Mergendoller, 1994; Waits & Demana, 2000). After all, the ‘video game’ generation has developed finely honed skills in interacting with machines having computer components, computers, interpreting visually displayed data, and ‘seeing the big picture’ (Montgomery, 1995). Montgomery’s assumption that students have developed an intuitive ‘feel’ for the new media, along with heightened impatience, is probably a well grounded one. Her analysis of learning styles versus lecture reveals that in all categories but ‘Sequential’, as shown in Table 2.2, the learning styles of today’s students favour teaching formats other than lecture.

Table 2.2 Students’ learning styles versus lecture characteristics

Learning Styles		Lecture Characteristics	
67%	Active	32%	Reflective (Passive)
57%	Sensing	42%	Intuitive
69%	Visual	30%	Verbal
28%	Global	71%	Sequential

2.6.3 Discovering teaching style and learning style: the development of instruments

The idealised vision for school mathematics as presented in Chapter 1 of NCTM (2000) Principles and Standards for School Mathematics*, is* suggesting that a teacher’s learning/teaching style has a bearing on a child’s learning. Hence, it has acted as a catalyst in encouraging researchers to look into teaching and learning styles, a move which has resulted in the development of teaching and learning style ‘tests/inventories/questionnaires’.

This study intends to use appropriate learning/teaching style instruments to check for the degree of correlation between teacher’s teaching styles and students’ learning style(s). It could be that a teacher’s learning style affects his/her teaching style, which in turn could favour students with certain categories of learning styles. What is needed is a solid theory which would uphold the claim for teaching which caters for individual learning styles (Swailes & Senior, 2001). To facilitate identification of learning style and teaching style a substantial amount of ground work has been done, resulting in the production of instruments such as inventories or questionnaires (Grasha, 1996), with most of these instruments dealing with learning style. There are numerous models of learning styles, but very few of them are geared towards primary school students. For instance, a paper by De Bello (1990), describes just 11 of them, with possibly two which could be used at primary school level.

Two of the most popular learning style inventories are those of David Kolb (1984) and Richard Felder (1993). Kolb’s Learning Style Inventory describes the way you learn and how you deal with ideas and day-to-day situations in your life. In 1984 Kolb presented his learning cycle model called *Experiential Learning,* which together with his learning style inventory and associated terminology are based on the work of John Dewey, Kurt Lewin, Jean Piaget, and J. P. Guilford.

Felder (1993), who carried out some studies on learning and teaching styles in college science, advised that if professors teach exclusively in a manner that favours their students' less preferred learning style modes, the students' discomfort level may be great enough to interfere with their learning. On the other hand, if professors teach exclusively in their students' preferred modes, the students may not develop the mental dexterity they need to reach their potential for achievement in school and as professionals. Felder (1993) is adamant that a key objective of education should thus be to help students build their skills in both their preferred and less preferred modes of learning. Learning style models that categorise these modes provide good frameworks for designing instruction with the desired breadth. The goal is to make sure that the learning needs of students in each model category are met at least part of the time. This is referred to as ‘teaching around the cycle’.

Maybe the teacher has always or most often taught through a problem-solving medium, and maybe he or she has followed researchers' suggestions as to how best to get children to develop number sense, but at the end of the day it could be that her learning/teaching style has favoured some pupils and disfavoured others (Dunn & Dunn, 1993; Fairhurst & Fairhurst, 1995).

In their book *Effective Teaching, Effective Learning: Making the Personality Connection in Your Classroom*, Fairhurst and Fairhurst (1995) illustrate teaching styles and learning styles. Applying the insights of both temperament theory and the Myers Briggs Type Indicator (MBTI) personality inventory, they describe the preferred teaching and learning styles for 16 personality types. In a bid to encourage teachers to incorporate awareness of teaching and learning styles in their teaching, the authors have filled their practical work with concrete examples, practical techniques and diagnostic aids, in an attempt to help teachers bridge the gap between their preferred teaching techniques and the needs of their students.

Nevertheless, the ever growing call for teachers to cater for their students' learning styles should be approached with cautious insight and informed decisions. Way back in 1984 Kolb saw a potential problem, which forced him to suggest that the learning process is not identical for all people and that identifying learning styles should not be used as a means of typecasting people, but as a way of determining choices, decisions and possibilities. He suggested that early educational experiences shape learning styles, and that we are taught (not always consciously) how to learn. Later on, educational specialisation, job roles, and professional careers will influence learning styles. Kolb also acknowledged that people adapt their learning styles to suit specific tasks and problems. Learning styles should not be perceived as fixed personality traits, but as adaptive states. Kolb suggested that experiential learning theory provides a system for managing the learning process in a way which can accommodate the different learning styles.

Research has shown that learning about number and operations is a complex process for children (e.g., Fuson, 1992), a situation which could be alleviated through an effective teaching approach. In the Australian context, increased emphasis is being promulgated towards more acceptance of a teaching approach which incorporates knowledge of teaching and learning styles. In tandem with this new tendency, the Australian Council for Educational Research (ACER) published a book by Owens and Barnes (1992), which offers an easy way to assess how teachers, students and parents respond to different ways of learning, by catering for students' learning styles. This calls for a totally new teaching approach.

While there are obvious drawbacks to typecasting and categorising people, the identification of a learning style can help learners to recognise some of the strengths and weaknesses of their current approach to learning (Boud, Cohen, & Walker, 1993). It can also help them to appreciate other styles and strategies, and to consider how to extend their range of learning styles (Dodds, 2004). Over emphasis on particular phases of the learning cycle can restrict learning capacity. For instance, taking Kolb's model as an example, the teacher must bear in mind that given a specific task, some learners will get on with it straight away and get through by trial and error. They are 'activists'. Some will enjoy the practical application of ideas in a common sense way. They are 'pragmatists'. Others prefer to stand back, observe and think things through analytically. They are 'reflectors'. Some people prefer to work systematically on a structured programme. They are 'theorists' (Kolb, 1984; Dunn & Dunn, 1999). Therefore, in the broader context a major role of the teacher is to help learners avoid concentrating only on their favoured style because certain learning situations may require different learning styles (Felder, Felder, & Dietz, 2002).

2.7 The Envisioned Ideal: Teaching to Promote Problem Solving and Number Sense

What sort of teaching will be observed during this study will depend on a lot of factors. Teachers who endeavour to develop the number sense and problem solving of their students have to consider the implications of these factors. It seems that there are three schools of thought and practice: those advocating reform, those who are suspicious of such reforms, and the teachers who find themselves in between these two extremes. This could make it very difficult for teachers to teach number sense through the desired problem solving approach. Then there is also the controversy caused through those who report that teachers are up-to-date in their teaching of mathematics and those who proclaim the opposite.

By now, it is expected that teaching strategies would have taken the results of the above-mentioned research and information into account. Swafford (1995) claimed that many teachers fail to use a problem solving approach in their teaching of mathematics. It could be that where problem solving is concerned it is now the case that teachers are becoming better equipped to implement the recommended approach. It helps to remember that although mathematical problems have traditionally been a part of the mathematics curriculum, it has been only comparatively recently that problem solving has come to be regarded as an important medium for teaching and learning mathematics (Stanic & Kilpatrick, 1988). In the past problem solving had a place in the mathematics classroom, but it was usually used in a token way as a starting point to obtain a single correct answer, usually by following a single 'correct' procedure. More recently, however, professional organisations such as the National Council of Teachers of Mathematics (NCTM, 1980, 1989) have recommended that the mathematics curriculum should be organised around problem solving. With the advent of emphasis on number sense, and given its camouflaged relationship with problem solving, it is an opportune time to investigate such a relationship and how teachers are implementing the relevant curriculum, through the suggested new teaching approach. The whole gamut of factors which contribute towards effective or non effective mathematics teaching would fit under the classroom environment umbrella.

2.7.1 Teaching and learning environment needs

The nature of both problem solving and number sense requires a teaching environment which is conducive to encouraging students to feel mathematically empowered. Classrooms in which students have ready access to materials such as counters, calculators, and computers, and in which they are encouraged to use a wide variety of strategies, support thinking that results in multiple levels of understanding (NCTM, 2000). Teaching and learning which promote development of number sense and problem solving have to incorporate certain codes of practice, which will necessarily permeate and rule the ethos of the resulting teaching and learning environment (Marshall, 1995; McIntosh & Ellerton, 1998; NCTM, 2000). The literature reveals that such codes of practice are observed through an amalgamation of certain influential factors. For this study the following factors are deemed important to observe: the impact of teachers' and students' beliefs; the type of assessment which develops mathematical power; the importance of monitoring children's learning habits; the influence of emotional factors; availability of, access to and appropriate use of calculators; how to teach for number sense development in a problem-centred maths curriculum; what teaching through a balanced curriculum entails; what it means and how to empower the students; why teaching should incorporate knowledge of students' interest; catering for the differences in learning styles. One of the greatest contributors in students' learning is the quality of the teacher's amalgamation of the influential factors (Borich, 1999; Good & Brophy, 1997). The desired environment is one which will encourage discovery learning to proliferate. As suggested by Baturo (2004):

> good teachers are not necessarily those who know, but are those who can recognise and utilise powerful ideas when they see them. The support they need is the inclusion of these ideas in their daily teaching. (p. 100)

Since the idea of constructivism is more important in discovery learning, many of its principles would fit into the reformed practice of teaching being advocated. According to Li (1996) the key points of the principle of instructional design, based on constructive learning theory, can be summarised as follows:

(1) emphasis on learner-centred

(2) emphasis on the important role of 'scene' in meaning construction

(3) emphasis on the design of learning environment (not teaching environment)

The teacher who is aware of the third point could be more disposed towards employing a teaching style which caters for differences in learning style (Hayes & Allinson, 1993). However, catering for differences in learning style should not be interpreted as matching of teaching style and learning style, for much controversy exist concerning the latter notion (Hayes & Allinson, 1996). For a long time it has been assumed that if teachers were able to diagnose the learning style of their students, then it would seem logical to assume that matching the characteristics of instruction to that style would make the instruction more effective (Pinto, Geiger, & Boyle, 1994). According to this assumption students tend to enter a learning situation with a style of learning already developed (Gregory & Butler, 1984). If they meet a learning environment at variance with that style, then it is likely the student will reject the learning environment (Kolb, 1976). In a later study Kolb (1984) concluded that there were potential long term benefits where there was an intentional mismatch between learning style and instructional style. So, the mathematics teacher who is aware of this would need to heed NCTM's (2000) call for striking a balance. Thus it is clear from the research evidence available that there remains much debate over the effectiveness of matching learning style and instructional style (Pinto, Geiger, & Boyle, 1994). Matthews (1991) argues that:

> While mismatching is appropriate for developmental reasons, students have more positive attitudes towards school and achieve more knowledge and skills when taught, counselled or advised through their natural or primary style rather than a style that is secondary or undeveloped, particularly when adjusting to a novel and new situation that creates stress... (p. 253)

Such beliefs persist although, as pointed out by Robotham (1999), “for each research study supporting the principle of matching instructional style and learning style, there is a study rejecting the matching hypothesis” (http://www2.glos.ac.uk/gdn/discuss/kolb2.htm).

2.7.2 The teacher’s role: catering for differences in learning styles

Teaching to cater for individual students’ learning styles is quite a tricky thing. Felder (1993) predicted that if professors (teachers) teach exclusively in a manner that favours their students’ less preferred learning style modes, the students’ discomfort level may be great enough to interfere with their learning. Thus many teachers think that the solution is to teach exclusively in their students’ preferred modes (Honey & Mumford, 1992), not realising that the students may not develop the mental dexterity they need to reach their potential for achievement in school and as professionals (Felder, 1993).

Thus, contrary to popular beliefs in the learning style-teaching style circle, which emphasises employing teaching styles which match students’ learning styles, Duch, Groh and Allen (2001) pointed out that the teacher should focus on problem-based learning. Since, problem-based learning itself takes students’ individual differences into consideration, teachers are advised to prepare meaningful learning situations so that each student would participate actively in problem solving. On the other hand proponents of Multiple Intelligence are urging teachers to focus on helping students build their skills in both their preferred and less preferred modes of learning (Goodnough, 2001). Such controversies leave many educators with the question, “Is it really possible to cater for students’ learning styles?” Nevertheless, those who support matching teaching and learning styles have proposed that one way of catering for individual learning style is for teachers to teach students to become aware of how they think (Colucciello, 1999). To do that the teacher must also understand how they as a teacher think (Guild, Guild, & Guild, 1986). Hence, teachers need to find ways to understand their own learning styles. With such knowledge they would be better equipped to teach students about their own learning styles (Felder, 1993).

The type of teaching being desired is one which is relying on the use of already known motivation enhancers such as realistic materials, visual aids and ‘hands on’ materials. According to Honey and Mumford (1992), competent trainers/developers [teachers] are characterised by the following four points:

1. They place emphasis on the learning process - their own and other people's
2. They know their own learning style preferences
3. They understand how their learning style ‘contaminates’ their training/teaching style
4. They are alert to the learning styles of their ‘customers’/trainees [students] and adjust their approach accordingly. (p. 15)

Furthermore, it is thought that poor academic performance, lack of motivation and failure to participate actively in class signal that learners differ in their styles of learning — and students who previously had difficulty in math may have been taught using a style which did not resonate with them (Claxton & Murrell, 1987; Felder, Felder, & Dietz, 2002; Owens & Barnes, 1992). Moreover, Baturo (2004) made a very pertinent observation about teachers’ beliefs which focussed solely on academic achievement while failing to consider differences in learning style. In reference to the beliefs of teachers who taught Aboriginal students Baturo (2004) pointed out that:

> Interestingly, when the teachers in this study were asked whether or not there were any mathematics learning differences between Aboriginal and non-Aboriginal students, many of the teachers spoke of achievement related issues such as attendance and school readiness rather than actual learning styles. (p. 245)

The literature pertaining to teaching style and learning style indicates that the benefits to learners of having correct knowledge of their learning style, and putting into practice such knowledge, greatly outweigh the possible misconceptions pointed out so far. The identification of preferred learning styles raises the individual learner’s awareness of his or her own approach to learning (Honey & Mumford, 1992). One aspect of this study intends to check upon any progress or transformation which could occur as a result of teachers and students knowing about their teaching and learning styles. It is well documented in the literature that when a group undertakes the exercise, they are likely to benefit from the awareness of different learning styles (Schroeder, 1996). Moreover, a teacher who clearly understands the possibilities and limits of his or her teaching style can make more consistent judgments about how best to use this medium (Grasha, 1996). Butler (1986) asserts that the teacher must be able to identify, appreciate, and explore his or her own teaching style in order to achieve style-differentiated instruction. Teachers are expected to introduce their students to the notion that the people who will cope most effectively, and gain from the greatest variety of opportunities, relationships and events, will be the people who can operate to some extent in all styles, but who can also decide which is the most effective style to adopt given a particular situation (Wagener, 2002). When the preferred learning styles have been identified and taken into account along with other learner characteristics, the learning styles can be developed to accommodate purpose, strategy, outcomes and review. Thus, teachers who have an awareness of the impact of teaching style upon learning style, and who believe in and practice teaching number sense through a problem solving approach, would necessarily see their role in a totally different light compared to those who are of the opposite view.

The decisions that teachers make about number sense through problem-solving opportunities influence the depth and breadth of students' mathematics learning. Teachers must be clear about the mathematics they want their students to accomplish as they structure situations that are both problematic and attainable for a wide range of students (Bransford & Stein, 1993). They make important decisions about when to probe, when to give feedback that affirms what is correct and identifies what is incorrect, when to withhold comments and plan similar tasks, and when to use class discussions to advance the students' mathematical thinking. By allowing time for thinking, believing that young students can solve problems, listening carefully to their explanations, and structuring an environment that values the work that students do, teachers promote problem solving and help students make their strategies explicit. The NCTM (2002) states:

> The teacher's role in choosing worthwhile problems and mathematical tasks is crucial. By analyzing and adapting a problem, anticipating the mathematical ideas that can be brought out by working on the problem, and anticipating students' questions, teachers can decide if particular problems will help to further their mathematical goals for the class. There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time. Choosing problems wisely, and using and adapting problems from instructional materials, is a difficult part of teaching mathematics. (p. 53)

The ability of the teacher to fulfill the role of an effective teacher who caters for differences in learning style will depend a lot on the wisdom of that teacher. Grasha (1996) advises that a teacher who clearly understands the possibilities and limits of his or her teaching style can make more consistent judgments about how best to use this medium to get children interested in learning mathematics.

2.7.3 Knowledge of students' interests

Knowing students' interests allows teachers to formulate problems that extend the mathematical thinking of some students and that also reinforce the concepts learned by other students who have not yet reached the same understandings. Furthermore, the teachers' role in teaching number sense requires that they endeavour to create in such students a spirit of flexibility in interaction with non-routine situations and problems. McIntosh et al. (1997, pp. 43-44) discussed the importance of the teacher's role in the development of number sense, including:

- to promote the development of sense-making;
- to create classroom settings where why (the meaning) is as important as what (the answer) or how (the method);
- to present activities that challenge and engage students to re-invent conceptions from different perspectives;
- to promote students' reflection on their own learning.

This discussion of the teacher's role clearly requires of the teacher a keen sense of monitoring the students' interest, since development of both number sense and problem solving depend a lot upon such interest (Turner, 1996). Furthermore, having good number sense and being proficient in 'novel' problem solving are important ingredients in the empowerment of learners (Thiessen & Trafton, 1999). Thus, teaching which takes into consideration students' interest has a better chance of empowering the students (Harvey & Burrows, 1992).

2.7.4 Empowering the students

According to the National Assessment of Educational Progress (NAEP, 1988) mathematical power is characterised as a student's overall ability to gather and use mathematical knowledge through exploring, conjecturing, and reasoning logically; through solving non-routine problems; through communicating about and through mathematics; and through connecting mathematical ideas in one context with mathematical ideas in another context or with ideas from another discipline in the same or related contexts.

To Leder (1992), mathematical power requires that students be able to discern relations, reason logically, and use a broad spectrum of mathematical methods to solve a wide variety of non-routine problems. Furthermore, he states that "the repertoire of skills which now undergird mathematical power includes not only some traditional paper-pencil skills, but also many broader and more powerful capabilities" (p. 82). As highlighted by the National Research Council (1989a) today's students must be able to:

- perform mental calculations and estimates with proficiency;
- decide when an exact answer is needed and when an estimate is more appropriate;
- know which mathematical operations are appropriate in particular contexts;

- use a calculator correctly, confidently and appropriately;
- estimate orders of magnitude to confirm mental or calculator results;
- use tables, graphs, spreadsheets, and statistical techniques to organise, interpret, and represent numerical information; judge the validity of quantitative results presented by others;
- use computer software for mathematical tasks; formulate specific questions from vague problems;
- select effective problem-solving strategies. (pp. 82-83)

Up to 1998 there was still this problem of many students not learning the mathematics they needed or were expected to learn (Kenney & Silver 1997). By the year 2000 this situation apparently still persisted. NCTM (2000) suggests that there are many reasons for this deficiency.

> In some instances, students have not had the opportunity to learn important mathematics. In other instances, the curriculum offered to students does not engage them. Sometimes students lack a commitment to learning. The quality of mathematics teaching is highly variable. (p. 5)

Hence, the underlying implication is that teachers need to address these concerns in their teaching to empower the students.

Baturo (2004) suggested that it would make it a lot easier for a teacher to be able to empower students mathematically if the teacher in question was himself or herself mathematically empowered. In a paper presented at the 28th Annual Conference of the International Group for the Psychology of Mathematics Education, Baturo (2004) highlighted the positive effect on student learning as a result of empowering a 20-year old classroom teacher with subject matter knowledge which was relevant to developing fraction understanding. Other researchers such as Goldman and Hasselbring (1997) point out that if students are to use "mathematics to solve complex mathematical problems that arise in day-to-day life, they need opportunities to learn in these contexts" (p. 202).

The value of fostering learning within a meaningful context is firstly that students are more likely to make links between their informal mathematical knowledge developed in the world outside school with the more formal or scientific knowledge required at school (Noble & Bradford, 2000). Secondly, students are involved in developing mathematical sense and skills in contexts that make sense to them and thirdly, authentic and meaningful tasks are more likely to be motivating to students (Hiebert et al., 1997).

2.7.5 Teaching through a balanced curriculum

Different educators hold different views of the type of balance we should have in a mathematics program. According to Mokros and Russell (2000) some seek a balance that is usually a response to a political, rather than a pedagogical problem. Others, such as The Massachusetts Mathematics Curriculum Framework (2000) attempt to answer the question by balancing different philosophies of learning. In the document, the major principles of mathematics learning, problem solving, communicating, connecting, and reasoning are right alongside standards of mastery for learning specific procedures at an early age.

Consequently, calls for teachers to teach through a balanced curriculum, have generated some opposition, from those who fear of the consequences that this may have on classroom teaching. Although they agree that it is possible to achieve balance in a mathematics program Mokros and Russell (2000) warn that we must be wary of advocating certain kinds of balance, especially the 'mixed program'. Recent research is showing that mixed programs may be confusing to students (Goodrow, 1998). Mokros and Russell suggest that too often, mathematics curricula lead children through a whirlwind tour of as many mathematical topics as possible at each grade level in an attempt to balance content areas and pedagogical approaches. Typically, elementary students using textbooks are exposed to one new mathematics concept in class every 30 minutes. For example**,** Mokros and Russell observe that in one study by Goodrow (1998), second graders whose mathematics programs focused on building understanding of number relationships were much more accurate in solving addition and subtraction problems than children whose mathematics programs involved a mixture of conceptual understanding and learning procedures. Goodrow (1998) concludes that children in the mixed group almost always performed less well than children who had either a curriculum built on understanding number relationships or a curriculum built on learning procedures. This was particularly true on more difficult problems. Goodrow suggests that children in mixed programs may be confused about when to use a taught procedure and when to rely on one's own number sense. Hence, it is important to find out what type of teaching is employed by mathematics teachers to deal with this problem of finding the right balance.

The claim by NCTM (2000) that a balanced curriculum includes number sense as a component along with practical applications, theoretical development, and problem solving, clearly places equal emphasis upon the importance of both number sense and problem solving in the teaching and learning of mathematics. The NCTM (2000) statement, "Through problem solving, students can explore and solidify their understanding of number" (p.32) is an epitome of the close association of problem solving and number sense; a theme which is prevalent throughout NCTM's *Standards* (2000). The emphasis upon number sense as the cornerstone of the entire mathematics curriculum (Reys & Nohda 1994) and its relationship to problem solving is given even more credibility through the following statement of NCTM (2000):

> In these standards, understanding number and operations, developing number sense, and gaining fluency in arithmetic computation form the core of mathematics education for elementary grades. As they progress from prekindergarten through grade 12, students should attain a rich understanding of numbers – what they are; how they are represented with objects, numerals, or on number lines; how they are related to one another; how numbers are embedded in systems that have structures and properties; and how to use numbers and operations to solve problems. (p. 32)

Therefore, the extent to which teachers are aware of such a call and how they ensure that students are being given the opportunity to reach such standards is a major concern (Center for Applied Research and Educational Innovation, 2001; Hiebert, 2003; Kilpatrick, Swafford, & Findell, 2001), which needs to be addressed through research.

Although learning with understanding has been advocated as essential in enabling students to solve the new kinds of problems they will inevitably face in the future (Schoenfeld 1989, 1992; Skemp 1987; NCTM 2000) many questions still need to be answered about teaching and learning with respect to children's number sense and related problem solving ability (Dougherty & Crite, 1989; Illinois State Board of Education, 1997). The attention of the proposed study has been fixed by the ensuing notion that, since number sense involves people being in control of their use of number and operations, as opposed to just remembering procedures, this could have an effect on that person's problem solving proficiency.

2.7.6 Number sense in a problem-centred mathematics curriculum

For teachers to develop number sense they must expose students to relevant number sense activities and problems on a daily basis. Thiessen's and Trafton's (1999) description of number sense emphasises the fact that number sense is not a discrete set of skills to be taught for three weeks in a certain month of the year or something that only those that are 'good at math' have. To Wilson-Carboni (2001) number sense is more than that.

> It is a part of children's daily mathematical lives and slowly grows and develops over time. In a problem-centered mathematics curriculum, number sense is closely tied to problem solving. Children used to a problem-centered mathematics curriculum are capable of solving problems and they can play with numbers to make sense of a problem. They use their growing number sense to develop strategies to help them solve problems. (http://www.learnnc.org/index.nsf/doc/numsense0402-1)

In response to the question "How can instruction be organised to facilitate the development of number sense?" McIntosh et al. (1997) highlight the following:

> Brownell (1945) characterised his theory of meaningful arithmetic as, "instruction which is deliberately planned to teach arithmetical meanings and to make arithmetic sensible to children through its mathematical relationships". Markovits and Sowder (1994) characterise instruction focused on developing number sense as "…instruction designed to provide rich opportunities for exploring numbers, number relationship, and number operations and to discover rules and invent algorithms". (p. 4)

The overriding suggestion is that instead of teaching problem solving separately, teachers should embed problems in the mathematics-content curriculum. When teachers integrate problem solving into the context of mathematical situations, students recognise the usefulness of strategies. Teachers should choose specific problems because they are likely to prompt particular strategies and allow for the development of certain mathematical ideas. For example, the problem "I have pennies, dimes, and nickels in my pocket. If I take three coins out of my pocket, how much money could I have taken?" can help children learn to think and record their work. Teaching and learning mathematics should reflect what goes on in real life, and since we live in a world which employs technology on a day-to-day basis, the problem solving approach must make use of available technological tools, such as calculators. This calls for a specific type of teaching environment.

2.7.7 Availability of, access to, and appropriate use of calculators

Calculators should be available at appropriate times as computational tools, particularly when many or cumbersome computations are needed to solve problems (Groves & Cheeseman, 1992). However, when teachers are working with students on developing computational algorithms, the calculator should be set aside to allow this focus. Today, the calculator is a commonly used computational tool outside the classroom, and the environment inside the classroom should reflect this reality. A look at research findings reveals that the effective teaching of number sense and problem solving requires a teaching style which aims to cater for all students. Such a teaching and learning ethos would necessarily involve the use of the calculator, as supported through the following.

i. Research has shown that calculators can aid in "stimulating problem solving, in widening children's number sense, and in strengthening understanding of arithmetic operations." They can also help students learn basics, such as numbers, counting, and the meaning of arithmetic operations (Campbell & Stewart, 1993, p. 14)

ii. Students also show greater ease in problem-solving when using calculators, since they focus less on computational recall and algorithmic routines and more on the other parts of the problem-solving process. Appropriate calculator use also "promotes enthusiasm and confidence while fostering greater persistence in problem-solving. (Campbell & Stewart, 1993, p. 14)

iii. Children often learn better and retain more information when they use calculators. And frequently students who use calculators rather than paper and pencil for much of their class work score higher on paper-and-pencil tests than do their non-calculator-using counterparts. Studies have also shown that some students learn basic computational facts better through the use calculators, even when learning the basic facts was not the specific reason for using the calculators. (Suydam, 1987, p. 31)

iv. Research from over 100 studies indicates that the use of calculators (a) promotes achievement, (b) improves problem-solving skills, and (c) increases understanding of mathematical ideas" (Suydam, 1987, p. 31).

v. Students using calculators possess a better attitude toward mathematics and an especially better self-concept in mathematics than noncalculator students. This statement applies across all grades and ability levels. (Hembree & Dessart, 1986, p. 84)

vi. When graphing calculators are incorporated, female performance improves in the areas of confidence, spatial ability, algebra skills, and classroom environment (Dunham, 1995).

vii. What is needed is neither a generation of techno-dependent nor one of techno-ignorant citizens, but one that is techno-literate and able to use the power of sophisticated machines in sensible ways when it is appropriate to do so. (Sparrow & Swan, 2004, p. 53).

2.7.8 The influence of emotional factors

While mathematics has a reputation for being rational and logical, it is also very much affected by emotion: when people feel confident and resourceful, they can do their best; when they feel anxious and inadequate, they find it difficult to handle the mathematics. For many, 'math anxiety' inhibits their learning (Arem, 2003; Buxton, 1981; Immergut & Burr-Smith, 2005; Newstead, 1998; Tobias, 1995), which would necessarily affect their motivation. Such a situation has resulted in various implications for teaching with many suggestions being provided as a means of encouraging teachers to find ways to motivate their students through meaningful problem solving activities.

For example, Blumenfeld,Soloway, Marx, Krajcik, Guzdial, and Palincsar (1991) described how the incorporation of long-term projects into the teaching-learning experiences encourages students to engage in the solving of “authentic” problems and increases students’ investment in classroom learning.

Turner, Thorpe and Meyer (1998) investigated fifth- and sixth-grade students’ reports of motivation and negative affect in mathematics classes. The results revealed four clusters of motivation and affect among participants, which appeared to be related mainly to the relative strength of the participants' self-regulatory beliefs and behaviours. These clusters were learning oriented, success oriented, uncommitted, and avoidant. The learning oriented and success oriented clusters showed adaptive patterns of learning strategy use and a challenge orientation. In comparison, the uncommitted and avoidant clusters indicated support for more maladaptive responses to challenging work. When learning goals were lower and ability goals were higher or when both were relatively equal and low, there was a relationship between these patterns and more negative patterns of affect. Results supported the idea that affect was a key factor in the implementation of motivational goals. The investigation envisaged through this study could encounter situations where students’ motivation and affect impact on learning. Although this is not a study into cognitive and affective domain issues per se, it would be interesting to observe how teachers react in such circumstances, as they endeavour to mathematically empower the students.

2.7.9 Assessing for mathematical power: Monitoring children’s learning habits

One of the major foci of the NCTM *Standards*, and hence their recommendations for assessment, is the notion of mathematical power. In his article *The 12 Good Habits of Effective Learners*, Wagener (2002) observes that in developing good learning habits, teachers should be interested not only in what a student knows but also how the student responds when he or she does not know the answer. Wagner provides educators with some guidelines as to what behaviour indicates the development of good learning habits. He proposes discussing perseverance, taking time, listening, alternatives, being aware, checking, habitual questioning, seeking accuracy, using all the senses, being creative, enjoying problem solving, and letting the past help the present as pointers of good learning habits. Although not explicitly stated by Wagner, students who exhibit such behaviours have a greater chance of acquiring and developing mathematical power (Lowrie, 1999).

As already indicated, central to the NCTM *Standards* description of the features of students' performance that should be assessed is ‘mathematical power’ (NCTM, 2000). The aim of teaching problem solving and number sense is mainly to empower the students so as to apply mathematics learnt in novel situations. Hence, developing students’ mathematical power calls for assessing their number sense and problem solving in a creative way. Mathematical power includes being able, and predisposed, to apply mathematical understanding in new situations, as well as having the confidence to do so. This calls for teachers to apply a comprehensive program of mathematics assessment, which includes opportunities for students to show what they can do with mathematics that they may not have studied formally but that they are prepared to investigate. Some assessments may be designed to determine how well students, presented with an unfamiliar situation, can use what they have learned previously. Other assessments may require that students learn a new mathematical concept or strategy during the assessment and use this knowledge to solve problems (NCTM, 2000).

Assessing students’ abilities to solve problems is more difficult than evaluating computational skills. However, it is imperative that teachers gather evidence in a variety of ways, such as through students' work and conversations, and use that information to plan how to help individual students in a whole-class context. Knowing students’ interests allows teachers to formulate problems that extend the mathematical thinking of some students and that also reinforce the concepts learned by other students who have not yet reached the same understandings (Boaler, 1998; Cobb et al. 1992; Kilpatrick et al., 2001).

In its 1992 publication *Assessment and Learning of Mathematics,* written for mathematics educators, ACER offers various perspectives on the links between teaching, learning, and assessment of authors from Australia, the United States, and the Netherlands. A recurring theme in the book is that narrow assessment procedures may explain why classroom practices lag behind current views of how mathematics should be learned. Through this study vital information could be obtained as to whether teachers are aware of and are applying these new assessment ideas, since they form part and parcel of the whole teaching through a problem-oriented curriculum (Leder, 1992).

2.7.10 The impact of teachers’ and students’ beliefs

An additional factor which could easily upset or enhance the whole teaching approach being advocated is the impact of the teacher’s and students’ beliefs upon their teaching and learning, respectively (Cooper, Baturo, Warren, Doig, & Shani ,2004;

Middleton, 1992; Stipek, Givvin, Salmon, & MacGyvers, 2001). Although this study does not espouse teachers' and students' beliefs as one of its major research factors, it does intend to gauge teachers' and students' perceptions, as a springboard from which other data collection will be informed.

Like most other issues directly related to the learning of mathematics, the reverse is also true; problem solving and number sense influence students' perceptions. For example, Curtis (1995) discovered that incorporating problem-solving activities into a developmental studies mathematics class influenced the students' beliefs about the nature of mathematics or their mathematics self-efficacy.

A teacher's beliefs affects the way that teacher practices the teaching of materials and concepts related to those beliefs (Thompson, 1992). Moreover, students' beliefs add another dimension to the whole process of teaching and learning of mathematics (Beswick, 2002) For instance, part of the uneasiness felt about the use of calculators in classrooms is a result of the belief that mathematics is and should be hard work, work that is normally associated with manual computations and manipulations (Campbell & Stewart, 1993).

Ernest (1989) argues that, for example, a teacher with beliefs about mathematics that reflected an instrumental view (in which mathematics is seen as an accumulation of facts, rules and skills) was likely to have an instructor view of teaching that taught skill mastery. Similarly, a teacher with belief about mathematics that reflected a Platonist view in which mathematics is discovered was likely to have an explainer view of teaching following a conceptual model of knowledge. And finally, a teacher with a belief about mathematics that reflected a problem solving view, seeing mathematics as a field of human creation and invention, was likely to have a facilitator view of teaching that would allow active construction of understanding (Handal, Bobis, & Grimison, 2001). The same could be said of students' beliefs versus how they deal with mathematics (Stage, 1991; Handal & Herrington, 2000).

Hence, knowing about the participants' beliefs could yield very important information about their teaching and learning of problem solving and number sense. Ernest (1989) concluded that "mathematics teachers' beliefs have a powerful impact in the practice of teaching" (p. 249). He also argued that this impact has significant implications to bring about change in teaching. In considering the impact of both the teacher's and students' beliefs Bonotto (2001) reminds us that "we must endeavour to change students’ conceptions of, beliefs about and attitudes towards mathematics; this means changing teachers' conceptions, beliefs and attitudes as well” (p. 75).

2.8 The Impetus for this Study

Although much still needs to be done in the area of research on children’s related number sense and problem solving ability, some relevant articles have been written and a few related researches have been carried out. For example, in an endeavour to raise standards in mathematics at Key Stage 2 (KS2), Gary Gornell of Temple Primary School and Rob Halsall of Manchester Metropolitan University involved a cohort of 54 Temple Primary School pupils from Year 4 through to Year 6, together with the class teachers in each year, and the school's research co-ordinator in a case study. Pilot testing of the KS2 mental arithmetic tests had revealed obvious shortcomings in the children’s abilities to use mental strategies to solve mathematical problems. At the end of the project they summarised their findings as follows:

- National test results in maths at KS 2 for the research cohort of pupils represented a considerable improvement on the previous two years, with results at level 4+ moving from 49% in 1998 to 58% in 2000.
- Pupil attainment, as measured through pre and post intervention tests, improved in each phase of the three-year project, with greater improvements for individual pupils in each year.
- A close correlation emerged in Year 5 between pupils' mental and general maths improvement and their pre-intervention Stage of English Acquisition (SELA) grades, suggesting that some of the changes in classroom practice introduced by the research may have favoured those children most proficient in English language.
- Teachers found that they did spend much longer than usual in planning lessons in the light of video coverage, observations and reflection on what was and wasn't effective in previous lessons.
- Opportunities for pupils to create and explain their own methods of mathematical calculation prompted most enthusiasm on their part and were linked to score gains in post-testing.
- Pupil enjoyment of the lessons and a feeling that they were progressing, served to help develop *self-esteem* and *self-confidence* in maths; this, together with careful planning and sequencing of the lessons dramatically reduced the need for behaviour management.

(http://www.canteach.gov.uk/community/research/conference/templegoesmental/full/fullsummary.htm, 1997)

Such encouraging results warrant further research in the area of how teachers teach for development of number sense and problem solving. Moreover, in pointing out that “…the opportunity to view and discuss their own performance and pupil interactions helped greatly in evaluating teaching and learning and then planning future lessons”, Gornell and Halsall’s (2001) final reflection concerning teachers goes to show the benefits that the latter derive from such research. However, research focusing simultaneously on students’ styles and problem solving are very few (Heppner & Baker, 1997), if any, and one possible reason for this could be the lack of research in the development and implication of a personal problem-solving inventory (Heppner & Baker, 1997; Heppner & Pettersen, 1982; Lim, 2000). One such endeavour is Heppner's (1988) exploration of whether technological problem solving is similar to, or different from, personal forms of problem solving. The purpose of Heppner's study was to better understand the problem solving style dimension of problem solving. Another related study was undertaken in 1998 by Lynn English. The following extract from English's report has some relevance to this present study:

> As Lesh and Lamon (1992) noted, students who are competent in mathematics "often have exceedingly different profiles of strengths and weaknesses," with their learning progressing along a variety of paths and dimensions (p. 7). When presented with problem-posing activities, children who possess strong number sense but are weak in novel problem solving, for example, might display patterns of response different from children who display the reverse profile (i.e., weak in number sense but strong in novel problem solving). Children who are competent in both domains might show other patterns of response and perhaps display superior problem-posing skills. (p. 83)

This extract from English (1998) proposes two categories of students when it comes to comparing their proficiency in number sense and problem solving. Those who display different profiles of achievement in these two domains, and those who are competent in both domains. The first group could be further divided into (a) children who possess strong number sense but are weak in novel problem solving and (b) children who display the reverse profile (i.e., weak in number sense but strong in novel problem solving). Hence, there is a need to know:

- the extent to which the above notion is true and if yes, what is the proportion from each category;
- whether there is an acceptable balance or a marked discrepancy. If no then, whether number sense is a pre-requisite for problem-solving or whether the former is inherent in the latter;
- if number sense makes a person more able to use flexible procedures to meet new situations (i.e. solve real life/contrived mathematics problems) in daily life; and
- how students adapt to new problems involving number sense concepts in new contexts (e.g. If on a certain planet 3 + 5 = 15 and 27 – 3 = 9, then (i) 2 + 8 = ? (ii) 20 – 5 =?)

More recently an interesting step has been taken, through an article in the *Australian Primary Mathematics Classroom*, which could pave the way for a joint study of numeracy [number sense] and problem solving. This article poses several questions about problem solving and provides six reasons to support the belief that problem solving enhances the development of numeracy and mathematical thinking (Gervasoni, 2000).

Research that aims to examine the conditions for classroom practice which promotes students' learning and problem solving has been gaining ground (Gallagher, Rosenthal & Stepien, 1992; Jones et al., 1997). For instance, Williams (2000) studied the links between sustained engagement and conceptual development when her own senior secondary mathematics students worked collaboratively to solve an unfamiliar challenging problem. Such studies provide some examples of how real progress is being made into creating classroom environments that can effectively use a problem solving approach to learning mathematics (Sternberg, 2001; Trismen, 1988), but the issue of teaching style and learning style are not specifically addressed.

Hitherto the literature provides very little evidence of studies to investigate the relationship between teaching/learning style and problem solving ability in the learning of mathematics. Furthermore, many questions asked more than a decade ago are mostly still unanswered. Silver (1985) made the following observations:

> Noticeably absent from the literature are characterisations of current practice in the teaching of problem solving in classrooms. We do not know how teachers conceptualise problem solving, or how they attempt to teach it. We do not have answers to such questions as the following:
>
> - How much time is devoted to instruction in problem solving?
> - Are special problems or materials used to supplement basic textbook work?
> - Do teachers stress tool skills?
> - What types of questions do students ask?
> - Do teachers give attention to general heuristics?
> - Are reflection and post hoc analysis of problem situations an integral part of problem-solving instruction?
> - Are alternate solution methods solicited?
> - How long do students persist before giving up on a problem?
> - Do students tend to seek the advice or assistance of their peers?
> - Is that their preferred source of help?
> - What is done to convey the importance of problem solving? (p. 297)

Silver remarked that those questions and related ones are important if one wishes to improve the problem-solving ability of students, but at that time they were unanswered, and most of them are still waiting to be answered. This present study intends to address some of these questions.

Within the research community, problem solving has received further impetus from the study of small group learning processes (Johnson & Johnson, 1996), although such on-going research into the use of collaborative learning (of various types) in mathematics classrooms has hitherto failed to address the issue of problem solving as a medium for developing number sense. The continued highlighting of the way in which students solve problems, how they come to construct knowledge through this process and the need for metacognitive awareness of problem solving processes (Nisbet & Putt, 2000; Hacker, Dunlosky, & Graesser, 1998), albeit a necessary one, makes very little mention of the relationship between number sense and general mathematical problem solving. Encouragingly, availability of technological instruments is one major contributing factor towards increased interest in number sense and problem solving research.

The introduction of new technology, especially graphics calculators, has been a major concern in recent years and there has been a great deal of exploration about the possibilities that this offers for new types of problems to be tackled (Stacey, 2000). This has opened new possibilities for developing students' number sense (Anghileri, 2001; Fuson, 2003).

At present, research in number sense and problem solving is being addressed quite strongly through numeracy research projects. A number of cross-sectoral numeracy research projects are currently being funded by the Department of Education, Training and Youth Affairs (DETYA) in each state and territory of Australia. The Western Australia project, based at Murdoch University, is looking to find the extent and character of the relationship between student mathematics achievement and their capacity to use mathematics in context and also has a particular focus on the problems students have in dealing with the numeracy demands of each learning area and developing strategies for addressing these problems. In contrast, this present research intends to focus on number sense and problem solving, two major mathematical aspects, which form the core of numeracy, and how the teacher caters for the students' learning preferences. It is worth noticing that the role and beliefs of the mathematics teacher, vis-à-vis the idea of developing students' numeracy, has been documented quite extensively (Askew et al., 1997), while the impact of their teaching style upon students' learning style has been given less attention (Hodges, 1983; Zaslavsky, 1994). This situation could soon change as researchers and educational authorities, such as NCTM, urge teachers to employ a variety of teaching approaches and strategies which will be beneficial, not only to a particular group of students, but to all types of learners in the classroom (NCTM, 1989, 2000).

McIntosh et al. (1997) observe that "while agreement exists that the development of number sense is an important goal for all children, many questions remain unanswered about routes to achieve this goal" (p. 5). They feel that better information is needed to guide curriculum and instruction efforts in this area. They give the following examples:

> - Are students developing number sense in the current curriculum oriented toward developing proficiency in standard paper/pencil algorithms?
> - If some students are developing number sense within this environment, what thinking, what thinking and learning strategies are they employing?
> - If some students are not developing number sense in this environment, what changes in curriculum and instruction would support the development of number sense?
> - Do mathematical tasks such as inventing strategies to estimate and mentally compute utilize and/or support the development of number sense?
> - Do teachers perceive the development of number sense as an important instructional goal?
> - Do they purposefully pursue its development? What type of curricular and instructional approach will best foster the development of number sense? (p. 5)

Through this study, attempts will be made to consider the first and last two questions, as they lend themselves appropriately to gathering data about the link between number sense development, teaching and learning.

Some of the interest for this study, in regard to learning style and teaching style, have been generated partially from a paper presented at the Annual Meeting of the Association for Educational Communications and Technology, Research and Theory Division, in Dallas 1982 by Carrier and Melvin. This study examined the relationship of teaching style orientation, expressed perceptions of the teaching-learning process, actual classroom behaviour, and learning styles of six full-time faculty in a dental auxiliary program at a large teaching institution. Data collection instruments used to assess this relationship included the Teaching Style Q-Sort, an interview protocol, classroom observations modified from Goldhammer's (1969) note-taking procedure, and the Learning Style Inventory. The subjects' teaching styles were identified as either social interaction, information processing, personal, or behaviour modification. According to learning style, 163 dental hygiene students were categorised as accommodators, assimilators, convergers, or divergers. Results showed a positive relationship between teachers' perceptions of their teaching style and their classroom behaviours. No relationship was determined for the teaching style and learning style inventories, although three teachers accurately predicted students' learning styles;

teachers did not perceive their students' learning style to be like their own (Carrier & Melvin, 1982).

Elsewhere part of the impetus for this study had been provided by the following extract from an article, about The Australian National Schools Network (Young, 1999).

> The common goal of this network community is to support a particular group of schools in rethinking the way they do their work in order to improve learning. All countries are being challenged to rethink the educational experience of the young people in schools and colleges as they attempt to lift the learning outcomes for all students with a view to meeting the emerging demands of a knowledge-based economy. At the same time there is a growing awareness that many groups of young people do not fare as well as others and these persistent inequalities challenge educators to search for new ways to build socially just schools.
>
> (http://www.google.com.au/search?q=cache:7Kut18CsnWgC:www.nsn.net.au/downloads/ANSN_know_process.doc+Australian+National+Schools+Network+(1999).&hl=en&ie=UTF-8)

By shedding some light on relationships between teaching and learning styles, and the number sense and problem solving abilities of year seven students, the present study might be able to help in meeting some of the demands expressed in Peter Young's statement.

This study also espouses the principle of Flick and Lederman (2002), that the heart and soul of reform is what happens in the classroom. A recurring theme in Flick's and Lederman's article *Finding Opportunity to Learn* is the notion that "…mathematics education need a vigorous scholarship in documenting and analysing the instruction of teachers in typical classrooms as they seek to provide students with opportunities to learn standards-based curricula" (p. 337). While acknowledging that they are not denigrating the significant work being done in research and reform endeavours, they stress that there are also other challenges which are central to implementing reform which are not met by the current research design focus. For instance, two such challenges are adjusting the content of instruction and the relationships among components of instruction to provide all students the opportunity to meet new learning standards. There is a substantial amount of truth in their claim that "observing and interpreting how the new content of…, mathematical problem solving, and the nature of…mathematics is meaningfully integrated with requisite discipline knowledge is not easily investigated with research designs that focus on parts rather than the whole" (p. 338). Due to the documentation of classroom instruction being labour intensive, and evaluating the results being time consuming, Flick and Lederman point out that, typical research designs often involve only one classroom, and they go on to state:

> Even then, it is rare that the reader gets a complete picture of reform implementation from teacher intentions, to execution, to student response, to student outcome. Common approaches to research on classrooms often leave out important elements of instruction, especially those teaching strategies designed to reach all students in the classroom; for example, homework and classroom assignments. Research on classroom practice, at best gives the reader a look at small portions of student-teacher activity. At worst, the author reports on a few exemplary students and little of what the teacher is doing. We still struggle to address today the challenge implied by an observation of Romberg and Carpenter (1986) 15 years ago: "Much of the research directly addressing questions of instruction remained untouched by the revolution in cognitive science" (p. 851). (p. 337)

Nevertheless, there are well meaning projects currently examining what indicators of classroom practice best measure on a large scale what is going on in classrooms (Aschbacher, 1999; Clare, Pascal, Steinberg, & Valdés, 2000). Effort is also being made in examining the nature and function of formative assessments important for feedback to students and communicating instructional objectives (Black & Wiliam, 1998). In addition there is the recent publication by the National Research Council's Committee of a review and synthesis of advances in the cognitive sciences and measurement to explore, among other things, classroom practices and assessments that make progress of learning clear to students, teachers, and other important stakeholders (Pellegrino, Chudowsky, & Glaser, 2001). Although Flick and Lederman (2002) proclaim such projects as representative of efforts to capture data critical to understanding opportunity to learn in classrooms, their concern with assessment could lead us back into the quagmire of distrust from teachers, which in the past has been the hallmark of resistance to reform which aimed at 'judging' teachers' performance through assessment. It is therefore, the intention of this study to avoid this path, and focus mainly on what actually happens in the classroom as a whole, with focus on assessment taking only a minor role as one of the factors which bear upon students' number sense and problem solving ability.

The focus of this study has also taken into consideration the controversial issue of what is going on in classrooms and who to believe. Although most of the literature paints a picture in favour of constructivist approaches, one which blends well with the concept of open classroom and problem solving approach, there are others, especially teachers, who feel that we could be throwing the baby out with the bath water, as expressed through the following article:

> At the same time as constructivist approaches have been promoted, direct teaching methods have been overtly or covertly criticised and dismissed as inappropriate, with the suggestion that they simply don't work and are dull and boring for learners. The message that most teachers appear to have absorbed is that all direct teaching is old-fashioned and should be abandoned in favour of student-centred enquiry and activity-based learning. It will be argued in a moment that this view is dangerously extreme, and that a more effective approach to teaching and learning mathematics involves combining direct teaching with student-centred activities.
>
> (http://www.acer.edu.au/acerpress/PDF/education/Numeracy_ch1.pdf)

Moreover, Vaughn, Bos and Schumm (1997) warn that there are probably many reasons for students' failure in mathematics, and most of them are likely to be based within the curriculum and the teaching method, rather than within the learner.

Others feel that recent calls for reform in mathematics education have failed to consider the impact of other stakeholders on such a venture. They remonstrate that such reforms have focused too much on enriching teacher knowledge and classroom activity to make students more active participants in the construction of knowledge. Their contention rests on the argument that such an approach underestimates the power of institutional histories to frame participants' views of schooling. They point out that in particular, this approach ignores the perspectives students and parents bring to reforms that are often implemented in their schools without their knowledge or assent (Graue & Smith, 1996). Hence, the two ends of the spectrum seem to be antagonistically at odds with each other. Moreover, there are reports of misuse of word problems in the teaching of mathematics, a situation which has created antithesis to what the reform movement aims to promote (NCTM, 2000; Anghileri 2001).

Bonotto (2001) observes that in normal practice establishing connections between classroom mathematics activities and everyday-life experiences still regard mainly word problems. Word problem has its place in the teaching of mathematics since besides representing the interplay between formal mathematics and reality, word problems are often the only means of providing students with a basic sense experience in mathematisation, especially mathematical modeling (Reusser & Stebler, 1997). But teaching which focuses too much on word problems has its fair share of disadvantages.

Evidence from recent research has revealed that the practice of word-problem solving in school mathematics actually promotes in students a "…suspension of sense-making…" (Schoenfeld, 1991) and the exclusion of realistic considerations. An alarming discovery is that "Primary — and Secondary — school students tend to ignore relevant and plausible familiar aspects of reality and exclude real-world knowledge from their mathematical problem solving” (Bonotto, 2001, p. 75).

This lack of use of everyday-life knowledge could be due to two reasons. There is first of all the textual factors relating to the stereotypical nature of the most frequently used textbook problems. Wyndhamn and Säljö (1997, p. 364) remark that “when problem solving is routinised in stereotypical patterns, it will in many cases be easier for the student to solve the problem than to understand the solution and why it fits the problem”. Secondly, there is the presentational or contextual factors associated with practices, environments and expectations related to the classroom culture of mathematical problem solving (Bonotto, 2001, p. 75). The result is a “…classroom climate that endorses separation between school mathematics and every-day life reality” (Gravemeijer, 1997,p. 389), which in turn affects the teacher’s perceptions of the standards and principles being advocated through the reform movement.

Recent research has documented that the use of stereotyped problems and the accompanying classroom climate relate to teachers’ beliefs about the goals of mathematics education (Verschaffel, De Corte, & Borghart, 1997). Such a situation has resulted in a difference in views on the function of word problems in mathematics education. There are now mainly two groups with extremely different views: “the researchers, relate word problems to problem solving and applications” (Bonotto, 2001, p. 75), while teachers, especially pre-service or new initiates, see the role of word problems simply as “exercises in the four basic operations which also have a justification and suitable place within the teaching of mathematics” (p. 75). The teaching which results from this does not seem to favour realistic mathematical modelling, which is both real-world based and quantitatively constrained sense-making (Reusser, 1995). Bonotto and Basso (2001) note that changes must be made if situations of realistic mathematical modeling in problem solving activities is to be established. The following extract from Bonotto’s (2002) article *Suspension of Sense-making in Mathematical Word Problem Solving: A possible remedy* highlights certain factors which should be changed:

> 1. The type of activity aimed at creating interplay between reality and mathematics must be replaced with more realistic and less stereotyped problem situations, founded on the use of concrete materials.
>
> 2. We must endeavor to change students' conceptions of, beliefs about and attitudes towards mathematics; this means changing teachers' conceptions, beliefs and attitudes as well.

> 3. A sustained effort to change classroom culture is needed. This change cannot be achieved without paying particular attention to classroom socio-mathematical norms, in the sense of Yackel and Cobb (1996). (p. 314)

Such concerns as raised above, when juxtaposed with positive reports about the teaching of problem solving and sense making in mathematics (Anghileri, 2001), suggests that teachers are approaching the teaching of problem solving and sense-making, which would entail number sense, from various perspectives. Hence, part of the interest of this study is to observe the sort of socio-mathematical norms through which the number sense/problem solving and teaching/learning discourse takes place.

Evidently such persistent antagonism warrants that research of the type to be undertaken in this study must consider both sides of the argument. Such a concern has eventually prompted the researcher to adopt a quasi ethnographic case study, whereby he will allow the situation to speak to him as much as possible, in an attempt to come up with as much an authentic picture of what is happening in the teaching of number sense and problem solving, with the intention of discovering how these two could be related.

2.9 Synthesis of Research

Research in mathematics education around the world has acted as a catalyst towards the present emphasis upon the problem solving approach and mathematical sense making. The current climate favours a teaching approach which is flexible enough to use appropriate and relevant teaching styles to cater for the differences in students' learning styles. A major influential factor in this mathematics teaching and learning revolution has been the search for mathematical literacy. The call for a quantitatively literate society has resulted in the coining of the term numeracy, which has paved the way for greater emphasis on the two central mathematical aspects o f number sense and problem solving. Hence, the need to investigate the relationships between the four important variables of number sense, problem solving, teaching style and learning style, in the teaching and learning of mathematics as a means of empowering the learner.

Problems and problem solving have a long history in mathematics education (Dewey, 1910; National Council of Teachers of Mathematics [NCTM], 1980; Pólya, 1945; Schoenfeld, 1992; Stanic & Kilpatrick, 1988). The *Standards* asserted, "Problem solving should be the central focus of the mathematics curriculum" (p. 23) and placed it as Standard 1 (NCTM, 1989). The 1990s saw the development of school mathematics curricula based on various interpretations of these Standards. In most of these curricula, the mathematics emerges from the solution of problems, and there is a growing body of research evidence supporting the effectiveness of these curricula (Senk & Thompson, 2003). Teaching mathematics through problem solving also continues to be a focus of mathematics educators independent of the curriculum that is used (Schoen & Charles, 2003).

The start of the 1980s witnessed a renewed call for reform in mathematics education with major documents placing increased emphasis upon the promulgation of number sense (Australian Education Council, 1991; Cockcroft, 1982; Emanuelsson & Johansson, 1997; Japanese Ministry of Education, 1989; NCTM, 1989). Unfortunately, ambiguities in the meaning and practice of terms such as numeracy, number sense and problem solving could be hindering the much advocated progress anticipated by the reform movement. For instance, albeit the call for teachers to develop number sense among their students, recent research suggests that teachers vary greatly in their own understanding of what number sense is or what it means to design instruction that focuses on understanding and sense-making (Turner, 1996). Moreover, according to the NCTM's (1989) *Curriculum and Evaluation Standards,* one of the five goals for all students is that "they become mathematical problem solvers" (p. 5), which entails a new way of looking at and practising the teaching of mathematics. Koehler and Prior (1993) state that if students are to realise these goals, they must have the opportunity to practise them in daily classroom interactions. This study intends to investigate such classroom interactions with respect to how number sense and problem solving are taught and valued.

In developing good learning habits, teachers should be interested not only in what a student knows but also how the student responds when he or she does not know the answer. It is crucial for teachers to be aware of the behaviours which indicate the development of good learning habits, in order to appreciate the students' need to be able to enhance their number sense and problem solving abilities (Wagener, 2002).

To help teachers in their quest to implement the reform requirements researchers have collaborated to develop instruments and materials, such as a number sense framework (Emanuelsson, Johansson, Reys, & Reys, 1996). Work has also been undertaken in identifying and/or generating items to assess the number sense of children aged 8 to 14 years (McIntosh et al., 1997). One important goal of such endeavours is to provide teachers with a tool to gauge their students' number sense as a basis for making decisions about appropriate curricular actions. Various tools also exist for helping teachers teach and assess students' problem solving. How teachers are coping and making use of such tools warrants to be looked into. Teachers face another challenge as they are being encouraged to teach to all abilities and variations in background of the students.

The principle of equity as expounded through the research of Griffin, Case and Siegler (1994), Secada, Fennema, and Byrd (1995), and Silver and Stein (1996), albeit subliminally, places equal emphasis upon adaptation of teaching style to cater for differences in learning styles. It is now an accepted fact in educational circles that students learn in a wide variety of ways mainly because they have different learning styles. Experienced learners are often able to take many different approaches to learning, but most still have some preferences. Although "there is no one 'right way' to teach" (NCTM, 2000, p. 18), there is no disputing the fact that "teachers have different styles and strategies for helping students learn particular mathematical ideas" (NCTM, 2000, p. 18). It is generally agreed that it is not necessarily the teacher's job to teach in all ways for all students, since some material by its very nature requires a certain approach to teaching and learning. However, taking a single approach to teaching, which entails teaching through only one personal style, that unnecessarily leaves out other approaches, is likely to exclude some students from opportunities to learn (Honey & Mumford, 1992).

Since the language of mathematics is based on rules that must be learned, teachers have tended to focus on rote learning (Dahlin & Watkins, 1997; Miura, Kim, Chang, & Okamoto, 1998). To remedy this situation publications such as Cockcroft's *Mathematics Counts* (1982) requires teachers to employ a problem solving approach to teaching mathematics. This view is given impetus through themes in the NCTM *Standards* (NCTM, 1989, 2000) and *Reshaping School Mathematics* (National Research Council, 1990), which place great emphasis on reshaping the teaching and learning of mathematics, especially problem solving and number sense. The new approach being advocated wants teachers to incorporate in their practice the notion that for student motivation to occur they must move beyond rules to be able to express things in the language of mathematics. Such a transformation suggests changes both in curricular content and instructional style. Teachers must now train students in:

- Seeking solutions, not just memorising procedures;
- Exploring patterns, not just memorising formulas; and
- Formulating conjectures, not just doing exercises.

It is expected that as teaching begins to reflect these emphases, students will have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be memorised. They will be encouraged to see mathematics as a science, not as a canon, and to recognise that mathematics is really about patterns and not merely about numbers (National Research Council, 1989). The task of the teacher is centred around producing mathematically powerful students who are quantitatively literate. It is therefore appropriate at this point in time to study the kind of teaching being practised when it comes to number sense and problem solving.

This study rests on the assumption that the relationship between number sense and problem solving will necessarily depend upon how teachers' teaching styles accommodate the students various learning styles. Teachers' and students' beliefs also play a great role in how the teaching and learning of number sense and problem solving are viewed. This of course will depend greatly upon the teachers' and students' definition and practice of mathematics. It is anticipated that if acceptance of a broadened view of mathematics has grown, traditional instructional patterns and roles of both students and teachers would have changed.

Although the literature has highlighted certain discrepancies which have tended to retard teachers' attempt to implement the teaching advocated through the reform emphasis, the other side of this coin is temptingly positive. Research focusing on reform-minded classrooms, has discovered that in such classrooms emphasis is shifting from a curriculum dominated by memorisation and paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations, mathematical modelling, and problem solving (Black and Atkin, 1996). Instructional emphasis is shifting away from teacher-dominated lecture and demonstration techniques toward small-group work, individual exploration, and discussions in which the role of the teacher is that of moderator, facilitator, and assessor rather than that of dispenser of knowledge (*Hiebert et al., 1997;* Nathan & Knuth, 2003; Wood & McNeal, 2003*)*. Assessment techniques are shifting from the dominant use of objective measures to include alternative means such as open-ended questioning, oral and written reporting, projects, interviews, and portfolios (Black & Wiliam, 1998).

In Western Australia, mathematics education has been changing. Developments in the 1980s brought about an increased focus on problem solving, investigations and activity oriented teaching (O'Brien, 2002). Similar developments have occurred elsewhere in both Australia and other parts of the world, with a growing attention to processes of mathematical thinking.

Mathematics education has changed towards a more open- ended, problem-solving approach that emphasises the process. To facilitate implementation of this approach teachers have been provided with examples of this type of mathematics and suggestions for relevant mathematical activities for children (Smith, 2002).

2.9.1 The relationship between number sense and problem solving

Research focussing on the relationship between number sense and problem solving is virtually non-existent. Yet the relationship between students' number sense and problem solving ability, although not overtly acknowledged by most researchers and mathematics educators, is becoming more and more evident through various modes and endeavours.

2.9.2 The central role of number sense and problem solving

This relationship is being advertised on a subliminal level through the call for reforms in the teaching and learning of mathematics (Carpenter, Fennema, & Franke, 1996; Reys & Reys, 1997), which has culminated in greater emphasis being placed upon teaching and learning through a problem-centered curriculum (Lester, Masingila, Mau, Lambdin, dos Santon, & Raymond, 1994). Given the recent emphasis upon mathematical sense-making (Bana & Korbosky, 1995; Markovits & Sowder, 1994; Resnick, 1988; Romberg, 1994), the importance of developing students' number sense has been given extra impetus through documents calling for reforms in school mathematics (Askew, 2002; Australian Education Council, 1991; Cockroft, 1982; Emmanuelsson & Johansson, 1996; Japanese Ministry of education, 1989; National Council of Teachers of Mathematics, 1989, 2000; Prais & Luxton, 1998; Reys, Reys, Barnes, Beem, & Papick, 1998), most of whom are suggesting teaching it through a problem-oriented approach (Olkin & Schoenfeld, 1994; O'Rourke, 1999; Thiessen & Trafton, 1999), a practice which surely relies upon linking number sense and problem solving (Anghileri, 2000; Flewelling, 2002; Schoenfeld, 1992; Thiessen & Trafton, 1999).

At one time it was accepted that problem solving be taught first as a separate area of mathematics, then the skills developed would be incorporated within the whole program (Smith, 1989). This view has been abandoned for a more holistic approach where "instead of teaching problem solving separately, teachers should embed problems in the mathematics-content curriculum" (NCTM, 2000, p. 119). The argument from

NCTM is that “when teachers integrate problem solving into the context of mathematical situations, students recognise the usefulness of strategies” (p. 119). The reform movement has promulgated the idea of teaching through a problem solving approach (Carnine, 1997; Hofmeister, 1993; Mercer, Jordan, & Millet, 1994; Prais & Luxton, 1998; Rivera, 1997). Hence, NCTM (1989) confers upon problem solving the central focus of the mathematics curriculum. It is thus a primary goal of all mathematics instruction and an integral part of all mathematical activity. To allow problem solving to play this central role the teacher must not treat it as a distinct topic but rather as a process that should permeate the entire program. Furthermore, problem solving should provide the context in which concepts and skills can be learned (Billstein, Libeskind, & Lott, 2001; NCTM, 2000). This necessarily entails the teaching and learning of concepts and skills that would develop number sense (Dehaene, 1999; Markovits & Sowder, 1994).

NCTM (2000) and other interested parties have also conferred upon the development of number sense a central role (Sowder, 1992) as well as one upon which other aspects of mathematics are built — the cornerstone of the entire mathematics curriculum (Reys & Nohda, 1994). Such acclamations are backed through NCTM’s (2000) statement that “all the mathematics proposed for prekindergarten through grade 12 is strongly grounded in number” (p. 32), which points, albeit indirectly, to the importance of number sense in the curriculum.

2.9.3 Definitions of number sense and the embracement of problem solving

Moreover, reference to problem solving ability permeates most definitions of number sense (Sowder, 1992; Denvir & Bibby, 2002; Ritchhart, 1994), which serves to show that there is a link between the two. Although in these definitions the intention weighs more towards number sense inherent problems, there is no question that number sense ability is also intricately linked to problems which are devoid of number sense (Lyon, 2001; Anghileri, 2000). Bolster’s and Reys’ (2002) suggestion that problem solving is the vehicle through which number sense is transported, further cements the link between the two. In general most definitions of number sense share elements pertaining to definitions of problem solving in mathematics. Hence, the relationship between number sense and problem solving ability is not only evident in the nearly similar role imputed to both of them, but also in the mention of the latter in definitions of number sense. Yet there is a lack of research to elucidate this relationship.

2.9.4 Teaching for number sense and problem solving

Given such constant and consistent linking of number sense and problem solving emphasised so far, it could be expected that a student with good number sense would be able to transfer such knowledge into solving given problems involving related concepts (Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Human, & Olivier, 1997). To take a specific topic as an example, some number sense research (Markovits & Sowder, 1994; Reys & Yang, 1998; Sowder & Wheeler, 1989) has shown that, when given two fractions, students have far more difficulty figuring out which one is closer to a third fraction than they do simply comparing them to each other. Inherent in this observation is that those who are weak in fraction number sense would find it very difficult to solve problems involving fractions as compared to those who have good fraction number sense. This final statement could apply to number sense ability in other topics such as decimals, integers and so on. This could mean that students who are weak in a particular number sense area might not be able to solve problems involving the same topic, although this does not necessarily imply that they will not be able to solve problems devoid of such topics (Lesh & Lamon, 1992; English, 1998). This definitely warrants further research to discover what sort of relationship exists, and how teachers teach for development of number sense and problem solving.

Independent studies of teaching for number sense and problem solving have revealed that teaching for either of them separately poses a great challenge for the teacher. McChesney and Biddulph (1994) noted the difficulty with the notion of number sense in that "it is not something that can be taught directly. Rather it is something that emerges from mathematical activity and exploration" (p. 10). McIntosh et al. (1997) report that in their discussions with teachers the latter agreed "that their teaching has not traditionally focussed on *the important* aspects" of number sense (p. 53). In tandem with McChesney's and Biddulph's (1994) proposition, McIntosh et al. have supported this belief by providing some practical help in this regard through the publication of four books, which focus on activities to develop number sense at different age levels (McIntosh et al., 1997).

Since problem solving is neither a series of steps to follow, nor a mathematical algorithm, it poses a great challenge for teachers (Mayer & Hegarty. 1996), but unlike number sense which is virtually impossible to teach (McChesney & Biddulph, 1994), it can be taught and is already being advocated as a medium through which all aspects of mathematics should be taught (Gill, 2001; Goos, 2000; Nisbet & Putt, 2000; Smith, 2002). Moreover, researchers have identified key behaviors associated with successful mathematical problem solving (Charles, Lester, & O'Daffer, 1988; Geary, 1994; Kintsch & Greeno, 1985; Lewis & Mayer, 1987; Mayer & Hegarty, 1996; Schoenfeld, 1987), which could be integrated into the solving of number sense problems (Anghileri, 2000). This difficulty in teaching number sense has further cemented the relationship of number sense and problem solving, in that the former can emerge from teaching and learning of the latter (Anderson, 1997). Thus, implications for teaching stemming from number sense research usually focus on promoting development of sense-making through various ways, of which problem solving plays a major role (English, 1998; McIntosh, et al., 1997). This is further evidence that there is a relationship between the two, which would be mainly apparent in a teaching context.

The call is now for teachers not only to teach through a problem solving approach, in which number sense is part of the broader construct of problem solving (English, 1998; NCTM, 2000), but to also adapt their teaching styles, and/or even adopt new ones to facilitate development of students' number sense and problem solving ability (Vance, 1999). Albeit efforts to get teachers to develop number sense among their students, it seems that teachers are finding it hard to facilitate children's number sense development (Turner, 1996). Recent research by Turner (1996) suggests that teachers vary greatly in their own understanding of what number sense is or what it means to design instruction that focuses on understanding and sense-making. Recommendations from research insist that if number sense is globally defined as "the foundation from which all other mathematical concepts and ideas arise" (Fennell & Landis, 1994, p. 187), it necessitates a more open-ended, innovative and problem-oriented teaching approach (Anderson, 1996, 1997; Thiessen & Trafton, 1999), as required by the calls for reform (Askew, 2002; Haynes, 1997; NCTM, 2000; Whitehead, 1997).

To help teachers in their quest to implement the reform requirements researchers have collaborated to develop instruments and materials, some of which will be utilised in this study. There have been progressive contributions towards the development of the number sense framework (McIntosh, Reys, & Reys, 1992; Emanuelsson, Johansson, Reys, & Reys, 1996; McIntosh et al., 1997), of which an important goal is to provide teachers with a tool to gauge their students' number sense as a basis for making decisions about appropriate curricular actions (McIntosh, et al., 1997). To facilitate identification of learning style and teaching style a substantial amount of ground work has been done, resulting in the production of instruments such as inventories/questionnaires (Grasha, 1996), with most of these instruments dealing with learning style. There are numerous models of learning styles, but very few of them are geared towards primary students. For instance, a paper by De Bello (1990), describes just 11 of them, with possibly two which could be used at primary school level. Various tools also exist for helping teachers teach and assess students' problem solving (Wilson, 1993; Mathematical Sciences Education Board, 1993; Randhawa, 1994; NCTM, 1995), although research on teachers' use of such tools and the challenges they face in doing so is lacking. Teachers face another challenge as they are being encouraged to teach to all abilities and variations in the backgrounds of the students.

2.9.5 Research on problem solving, number sense, teaching style and learning style

Research on problem solving in mathematics education has focussed mainly upon problem solving strategies (Lesh, 1981; Wilson, 1993), problem posing (Lowrie, 1999; Gonzales, 1996; Silver & Cai, 1996; Stoyanova, 2000), teachers' and students' beliefs (Anderson, 1998; Curtis, 1995), solving specific topic-related problems in areas such as algebra (Dougherty & Matsumoto, 1995), percentages (Dole, 1999), the affective domain (DeBellis & Goldin, 1997) and assessment of problem solving proficiency (Charles, Lester, & O'Daffer, 1987; Collis & Romberg, 1992; Schoen & Oehmke, 1980; Stacey, Groves, Bourke, & Doig, 1993). Recently within the research community, problem solving has received further impetus from the study of small group learning processes (Johnson & Johnson, 1996), but the continued highlighting of the way in which students solve problems, how they come to construct knowledge through this process and the need for metacognitive awareness of problem solving processes (Hacker, Dunlosky, & Graesser, 1998; Nisbet & Putt, 2000), albeit a necessary one, makes very little mention of the relationship between number sense and general mathematical problem solving, let alone relating their development to the matching of teaching and learning style.

Research findings related to number sense have resulted in some vital discussions, which according to McIntosh et al. (1997) have included:

> A listing of essential components of number sense (Resnick, 1989; Sowder & Schappelle, 1989; Willis, 1990; Sowder, 1992; McIntosh, Reys, & Reys, 1992), descriptions of students displaying number sense or the lack thereof (Howden, 1989; Reys, B. J., Barger, R., Dougherty, B., Hope, J., Lembke, L., Markovits, Z., Parnas, A., Reehm, S., Sturdevant, R., Weber, M., & Bruckheimer, M., 1991), a theoretical analysis of number sense from a psychological perspective (Greeno, 1991), and discussions of instructional strategies which promote the development of number sense (Brownell, 1945; Kamii, 1989; Reys et al., 1991, Burton, 1993; Burns, 1994). (p. 4)

Research findings indicate that certain teaching strategies and methods are worth careful consideration as teachers strive to improve their mathematics teaching practices (Kilpatrick, 1992). Unfortunately, studies on teaching style and learning style have produced some contrasting (Hayes & Allinson, 1996) and sometimes conflicting (Coffield, Moseley, Hall, & Ecclestone, 2004a, 2004b) perspectives, which have contributed towards a feeling of scepticism against such concepts (Noble & Bradford, 2000). In considering the implications of such a controversy, Hadfield (2006) referred to the results of a review of learning style inventories by Coffield *et al.* (2004b). According to Hadfield (2006):

> Coffield *et al.* (2004b) consider learning styles in five categories, from those at one end who consider learning styles to be fixed, even genetically determined, to those at the other end who consider styles to be mutable and learners as having the option to move between styles. This is a crucial distinction, since many implications for classroom practice hang on the question of whether we consider learning styles to be fixed or mutable: namely how far we should match teaching techniques and tasks to learning style and how far we should individualize instruction for different types of learner. These are questions that need to be addressed, so I will preserve the order in which these styles are considered—from fixed to mutable. (p. 372)

Nevertheless, when all these findings are put together a more congruent picture emerges, which brings down many of the fallacies attached to the concept of teaching style and learning style (Montgomery, 1995; Noble & Bradford, 2000). Hence, to alleviate the discrepancies resulting from such controversial views, most proponents of the teaching style and learning style theories have insisted that catering for differences in learning style should not be interpreted as matching of teaching style and learning style, for much controversy exists concerning the latter notion (Hayes, & Allinson, 1996). Biased interpretation of past research had thwarted the purpose of matching teaching and learning style. For a long time it has been assumed that if teachers were able to diagnose the learning style of their students, then it would seem logical to assume that matching the characteristics of instruction to that style would make the instruction more effective (Pinto, Geiger, & Boyle, 1994). Moreover, certain researchers, like Garner (2000), still tended to treat learning style as a fixed personality trait. According to this assumption students tend to enter a learning situation with a style of learning already developed (Garner, 2000). Although Kolb is one of those who opposed such an idea, he still maintained that if a learner meets a learning environment at variance with his or her preferred style, then it is likely the student will reject the learning environment (Kolb, 1976). Nevertheless, in a later study Kolb (1984) concluded that there were potential long term benefits where there was an intentional mismatch between learning style and instructional style. So, the mathematics teacher who is aware of this would need to heed NCTM's (2000) call for striking a balance. Such a balance could be achieved in partiality through matching teaching style and learning style, so it is clear from the research evidence available that there remains much debate over the effectiveness of matching learning style and instructional style. Matthews (1991) argues that:

> While mismatching is appropriate for developmental reasons, students have more positive attitudes towards school and achieve more knowledge and skills when taught, counseled or advised through their natural or primary style rather than a style that is secondary or undeveloped, particularly when adjusting to a novel and new situation that creates stress... (p. 253)

Research literature pertaining to teaching style and learning style indicates that the benefits to learners of having correct knowledge of their learning style and putting into practice such knowledge, greatly outweigh the possible misconceptions pointed out so far. The identification of preferred learning styles raises the individual learner's awareness of their own approach to learning (Honey & Mumford, 1992). One aspect of this study intends to check upon any progress or transformation which could occur as a result of teachers and students knowing about their teaching and learning styles. It is well documented in the literature that when a group undertakes the exercise, they are likely to benefit from the awareness of different learning styles (Schroeder, 1996). Moreover, a teacher who clearly understands the possibilities and limits of his or her teaching style can make more consistent judgments about how best to use this medium (Grasha, 1996). Researchers assert that the teacher must be able to identify, appreciate, and explore his or her own teaching style in order to achieve style-differentiated instruction (Griggs, 1991; Schifter & Fosnot, 1993; Pinto, Geiger, & Boyle, 1994).

2.9.5 Related research and the impetus for this study

Much still needs to be done in the area of research on children's number sense and problem solving ability, and the style of teaching and learning which facilitate such development, even though some relevant articles have been written and a few related studies have been carried out. This study intends to take certain issues, from such related literature and research, into consideration.

Where problem solving is concerned Silver's (1985) questions, Schoenfeld's (1992) suggestions about sense-making and problem solving, as well as English's (1998) propositions, have had great impact on this study. Silver (1985) posed eleven questions after he had expressed the fact that:

> Noticeably absent from the literature are characterisations of current practice in the teaching of problem solving in classrooms. We do not know how teachers conceptualise problem solving, or how they attempt to teach it. (p. 297)

He remarked that those questions and related ones are important if one wishes to improve the problem-solving ability of students, but at that time they were unanswered, and most of them are still waiting to be answered. It is worth noticing that Silver's (1985) questions revolve around the issue of 'what type of teaching style is being employed', while Schoenfeld (1992) suggested that sense-making is a crucial aspect of effective problem solving practices.

Lesh and Lamon (1992) noted that students who are competent in mathematics "often have exceedingly different profiles of strengths and weaknesses, with their learning progressing along a variety of paths and dimensions" (p. 7). While Lesh and Lamon provided fodder for English's (1998) proposition, the latter in turn is the nearest anyone has been in regard to the present study's focus, since it attempts to relate problem solving and number sense. English (1998) observed that:

> When presented with problem-posing activities, children who possess strong number sense but are weak in novel problem solving, for example, might display patterns of responses different from children who display the reverse profile (i.e., weak in number sense but strong in novel problem solving). Children who are competent in both domains might show other patterns of response and perhaps display superior problem-posing skills. (p. 83)

This extract from English proposes two main categories of students when it comes to comparing their proficiency in number sense and problem solving — those who display different profiles of achievement in these two domains and those who are competent in both domains. The first group could be further divided into (a) children who possess strong number sense but are weak in problem solving and (b) children who display the reverse profile (i.e., weak in number sense but strong in problem solving).

Such issues also relate directly to the type of teaching approach(es) used by the teacher to facilitate such learning, which leads to the question of teaching style versus learning style. Very little research in mathematics education pertaining to teaching style and learning style per se has been carried out (Gill, 2001), and the meagre endeavour in this area has concentrated mainly on gender, learning style and teaching style (Keast, 1999; Boaler, 1997), with some endeavour in a few other areas such as cognitive learning style and achievement in mathematics (Mrosla, Black, & Hardy, 1987); and most recently Sloan, Daane, and Giesen (2002) investigated the relationship between elementary pre-service teachers' mathematics anxiety levels and learning style preferences. Nevertheless, the most recent research tends to support the findings from previous related research. For instance, three related studies revealed that the learning style preferred by female students is based on cooperation rather than competition, which is favoured by males (Keast, 1999). An interesting outcome from Keast's (1996) study is that from observations of the teachers' classes and from conversations with them, the teachers found they changed their teaching methods to reflect the style preferred by the students.

Gornell and Halshall's (1997) endeavour to raise children's abilities to use mental strategies to solve mathematical problems, has resulted in a close correlation between pupils' number sense and general mathematics improvement. In addition teachers found themselves adapting their teaching styles to facilitate student learning. Such findings are supported by Hembree's (1992) meta-analysis of research on problem solving which suggests that successful problem solving was associated with high mental ability, especially in forming analogies; with attitudes towards mathematics and mathematical self-concept; with high socio-economic status and being male from grade 9 onwards. Better performance on problem-solving tasks was also linked to the problem being set in a familiar context or including a picture, and lower performance with the inclusion of extraneous data (data not relevant to the problem). Other factors (such as concrete contexts and readability) were not significant. Receiving instruction in problem-solving skills was positively related to performance, as was training in specific problem-solving sub-skills.

The introduction of new technology, especially graphics calculators, has been a major concern in recent years and there has been a great deal of exploration about the possibilities that this offers for new types of problems to be tackled (Stacey, 2000). Although this has opened new possibilities for developing students' number sense (Markovits & Sowder, 1994), recent research on number sense has focused mainly on investigation of in-service teachers' beliefs about number sense in elementary school mathematics and the teaching and learning of that sense (Simmt, 2000; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997), which implies that there is a lot of scope for number sense related research, particularly in primary schools.

McIntosh et al. (1997) observe that, "while agreement exists that the development of number sense is an important goal for all children, many questions remain unanswered about routes to achieve this goal" (p. 5). They feel that better information is needed to guide curriculum and instruction efforts in this area. In the Australian context there are encouraging signs that the use of a problem solving approach to teaching is becoming much more sophisticated (Williams, 2000). Recently research and literature is showing signs of encouraging the teaching and learning of number sense through problem solving. A typical example is the case study of three teachers stemming from the 1994 "Numeracy Strategy Project" (Gervasoni, 1999). In describing the insights the teachers gained about aspects of children's early number learning, Gervasoni's (1999, p. 236) list includes the statement that "problem solving activities support the development of children's number sense". The respective teachers' recommendations advise that "it is important to encourage children to estimate and predict when solving problems and then confirm their predictions" (p. 237) — a recurring theme in the literature, which obviously links number sense and problem solving, in that it does not specifically mention 'number problems'. Both number sense and problem solving have become increasingly important research areas as more effort is spent in studies pertaining to teaching practices that enhance numeracy achievement (Stephens, 2000).

Numeracy is concerned with teaching, which develops students' learning ability to make sense of mathematics and solve problems (Bobis, 2000; Gervasoni, 2000). Since learning about number is central to the development of numeracy and underpins later success in mathematics (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997), it is appropriate that a link is made to research done on numeracy. According to Gervasoni (2000) problem solving enhances children's numeracy learning, which entails the development of their number sense. Albeit its claim to encompass all mathematics topics, numeracy as proposed by the mathematics community focuses exceedingly upon number problem solving, at the expense of the wider context of mathematical problem solving (Lord & Lester, 1990). This could be confusing to teachers who are endeavouring to upgrade students' number sense and problem solving ability in the context of getting the latter to work mathematically, since numeracy is a very influential factor in recent mathematics educational agendas (Willis, 2000).

One of the most comprehensive and recent studies, pertaining to numeracy, was carried out in England by Askew et al (1997). Unlike the research being proposed, this study explored the knowledge, beliefs and practices of a sample of effective teachers of numeracy. The present research will make use of such findings by gauging the teachers' perception, and comparing that to what happens in actual practice as they attempt to get students to work mathematically.

At present, in Australia, research in number sense and problem solving is being addressed quite strongly through numeracy research projects. A number of cross-sectoral numeracy research projects are currently being funded by the Department of Education, Training and Youth Affairs (DETYA) in each state and territory (Department for Education and Employment, 1999). The Western Australia project, based at Murdoch University, is looking to find the extent and character of the relationship between student mathematics achievement and their capacity to use mathematics in context and also has a particular focus on the problems students have in dealing with the numeracy demands of each learning area and developing strategies for addressing these problems.

In-depth research is providing guidance on how to replace traditional teaching with approaches that engage students more fully in mathematical thinking (Nisbet & Putt, 2000). Recently, most emphasis has been placed on monitoring achievement on basic number sense skills, but against all expectations this has not excluded any emphasis on problem solving (Owens & Mousley, 2000). Albeit such commendable efforts, most problem solving research being undertaken in Australia are focusing on the way that students solve problems in different content areas of mathematics (Nisbet & Putt, 2000). Research into students' solving of number sense problems has so far failed to deal with the general mathematical problem solving aspect per se (Keeves & Stacey, 1999).

Wagener (2002) reiterates the fact that number sense and problem solving are related. Yet, despite the rising popularity of problem solving and number sense as research foci, there is still a seemingly unconscious reluctance to link the two, apart from their expressed relationship in definitions of number sense. Extensive reviews of Australian research on problem solving for four-year intervals provide no indication that much attention will be paid to studies of the relationship between number sense and problem solving (Nisbet & Putt, 2000). Even Keeves' and Stacey's (1999) reporting of other aspects of problem solving, in the context of broad developments in research in mathematics education in Australia since 1965, do not indicate any former or recent attempt to go in this direction. Yet problem solving and number sense are two intertwined concepts, central to the call for reform in the teaching of mathematics (Shaw & Blake, 1998).

Although the literature has highlighted certain discrepancies which have tended to retard teachers' attempt to implement the teaching advocated through the reform emphasis, the other side of this coin is temptingly positive. Research focusing on reform-minded classrooms, has discovered that in such classrooms emphasis is shifting from a curriculum dominated by memorisation and paper-and-pencil skills to one that emphasises conceptual understanding, multiple representations, mathematical modelling, and problem solving (Black & Atkin, 1996). Instructional emphasis is shifting away from teacher-dominated lecture and demonstration techniques toward small-group work, individual exploration, and discussions in which the role of the teacher is that of moderator, facilitator, and assessor rather than that of dispenser of knowledge (Kilpatrick, Swafford, & Findell, 2001). Assessment techniques are shifting from the dominant use of objective measures to include alternative means such as open-ended questioning, oral and written reporting, projects, interviews, and portfolios (Black & Wiliam, 1998).

Therefore, the notion of the existence of a relationship between number sense and problem solving permeates most, if not all, definitions of number sense, and the role given to both by proponents of the reform movement and curriculum developers. Recognition of such a relationship would help in facilitating the development of number sense through a problem-oriented curriculum.

Hitherto relevant research has not addressed the issue of the existence and nature of such a possible relationship, although certain research results have provided the foundation for the present study. Due to the difficulty of teaching problem solving and the impossibility of teaching number sense, and the fact that teachers are being challenged to cater for the learning of different students, it is also appropriate to investigate the teaching style through which number sense development and problem solving ability are being facilitated, and how this relates to students' learning styles.

Given that the shift towards problem-focussed teaching of mathematics is slowly gaining ground, both internationally and in Australia, it is appropriate that research effort be spent on the documentation of the dynamics of the relationships between the respective teaching style and learning style, and the number sense and problem solving ability of students. The following chapter will outline the methodology for this research.

Chapter 3: Methodology

3.1 Introduction

Since this study relied upon data gathered from participants in a specific environment, this researcher developed a design which not only combined aspects from various research methods and theories, but also adapted these aspects to fulfill the purpose of this study. Hence, the following were deemed most appropriate: an ethnographic case study approach (a combined qualitative and quantitative design) incorporating certain grounded theory principles were used in three Year 7 classes. The sampling approach used was rather deliberate 'purposive sampling' and was partially subject to convenience. This is in line with Punch's (1998) analysis of sampling in qualitative research in that:

> Qualitative research would rarely use probability sampling, but rather would use some sort of deliberate sampling: 'purposive sampling' is the term often used. It means sampling in a deliberate way, with some purpose or focus in mind. (p. 193)

The study took place for the duration of four primary school terms and it examined the teaching of the teachers of these classes, the chosen students' number sense and problem solving ability, the compatibility of the teacher's teaching style and the students' individual learning style, and any form of relationships that exist among the four major variables comprising the teaching style of three Year 7 teachers and their students' learning style, and the number sense and problem solving ability of the selected Year 7 students. Initially an inquiry was carried out in collaboration with educational organisations, institutions and personnel such as the Mathematical Association of Western Australia (MAWA), the Western Australian Department of Education and Training, and primary school principals as a means of identifying 'effective' teachers of mathematics who had a proven record of teaching number sense and problem solving through the context of working mathematically. Then three teachers (and their students) in three different schools were selected as the main participants in this study, according to criteria described below. Data analysis began with the first set of data collected and continued throughout and after the other data collection phases ended. The main data collection employed three phases:

1) Pre-testing, open and focused observation phase: all lessons taught by each teacher participant and/or attended by the respective student participants. Focused face-to-face observation of mathematics lessons only, plus teacher and student interviews;
2) Selected topics for classroom observation: Continuation observation of mathematics lessons and interviews. Observation of four lessons which are based on topics selected by the researcher; and
3) Post-observation and wrap-up phase: post-testing, validation of data collected, and teacher and selected student interviews.

During the last decade researchers and writers such as Punch (1998), Bryman (1992) and Hammersley (1992) have written at length on the benefits of combining quantitative and qualitative approaches. According to Punch (1998) "Each approach has its strengths and weaknesses, and over-reliance on any one method is not appropriate" (p. 241). Both of these approaches are needed in social research. A most convincing argument from Hammersley (1992) points out that the seven dichotomies typically used to differentiate quantitative and qualitative approaches are highly exaggerated. Although many researchers still tend to stick to either one or the other, postmodernist proponents, such as Punch (1998), encourage the combining of both approaches.

> The quantitative and qualitative approaches to research have important differences …Despite this, the two approaches also share many similarities. Indeed…some of the same logic drives both types of empirical inquiry…The main differences emphasized here between the two approaches lie in the nature of their data, and in methods for collecting and analysing data. However, these differences should not obscure the similarities in logic, which makes combining the approaches possible. (pp. 39-40)

In employing pre-tests, post-tests and questionnaires to gather data, this study adopted a quantitative feel. The pre- and post-tests gave a slight quasi-experimental flavour to the research design, as a means of providing information which would guide the formulation of propositions and conceptualisation. But the fact that other data which formed the bulk of all the data collected were largely qualitative in nature, tips this research more towards a qualitative study. Nevertheless, an important aspect of this study is that it attempted to combine both quantitative and qualitative approaches. One intention of this study was to arrive at some propositions and conceptualisations, which implied that the data would be analysed not only through describing but also by conceptualising. The intention was not to prove any of the generalisations and/or propositions, but to "suggest such generalisability, putting forward concepts or propositions for testing in further research" (Punch, p. 154). This necessarily called for an amalgamation of a case study technique and some grounded theory principles (Glasser & Strauss, 1965; 1968). The latter method would also become handy, as some of the directions for this study would emerge from the early analysis of data. Hence, certain literature coverage and design of instruments were still going on until such directions emerged (Denzin & Lincoln, 1994). Hitherto the literature had not revealed any existing theory in regard to the relationship of Year 7 students' number sense and problem solving ability, and how these are related to teaching style and learning style. Hence, the aspect of conceptualisation and proposition was guided by a grounded theory approach, in an attempt to come up with suggestions, which would be beneficial in informing further research. As pointed out by Punch (1998) "the rationale for doing a grounded theory study is that we have no satisfactory theory on the topic, and that we do not understand enough about it to begin theorizing" (p. 168).

> Furthermore, the pre- and post-test aspect was important in providing some insight into whether or not awareness of one's teaching/learning style has significant bearing upon one's teaching and learning.

3.2 Subjects

The main participants in this study were three Year 7 teachers of mathematics and seventy-one of their students, comprised of twenty-eight males and forty-three females, from three Perth metropolitan primary schools. Some key descriptive aspects of each school are provided in Table 3.1, using pseudonyms. The principal and deputy principal were also involved, albeit less frequently than the main participants, to answer some interview questions, which helped in verification and triangulation of data. Depending on their results on the number sense and problem solving pre-tests, students from each class were specifically observed and informally interviewed, at least twice per term, for the study. After the post-tests were administered, 70 percent of students from each class were selected to be interviewed through a Think Aloud Stimulation Recall Interview (TASRI) protocol. As much as possible each group of students was heterogeneous and representative of each class. The students' and their teacher's learning styles were determined through a standard Learning Style Inventory (LSI). The three teachers' teaching styles were also determined through a standard Teaching Style Inventory (TSI).

Table 3.1 Key descriptive aspects of each of the three participating schools

Aspect	Arlenta	Baden	Cottonfield
School type	Private girls' school	Private boys' school	Coeducational State school
Year levels	K - 7	K-7	K-7
Number of students enrolled up to Year 7	295	380	430
Socio-economic status	Middle to upper	Middle to upper	Middle to upper
Number of Year 7 students in whole school	75	60	47
Classes	3	3	2
Basis for allocation of Year 7 students to classes and teachers	Mix of academic and social backgrounds	Split evenly across the classes – mixed ability groups	Split evenly across the classes – mixed ability groups
Number of students observed in participating classes*	24	14	30

Note: A total of 64 students completed all the assessments.

3.3 Design

A combination of qualitative and quantitative methods were used to capture data on multiple planes within each community (class), including observation and audiotaping classroom interaction, collecting student work, teacher reflection and ratings of the 45 students who were selected for the Think Aloud and Stimulated Recall Interview (TASRI), school and student background questionnaires, individual and group interviews/discussions, number sense achievement testing, problem solving performance assessments, observation and analysis of artefacts and school records and other documents.

Since the research questions guiding this study involved an in-depth examination of the (a) nature and relationships of certain mathematics teaching and resultant learning phenomena; (b) interaction between children's number sense and problem solving abilities; and (c) impact of teachers' teaching styles and their students' learning styles upon the teaching and learning for development of number sense and problem solving ability through the context of working mathematically; the data needed for this study should be mostly of a qualitative nature (Creswell, 1998; Pirie, 1998). Hence, it was decided that the research strategy best suited to helping the researcher understand the perceptions, actions and interactions of the teachers and students was the case study with mostly a qualitative methodology (Goetz & LeCompte, 1984; Yin, 1994; Huberman & Miles, 2002; Merriam, 1998). The case study as a research strategy was based upon an ethnographic paradigm, backed up by a quantitative approach, with some elements, especially for the coding of information, consistent with its orientation, gleaned from grounded theory (Punch, 1998, p. 162). The time limit for this study would not support full scale ethnography since this would entail "carrying out a detailed and demanding study, with fieldwork and data collection running over a long period of time" (Punch, 1998, p. 162). Instead the 'borrowing ethnographic technique (Wolcott, 1988), was used, as this study was dealing with something new (relationship between students' number sense and problem solving ability, and implications of relevant teaching styles and learning styles), which is quite different to research carried out in all four areas, and therefore contains many unknowns (Punch, 1998).

Expanding upon Chambers' (1991) support for the appropriate use of qualitative research in language and mathematics, McIntosh and Ellerton (1998) state that "…methods employed for the natural sciences — such as holding certain factors constant while varying others—may not be appropriate for dealing with human behaviour that may only be described by ambiguous and overlapping meanings" (p. 30). Moreover, "… it has become increasingly evident that over-concern with quantitative data may miss significantly important links and relationships within an educative process" (Burns, 1994, p. 247). Since a case study can be applied to both quantitative and qualitative research methods it is well suited for this study; the qualitative component of the study was attained mainly through a semi-ethnographic approach calls for the researcher to be on site (Stake, 1994). A "qualitative case study is characterised by the main researcher spending … time, on site, personally in contact with activities and operations of the case, reflecting, revising meanings of what is going on" (Stake, 1994, p. 242).

Although it is not explicitly stated in the literature, there is a strong link between the case study and the ethnographic approach in that both rely upon the researcher allowing the culture and its subjects to be the immediate source of unadulterated data, which was the intention of the researcher for this study. The nature of the present research lent itself appropriately to a quasi-ethnographic approach in that ethnography is defined as the art or science of describing a group or culture (Taylor, 2002), and with respect to this study the classroom culture, which involves teachers and students teaching and learning about number sense and problem solving, as the main focus.

A strictly linear approach was not used in this study, since the data collected from the observations and interviews were utilised in informing later data collection opportunities. This position is in accord with the notion that "ethnography research relies on what we, as observers, see and what we are told by the participants in our research studies" (Brayboy & Deyhle, 2000, p. 163). According to Agar (1986), ethnography is neither subjective nor objective, but interpretative. Hence, the researcher became immersed in the 'community/class/group' (Fetterman, 1998), so as to present a "holistic depiction of an uncontrived group interaction over a period of time, faithfully representing the participant views and meanings" (Goetz & LeCompte, 1984, p. 51). Since "an ethnography is likely to be an unfolding and evolving sort of study, rather than pre-structured (Punch, 1998, p. 161), some of the questions and instruments were developed as the researcher learnt about the community, gained their confidence and was accepted. The aspect of learning about the community was further strengthened through the use of a grounded theory cycle of collecting data, analysing it and then collecting further data, and continuing the cycle until saturation was achieved (Strauss & Corbin, 1990; Glaser, 1994), which was in accord with this study's use of an instrumental exploratory case study (Anderson, 1998).

The purpose of this research was to explore the relationships between teaching styles, learning styles, number sense and problem solving, with the main focus on the impact of the teaching-learning experience upon students' number sense and problem solving ability. In addition, the researcher also considered how teaching style and learning style compatibility can be utilised to enhance the students' number sense and problem solving ability. It was an object of this study to construct questions during the course of the research phases, for purposes of triangulation of data from other instruments. Data received from respondents were analysed to determine the relationship between number sense and problem solving ability of the students, the extent to which the development of number sense and problem solving were considered in the teaching and learning of number sense and problem solving, and the extent to which there were underlying relationships across teaching and learning styles as they related to number sense and problem solving.

The main participants, consisting of three Year 7 teachers and their students, were observed and interviewed, both formally and informally, on a regular basis; and all interviews were audio-taped. More specific details are provided when presenting information pertaining to instruments.

3.4 Instruments

While acknowledging that "data collection in ethnography may use several techniques…" Punch (1998, p. 161) warns that "any structuring of the data or of data collection instruments will most likely be generated *in situ*, as the study unfolds. Certain instruments were developed as the study progressed, in tandem with the grounded theory cycle mentioned earlier. After coding responses from informal interviews with about 50 percent of students from each class, a semi-structured 'interview-questionnaire' was designed and administered to all 64 students of the three classes to validate: the responses obtained through the informal interviews vis-à-vis the researcher's codes; and the results obtained from classroom observation, inventories and other interview data. Since this questionnaire was created *in situ* without any prior piloting, and due to its purpose being solely to validate information gathered previously, no attempt was made to assess its reliability. Hence results stemming from this questionnaire should be treated with caution.

A range of data collection methods were utilised in this study. According to Nickson (1992) studies such as this one use four typical methodologies for data collection, of which she listed "participant observation, ethnographic interviewing, a search for artefacts (available written or graphic materials related to the topic of study), and researcher introspection" (p. 107). She points out that other data could be obtained by surveys and questionnaires. Anderson (1998) advises that "in conducting case studies, one typically uses seven sources of evidence: documentation, file data, interviews, site visits, direct observation, participant and physical artefacts" (p. 155). Except for questionnaires, all these data collection methods were applied as the researcher collected both qualitative and quantitative data, through tests, classroom observations, log books or diaries, surveys, interviews, and inventories. The instruments and methods of data collection are detailed in the following sections.

3.4.1 Inventories

For the purpose of this research the revised 44-item Web-based version of the 'Index of Learning Styles' (http://www.ncsu.edu/felder-public/ILSdir/ilsweb.html), which was scripted by Benjamin Heard of North Carolina State University, was used to identify students' preferred learning styles. This is an instrument used to assess preferences on four dimensions (active/reflective, sensing/intuitive, visual/verbal, and sequential/global) of a learning style model formulated by Richard M. Felder and Linda K. Silverman. The instrument is being developed by Barbara A. Soloman and Richard M. Felder of North Carolina State University. A preliminary version of the ILS was tested by its creators, the responses were subjected to factor analysis, and some items that were not providing noticeable discrimination were replaced.

The teachers' teaching style preferences were identified through the 'Teaching Style Inventory' (http://snow.utoronto.ca/Learn2/mod3/tchstyle.html), which has been adapted by Greg Gray from Dunn and Dunn (1993). This instrument is comprised of eight categories: instructional planning, teaching methods, student groupings, room design, teaching environment, evaluation techniques, teaching characteristics and educational philosophy. These categories are further divided into 67 items.

Both the teaching style and learning style inventories were used to gather baseline data about the three teachers' teaching styles and the learning styles of the 64 students respectively. These inventories were administered during the first week of the fourth term's data collection and the resulting data was used to inform the focus of subsequent observation, interview and questionnaire items.

3.4.2 Number sense pre/post tests and problem solving pre/post tests

Separate pencil-and-paper pre- and post-tests of Number Sense (Appendix II) and Problem Solving (Appendix III) were given to a total of 64 Year 7 students in three Perth metropolitan primary schools. The Problem Solving Test (PST) comprised two components: (a) Number Sense-Inherent Problems (NSIP) and (b) Devoid-of-Number Sense Problems (DNSP).

The tests were administered during the students' usual mathematics time, over two days; one test per day. The NST is a validated test borrowed from McIntosh et al. (1997), while the PST items have been selected from the Mathematical Association of Western Australia (MAWA) Have Sum Fun Competitions (2000, 2001, & 2002) for students of Years 5 to 7. Validity of the PST was obtained by subjecting it to the scrutiny of two 'expert' (Black, 1999) mathematics educators and by agreement of the three class teachers involved to ensure that the test reflected the range and level of mathematics being taught in their classrooms. After the post testing phase, results of reliability tests (Cronbach-Alpha) for internal consistency of number sense (45 items) and problem solving (8 items) were estimated as 0.92 and 0.63 respectively.

On the basis of the number sense (NS) and problem solving (PS) test results, 45 students (70% from each class) were selected for individual interviews to explore their solution strategies and thinking. Initially the intention was to interview only students from the high and low number sense proficiency bands (Table 3.2). Eventually it was deemed necessary to include students who were in the medium proficiency band so that the sample of students selected would be representative of the distribution of students' performances in the class, as it will be shown in Table 4.14.

Table 3.2 Categorisation of students according to performance on number sense and problem solving tests

		Type of problem			
		Number sense inherent		Devoid of number sense	
		High	Low	High	Low
Students	High number sense	Category 1_H H_{NS}-H_{NSIP}	Category 1_L H_{NS}-L_{NSIP}	Category 2_H H_{NS}-H_{DNSP}	Category 2_L H_{NS}-L_{DNSP}
	Low number sense	Category 3_H L_{NS}-H_{NSIP}	Category 3_L L_{NS}-L_{NSIP}	Category 4_H L_{NS}-H_{DNSP}	Category 4_L L_{NS}-L_{DNSP}

Note: H = High; L = Low; NS = Number Sense; NSIP = Number Sense Inherent Problem; DNSP = Devoid of Number Sense Problem

In order to ensure that the testing environment was as 'normal' as possible for the students, the following measures were applied:

1. The researcher met beforehand with each teacher-participant and discussed the NST and PST protocol and how it should be followed, when the test was to be administered and other relevant issues;
2. Since students were used to having tests in their home class, both tests were done in that particular environment;
3. Since it was expected that students were used to the teacher-participant's presence, the latter was present during the tests' administration;
4. The tests were administered in the morning when students were as fresh as possible;
5. The lighting condition was discussed with students prior to the test to ensure that they all felt comfortable in this sort of lighting environment;
6. The teacher-participant read the protocol to the class as a means of getting them to hear it in a familiar voice; and
7. Each item was read to the whole class, so as to minimise the risk of misreading of the items.

Both the NST and the PST had a practice item which the examiner took the students through. This gave each examinee a chance to ask questions and to better understand what was needed of them, although it was necessary to stress that they were not expected to solve the problems presented in this way; this served only as an example of how a particular student could go about solving a problem.

Number Sense Test

The number sense test (NST) consisted of 45 items. After going through the protocol the items were read one at a time. After an item was read the students were allowed 30 seconds to complete it. No one was permitted to spend more than 30 seconds on an item and if anyone had already found the answer to an item before the 30 seconds was over, that student had to wait for the test administrator to read the next item before turning to it or solving it. If the test was completed before the last 30 seconds was over, students were not permitted to work on any unfinished items. The NST protocol is presented in Appendix II.

Problem Solving Test

The PST was comprised of Number Sense Inherent Problems (NSIP) and Devoid of Number Sense Problems (DNSP). An NSIP is a problem which definitely requires number sense as a means of solving it, while a DNSP is a problem which could be solved without any recourse to number sense. The difference was that for the NSIP anyone wanting to solve it would definitely have to make use of numerical reasoning principles to reason about or to predict the outcome of the numerical operations embedded in the problem, whereas numerical reasoning is not a priority requirement in solving a DNSP. Moreover, NSIP problems explicitly require a numerical solution and answer, while a DNSP would necessarily require a non-numerical answer.

Originally the PST was made up of six items; three number sense inherent problems (NSIP) and three devoid of number sense problems (DNSP), but since it was felt that something needed to be done to increase the possibility of differentiating between students who preferred NSIPs more than DNSPs or vice versa, after the pilot study discussions with other personnel resulted in two more items being added, which resulted in an eight-item test paper. The PST protocol is presented in Appendix III.

Think Aloud Stimulated Recall

As explained previously, 45 students were selected to participate in a Think Aloud Stimulated Recall Interview (TASRI). During the TASRI students were given four mathematics problems to solve, and asked to verbalise what they were doing and thinking as they solved these problems. After solving a problem they were further interviewed about their work and what they had said while solving the problem. The TASRI problems were designed in such a way that items one and two would be number sense inherent (NSI), while problems three and four would be devoid of number sense (DNS). Both DNSP items could be solved either through logical reasoning, which involved non-numerical ordering, or the drawing of shapes without much knowledge of numerical concepts. The TASRI protocol is presented in Appendix VII.

3.4.3 Interviews

As the researcher went through the period of being accepted in the 'community' other interview schedules were drawn up depending upon the cooperation of the participants, which in turn depended greatly upon time available and other factors. But as much as possible the researcher aimed at a least-possible scenario in which brief 5- to 10-minute informal-unstructured interviews with teachers and students took place once every four weeks. There were individual face-to-face verbal exchanges, and also informal interviews/discussions. In each term there was a structured formal interview with teachers; whereas there was only one with students, during term four. Except when they were informal, all interviews took place after an observed session. All of the formal student interviews were of a mixed semi-structured and unstructured nature to ensure guided flexibility and to allow for open flexibility of discussion respectively, in eliciting responses from the participants and also to get as authentic an account as possible. The teachers' interviews used in phase one of the study (Appendix VI), were a mix of fully-structured and semi-structured, with the rest being unstructured so as to allow a form of natural conversation to take place. Hence, this study made use of the informal conversation interview, general interview guide approach and standardised open-ended interview (Patton, 1990; Minichiello, 1990; Lee & Fielding, 1996; Fontana & Frey, 1994).

The researcher had a short five- to ten-minute discussion (Informal interview) with each teacher and at least one student per school each week. An informal interview format was designed so as to maximise the collection of relevant data around pre-discovered themes; to this end the first interviewee was used more or less like a guinea pig. With regard to the students about 50 percent of students from each of the three classes were informally interviewed. For the students, informal interviews questions were asked to:-

i. check students':
    a) ability to solve the different problem categories;
    b) ability to notice relationship between problems;
    c) perception of the relationship between number sense and problem solving;
    d) views of what the lesson was about, re involvement of NS and/or PS;
    e) evolving comments about their NS and PS performance;
ii. check how much students' number sense assisted them with problem solving;
iii. the level of link of students' NS and PS ability;
iv. check students' notion of their learning styles; and
v. check their views of their teachers' teaching style and how these impact upon their NS and PS ability development.

With respect to teacher informal interviews, due to the nature of the design of this sort of interview, as much as possible, visit days to each school were rotated so as to ensure that each teacher got a chance to be interviewed first. The informal interviews usually revolved around no more than three questions. In most cases only one main question was used to start off the discussion with other subsequent questions being formulated as per the direction of the discussion. Hence, it was normal for only one major question theme to be prepared and put to the first teacher interviewed on a particular week, with all other questions being formulated according to the flow of the answers, explanations and other statements made by this first teacher, as the discussion progressed. When the first participant was interviewed, it was quite usual for a particular pre-formulated question to be followed by other spontaneous questions ensuing from this first discussion. Once the first interview for a particular pre-formulated question was over, a simple trend analysis was done to ascertain certain relevant directions that the subsequent questions had followed throughout this first discussion. Consequently the next two teachers were interviewed in such a way that question themes explored in the first interview would be more or less preserved. So, although most subsequent questions were not pre-formulated for the first informal interview, some form of pre-coded question themes were used in the next two interviews. In this way it was ensured that:

- the discussion would flow as freely as possible;
- these teachers were made to reflect and discuss nearly the same issues;
- common themes could be explored in a semi-flexible interaction format;
- the element of irrelevancy of information would be reduced considerably; and
- coding of data would be more controlled and focused on relevant issues

Through each teacher interview, baseline data was gathered which informed the formulation of instruments and questions in later data gathering events. Teachers were interviewed to ascertain their beliefs in relation to number sense, problem solving and mathematics, and to teaching style and learning style. Questions which probed each teacher's understanding of, and their view of mathematics education, number sense and problem solving principles, were asked. The interview questions were also designed to give insights into the match and/or mismatch between a teacher's beliefs and practice. Hence, the formal interview schedule (Appendix VI) also included questions which asked the teachers about their choice of content, their perceptions about mathematical abilities, and their own confidence, as teachers, in the area. Interviews with teachers allowed the researcher to gain more information about teachers' experiences, practice and perceptions of impact of teaching style and learning style upon each other, their philosophy about the teaching/learning and relationship of number sense and problem solving, and opinions about the use of teaching style and learning style inventories. The interviews were audio-taped and transcribed. It should be noted that some questions are adapted versions of Bickmore-Brand's questions (1997, pp. 133–137).

3.4.4 Observations

Direct and indirect observation techniques were used both during and after direct teaching (Punch, 1998). Indirect observations were in the form of participant-self-observation and also when asked to recall their experience. This form of observation was used for comparison with the directly observed data, as a means of validation of data gathered from direct observation, and also to assess the compatibility of what was being observed directly and the perceptions of the participants. Hence, after each observation the researcher would talk to the teacher about the lesson, and if time was not available for this, at most five questions were left with the teacher; the teacher's comments were most often emailed to the researcher, and any validation of information was done either immediately through email or during the next visit/observation. The researcher observed each of the three teachers and their students in the classroom, once per week for four school terms (Table 3.3).

Table 3.3 Actual number of classroom observations

Term	Terms' beginning and ending dates	Number of school days	Number of school weeks	Number of observed lessons per teacher T1	T2	T3	Tot
Term 1	Monday 2 February - Thursday 8 April	54	11	7	7	6	20
Term 2	Tuesday 27 April - Friday 9 July	54	11	9	8	9	26
Term 3	Monday 26 July - Friday 1 October	50	10	8	8	8	24
Term 4	Monday 18 October - Thursday 16 December	43	9	7	7	7	21
Total		201	41	31	30	30	91

Note: Only the number of mathematics lessons observed is recorded. Hence, the number of lessons pertaining to other subject areas, which were observed during terms 3 and 4 are not represented.

One goal of the classroom observation was to describe and characterise the teacher's teaching style, how well it conformed to the problem-based teaching as advocated in many recent reform documents, how and when the teachers teach for development of number sense and problem solving, and how they catered for individual learning styles. Attempts were made to also observe and evaluate the effectiveness of the teachers' teaching program — how well students and teachers are able to use the materials, what experiments/activities they do, what kinds of questions students ask, and so on. The students were also observed; the main focus of the observation being how they solved problems, what questions they asked, how they responded to questions, and to which mode of teacher presentation/explanation/exposition/activity did particular students tend to be more active.

In this way information was gathered about how the teacher catered for individual differences, and taught for the development of number sense and problem solving. In the case of the students this method of observation provided information about how they preferred to be taught and learn, what sort of activities they liked to engage in, and what learning modality seemed to suit which particular students. To accomplish this each observed session was radio-recorded, while the researcher took field notes: of what was going on; on what was being said; about the body language of immediate interactors; and pertaining to the environment itself. Since it would not have been possible to always identify speakers from listening to the recordings, each person in the class was assigned a code and each time someone spoke the time was noted along with that person's code. The recordings of observed lessons were transcribed the same date of the respective observation and each speaker identified immediately in the transcript through comparison with the field notes. The field notes were used in conjunction with the transcribed observation data, and then these were compiled into one set of information, and finally coded.

3.4.5 Documentary data

Institutional Documents

In this category the range of documents of interest to this study were institutional memoranda and reports, government/department of education teaching programmes, pronouncements and proceedings (Jupp, 1996), relevant files, statistics and records (Hammersley & Atkinson, 1995). Information gathered from such documents helped the researcher in gaining a better understanding of what could have happened during unobserved lessons, transformations in the thoughts and philosophies of the participants in relation to issues pertaining to this study, and the school's policy towards the development of number sense, problem solving, learning style and teaching style. These documentary data were collected in conjunction with the interviews and observations, and they were analysed immediately after collection, since the resulting information was used to inform new subsequent data collection.

Students' Work and Teachers' Notes

Since it was felt that asking the participants to keep a journal, as originally planned, would be asking too much of them, a decision was made to gather similar information from lesson plans, portfolios, exercise books and mark books. After discussions with teachers and students an agreement was reached for the researcher to have access to the teachers' lesson preparation notes and students' exercise books, although for ethical reasons both researcher and teachers came to an agreement that the portfolios would not be made accessible to the researcher. Information gathered from these were used to gain an idea of what mathematics and teaching-learning activities students and teachers were engaged in while the researcher was not observing. This further served to provide an overall picture of how much time was spent on problem solving and number sense as compared to time spent on other strands and approaches. This exercise was carried out once a week when the researcher visited the school. The qualitative data gathered were coded and/or quantified on a weekly basis, and all such information were eventually brought together before the last teacher-interview and student interview-questionnaire were administered, since the items pertaining to these instruments were partially informed from these data.

3.5 Procedure

The first five weeks of term three 2003 were devoted to searching for effective Year 7 teachers of mathematics of the metropolitan area through MAWA, the Western Australian Department of Education and Training, and Perth metropolitan primary school principals. The feedback obtained from these sources provided the researcher with a list of possible teachers who could be used for the study.

This study was carried out through a sequence of four phases; starting with the pilot study phase, from the last two terms of 2003, and the three main data collection phases, from term one to term four in 2004. It was anticipated that the first few weeks of the first term, in 2004, would be a very busy time for the teacher as there would be new students to deal with. Hence, no observation was done during the first two weeks. The study pre-tested and post-tested students with NS and PS tests. The number sense and problem solving tests were given to students both at the start of the research, in 2004, during the first term, to assign them to categorical groups and, at the end of the fourth term in order to evaluate change in their level of performance, and also to select students for the Think Aloud Stimulated Recall Interview (TASRI) protocol. Collection of data through other means was carried out between the pre- and post-tests of instruments mentioned above.

As stated previously, the study was spread over four phases. The first phase pertained to the piloting of instruments and data collection methods. The main data collection was started in phase two when students were also pre-tested. Focussed mathematics lesson observations also started in phase two. Phase three saw the continuation of observations and interviews, in addition to the observation of four lessons of which the topics had been selected by the researcher. The collection of data ended in phase four which involved both post-testing and validation of data collected through the four previous phases.

Phase 1: Pilot Study

The first phase of the study was divided into three parts. The first part of phase one involved the selection of teachers for the pilot study and those for the main study. Once teachers were identified three were approached to take part in the pilot study, which implied that their 70 students also participated in that phase of the study. The piloting of the Teaching Style Inventory (TSI), the Learning Style Inventory, and the NS and PS tests also took place during this phase. The latter were done in the following order:

i. administration of NS and PS pre-tests;
ii. selection of eight students per each of the three selected classes, based on performance on NS and PS tests. These students would be the ones to be focus-observed during the pilot observation sessions;
iii. selection of three students, from the eight in each class, to be interviewed through the Think Aloud Stimulated Recall Interview (TASRI) protocol;
iv. the selection of three more students per class;
v. interviewing of the nine original students and the other nine who were newly selected; and
vi. finalising the set of coded utterances which would be used as conditional probes in the final phase of the main data collection.

From the list of suggested effective teachers of mathematics three 'volunteers' were selected to be involved as participants in the main study, which took place in 2004. Part two of phase one, involved the interviewing of the three effective teachers and students from each class. The teacher interview concentrated more on each teacher's beliefs about teaching style and learning style vis-à-vis number sense and problem solving. A major question which was posed to the teachers was to get them to express their views about the relationship among these four variables.

Part three of phase 1 was devoted solely to observation. This part spanned week 3 to week 8 of term 4, and involved observations of all the classes/subjects as normally taught by each of the three teacher-participants. The researcher took this opportunity to hone his observation skill in the specific circumstance of observing the teaching of three effective teachers of mathematics, in the Perth Metropolitan area, and the learning of their students. Such observations were done weekly on a one day per teacher basis. Sometimes the researcher did not focus on any student and at other times any student observed had to be one of the eight participants in the teacher participant's class. If the teacher participant teaches another group of students the observation of the latter was only for comparison purposes, but not focussed. These observations provided the researcher with some baseline data as to what the teacher did, how often he/she employs a problem approach to his/her teaching, any major variation in his/her teaching style, and to get more acquainted to the teacher participant. Some informal interviews, about relevant issues arising out of the observations, also took place during that time.

More specifically part 3 of phase 1 was devoted to observation of:

1. these three pilot study teachers teaching any lesson. Valuable information was collected about what to expect, what to be attentive to during such an observation stage and how to record relevant field note observations. It was also through this practice that the researcher obtained a better idea of how these teachers could vary their teaching style depending on the type of teaching and learning being experienced;
2. the three pilot study teachers as they engaged in the teaching of mathematics only. This presented the researcher with an opportunity to compare the teaching style of the teacher when teaching mathematics versus teaching subjects other than mathematics;
3. the students of these teachers both during mathematics lessons and lessons other than mathematics. It was during this stage that the researcher saw the advantage of sometimes observing only the eight selected students and at other times doing some free-open observation according to what was happening in the class as a whole. The experience provided the researcher with ample evidence that it would be more worthwhile to record as much as possible nearly everything that was going on in the class, while the audio recorder was recording whatever verbal interaction took place. The researcher used this setting to refine his observation techniques:
    - quickly sketch a plan of the seating arrangement in the class;
    - allocate a code for each participant;
    - start both the timer and the audio recording device simultaneously;
    - each time a participant made an utterance both the respective code and the time of utterance were noted; and
    - while the audio recorder would be recording sound exchanges the researcher would quickly record cues pertaining to body language, facial expression, voice tone and sketches of what is written/drawn (the object on which it is written/drawn, colour of letters/drawings, and location of writing/drawing on the object).

The data and experience gained through phase 1 were used to inform new data collection procedures and construction of instruments in the 2004 main data collection procedure.

Phase 2: Introductory Collection of Data

The first part of this phase covered weeks 1 and 2 of term 1. The first week was used to inform the students of what the NS and PS data gathering instruments were about, and why they were important. In this way both students and teacher were methodically and systematically prepared for the coming tests. It should be noted that during this discussion the PST and NST were referred to as 'data gathering exercise' instead of 'tests', in an attempt to reduce any adverse effect which could cause test anxiety. Sixty-eight students from these three teachers' classes completed the pre-NST and the pre-PST. During the second week of term 1 — the pre-testing week — each test was administered on a different occasion, with a space of two days between administration of the NST and PST, as advised by these students' teachers.

As a result of what was observed in the pilot study it was decided that instead of having eight students per class, for initial focussed observation, a selection of 50 percent of students best suited the intention of this study. The results of the students' performance on the NST and the PST were used to select 50 percent of students from each class for a two-week focussed observation. As discovered through the pilot study, it was deemed more appropriate to alternate focussed observation of these students with whole class observation on every second observation day. This practice was started in the fourth week, when the focus was on students belonging to the selected 50 percent group; during the fifth week whole class observation prevailed, and so on. It should be noted that students were selected anonymously. Hence, no student was aware that they belonged to the selected 50 percent. The second structured teacher interview was done during week eight of the third phase.

During weeks three and four of phase two the researcher observed all classes taught by each teacher participant. An average of about three days per class was spent at each school. With the exception of the first two weeks' average of fifteen lessons per class, all the other observations were done on a one lesson per week basis. This phase was used by the researcher to get acquainted with the participants and to be 'accepted' by them so that the former could eventually be seen as an 'unobtrusive' observer who could be present but go unnoticed while amongst the participants. The three teacher-participants were formally interviewed during week 5, using the first in a series of four fully structured interview schedules (Appendix VI).

The third part of this phase saw the beginning of observations of mathematics lessons only, starting in week 5 of term 1. The first week of this part entailed general observation, opportune discussions/interviews focussing upon the issues of the relationship between number sense and problem solving ability and how these are/should be implemented through the context of working mathematically, and selection of data, relevant to answering the research questions, from various documents. Hence, the third part of phase 1 involved:

1. Observation of each of the three teachers as they taught children mathematics only, so as to get better acquainted with them and their teaching of mathematics. Student observation ran concurrently with that of the teacher as both parties interacted with those in the classroom environment. This took place on one teaching day per week up to the last week of term four.
2. Informal interviewing of teachers and students from these three classes for brief 5- to 10-minute interviews based on information from the analysis of data from part 1 of phase 1. Both teachers and students were interviewed before or after the lesson, depending on time and teacher availability. There were also the occasional informal chat during lessons, but this was usually with students concerning the work they were doing, so as not to distract them. Whenever an opportunity presented itself, the researcher engaged the students and/or teachers in informal 'chats' to:
    - clarify certain issues from the observations;
    - check upon the participants beliefs about teaching, learning, number sense and problem solving; and
    - to better understand the personality of the participant.

The grounded theory approach was used to analyse the information gathered from parts one and two, and the results were used to inform the construction of further interview questions for the three teachers (teacher-participants) and for the 24 selected pupils (pupil-participants), and to refine the observation focus.

Phase 3: Selected Topics for Class Observation

Just before the end of term break each teacher was briefed about the teaching of four lessons each based on a task or idea selected and proposed by the researcher, so that these teachers would be prepared to start teaching the first of these four lessons in week three of the third term. Hence, during the fourth phase the researcher continued to engage in participant observation as a regular visitor to the classroom, with the exception that at this stage four of the classroom observations of mathematics lessons were based on tasks selected by the researcher from some mathematics literature or text books. This exercise had been tried before during the pilot study. Each teacher was asked to prepare and teach four lessons based upon certain specific tasks the researcher had selected from different sources, which incorporated an element of number sense and problem solving. It was hoped that this would act as a form of control in that:

- the researcher would be able to observe each of these three teachers teaching the same topic;
- it would ensure that the teaching to be observed would involve some aspect of number sense and problem solving;
- although they would necessarily employ different teaching techniques, approaches and strategies, the lessons would be similar in the sense that the topic would not vary.

In his observations the researcher picked on the teaching style being employed, what sort of learning style students were using and how the teacher's teaching style catered for these students' learning styles. Other points of interest were:

- whether or not these teachers taught as they normally did;
- what aspect/topic/strand they focused upon; and
- which aspect of the working mathematically strand they focused most upon

In week seven, after these four lessons had been taught, four students were interviewed (semi-structured) around the theme what impact the teacher's teaching had upon;

- their learning style;
- their problem solving and number sense development?

Other situational observations were carried out one day per week in each school, during the whole of term three and up to the end of term four. Informal interviews with the participants helped in preparing for the final validation interview. An 'interview guide approach' (Best & Khan, 1998, p. 201) was used. Considerable time was invested in observing and interacting with students chosen as 'key informants' (Goetz & LeCompte, 1984, p. 119).

Phase 4: Post Testing and Data Validation

It was during this phase that the final confirming of data was done through consultation with the participants. By then the themes already identified were scrutinised and validated through further participant interviews, and resultant information used to modify the themes as necessary. Hence, the participants' 'validation' of any emergent concepts of the framework was sought as a means of ironing out any discrepancies between the observed data and participants' perceptions. This phase started from the first week and ended on the last week of the fourth term. This involved the final collection of data through administration of the post number sense and problem solving tests, the interviewing of the school managements personnel, the three teacher-participants, the eight informant student from each of these classes, and the Think Aloud Stimulated Recall Interviewing (TASRI) of 45 selected students.

Four weeks before the end of term four the students were post-tested through the NS and PS tests. Three weeks before the end of term four the results of the post-tests were used to select 45 students for the TASRI. The data collected from the TASRI was used as the basis for emerging theories related to the efficacy of the learning environment to enhance students' development of the number sense/problem solving relationship. Originally it was decided that the TASRI protocol should be kept reasonably brief, from 15 to 20 minutes, to avoid incidence of fatigue on the parts of both interviewer and students, while with the teachers a balance will be struck before the interview starts. This view was changed after the pilot study due to the discovery that if students were aware that they could stop anytime they wanted while solving a problem, and that they could go to another problem if they so wished, they tended to feel less pressure than they normally would under a strict exam-type regime. During the pilot study it was also discovered that under such circumstances, although they were allowed to work on a problem for as long as they wanted to, in the most extreme cases students spent no more than 40 minutes solving all four TASRI problems.

It was also during this phase that the principal and deputy principal were formally interviewed about the school's policy in relation to number sense, problem solving and working mathematically, and also based on analysis of documents and other data collected up to now. This interview helped in forming a picture about the school's policy on teaching style and learning style, and the development of problem solving ability and number sense. Views about the relationship of such variables were also sought, which helped the researcher to gain more insight regarding the impact of such policies upon the type of teaching practised with respect to the four main research variables.

Although it was set in four stages, the procedure proposed was rather of a spiral form. Each level of the spiral merged one into and overlapping each other. This spiral model lent itself well to the research design of this study, since data collection, analysis, and construction of certain instruments were an ongoing cyclic process throughout the duration of the study. The advantage of such a system was that data could be coded, indexed, compared and evaluated at various intervals of the study, allowing further data collection to be better informed than former ones — an exercise which enhanced the validity and reliability of the data collected, its analysis and eventual reporting.

In regards to the possible lack of quality criteria noticed in Chapters Three and Four, due consideration was given to Guba and Lincoln's (1989)' naturalistic' approach, as presented in Chapter Eight of *Fourth Generation Evaluation*. This was done in accordance with Guba and Lincoln's (1989) outlined methodological guidelines for conduct of naturalistic evaluations; in instances where it was found that techniques and methods from the scientific/positivist paradigm of research could not be used effectively to help the researcher understand his own position as to the interpretation of the qualitative data. Hence, the researcher attempted to impose certain quality criteria as a means of overcoming certain potential hazards of analysing and interpreting the qualitative information. The most common method employed was peer debriefing in which the researcher engaged in discussions with at least five colleagues who seemed to have no interest in the nature, context and content of this study. These colleagues' suggestions helped the researcher to understand his own posture and interpretations of the data from the interviews and observations.

The analysis of data, as described in the next chapter, started immediately after that and continued up to the end of school term four in 2004, after which most of the time was spent in the final writing up of the thesis.

Chapter 4: Analysis and Results

The previous chapter introduced the *raison d'être* of using a research paradigm comprising a combination of qualitative and quantitative data collection methods. The schools and participants were introduced, with emphasis placed upon the student-participant. In this chapter a brief overview of the context will be set, followed by the analysis of the qualitative and quantitative data. This chapter is concerned mainly with presentation of data analysis and subsequent results, the latter taking the form of specific assertions resulting from analysis of relevant quantitative data, culminating in elaborations on these assertions supported through evidence collected as qualitative data. To begin with there will be discussion of the main features of the data. Then each research question will be examined and answered through the assertions extrapolated from first the quantitative data, then the qualitative data. Finally the assertions are brought together and the ensuing discussion geared towards answering the main research question.

4.1 The Context

To ensure anonymity of the schools and participants pseudonyms were used as per Table 4.1. In most cases a full name pseudonym is used as a means of preserving the authenticity of the information as closely as possible. In cases where, for statistical purposes, it is found necessary to link specific data such as gender, school and student in a concise manner, the subscripted or alphabetical code have been used.

Table 4.1 Pseudonyms and codes used in the thesis

School		Teacher	
Pseudonym	Type	Pseudonym	Subscripted code
Arlenta Primary School	Private	Amanda	T_1
Barden College	Private	Bob	T_2
Cottonfield Primary School	Public	Chantal	T_3

The three teachers and sixty-four Year 7 students who participated in this study were from three schools with the following backgrounds:

- School One (Arlenta Primary) is an all-girls private school taught by a female teacher (Amanda). Students come from upper middle class to high social class families;
- School Two (Barden college) is an all-boys private school taught by a male teacher (Bob). Students come from upper middle class to high social class families;
- School Three (Cottonfield Primary) is a mixed-sex public school, also taught by a female teacher (Chantal). Students come from middle class to high social class families.

As mentioned earlier, these three teachers were selected as a result of having been identified as effective mathematics teachers. A comparative observation of these teachers revealed that there were many similarities among them. For instance, all three teachers encouraged their students to take part in various mathematics competitions, and the training of selected students was done by these teachers. The only major areas for which they exhibited marked differences were in some of their beliefs and the way they prepared learning experiences and taught their students. Nevertheless, each taught through a problem-centred approach. In regard to lesson preparation those from the two private schools did a lot of mental preparation, accompanied by a few sketches of major 'lesson points', but always keeping a record of what they intended students to learn, what students really learnt and what action could be taken after that. The public school teacher undertook much mental preparation, although she seemed a bit more disposed towards preparing detailed written lesson plans. An interesting discovery made through analysis of interview data gathered from management personnel such as the principal, deputy principal and/or curriculum coordinator, is that the more that people from management believed in detailed written lesson preparation, the greater the emphasis placed on such preparation by the respective teacher; and the converse was also apparent.

4.2 Quantitative Data Analysis Mode

Quantitative data was of two forms:

1. Those directly obtained from quantitative data collection modes; number sense (NS) and problem solving (PS) pre- and post- tests scores; and

2. Those indirectly obtained through quantification of qualitative data; teaching style inventory and learning style inventory, and coding of interview responses and the student-informal interview-validation questionnaire.

Since the NS test, being of a purely multiple choice format, was an objective one, a simple marking scheme was employed, whereby each correct or wrong NS test item answer was awarded a basic score of 1 or 0 respectively. The marking scheme for the PS test items was designed in such a way that it would lean to a large extent towards being an objective scoring system. Hence, two schemes were employed — a basic-scoring system and a process-scoring system — as a means of comparing how far apart respective students' marks would be from one marking scheme to the other. The basic scoring system was purely objective; a score of 1 for a correct answer, regardless of whether written working was shown or not, and 0 for a wrong answer. The process scoring system employed an Analytic Scoring Scale (ASC) adapted from Charles, Lester and O'Daffer. (1987). Table 4.2 shows how the scores were allocated based on the criteria Understanding the Problem (U), Planning a Solution (P) and Getting an Answer (A); where a score of 2, 1 or 0 was respectively awarded for fully, partially or not satisfying each of the three criteria in turn.

Table 4.2 Analytic scoring scale adapted from Charles, Lester and O'Daffer (1987)

Criteria	Mark	Description
Understanding the Problem	0:	Complete misunderstanding of the problem Or did not even start solving the problem
	1:	Part of the problem misunderstood or misinterpreted
	2:	Complete understanding of the problem
Planning a Solution	0:	No attempt, or totally inappropriate plan
	1:	Partially correct plan based on part of the problem being interpreted correctly
	2:	Plan could have led to a correct solution if implemented properly
Getting an Answer	0:	No answer, or wrong answer based on an inappropriate plan
	1:	Copying error; computational error; partial answer for a problem (with multiple answers)
	2:	Correct answer and correct label for the answer

The pre-test scores for NS and PS were compared and the same was done to the post-tests scores, after which respective NS and PS pre-test and post-test scores were also compared. After discussions with other researchers, the teachers and supervisors, it was deemed appropriate to also combine the pre- and post- NS test scores and PS test scores, as a means of better representing and comparing students performance. Hence, unless specified, any reference to the test scores pertains to the combined scores.

4.3 Qualitative Data Analysis Mode

Learning experience observation data was gathered through field notes and audio recordings. These were transcribed and coded, and the results used to inform the design of subsequent data collection items and instruments to collect further information. From the four teacher interviews, informal short interviews, Think Aloud Stimulation Recall Interview (TASRI) and the Teaching Style Inventory (TSI), information was gathered concerning each teacher's beliefs and preferred teaching style, and the students' perceptions with regard to how they solved problems. Such data were compared to the field notes and observation data. Eventual quantification of these data revealed estimates of time spent on teaching for NS and PS, and factors which could have a bearing upon students' NS and PS ability enhancement. Fifty percent of students from each of the three classes were selected, according to their NS and PS post-tests scores (see Chapter 3), to take part in the combined think-aloud and non-videotaped stimulation recall. After coding and quantification of these data, a comparative check was done as a means of providing back-up evidence for the support of suggestions resulting from analysis of the direct quantitative data. As described in Chapter 3 a cyclic grounded theory approach, based on Glaser and Straus's (1967) method, was applied. Hence, the aspects of collecting, coding, analysing, and theory generation were done on a nearly simultaneous basis. Since a cyclic model was employed the process was iterative and progressive in application. From day one of data collection in 2004 data collected through various means were coded into categories. If these categories were confirmed through subsequent coding then they were used to start the building of the theoretical model. In cases where subsequent coding failed to confirm these categories the latter were refined, extended and modified to accommodate subsequent data. Many new categories emerged throughout the different phases of data collection. According to Pak and David (2004) this data collection procedure is:

> governed by a process known as *theoretical sampling* where the coding and analysis done at the initial stages determines the subsequent data to be collected. Theoretical sampling, unlike statistical sampling, is the process of collecting data for comparative analysis and it is especially useful to facilitate theory generation. (http://InformationR.net/ir/9-4/paper195.html)

4.4 Test Results

As mentioned in Chapter 3, for each correct response on the 45-item Number Sense Test (NST) a score of 1 was awarded; this was in tandem with the original marking scheme used by the authors of the NST (McIntosh et al (1997), making it possible to compare the results with other studies which have used this same NST. The Problem Solving Test (PST) was made up of eight items, four of which were Number Sense Inherent Problems (NSIP) while the other four were classified as Devoid of Number Sense Problems (DNSP). For comparative and logistic purposes two different marking schemes were used. The first one was a basic marking scheme of 1 for a correct answer and 0 for a wrong answer which was used to identify the number of students who scored full marks — item number 7 was the only one asking for two separate answers which, resulted in half a mark being given for each answer so that no item would carry more than 1 mark. Since the researcher was also interested in the strategies used by students, a process scoring system, adapted from Charles et al. (1987) was used where the scheme presented in Table 4.2 was used.

An examination of the summary of results presented in Table 4.3 reveals some interesting information about the students' performance in regard to the NST and PST. The NS and PS ability range of the students for both tests are wide across pre-test, post-test and aggregate score. The maximum possible score was achieved for problem solving and nearly achieved for number sense. The fact that the mean is above fifty percent throughout is indicative of a high level of performance in all three classes which may be due to their being taught by an effective mathematics teacher. It is also worth noting that the mean of means for the four pre-tests is 68 percent with an approximate standard deviation of seven. The fact that these means range from 59 percent to 76 percent, in the direction of pre-tests to post-tests, and that the mean percent change is quite consistent at six percent and four percent for NS and PS respectively, indicates that there is an existing relationship between NS and PS performance. The statistical direction and strength of this relationship will be briefly introduced through Table 4.3 and will be explored in more detail later.

Table 4.3 Summary of NST and PST process score results (N = 64)

Mathematics Content	Number Sense Pre-test	Number Sense Post-test	Problem Solving Pre-test	Problem Solving Post-test	Aggregate Score NS	Aggregate Score PS
Max Possible Score	45	45	48	48	90	96
Range (%)	24-93	36-98	35-100	40-100	30-93	38-100
Mean (%)	59	72	65	76	66	70
SD (%)	17	17	17	18	16	16
Error of Means (%)	2	2	2	2	2	2

Table 4.4 and Table 4.5 present frequencies of correct pre- and post-tests responses for PS and NS separately as a means of gaining information as to how the scores are distributed per number of students. It is interesting to note that in terms of the modal score (highlighted in Table 4.4 and Table 4.5), while this obviously increases from pre-test to post-test for NS — with both modal counts being constant at six students —the reverse seems to occur for PS with 14 students. Yet on close examination it becomes apparent that compared to six students on the pre-test, there are now 12 students who are scoring at 87.5 percent, indicating a significant improvement in PS performance. Furthermore, when these frequencies are observed from one variable to another, it is apparent that although there is some form of correlation between pre-PS and post-PS, and pre-NS and post-NS performance, the number sense test scores seem to be nearer to a linear relationship than the scores for problem solving. Nevertheless, an examination of the combined pre- and post-test scores for both PS and NS presented in the analysis and results pertaining to the first research question, and introduced through Table 4.7, indicates that there is a very strong linear relationship between NS and PS.

Table 4.4 Frequency of pre- and post-NS scores for all three classes

Pre-test						Post-test					
Score			Score			Score			Score		
Out of (45)	%	f	Out of (45)	%	f	Out of (45)	%	f	Out of (45)	%	f
11	24	2	28	62	5	16	36	1	32	71	2
14	31	1	29	64	3	17	38	2	33	73	4
15	33	5	30	67	2	20	44	3	34	76	3
18	40	3	31	69	4	21	47	1	35	78	3
19	42	1	32	71	5	22	49	1	36	80	3
20	44	1	33	73	2	24	53	3	37	82	6
21	47	2	34	76	1	25	56	4	38	84	4
22	49	3	36	80	2	26	58	2	39	87	4
23	51	4	37	82	1	27	60	2	4	89	3
24	53	6	38	84	3	28	62	1	42	93	4
26	58	4	39	87	1	29	64	3	43	96	3
27	60	1	41	91	1	31	69	1	44	98	1
			42	93	1						

Note: N = 64

Table 4.5 Frequency of pre- and post-PS basic scores for all three classes (N = 64)

Pre-test			Post-test		
Score out of (8)	%		*Score out of* (8)	%	
1.0	12.5	1	2.0	25.0	2
1.5	18.8	1	3.0	37.5	4
2.0	25.0	4	3.5	43.7	4
2.5	31.3	2	4.0	50.0	14
3.0	37.5	6	4.5	56.2	1
3.5	43.8	4	5.0	62.5	9
4.0	50.0	11	6.0	75.0	6
5.0	62.5	17	6.5	81.2	2
5.5	68.8	2	7.0	87.5	12
6.0	75.0	6	7.5	93.8	2
6.5	81.3	1	8.0	100	8
7.0	87.5	6			
7.5	93.8	2			
8.0	100.0	1			

Note: Pre-Test Mean = 57.6%, SD = 20.2%; Post-Test Mean = 67.9%, SD = 21.6%; Combined Mean = 63%, SD = 18%

Initially the relationship between NS and PS performance was explored through categorisation of students as per three extrapolated proficiency levels resulting in six proficiency bands across NS and PS performance. Before organising the cross-tabulated information presented in Table 4.7, students' number sense and problem solving proficiency levels were determined, so that the cut off points were fixed at the

30th and 70th percentiles. To obtain uniform proficiency levels across problem solving and number sense, students' performance was categorised on the basis of their NS and PS scores falling in one of three NS and PS proficiency bands, as illustrated through Table 4.6. This method ensured that the cutting points were determined by the way the sample was distributed. De Vaus (2002) highlighted the statistical integrity of such an approach when he wrote that it "…allows the distribution to define what is high or low rather than the researcher imposing their own views as to what a low or high [PS or NS score] is"(p. 38).

Table 4.6 NS and PS proficiency codes and categorisation of scores (N = 64)

Grade	Proficiency Band		Class Percentage		Grade Code		Proficiency Code
NS	Top 30%	=	76 - 100	=	1	=	H_{NS}
	*Middle 40%	=	58 - 76	=	2	=	M_{NS}
	Bottom 30%	=	0 - 58	=	3	=	L_{NS}
PS	Top 30%	=	80- 100	=	4	=	H_{PS}
	*Middle 40%	=	61 - 80	=	5	=	M_{PS}
	Bottom 30%	=	0 - 61	=	6	=	L_{PS}

Note: H = High; M = Medium; L = Low; NS = Number Sense; PS = Problem Solving
*To maintain normality of distribution The Middle Band (Medium) has 10% more than the top and bottom bands to cater for the greater number of scores falling in this band.

An important reason for having proficiency levels was to gain insight into the nature of the relationship between NS and PS, in relation to level of performance. Hence, the first relationship to be gauged through observation was the linearity of the students' NS and PS performance. Observation of Table 4.7 shows that there is a fairly constant rate and direction of change between the numbers (percentages) of students in both the first and third rows; with the directions in the first row being in reverse to those in the third row. According to De Vaus (2002), "given these two facts (constant rate of change and a constant direction of change within a row) we can conclude that the relationship between X and Y is linear" (p. 84). Hence, as it will be graphically shown when answering research question 1, that there was a linear relationship between students' NS and PS performance.

Table 4.7 Number and percentage of students within each category

	H_{PS}	M_{PS}	L_{PS}	Total
H_{NS}	12 (18.8)	6 (9.4)	1 (1.6)	19 (29.7)
M_{NS}	5 (7.8)	15 (23.4)	6 (9.4)	26 (40.6)
L_{NS}	1 (1.6)	6 (9.4)	12 (18.8)	19 (29.7)
Total	18 (28.1)	27 (42.2)	19 (29.7)	64 (100)

Note: Percentage of students out of 64 is shown in parentheses

Another very interesting observation, stemming from Table 4.7, is the strong symmetrical pattern of numbers of students falling in each subcategory, suggesting a very strong relationship between students' NS and PS performance.

4.4.1 Number sense test results

What follows is a presentation of patterns occurring in the number sense data, which will be used to gauge the occurrence of problem solving performance in relation to number sense performance. As mentioned above, students' performance on the Number Sense Test (NST) indicated that there was significant gain from pre-test to post-test, an aspect which will be examined in research questions two and three.

Table 4.8 was drawn as a means of gaining more insight into the relationship between NS and PS. Since the students' NS and PS proficiency levels had been set so that performances falling in the top 30 percent, the middle 40 percent or the bottom 30 percent would be considered as high, medium and low respectively, only items for which 80 percent of the students were getting a correct answer were considered to be highly rated. Items for which less that 58 percent (below the lower boundary of the medium proficiency class (Table 4.5) of the students were getting a correct answer were deemed to belong to the low rated performance items. In order to better access such information, both the DNS and NSI components of the PS tests were designed around the three components of number sense (numbers, operations and computational settings). These three components were cross-matched with the six strands of number sense listed by McIntosh, Reys, and Reys (1992).

Table 4.8 Number of students with correct NST answer per Item (N = 64)

								Pre-test									
	Item	40	5	14	19	12	7	39	3	17	43	Less than 80% Correct on Pre-Test					
Correct	Count	62	60	60	57	55	54	54	53	52	51						
	%	97	94	94	89	86	84	84	83	81	80						
	Rank	*1*	*2*	*2*	*4*	*5*	*6*	*6*	*8*	*9*	*10*						
								Post-test									
	Item	12	14	40	9	39	5	3	11	17	19	7	43	6	10	15	13
Correct	Count	63	63	62	61	59	58	57	57	57	57	56	56	54	54	54	52
	%	98	98	97	95	92	91	89	89	89	89	88	88	84	84	84	81
	Rank	1	1	3	4	5	6	7	7	7	7	11	11	13	13	13	16
								Combined									
	Item	40	14	5	12	19	39	3	7	17	43	Less than 80% Correct on Aggregate					
Correct	Count	124	123	118	118	114	113	110	110	109	107						
	%	97	96	92	92	89	88	86	86	85	84						
	Rank	1	2	3	3	5	6	7	7	9	10						

Note: The maximum ‘count’ for the combined data is twice 64.

This analysis revealed that, at the pre-testing stage, ten of the 45 NS items were answered correctly by 80 percent or more of the students. Item 40 was the most successfully answered, followed very closely by items 5 and 14. The students’ performance at the post-testing stage produced six more items for which 80 percent or more of the students got a correct answer. The post-test results also indicate that there was an increase in the number of students getting correct answers for the same ten items observed for the pre-test, although by this stage the increase in the number of students getting perfect scores on items 11 and 9 caused items 7 and 43 to occupy ranks outside the first ten.

It is worth noting that both items 39 and 40 are concerned with the number strand component of ‘effect of operations’ and the mathematical content ‘percentages’ and ‘whole numbers’. Analysis of the items for which 80 percent or more of the students were getting correct answers (10 items for the pre-test and 16 items for the post-test) also reveal that students were quite proficient in working out solutions to items generally incorporating whole numbers. Of the 16 post-tested NS items falling in this category, about 38 percent relate strictly to whole numbers (Items 3, 5, 6, 7, 17 and 19), 25 percent being strictly of the ‘multiple representations’ and ‘decimals’ strand components. The above focus was mainly on high performance, and will be used later to discuss issues pertaining to the occurrence of problem solving vis-à-vis number sense.

In order to understand how NS and PS were related it was deemed necessary to analyse not only the NS strengths, but also the NS weaknesses of the students based on how they performed on the NST. Of the 18 items for which less than 58 percent of the students got a correct answer, only 11 stayed in that same category at the post-test stage (Items 1, 25, 26, 28, 30, 33, 35, 36, 37, 38 and 41). In fact for items 35, 36, 37, 38, and 41 students' performance level was lowered from pre-test to post-test. Of the 11 items for which less than 58 percent of the students managed to get a correct answer, more than half (55%) involved working with fractions. In fact, data obtained from the observations and interviews also point towards fraction-in-context as being a major mathematics element which presented an obstacle for students when it came to problem solving.

In answering research questions one, two, three, and four, some suggestions will be made pertaining to what has been presented up to now, with regard to students' number sense performance. An interesting aspect of this study revolves around the issue of comparison with other studies. When the results of students participating in this study are compared to those carried out through virtually the same number sense test in Australia and the United States, the item by item mean scores of students participating in this study were in most cases higher than same-age US and Australian students. This could be due to the fact that all three teachers in this research were identified as effective teachers of mathematics.

4.4.2 Problem solving test results

The overall problem solving results, based on the basic scoring system, presented in Table 4.9, reveal that in the pre-testing stage students were getting more correct answers on number sense inherent problems (NSIP) than they did on devoid of number sense problems (DNSP) — the difference being 14 percent. By the time they did the post-test this tendency was reversed, with the number of correct DNSP answers being greater than that of NSIP by four percent, although there was a slight percentage increase of one percent on the number of correct NSIP answers. Another interesting observation pertains to the percentage of incorrect NSIP answers staying constant at 32 percent, while there was a decrease of 20 percent in incorrect DNSP answers. There seems to be strong evidence in support of the suggestion that the teaching emphasis played a major role in the occurrence of such a pattern. At first it was thought that such a pattern existed due to focus being placed solely on how many students got the correct answer. Hence, a comparative analysis was done pertaining to data obtained through the process scoring system (Table 4.10). This exercise confirmed what was discovered from data obtained through the basic scoring system; namely, that students' performance did improve for both NSIP and DNSP, with greater improvement in the latter.

Table 4.9 Number of students with correct, partially correct or incorrect process scored answers across problem solving items

			Number of Students			Percentage of Students		
Item			Full	Partial	None	Full	Partial	None
Pre-Test	1	NSIP	41	15	8	64	23	13
	2	NSIP	26	28	10	41	44	16
	3	DNSP	9	24	31	14	38	48
	4	DNSP	42	22	0	66	34	0
	5	NSIP	44	15	5	69	23	8
	6	DNSP	48	10	6	75	16	9
	7	NSIP	44	17	3	69	27	5
	8	DNSP	31	13	20	48	20	31
	Total NSIP		129	47	16	67	24	8
	Total DNSP		88	47	57	46	24	30
Post-Test	1	NSIP	42	21	1	66	33	2
	2	NSIP	33	23	8	52	36	13
	3	DNSP	30	23	11	47	36	17
	4	DNSP	53	9	2	83	14	3
	5	NSIP	38	20	6	59	31	9
	6	DNSP	50	9	5	78	14	8
	7	NSIP	51	10	3	80	16	5
	8	DNSP	43	14	7	67	22	11
	Total NSIP		131	51	10	68	27	5
	Total DNSP		123	46	23	64	24	12

Note: NSIP = Number Sense Inherent Problem; DNSP = Devoid of Number Sense Problem

From Table 4.10 a similar pattern is observed. At the pre-testing stage performance scores with regard to full marks favoured NSIP over DNSP, but this time the difference is lower (9.7%). Once again full marks for DNSP and NSIP post-tests performance are reversed compared to that of the pre-tests, with a difference of 5 percent in favour of DNSP. It is also worth noting that improvement in solving NSI problems was only 4 percent compared to 18 percent for DNS problems. Factors which could be responsible for influencing such a pattern of comparative change in NSIP and DNSP scores are explored when answering research question two, three and four.

Table 4.10 Percentage of students with correct, partially correct or incorrect process scored results across problem solving items

		Percentages			
	Student Scores	Pre-Test		Post-Test	
		NSIP	DNSP	NSIP	DNSP
	Full Marks (6)	60.5	50.8	64.1	68.8
Partial Marks	5	4.7	0.8	3.5	1.2
	4	1.6	1.2	1.6	2.0
	3	4.7	4.7	3.9	3.1
	2	4.7	6.6	10.5	5.1
	1	13.7	13.7	9.4	10.2
	Total Partial Marks	29.3	27.0	28.9	21.5
	No Score	10.2	22.3	7.0	9.8

4.4.3 Learning style inventory results

To examine the impact of learning style upon students' number sense and problem solving performance, data was collected through administration of a standard Learning Style Inventory, observation of students and teachers during teaching experience sessions, and interviewing of teachers and students. The revised 44-item Web-based version of the 'Index of Learning Styles' (ILS) (http://www.ncsu.edu/felder-ublic/ILSdir/ilsweb.html) was used to identify students' preferred learning styles. This instrument assesses preferences on four dichotomous dimensions spread across eight learning modalities (active/reflective, sensing/intuitive, visual/verbal, and sequential/global) of a learning style model formulated by Richard M. Felder and Linda K. Silverman. Explanations pertaining to how a student's learning preferences are determined are presented at http://lorien.ncl.ac.uk/ming/learn/ils_score.htm, and also in Chapter 3. To eradicate students' errors discovered during the piloting of the ILS, the students were asked to fill in a hard copy version of the ILS and then transfer that to an on-line version of the ILS, then print a copy of the completed on-line inventory before submitting for assessment on-line. The printed on-line inventory and the original hard copy versions were compared for any inconsistencies, and if any were detected students were questioned about their preference.

As introduced in Chapter 3, the ILS is comprised of 44 items each having two possible choices for answers. The possible score on each of the eight modalities have a range of zero to 11. The learner reference across a particular dimension is calculated as the difference (differential) between the scores obtained for the two respective polar modalities. The analysis which follows will use both the score obtained and the differential depending on the nature of the data being presented. The analysis will sometimes focus on individual (singular) learning modalities, balanced (bimodal) preference, pairs of modalities (four learning dimensions), and combinations of three or four modalities. By focussing on the relationship between learning style combination groups and number sense-problem solving performance more understanding was gained about learning combination differences when the preferences for individual learning dimensions were combined to produce the 16 combinations shown in Table 4.11.

Table 4.11 Combinations of four learning styles modalities

	Combination				Code
1	active	sensing	visual	sequential	ASnVS
2	active	sensing	visual	global	ASnVG
3	active	sensing	verbal	sequential	ASnVrS
4	active	sensing	verbal	global	ASnVrG
5	active	intuitive	visual	sequential	AIVS
6	active	intuitive	visual	global	AIVG
7	active	intuitive	verbal	sequential	AIVrS
8	active	intuitive	verbal	global	AIVrG
9	reflective	sensing	visual	sequential	RSnVS
10	reflective	sensing	visual	global	RSnVG
11	reflective	sensing	verbal	sequential	RSnVrS
12	reflective	sensing	verbal	global	RSnVrG
13	reflective	intuitive	visual	sequential	RIVS
14	reflective	intuitive	visual	global	RIVG
15	reflective	intuitive	verbal	sequential	RIVrS
16	reflective	intuitive	verbal	global	RIVrG

Note: A = Active; R = Reflective; Sn = Sensing; I = Intuitive; V = Visual; Vr = Verbal; S = Sequential; G = Global

It should be noted that in this study, whenever reference is made to an individual student's type, this is expressed as one of sixteen possible combinations of those preferences (Table 4.11). In addition to the above approach, in scoring and scaling the students' responses, the procedure used by Zywno (2003) were also adopted. According to Zywno (2003) when conducting research to validate the ILS, "one of the problems encountered was scoring of the scales" (p. 4). Zywno (2003) elaborated upon this problem as follows:

> The ILS scales are bipolar, with mutually exclusive answers to items, i.e. either (a) or (b). Because there is an odd number of items on each scale, if items are scored as +1 and –1, respectively, the total score on a scale from –11 to +11 shows an emerging preference for the given modality. However, the dichotomous nature of scales makes the use of standard statistic tests difficult17. Thus, only scales for either (a) or (b) should be considered, each consisting of 11 items. The responses were scored for the Active, Sensing, Visual and Sequential scales by assigning a value of 1 to (a) items, and 0 to (b) items. Scores for the respective opposite polarities, Reflective, Intuitive, Verbal and Global, can be found as a complement of 11 (i.e., if the average Active score is 6.5, the average Reflective score is 4.5). (pp. 4-5)

The 'Statistical Package for the Social Sciences' (SPSS) was used to analyse data pertaining to the learning style of the 64 students. Descriptive statistics were tabulated to determine each student's predominant learning style. The variances were analysed to determine the differences between learning styles and students' preference, number sense (NS) performance and problem solving (PS) ability. Correlation analysis and tests of repeated measures and independent samples were carried out to determine if there was any relationship between gender and learning styles. Since some students' learning styles scores indicated a balanced tendency for certain dichotomous dimensions, Chi-square analysis was conducted to determine if gender, NS score and PS score had an effect in which the learning styles of students were bimodal. Fisher's test of Least Significant Differences was conducted to make pair-wise comparisons among the means of the three proficiency level groups for both NS and PS. Since no similar research had previously been done with Year 7 students, comparison could only be made with existing results from research on tertiary education students.

4.4.4 Comparison of learning preferences

Singular Learning Style Preference

Results of preference for individual (singular) learning modality was obtained first through analysis of measures of central tendency and spread, and secondly through comparison of preference differentials. Table 4.12 shows the mean, median and standard deviation of the students' scores on the eight Felder-Solomon (2004) learning modalities, grouped according to the four dichotomous learning dimensions: (1) processing information; (2) perceiving information; (3) receiving (input) information; and (4) understanding information. Since 5.5 was both the mean of means and the median of medians, any mean preference score above 5.5 was considered high, and any such mean score equal to or lower than 5.5 was considered to be low. The greatest difference (5.12) in related means pertained to the 'receiving information' dimension; the visual modality was the most favoured learning modality with mean score slightly higher than eight.

Table 4.12 Students' learning style scores (0–11)

Dimension	Modality	Mean	Median	SD
Processing Information (A-R)	Active	6.1	6	2.3
	Reflective	4.9	5	2.3
Perceiving Information (S-I)	Sensing	5.9	6	2.5
	Intuitive	5.1	5	2.5
Receiving Information (V-V)	Visual	8.1	8	2.0
	Verbal	2.9	3	2.0
Understanding Information (S-G)	Sequential	5.5	6	1.8
	Global	5.5	5	1.8

Note: A-R = Active/Reflective; S-I = Sensing/Intuitive; V-V = Visual/Verbal; S-G = Sequential/Global

Although the mean scores are quite close for three of the four related pairs of modalities, it is worth noting that the students' preference scores were higher ($\overline{X} > 5.5$) for the Active, Sensing, Visual and Sequential modalities compared to their respective Reflective, Intuitive, Verbal and Global scores ($\overline{X} \leq 5.5$).

Consolidation of the above-mentioned results are presented, in Table 4.13, through further analysis and results pertaining to individual (singular) differences in the four dimensions of learning style and their respective modalities. With regard to individual preferences in the processing of information, the results presented in Table 4.13 indicate a greater preference for active learning with an overall frequency difference score of 21 percent. An examination of the second ILS dimension (perception of information) suggests that more than 60 percent of the students had a greater preference for sensing learning as opposed to intuitive learning. As already suggested with regard to the information input dimension, a very large majority (more than 90%) of students indicated a preference for receiving information through the visual modality instead of the verbal (only 8%). Of all the four dimensions, the least preference differential for an individual modality pertained to understanding information, where students showed a very slight preference of four percent for the sequential modality over the global modality. The latter observation is further highlighted below in the results of balanced (bimodal) learning style for three of the four dimensions.

Table 4.13 Comparative results of preference for each of the eight learning modalities

Differential		Processing Information ACT	Processing Information REF	Perceiving Information SEN	Perceiving Information INT	Receiving Information VIS	Receiving Information VRB	Understanding Information SEQ	Understanding Information GLO
Balanced	"1"	11	16	22	9	14	3	20	19
	"3"	17	13	14	5	20	3	16	16
Moderate	"5"	17	6	14	9	13	2	14	11
	"7"	5	6	11	8	20	0	2	2
Strong	"9"	6	2	0	5	11	0	0	2
	"11"	2	0	3	0	14	0	0	0
%		58	42	64	36	92	8	52	48
Count		37	27	41	23	59	5	33	31

Note: N = 64

Balanced (Bimodal) Learning Style preference

Median split analysis of data was performed in order to ascertain the degree of balance (bimodality) in the students' learning style preference (Figure 4.1). On the whole students' scores indicated relatively balanced (bimodal) learning preferences on the Processing Information (A-R), Perceiving Information

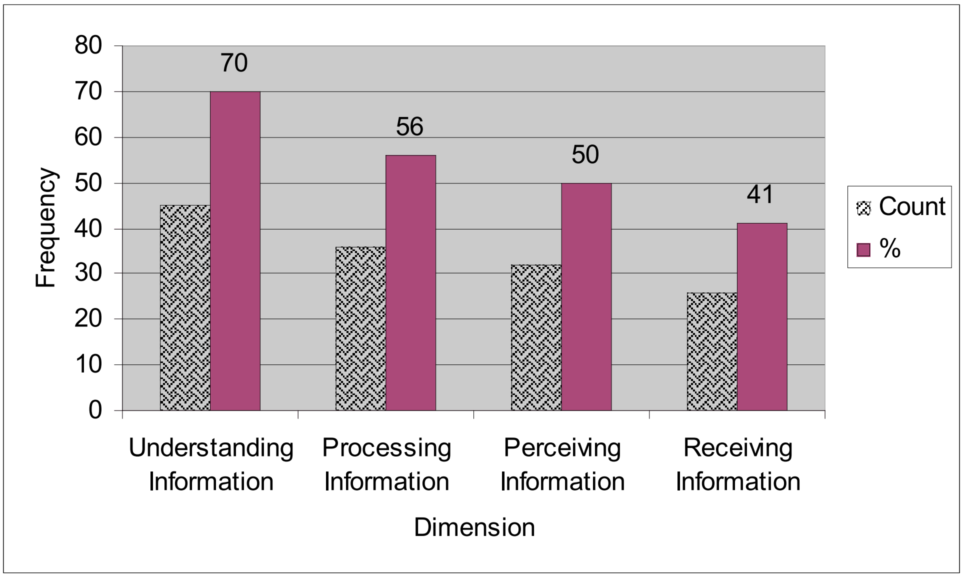
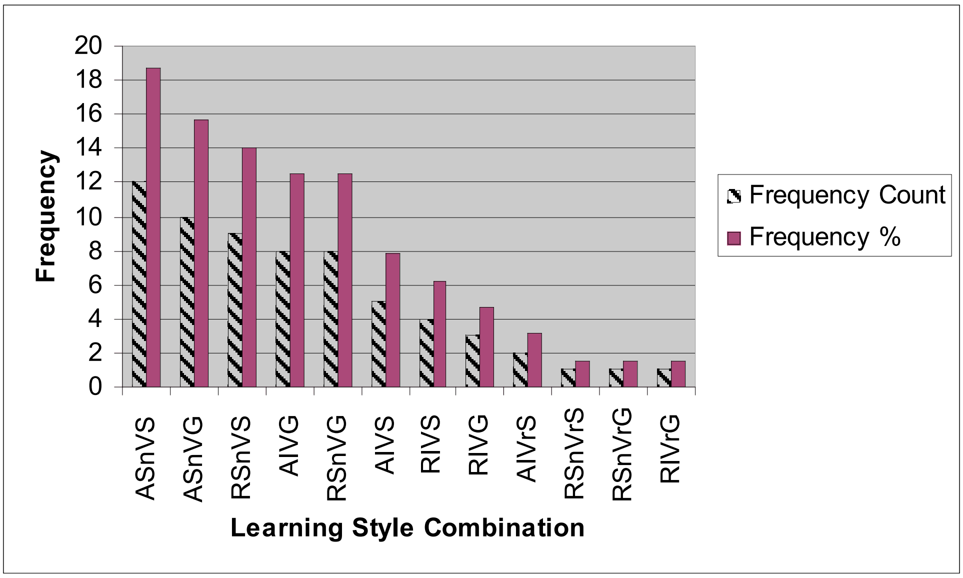


Figure 4.1 Students' bimodal learning dimension preference

(S-I) and Understanding Information (S-G) dimensions (median split 6-5 on the 0-11 scale), while showing a moderate-to-strong preference for the visual modality (median split 8-3 on the 0-11 scale); in tandem with comparison of the frequency of students' preference per differential for the different modalities (Table 4.12). The illustration presented in Figure 4.1 indicates that on the whole students' favoured a bimodal learning style with greatest balanced preference for Understanding information followed by Processing, Perceiving and Receiving information, respectively.

Preferred Learning Dimension Combinations (Types)

Since the nature of the ILS questionnaire implies that students would express a preference type in four of the eight modalities, an analysis was performed to ascertain such combinatorial preferences. When the degree of preference for a combination of singular learning modalities is considered, 16 appropriate types of combination of four learning style modalities are obtained (Table 4.11); 12 of which are shown in Figure 4.2. The ASnVS combination was the most popular, with approximately 19 percent of the 64 students expressing a certain degree of preference for it. The next four most frequent combination types were ASnVG (15.6%), RSnVS (14.1%), AIVG (12.5%) and RSnVG (12.5%), having a combined preference frequency of 47 students (73.4%). The range in the number of students (8 to 12), mean (9.4) and standard deviation (1.7) for the five highest frequency-ranked combinations indicate that most students' preferences are quite evenly distributed among these five combinations (namely ASnVS, ASnVG, RSnVS, AIVG,and RSnVG).



Note: A = Active; R = Reflective; Sn = Sensing; I = Intuitive; V = Visual; Vr = Verbal; S = Sequential ; G = Global

Figure 4.2 Students' learning type combinations of four learning modalities

As illustrated in Figure 4.2, only 12 of the 16 learning style combinations are presented since no students were identified for the four combinations of ASnVrS, ASnVrG, AIVrG and RIVrS. A striking result coming from this observation indicates that all active-sensing students who recorded either a sequential or global learning preference were more visual than verbal, while on the other hand all active-intuitive students who recorded either a sequential or global learning preference were more verbal than visual.

Compatibility of Learning Style

Figure 4.3 is a visual representation of comparative mean percentage preference scores for the three teachers and their 64 students. The chart in Figure 4.3 indicates that except for the understanding information dimension, the teachers' average learning style preference for the other three dimensions — processing, perception and reception (input) — were opposite to those of the students. The above observation was consolidated through Pearson correlation analysis which revealed that the teachers' mean percentage modality preference scores were very strongly and significantly inversely correlated to the students' mean percentage modality preference scores ($R = -0.934$, $p < 0.001$), confirming that for most learning modalities teachers and students had opposite preferences. The most striking of these pertained to the reception dimension, where the teachers indicated no preference at all for receiving information through the visual mode, compared to the students' visual preference average of 92 percent. Yet the learning experience observation data revealed that these teachers placed a lot of emphasis on getting students to learn through the visual modality.

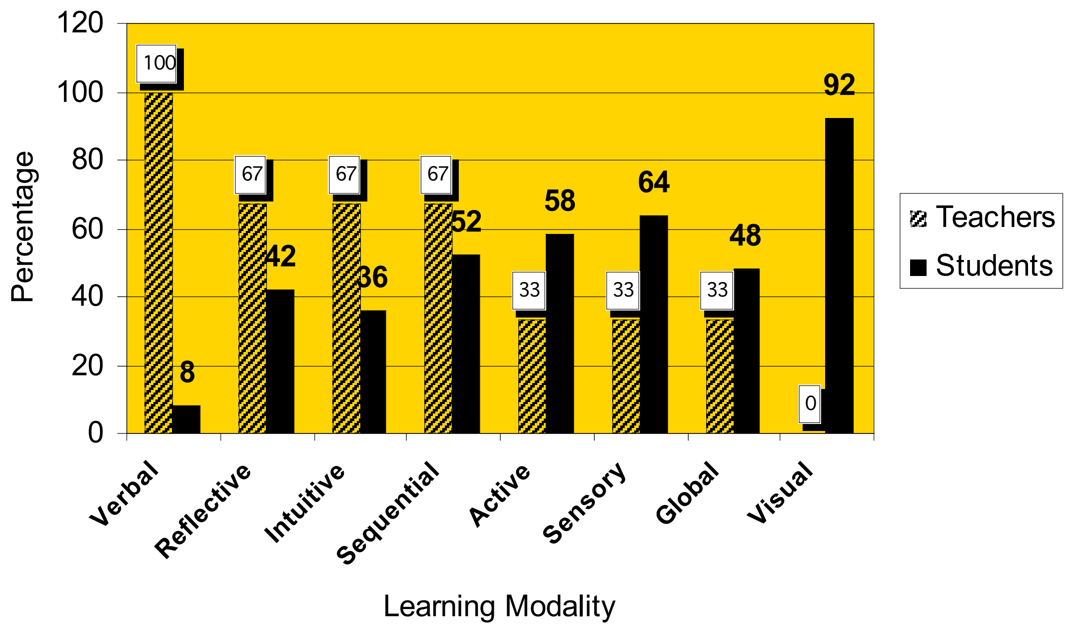


Figure 4.3 **Comparative distributions of teachers' versus students' mean learning modality preference**

4.4.5 Think aloud stimulated recall interview results

Forty-five students were selected for the Think Aloud Stimulated Recall Interview (TASRI). Table 4.14 shows the distribution of the 45 selected students according to their performance on the Number Sense Test (NST) results. It should be noted that since students were initially selected according to their number sense performance on the NST the inequality in the score ranges in Table 4.14 could not be avoided.

Table 4.14 Distribution of students selected for TASRI by NS proficiency band

	NS Proficiency Band			Number of Students		
Score Range (%)	Code	Band		TAANVSR-Selected		In Class
87 - 93	1	Top 30%	Above Average	6	13	6
76 - 84	1			7		13
66 - 74	2	Middle 40%		12	20	14
58 - 63	2		Below Average	8		12
50 - 57	3	Bottom 30%		6	12	8
42 - 47	3			5		6
29 - 39	3			1		5
		Total		45		64

Note: Mean = 64.7% [Ranked between 33rd (65.6%) and 34th (63.3%) on NS test]

The main reasons for selecting students based solely on their NST results were due to:

1. Information from the pilot study participants indicated that 92 percent of them were of the notion that students' problem solving performance depend mostly on their number sense proficiency; and
2. Previous distribution of pilot study students based on NST scores resulted in an acceptable balance of students according to their combined NS-PS proficiency level (Figure 4.4).

Figure 4.4 shows how the 45 students who participated in the TASRI were distributed as per their combined NS-PS proficiency score.

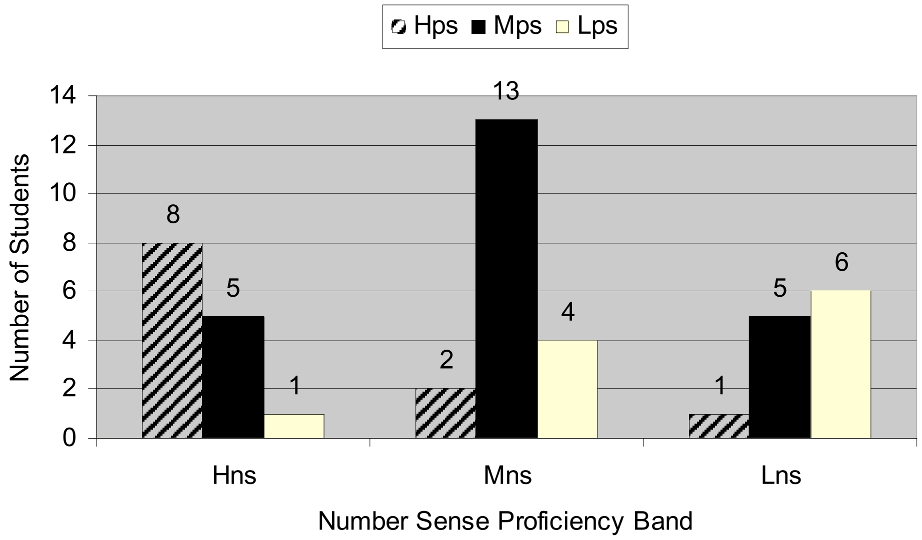


Figure 4.4 Distribution of TASRI students' by NS-PS proficiency bands

The TASRI test consisted of four problems; two NSIP and two DNSP. During the TASRI setting, the 45 students' comments were audiotaped as they thought aloud while solving a problem. They were also asked various questions, mainly after they had solved a problem, as a means of gaining information about the way they solved NSI and DNS problems, and how these related to their NS and PS performance, and learning style preference.

During the piloting stage the TASRI interview had two main phases in which: (i) nine students (Group A) were interviewed through one practice problem and two test problems, on the basis of one student per day for six days; and (ii) a new batch of nine students (Group B) plus the original nine (Group A) were all interviewed though four test problems, on the basis of six in one day. All of the first nine taped TASRI interviews conducted during the first phase of the pilot study were analysed, with each interview being replayed over and over again for identification of common themes and occurrences. In this way it was possible to note each student's verbal behaviour and from these the most common patterns which occurred repeatedly were grouped into various classes. This followed a grounded theory approach in which the first interviewee's words were transcribed and analysed and possible situations noted which might need probing. The next interviewee was then interviewed and probed according to results obtained from analysis of data from the first interviewee. This procedure was repeated from interviewee to interviewee as a means of identifying any manifestation of new probing possibilities. Before the second interviewing phase, 12 occurrences were identified as those which would be used for probing. During the second phase of the

TASRI pilot interviews the 18 students who were interviewed were probed, according to the ‘conditional probes’ previously identified, whenever they exhibited any of the situations “for which probing might uncover additional information without threatening validity” (Conrad, Blair, & Tracy, 1999, p. 12). The pilot exercise produced 51 codes, a checklist of which was used to inform the application of the TASRI during the main research itself.

During the main research another 49 codes were formulated. Hence, originally there were 100 major codes. These were finally collapsed into a more workable 14 categories. Table 4.15 gives an indication of the number of students who consistently exhibited verbal and/or non-verbal behaviour matching the final 14 codes. The first interesting observation is that most students preferred to work mentally on all four TASRI items. Such students used paper and pen mostly as a means of keeping a record of the end results of their mental processes. Hence, the conditional probes obtained from the pilot study were used to help students articulate their thoughts; they were free to write or not to write anything, although it had been explained to them, prior to the interview, that anything written down by them will be of utmost use in the analysis. Therefore, all participating 45 students wrote down something; some wrote all the algorithms used, others jotted down a few points, while some preferred to sketch as well.

Table 4.15 Number of students according to most common verbal and non-verbal behaviour patterns

	Code	Description	Count	%
1	LATP	Likes All Types of Problems	6	13
2	PNSIP	Prefers NSIP	24	53
3	ROPAE	Reliance on Prediction and Estimation	12	27
4	OMR	Ongoing Monitoring of Reasonableness	21	47
5	PRP	Perseveres Rest Perseveres	23	51
6	WMUNSA	Works Mentally and Uses Non-Standard Algorithms	30	67
7	LNSBI	Likes NSIP but most often Inconsistent	11	24
8	ROKS	Reliance On Known Strategies	7	16
9	UVRA	Uses Visual Presentation as an Aid	1	2
10	PDNSP	Prefers DNSP	15	33
11	PLPWS	Prefers Logic Problems and Works Sequentially	24	53
12	PCISAR	Partially Correct Interpretation, Solution and Result	14	31
13	PNSPSA	Poor Number Sense and Problem Solving Ability	3	7
14	ECPLOP	Easily Confused Plus Lack Of Perseverance	9	20

Note: N = 45

The codes in Table 4.15 have been organised so that comparison could be made between students who preferred NSIP and whose NS performance scores were high, and those who preferred DNSP or logic problems and also performed better on these types of items. A striking discovery pertains to the last code of Table 4.15. It was observed that six of the nine students classified as easily confused got perfect scores for TASRI item three, which was a logic problem, while two of the other three students could not understand or solve any of the four problems. All of the nine ECPLOP students formed part of those who expressed a preference for solving logic problems, although the other 21 students fared better in solving items 1, 2 and 4 as well. In Table 4.16 a more detailed view is presented of the final three codes which characterised students who:

(i) could only demonstrate partial understanding, interpretation, solution and result;

(ii) verbally expressed and showed signs of being easily confused and lack of perseverance; and

(iii) seemed to have a lot less number sense and problem solving ability than their other Year 7 colleagues.

Table 4.16 Number of students observed per detailed coded behaviour

Final Code	Component Codes	Description	Count Obs	Count Com	%
	MSE	Makes Simple Error	20	14	
	IOCDTP	Inserts Own Conceived Dimensions to the Problem	14	14	31
	PCI	Partially Correct Interpretation	15	14	
	UPbCCR	Partially Understands the Problem but Cannot Always Chart a Route	16	14	
PCISAR	PCIWWP	Partially Correct Interpretation of the Problem and Working from Wrong Premises	18	14	
	PUoT	Partial Understanding of Terms	16	14	
	UPbMI	Understands the Problem but Mixes Interpretations	15	14	
	LoP	Lack of Perseverance	12	9	
	GUVE	Give Up Very Easily	13	9	20
ECPLOP	Con	Confused	12	9	
	FPDS	Finds the Problem Difficult to Solve	14	9	
	PTQ	Proceed Too Quickly	14	9	
	WNP	Would Not Persevere	12	9	
	VPFWN	Very poor Facility With Number	3	3	7
	DWF	Difficulty With Fractions	3	3	
VPNSPSA	FVRS	Fails to Verify Reasonableness of Solution	3	3	

Note: Obs = Number of students observed per code; Com = Number of students who commonly performed according to a particular code.

Forty-three percent of students exhibiting behaviours pertaining to the three categories and their component codes (Table 4.16) stated that they neither liked nor hated solving number sense inherent problems. Another group of these students (43%) agreed that they sometimes like solving NSI problems. Very few of these students (14%) stated that they hated NSI problems, and they were among the three who did not manage to solve any of the four problems. The greatest factor causing students to either solve the problem partially or not at all was the number of simple errors made. Forty-four percent of the 45 students constantly made simple errors which prevented them from solving at least one of the four problems. Thirty-one percent of students consistently made simple errors in solving all four problems, although these did not always result in them not being able to completely solve the problem. What was interesting about this phenomenon is that if the error was detected, seemingly rectified and then yielded an incorrect or partially correct answer, most often these students seemed either too ‘irritated’ or lacked the necessary skills to notice that the result was incorrect. In contrast, students with high number sense who were in the habit of constantly monitoring their own progress had the ability not only to detect and rectify errors in NSI problems but also in DNS problems.

4.4.6 Classroom observations

Table 4.17 shows the number of students and number of lessons observed from school term one to term four of 2004. Compared to Chantal’s and Amanda’s schools, where the least number of students in a class was about 24 students, it was a policy in Bob’s school to have small classes of no more than 20 students per class. It would be shown while answering question 2 and question 4 that class size and duration of lesson played a great role in the amount, length and quality of one-on-one teacher-student interactions which lasted at least two minutes. Table 4.18 gives an indication of how many students got a chance to be involved in one-on-one work with their teacher. Very few students were given such attention on at least three occasions.

Table 4.17 Number of lessons observed

School	Teacher	Number of Students	Number of Lessons Observed
Arlenta	Amanda	24	31
Baden	Bob	14	30
Cotton	Chantal	26	30
Total		64	91

Table 4.18 Distribution of one-on-one teacher and student working together for at least two minutes

School	Proficeiency	Number of students	Number of One-on-one Interactions Once	Twice	Thrice	Total	Mean per student
Arlenta	Hns	9	26	0	0	26	3
	Mns	12	162	2	0	166	14
	Lns	3	55	11	2	83	28
Baden	Hns	7	94	5	0	104	15
	Mns	6	93	36	3	174	29
	Lns	1	13	10	3	42	42
Cotton	Hns	3	5	0	0	5	2
	Mns	8	28	0	0	28	4
	Lns	15	174	17	3	217	14
Total		64	650	81	11	845	

Note: Total = total number of individual one-on-one interactions.

In the following sections the research questions are answered through in-depth analysis and presentation of results of data pertaining to each research question. Each question will be summarised through a series of assertions. At the end of this chapter the theoretical framework presented previously will be revisited and brought up-to-date with the results and discussions presented in this chapter. This will serve as the basis for the construction of a theoretical model which reflects: (i) what was observed in the lessons; (ii) the beliefs of the teachers and their students; (iii) the relevance of the students' performance on both the number sense and problem solving pre- and post-tests; and (iv) data obtained from the Think Aloud and Stimulated Recall Interview (TASRI) pertaining to the students' behaviour and performance while solving Number Sense Inherent (NSI) and Devoid of Number Sense (DNSP) problems.

4.5 Analysis and Results of Research Question 1

What is the relationship between the number sense and problem solving abilities of Year 7 students?

To answer this question, inferences were made about the relationships between the variables elicited from both the quantitative and qualitative data. To this end a modelling process was used whereby the various variables and sources of data were drawn first from a correlation of problem solving performance and number sense performance, then further analysis through linear regression and thirdly comparing the claims advanced through data generated and analysed from the grounded theory approach mentioned earlier.

Both the Number Sense Test (NST) and Problem Solving Test (PST) were administered separately on two different occasions; at the start of term one the pre-tests were administered and the post-tests were given towards the end of term four. As explained previously in Chapter 3, a total of 64 Year 7 students from three different schools did the pre-tests and post-tests for NS and PS. Using Cohen's (1988) guidelines for interpreting the strength of a correlation, the results summarised and presented through Figure 4.5 indicate that the correlation coefficient is large, which in turn suggests that there is a strong relationship between NS and PS. Triangulation of data further revealed that this relationship might be explained by other factors, as will be further analysed and discussed in answering questions two, three and four.

4.5.1 Correlation of number sense and problem solving

An underlying aim of this research was to ascertain whether or not there was a relationship between number sense and problem solving. As observed through Table 4.7 the constant rate and direction of change in comparative NS and PS percentages, especially within the top and bottom rows of Table 4.7 was already indicative of the existence of a very strong relationship. The next step was calculation of the magnitude of this correlation as a means of confirming the direction and strength of the relationship. Figure 4.5 shows a scatter plot of the problem solving and number sense performance scores of the 64 students. Since the students' preference for solving either NSIP, DNSP or both seemed to be related to their number sense and problem solving performance the scatter diagram presented in Figure 4.5 also shows the distribution of the students' scores according to the type of problems they preferred to solve. Although there was no marked difference between the percentage of students preferring NSIP (45%) and those preferring DNSP (38%) in previous discussions it was shown that students preference for solving NSIP was more closely related to number sense performance ($R = 0.69$). Even more striking was the higher correlation of NSIP preference and problem solving scores pertaining to NSI problems ($R = 0.56$) as opposed to a very low correlation between NSIP preference and performance scores for solving DNS problems ($R = 0.29$). These results are graphically supported in the scattergram presented in Figure 4.5.

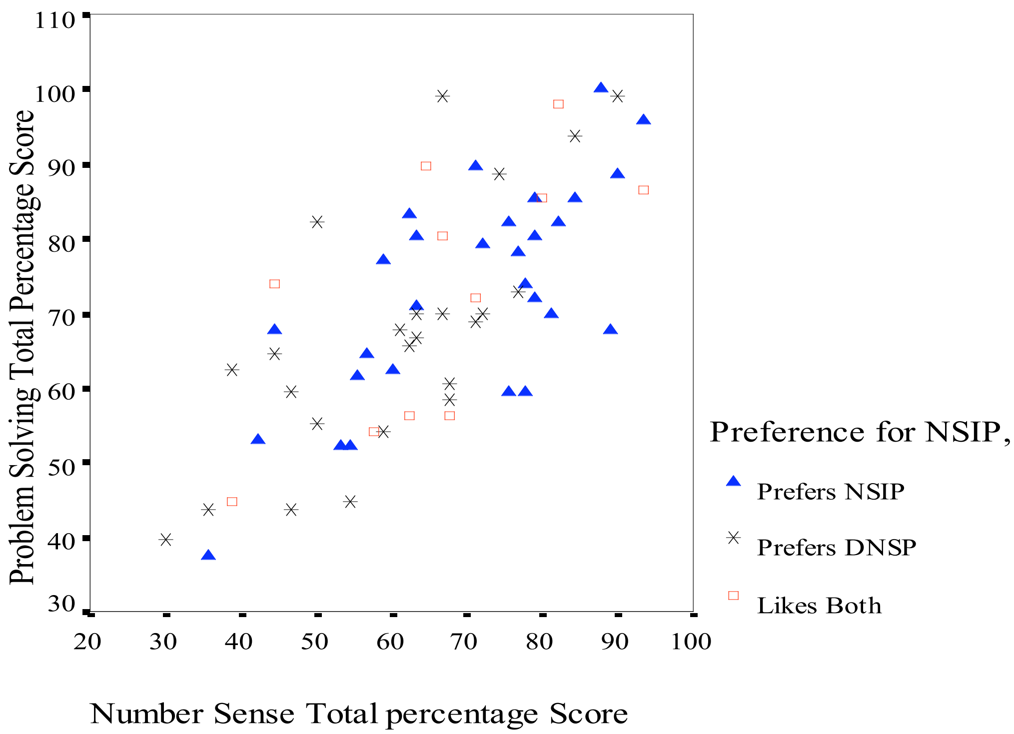


Figure 4.5 Overall correlation of number sense and problem solving scores

A two-tailed Pearson Correlation was applied to the pre- and post- PS and NS combined scores, resulting in quite a strong correlation of 0.77 at the 0.01 level. The coefficient of determination indicates about 60 percent shared variance which implies that number sense helps to explain nearly 60 percent of the variance in students' scores on the problem solving test. Although the converse could also be true, triangulation of data obtained from the various forms of data collected, especially those from the Think Aloud Stimulated Recall Interview (TASRI) protocol, show greater support for a theoretical framework in which problem solving ability level depends more on number sense than vice versa. For instance, there was a significant correlation, at the 0.05 level, between the 45 students' TASRI performance scores and their PST performance scores ($R = 0.31$, $p = 0.04$). There was also a significant correlation, at the 0.01 level, between the 45 students' TASRI scores and their NST performance scores ($R = 0.55$, $p = 0.005$).

Interestingly the correlation between these students' TASRI performance scores and their NST performance scores was not only higher than that of their TASRI scores and their PST performance scores ($R = 0.317$, $p = 0.04$), but the former was also more significant at a higher level ($p = 0.01$) than the latter ($p = 0.05$). This could explain why both teachers and students felt that the more number sense a person has the greater that person's problem solving performance.

When each of the 45 students' TASRI problem solving score is added to their respective PST score the correlation of NST and PST increases from 0.55 to 0.62 ($p = 0.000$), suggesting that assessing students both through a think-aloud and stimulated recall protocol, and written tests would produce a more realistic picture of a student's number sense and problem solving performance. It will be shown during the various discussions that although the NST and PST were designed to provide information about the relationship between NS and PS, it was only through the qualitative and semi-qualitative-quantitative data that the occurrence of problem solving performance, as a result of a student having or not having number sense, could be explored and discussed. From what has been presented so far the following assertion is appropriate.

Assertion 1

There is quite a strong correlation between the number sense and problem solving proficiency of Year 7 students. The evidence points towards a relationship in which problem solving performance depends upon number sense proficiency more than the latter depending on the former.

For instance, even Amanda, who insisted that "number sense is important, but not more important than any other mathematics content and/or processes", acknowledged that number sense is important for problem solving. The excerpt below shows that although she tried to identify both language and number sense as factors contributing towards some students' poor performance in solving mathematics problems, she ended up indicating that students who have number sense have an advantage when it comes to solving mathematics problems. The following was recorded in an informal interview prior to the first observation.

Excerpt 1

R:	In your opinion, why is it that some students find it hard to solve mathematics problems?
Amanda:	If you remove the aspect of number and words, then children who do not have good number sense could perform very well in problem solving.
R:	Do you think that these are the only factors which affect a student's problem solving ability?
Amanda:	Obviously no…I would say that there are other factors, but the language and number aspects are the main ones.
R:	So, it seems that the number sense of a student plays a role in her ability to problem solve.
Amanda:	Yes, to a certain extent, but some children who do not have, good facility with numbers can still solve problems if they understand the language and there are no numbers

After the first observation the same teacher was asked several questions to ascertain her belief about the impact of number sense and students' problem solving ability upon each other; this is treated in more detail when answering research questions

2 and 4. It was clear by then that although it seemed at first that all she wanted to do was to downplay the importance of number sense, she was in fact trying to point out that there are exceptions to the rule. The other two teachers, Bob and Chantal, felt very strongly about the important role that having good number sense plays in helping students solve mathematics problems, as indicated through Excerpts 2 and 3.

Excerpt 2

Bob:	There are many factors such as the nature of the problem itself, and…and, issues of language comprehension. But I believe that lack of number sense is a big factor…the major culprit.
R:	Why do you think so?
Bob:	In my experience, most problems in mathematics will contain some number…aspects of…yes, some numerical aspects. So, anyone who does not have number sense will have diff…a great deal of difficulty. They will find it difficult, maybe very challenging, to solve mathematics problems.

Excerpt 3

Chantal:	Well, there is always going to be some problems which will be solvable, to a large extent…by most students. But there are also those problems which require that students have a good grounding not only in their tables and number facts, but…as you will notice later on,…, some have a lot of problems when it comes to understanding what the numbers stand for, their relationships, er, erm…the underlying principle.
R:	I am thinking of someone who said that ‘students need number sense to solve problems just as much as they need good problem solving skills to develop good number sense’. What do you think of this person’s belief?
Chantal:	As much as we are all entitled to our own opinion, I think that this is a sort of circular reasoning.
R:	What do you mean by circular reasoning?
Chantal:	Anyone who has been teaching mathematics successfully…I insist on this successfully, effectively or you can say efficiently…Yes, anyone who has worked with kids, who has had a lot of experience teaching them would know…that those children who have a good understanding, very good grounding in their number concepts, skills and understanding of how they function in relation to each other…they will know that such students have a better chance…a far greater chance of being good problem solvers than others…especially those who do not have that good facility I would say with numbers.

From Excerpts 1, 2 and 3 emanated a common theme in the belief of all three teachers that students with good number sense have a greater chance of becoming good mathematics problem solvers. The students’ perceptions presented later on tended to echo their teachers’ beliefs that if one of PS or NS was responsible for progress in the other, then it must be number sense which would be needed for one to solve mathematics problems.

4.5.2 Problem solving: the vehicle driven by number sense

Due to the ethnographic-grounded theory-framework approach data collection methodology used, there were times when teachers were asked a few questions immediately after they had been observed teaching. On one such occasion during term 1, each teacher was shown the diagram in Figure 4.6 and asked to answer the question appearing above the drawing.

According to your perception, which of these eight diagrams is showing the relationship between problem solving (PS) and number sense (NS)? Why?

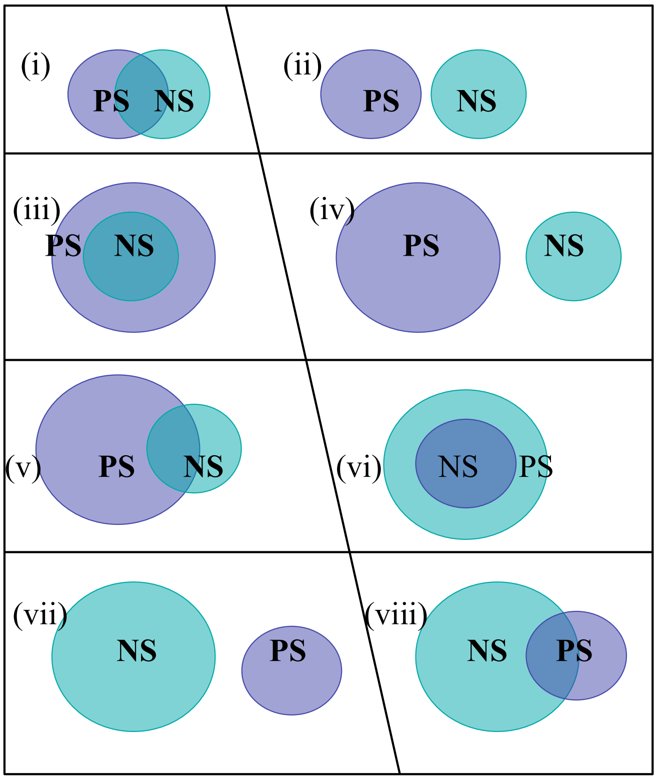


Figure 4.6 Possible relationships between NS and PS

All three teachers started off by selecting option (i), but when it came to answering ‘why’, they discarded this option and went for option (iii). When asked what this relationship implies about problem solving and number sense there was hesitation on each occasion and the following comments, presented through Excerpts 4, 5 and 6, were recorded.

Excerpt 4

Bob: It is true that number sense would be smaller than problem solving, but...yeah, I see a problem with this. I think what is bothering me is that not all number sense is problem solving. I’d prefer number five because it shows problem solving being greater than number sense and not all number sense included as problem solving.

Excerpt 5

Chantal:	Problem solving covers all mathematics content. I don't think that it covers all of number sense. Some number sense questions are not problems at all. I mean if a problem is unique, I…I mean for any question to be a problem for a child…it has to be new to the child. But sometimes there are number sense questions which do not require the children to solve… to find a solution like for a new problem. What is confusing me is the fact that, mmm, ur… I was thinking of number sense as being at the centre of problem solving, yeah, but, this cannot be the case. I'll have to settle for five, although I still believe that the number sense circle needs to be larger.

Excerpt 6

Amanda:	Number sense is not necessarily central to problem solving. No. The other components of mathematics are also very important and each of them could also claim to be part of the centre of problem solving…but maybe to a lesser degree…less than number sense. I'd say that sometimes we deal with number sense which does not require a lot of thinking. Problem solving is more like having to solve a problem by going through some stages, and, and, normally…nor…usually, yes number sense questions are like problems, but I think there are a few which do not require problem solving. I am not sure about this, but I think the fifth diagram or the…No, yes I will definitely choose the fifth diagram over this one [Referring to the third option].

When interrogated further through the question "If you were to justify whether number sense is necessary for problem solving, or problem solving is necessary for number sense, what would you say?", these teachers tended to focus more on problem solving as a "vehicle" driven by number sense (Excerpts 7, 8 and 9).

Excerpt 7

Amanda:	Number sense is very important, although I think that too much emphasis on developing number sense could result in neglecting other areas of mathematics. But if the teacher wants students to become successful problem solvers there is no way out of it, except to help them develop number sense.
R:	Why?
Amanda:	Most problems are unfortunately loaded with numbers. Whether we like it or not is irrelevant here. The reality is that those who have good number sense always have an advantage [That is, when it comes to solving problems]. Because to a certain degree…without number sense most mathematics problems would not be solved.

Excerpt 8

Bob:	I would say that problem solving relies heavily upon good number sense. Number sense provides…it equips the boys to…develop more profound, more in-depth problem solving awareness and skills. When you look at it…yes, very closely, number sense and problem solving are nearly the same, except that…in the case of the former, number sense, there will always be numbers involved. Whereas if you think…take any problem..er..and you'll find that it might have numbers or it might not. So, in that case number sense is sort of…like the vehicle which carries problem solving.

Excerpt 9

Chantal: In a situation like this there cannot be two answers, for I have just made this point by selecting drawing five…there is a relationship between the two and I'll say that…this…number sense pushes the children's problem solving. Obviously problem solving relies a lot more on number sense…most of the time I'd say…It relies a lot more on number sense than number [sense] relying on problem solving.

Further discussions with the three teachers and students clearly pointed towards this strong belief that lack of number sense could hinder development in problem solving ability. Sometimes after a teaching session the researcher would ask students a question, which was written on the board or printed on a slip of paper, and students would provide a written answer to the question. Table 4.19 presents the results obtained on one such occasion when students were asked to "state, in order of most responsible, three issues which could be responsible for poor performance in solving mathematics problems".

Table 4.19 Main factors identified by students as responsible for poor problem solving performance (N = 64)

Factor	Summary of students' most common answer	Count	Percentage
Lack of number sense	Lack of understanding of number facts and how to apply them	45	70
Lack of language proficiency	Not understanding the language; not being able to read properly	36	56
Mathematics anxiety	Afraid to solve any mathematics problems; lack of confidence	31	48

When pressed about why most of them thought that *lack of understanding of number facts and how to apply them* was the main reason which causes students to fail in solving mathematics problems, these students thought that this occurred mainly because:

- in most problems you have to be able to interpret the numerical aspect;
- if one fails to understand what operation is required and how to perform this operation then the solution is most often wrong;
- when one does not know one's tables and the fundamental or basic number facts it would be very difficult to solve most problems;
- if one cannot link the different number aspects to each other it would be very difficult to estimate the answer or be confident about how to solve it [the problem];

- failure to know whether the answer is in the ball park [reasonableness] could make it very hard to judge the accuracy of the numerical answer; and
- one who finds it hard to work with numbers would necessarily find it very hard to work mathematically most of the time.

A closer look at the students' suggestions revealed that, although they were not always using the term 'number sense', they were talking about some of its key aspects, such as:

- interpret the numerical aspect
- understanding of number operations
- mastering the fundamentals and basic number facts
- making connections
- estimating
- reasonableness of the answer
- working mathematically

The last point extrapolated from the discussion with the students seemed to be summarised quite succinctly through Bob's statement that "without number sense students would find it hard to work mathematically"; and according to Amanda, "it is extremely difficult to work mathematically if one has poor number sense…because this will make it even more difficult to solve most problems". Yet, as Chantal pointed out, the challenge to overcome this obstacle "is a very big one, given that mathematics is not only about number sense, but also about other concepts and mathematics sense". This notion of "making sense of the mathematics" was explained by Chantal as being "more prominent in making sense of number as it permeates all other strands of the mathematics curriculum". Such a notion was quite widespread in both practice — through the learning experiences observed — and theory, as expressed by Amanda: "since most problems require number sense, students with such ability have a great advantage over those with poor or no number sense, when it comes to successfully solving a problem".

Assertion 2
All three teachers and the majority (70%) of students believe that lack of number sense is a probable major cause of poor performance in solving mathematics problems.

4.5.3 Factors linking ns and ps

Number Sense Performance is Closely Tied to that of Problem Solving

Table 4.20 summarises the problem solving and number sense combined pre-test and post-test scores of all 64 students who participated in the research, indicating a mean of 70.4 percent for PS and a mean of 65.5 percent for NS. The main feature of the results from this table which is of interest to this research has to do with the degree of associative and comparative closeness between NS and PS statistics. It has already been shown that since the correlation between NS and PS is positive and strong, it is expected that most students with high number sense would also be in the high problem solving category.

Table 4.20 Summary of NS and PS Percent Scores

N = 64	Test	
Statistics	Problem Solving	Number Sense
Mean	68 (70%)	59 (66%)
SD	15	14
Max	96 (100%)	84 (93%)
Min	36 (38%)	27 (30%)

Note: Basic score statistic is shown outside parentheses.

Table 4.20 shows that on average students' performance scores were quite close for both tests. This is also true for both the standard deviation and the range. Hence, this made comparison of individual students NS and PS scores easier to analyse with respect to their ability in these two domains. Nevertheless, there were some important patterns in some students' performances in both the NST and PST, which made it even more important to use the qualitative data as a means of explaining some of the phenomena which were not that precise in the quantitative data. An important pattern emerging from the quantitative data — observation and analysis of students' written solutions — indicated that students with high number sense worked in a totally different way compared to those with lower number sense. Hence, analysis of data was directed towards discovering some of the most prominent factors as expressed by the teachers and students, and supported by data collected through other methods.

An area where the teachers felt that both number sense and problem solving are related was in the domain of assessment. What follows are examples of situations which highlight the importance of assessment as a role player in a theoretical framework where NS is seen as being as important as PS itself. It was thought that instead of focussing on all students, it would be best to highlight one or two students who have exhibited some typical examples of particular behaviour or performance.

Relationship between NS and PS through Assessment

In 40 percent of the interviews with the teachers they mentioned assessment as a major factor impacting upon the relationship between NS and PS. Throughout the data collection period all three teachers always assessed number sense through some problems. Moreover, all three teachers tended to teach mathematics mainly through problem solving. Hence, it was not surprising to learn that Chantal's comment "…, number sense should be, or maybe I should say must be assessed through problem solving, since it [number sense] involves mainly how students make sense of the number components of a problem", was a view shared by the other two teachers as well. Bob's view, that "assessing for number sense through a problem based method helps me not only to gauge the student's content knowledge, but also his thinking process and solution" was a prevalent one among all three teachers.

Assertion 3

Number sense and problem solving are related through sharing the same assessment strategies and tools.

Assertion 4

Number sense and problem solving are linked through assessment which incorporates consideration of both the thinking process and the final solution by a student.

Since all three teachers assessed most mathematics work as they would for problem solving, it was deemed appropriate to observe how they took into consideration the student's thinking process in the design and implementation of the marking rubrics. It was discovered that they preferred using the process scoring system instead of the basic scoring system. Chantal remarked that "one advantage of giving marks for students' strategies and other aspects is that you learn more about the student and your marking is more representative of the child's ability". In the case of Bob he felt that "unless the teacher attempts to understand the steps, strategies and thinking the student uses to solve the problem, … then it is not fair to judge his [student's] performance in mathematics". To Amanda, "assessment is an area which highlights how the two [number sense and problem solving] are closely related, and that's why it is important not only to assess them regularly, but also to use a method which will give a true picture of the child's ability". In using both the process and basic scoring system for the PST, it was possible to judge which system would yield scores which would be most representative of the students' performance. The process scoring system allowed the researcher to make better judgements about the students' "real ability", as Bob would call it, than just a mark based mainly on getting the correct answer. In responding to the question, "Which students are most at risk of being given an unfair mark if assessed only through the basic scoring system?", Amanda stated that it would be "those girls who are less self confident, especially those who have difficulty in solving number problems". In Chantal's thinking "those students who have tried very hard but failed to get the correct answer are definitely at a disadvantage if you use a basic scoring system to assess mainly their answer". The process scoring system was used and seen by all three teachers — as explained by Amanda — as one way of ensuring that "the marking was not only as fair as possible but also closer to reality", since as Chantal stated "it provides the teacher with more reliable and valid information about how much sense students are making of what they are learning". Bob reiterated this point when he stated that:

> Number Sense is very much like problem solving in the sense that you have to read the problem, try to understand it, plan a way to solve it and come up with a reasonably accurate answer. All these performance components must be assessed in both number sense and problem solving if I am to encourage the students to love working with numbers, and to make sense of what they have learnt.

Although most number sense items were multiple choice-formatted, which therefore required only a simple basic scoring system, it is interesting to note that the process score statistics, for the PST, are closer to those of the number sense ones than are the respective PST basic score statistics. In exploring the results obtained from the Think Aloud Stimulated Recall Interview (TASRI) students expressed their feelings and beliefs vis-à-vis either Number Sense Inherent Problem (NSIP) or Devoid of Number Sense Problem (DNSP) or both. The TASRI results highlight the importance of assessing what Bob called "students' real number sense" and problem solving ability through student-interview. For instance, taking a student whose written Number Sense (NS) and Problem Solving (PS) performances, assessed through the basic scoring system, were 46.7 percent and 59.4 percent respectively, it can be seen that these scores are well below the respective means presented previously in Table 4.3 and Table 4.20.

Some students, found it difficult even to start working on some number sense inherent problems. A typical example was $S_{(3,51,2)}$ who when she suddenly gave up on starting to find a solution to item 1 of the TASRI problem, simply said "I don't get this question!", and when asked why she explained that "there are quite a lot of numbers involved", and that it might take her quite a while to figure out what to do. After reading the problem twice, she spent only 30 seconds fidgeting with her pencil, seemingly thinking about how to start and then she said "I think I'll go to another problem and come back to this one later". As can be observed in Table 4.21, her basic score results, which was typical of students in that category, suggested that she was more proficient at solving problems which are devoid of number sense. Nevertheless, the basic score does miss out on certain important aspects of her work. Hence comparison is made with how she fared through the process scoring system

It is worth noting that during the TASRI and through the process marking students' strengths and weaknesses in both NS and PS were more apparent as they went through the process of solving the problem. The analysis revealed that there was a remarkable difference in performance between students who scored high and those who scored low on the number sense test (NST). The analysis which follows will first look at a typical example of a low number sense performer, then that of a typical high number sense performer.

As the analysis unfolded it became apparent that the response given by the low number sense students were quite similar. The results and response of student $S_{(3,51,2)}$ are presented in Table 4.21, because it was the most representative of such students; and the discussion which follows summarises the attitudes of students who solved both DNSP during the TASRI while they found it either difficult or preferred not to solve any NSIP.

Table 4.21 Student [$S_{(2,51,3)}$]'s summary of PS and NS scores

		Number of Items per Test	Pre-test (%)	Post-test (%)	Total (%)
Problem solving	NSIP	4	50 (58)	13 (42)	31 (50)
	DNSP	4	25 (38)	100 (100)	63 (69)
	Overall	8	37 (48)	56 (71)	47 (62)
Number Sense		45	40.0	53.3	46.7

Note: Number of Items per Test = Maximum Possible Score per Test. The process scores are shown in brackets on the right hand side of the respective basic score.

Through the basic scoring system it was revealed that for the problem solving pre-test she seemed to have solved two NSIP and only one DNSP. Her basic post-test results indicated that she successfully solved three more DNSP items while she could not completely solve any NSIP, scoring 0.5, compared to two in the pre-test. Hence, while she improved in solving DNSP items, scoring 100 percent for the post-test, she also retrogressed in her NSIP performance. When pressed further as to which type of problems she preferred to solve, her answer was a straightforward: "Those problems…like logic problems, drawings…like the third and last questions [referring to items 3 and 4 of the TASRI]". The 'preference' factor will be discussed later for research question 1 and also for question 3. To ascertain how comfortable she was when it came to appreciating number sense inherent problems, a short discussion ensued and Excerpt 10 highlights her discomfort vis-à-vis NSIP.

Excerpt 10

R:	What about items 1 and 2? Do you like them?
$S_{(3,51,2)}$:	No, not really. There's too many calculations to do.
R:	Are you bothered by the calculations only, or is there something else about these problems which bother you?
$S_{(3,51,2)}$:	I think there's a lot of numbers in it. I like doing problems where there's more drawings or tables, instead of numbers.

Obviously if assessed only through pen and paper and the basic scoring system such students would not succeed since, as it will be shown with regard to the relationship between error and success, analysis of the TASRI data indicated that in about 55 percent of cases where students failed to get the correct answer they had shown an understanding of the problem and appropriate planning of solutions. Such students usually seemed to lack sufficient concentration to keep track of what they were doing and monitor errors which crept into their work. Moreover, there is also the interference of the student's preference which could play a major role in motivating the student to solve certain types of problems. The results presented in subsequent sections will address the relationship between NS and PS through a learner's preference for solving either NSI problems or DNS problems or both.

Assertion 5

In 55 percent of cases where students were not able to get the correct answer, they did manage to show an understanding of the problem and also charted a route which could have helped them solve the problem and come to at least a partially meaningful conclusion.

Relationship between Error and Success in Solving a Problem

A propensity for error detection, identification and rectification was one factor which seemed to have contributed greatly in helping some students reach an appropriate conclusion, while preventing others from succeeding in their attempt to solve certain problems. This propensity seemed to be related to a student's number sense proficiency level. Good number sense being related to error rectification leading to appropriate solution was particularly evident in approximately 78 percent of the 180 TASRI problem solving attempts observed. Error detection, identification and rectification was discovered to be a very important distinction between high and low problem solvers based on their number sense proficiency.

Assertion 6

The higher a student's number sense the more disposed was that student towards detecting, identifying and rectifying errors in the solution process.

In 90 percent of cases students who preferred solving DNSPs did most of the work in their heads. Hence, they also did a lot of mental problem solving when confronted with a problem which did not require them to have number sense. However, only five percent of students with high DNSP proficiency attempted to solve NSIP mentally, compared to 95 percent of students who had good number sense and/or those who preferred to solve number sense inherent problems. One main difference between students who showed a preference for NSIP and those who favoured DNSP is that 80 percent of the former had cultivated a habit whereby they always checked the reasonableness and accuracy of their solutions whereas only 10 percent of the latter did so. This occurred whether students were solving DNSP or NSIP. Such a habit could be attributed to ‘checking the reasonableness of one’s answer’ being one aspect of having number sense; it is an inherent factor.

During classroom observations prior to the TASR interviews the researcher talked to students about their work and noted their reactions. Students were at that time identified by their NS and PS pre-test results, since the post tests had not been administered yet. One of the most striking discoveries was made about students’ perseverance, error detection, identification and rectification. When a student seemed to be on the wrong track, the researcher paid very close attention to how they proceeded, and it so happened that there seemed to be a pattern in the way different students reacted to errors committed along the way. Most students with high number sense seemed to frequently check the reasonableness of any partial results as they attempted to solve the problem. Hence, it was quite frequent to hear them talking to themselves and making comments such as “I am not sure that this is correct”. Compared to only 60 percent of the 19 low problem solving students who attempted to check for possible errors, 84 percent (16) of those with high number sense would usually revisit the problem and try to discover the error, regardless of whether the problem was number sense inherent or not. Students with low number sense performance scores found it difficult to concentrate upon finding the error. Usually they lacked the perseverance to keep going compared to those students with high number sense. This was consistent with what had been discovered from the pilot study.

Since it was observed that the appropriateness and accuracy of the solution to a problem depended greatly upon errors committed, their detection and rectification, the error patterns of students were studied. During the TASR interviews, as students attempted to solve a problem, their solution paths were closely monitored for how they dealt with errors. Patterns emanating from their words (thoughts), written work (algorithms and notes) and drawings were noted and later mapped sequentially so that an individual flow chart was obtained for each student. Common patterns were then grouped together until the number of different categories was exhausted. In cases where an individual student made more than one error, these were still counted as a single error, since failure to rectify any one of them still led to either an inappropriate conclusion or none at all (Student eventually gives up). The TASR interviews revealed different categories of students who inserted an error or errors into their working solution. Some surprising discoveries pertained to students who solved the problem although they failed to identify the error. This occurred only with two high number sense students who although not able to rectify the error allowed their “gut feeling” or intuition to guide them. This happened mainly when solving the first problem, where they started off with an estimate of Henry’s age and those of his daughters, and by using algebraic reasoning. Both of them correctly wrote an equation relating Henry McPenny’s age and those of his three daughters in the form $x + (x + 1) + (x + 2) = {}^{1}/_{5}y$, but then made an unrectified error along the way. For instance, the first student made a calculation error where he got $3x + 4 = {}^{1}/_{5}y$, and although he felt that the value of 5.5 years that he got for x was wrong, he failed to identify where he had gone wrong. Nevertheless, he still considered 102.5 years to be “too far from my estimate of Henry’s age and also it does not fit well with this one fifth and two third thing”, and he correctly estimated Henry’s age to be 30 years old. It should be noted that both students come from Bob’s class, and that they had just recently been introduced to working more with algebraic equations. Another important discovery is that a student who solves a problem might still make the wrong conclusion based on various factors such as: unawareness that the problem has been solved; wrong interpretation of another aspect of the problem; or failure to reject an incorrect hypothesis or prediction. Data pertaining to error analysis is presented in Table 4.22, which gives an idea of the different routes taken by various students towards obtaining or failing to obtain a correct answer as a result of whether errors could be detected or not.

Table 4.22 Common error pattern categories in solving TASRI problems (N = 45)

Error Pattern Category	Frequency
1. Does not feel that anything is wrong. Does not identify the error. Hence, does not rectify it. Got incorrect solution.	41
2. Feels that there is something wrong, but does not identify any error. Cannot rectify it. Got incorrect final solution.	30
3. Feels that something is wrong. Identifies the error but could not rectify it. Got incorrect final solution.	9
4. Feels that there is something wrong, but does not identify any error. Cannot rectify it, but still gets correct final solution.	2
5. Feels' that there is something wrong but does not identify the error. Manages to rectify it through starting all over again or through some other means. Only 3 got correct final solution. The other 10 solved the problem but could not conclude.	13
6. Does not feel that anything is wrong. Discovers the error while going over the whole work. Rectifies error and gets correct final solution.	20
7. Does not feel that anything is wrong. Makes no error and gets correct final solution.	22
8. Feels something is wrong. Identifies the error and rectifies it and gets correct final solution.	43
Total Number of Cases	180

Note: Frequency = Number of problem solving cases

The flow chart presented in Figure 4.7 is an attempt to trace the pattern of error detection, identification and rectification as discovered from the TASR interviews. It emerged that about 85 percent of students who were NSIP oriented tended to identify any error made regardless of whether they eventually managed to solve the problem or not. Whereas only 25 percent of those students who were DNSP oriented managed to identify any errors committed.

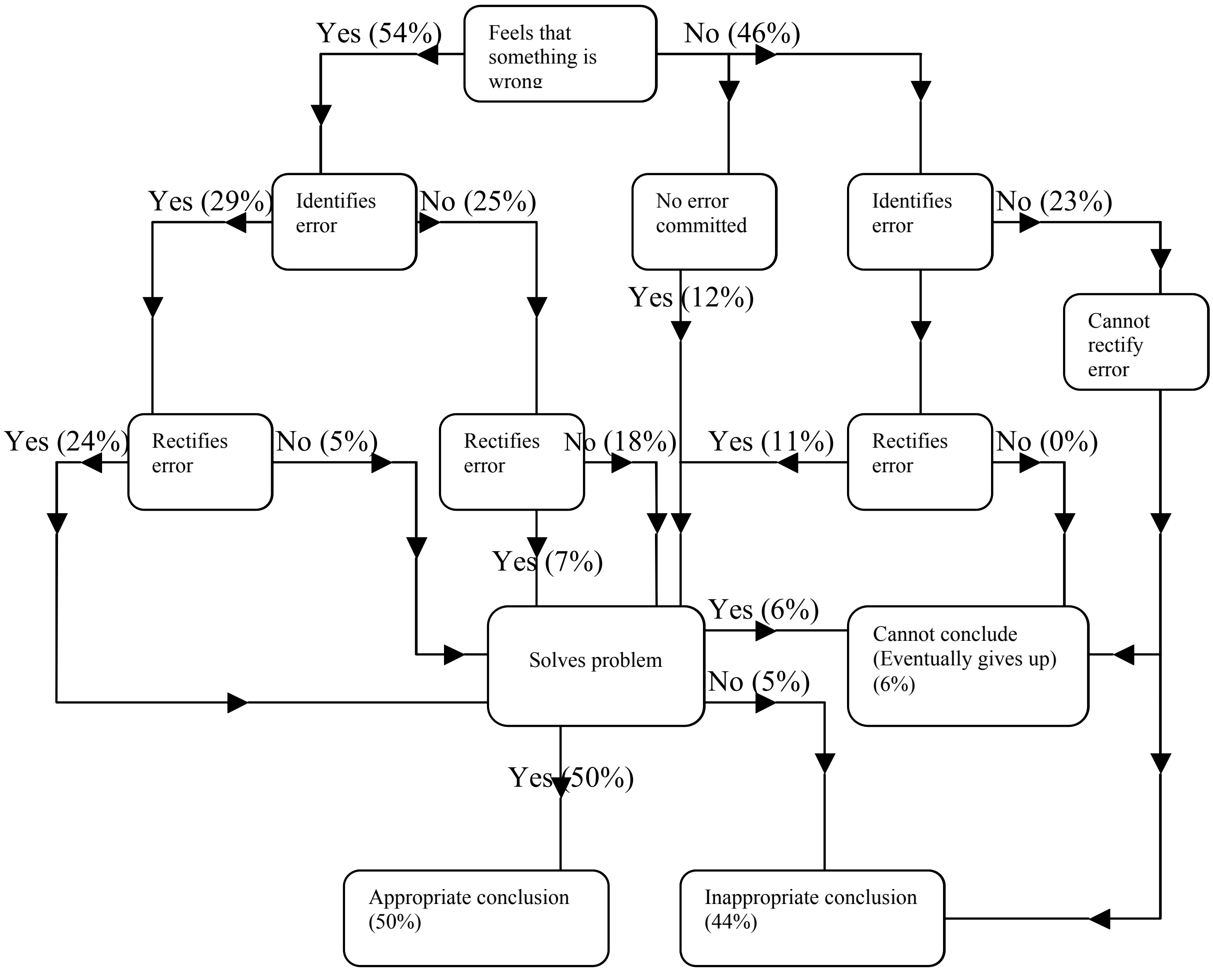


Figure 4.7 Flow chart of how students analysed errors when problem solving during the TASRI

In regard to error patterns impacting upon the relationship between number sense and problem solving, an important discovery was that 55 percent of students with below average number sense performance, who expressed high preference for being NSIP-oriented tended to identify and solve a different problem when it came to DNS problems; they tended to force a greater numerical dimension into the problem, than what the problem was actually requiring of them. For instance such students would think that TASRI problem number 4 was asking for a numerical area, and they found it hard to notice that the problem required a geometrical drawing solution.

Assertion 7

Preference for NSIP among students with below average number sense performance does not always necessarily indicate high success rate for solving DNS problems or in some cases even NSI problems.

Solving Number Problems helps Solving other Problems

It became obvious at the time of the pre-testing that many students (44%; mainly those in the low number sense category) were intent on trying to solve problems through the use of specific algorithms. Hence, after the pre-test, one of the main targets of the observation was 'the method or algorithm that these teachers encouraged their students to use'. What came out of this showed that application of certain number sense principles, such as free use of personal non-standard algorithms, were seen as a stepping stone or even a catalyst towards enhancing the students' problem solving ability. Throughout the observation each of these three teachers tended to discourage students from using the standard written algorithm unless they understood what they were doing and could fully explain the process. All three teachers confirmed that getting the correct answer through the standard written algorithm might be misleading, as pointed out by Bob who stated, "it is easy for many students to arrive to a particular answer by using a certain working and setting out [algorithm] of the steps without much understanding; such students find it difficult to make sense of their answers". When asked, "how successful are students who can fully work through an algorithm, at solving a given number sense inherent problem?", Chantal strongly expressed her view that many of them do get an answer, "but this does not mean that the child understands what the problem is about or whether or not the answer is correct; I've discovered that children who are able to follow an algorithm do not necessarily have good number sense", although she did confirm that "once they know and understand how it works and why it works, they find it easier to use, and usually it might help them solve a problem. Amanda's statement that "getting the girls [students] to use their own methods of calculation also helps them become more at ease,… a lot more adept at solving all sorts of problems…not only mathematics ones but many other problems". This summarises the other two teachers' sentiments about this issue. Excerpt 13 indicates that Bob went a bit further in supporting such a view in his analysis of why he employs a lot of numerical mathematics [number sense] in his teaching.

Excerpt 13

Mathematics is full of number. The numerical aspects provide a lot of opportunities for students to develop a sort of resilience, patience….and also perseverance in solving problems. I believe that my children can and they do,..Yes, to a large extent transfer their skills and knowledge of solving number problems…in many contexts…to the solving of other problems that they encounter in mathematics. With the current batch of students we are just getting to know each other. But I presume that by the second term they will be in a position to use what they've learnt about solving number problems, to…in solving other maths problems

By the time that students did the post-tests many of those who tried, at the pre-testing phase, to use specific written algorithms to work out a solution to a particular problem, were now showing a more deliberate attempt to employ a more free-style method. In fact 15 of the 20 students who used mostly the standard written algorithm at the pre-testing stage were now using their own algorithms or a combination of standard and non-standard algorithms. In this latter method students tend to follow a line of thought instead of a specific standard algorithm. In many cases (about 78% of the 45 TASR-interviewed) students' work tended to display a detachment from the conventional way of aligning the equal sign and conserving the balance of the equation through correctly recording the transposition of terms.

The researcher sometimes got the opportunity to informally interview a few students after an activity or lesson observation. Such interviews usually revolved around only one main question and any sub-questions which would stem from that. On one such occasion, discussion with 10 randomly selected students from each school, after administration of the problem solving post-test, revealed that the strategies they wrote down in the *Strategies Used* section was not necessarily the only strategy used, but rather the one they thought they were focusing most upon. Excerpt 14 highlight this common theme through a typical example presented in an informal chat with Sonia [$S_{(2,8,1)}$]. In stating which strategy she used to solve the second item on the pre-test and the post-test, she wrote 'Draw diagram' and 'guess and check' respectively.

Excerpt 14

R:	Have a look at how you worked out a solution to problem number 2 both for the pre-test and the post-test; is there a difference in the way you worked towards a solution.
Sonia:	Yes, I did try to calculate the answer here [Pointing to her working for the pre-test]. Yeah, I worked from top to bottom in this one [pre-test], and for this one [Pointing to the post-test] I worked all over the place.
R:	Why is it that you worked out your answers in those neat straight columns in the pre-test?
Sonia:	That's how we had been taught to work out [the answer].
R:	You mean, that's how Mrs [Referring to Amanda] taught you to do your working.
Sonia:	No, she didn't; this was before Year 6.
R:	Were you confident of getting the correct answer when you used this method of working?
Sonia:	Not really. Sometimes yes, but…there was no other way [of doing the calculation].
R:	Do you think that you worked in another way here [Pointing to her working for the post-test]?
Sonia:	Definitely. Yep, erm, I'm working more in…from my head and I'm jotting down stuff all over the place…as I get an idea I write it down and sometimes use the calculation [Standard written algorithm] to check my answer.
R:	Did you use the same strategy as you did the first time?
Sonia:	No [Hesitates and thinks silently for a few seconds]. Yes, no. In both times I used a diagram, but the second time… Yeah I,…I used another strategy this time.
R:	What strategy did you use to solve this problem the second time?
Sonia:	Like I've said here [Pointing to 'guess and check' that she had written on the post-test paper], I used guess and check.

Relationship of NS and PS and Language

Although some reports of research findings have tended to support the notion that students at the lower end of the problem solving performance scale fail to perform at a high problem solving level because they do not understand the language, analysis of the TASRI interviews reveal some other underlying constraint; it could be that such students are not able to tease out or identify the key elements and any other cues which might help them in knowing what to focus upon. This view is strongly supported by two of the three teachers and a bit reluctantly by one of them, as elucidated below.

Of the three teachers only one (Amanda), suggested that language "is…definitely a hindering factor". Chantal pointed out that "it's more to do with mathematical terms…the language of mathematics rather than the English language itself, although being good at English does play a role". This view was supported by Bob who claimed that "certain specialised mathematical words tend to create a challenge for students who are not mathematically oriented". When asked whether students with poor language proficiency level could be very good in mathematics, Bob replied that:

> Many of the students I have taught over the years who were good at maths were not necessarily very good at language. In fact some of them hated language lessons and assignments. I still have students who are like that in this year's group. Take for example Arnold [$S_{(1,34,2)}$], Joseph [$S_{(1,31,2)}$], George [$S_{(1,34,2)}$]. These students have problems when it comes to language assignments such as reading, comprehension and so on. Yet they are three of the best in class, where mathematics is concerned.

The three teachers were asked to rate their students' English proficiency level as per those who were below average, average and above average. When this rating was compared to student performance on the number sense and problem solving tests the correlation was not consistent. For instance, 25 percent of the students who scored high on both number sense and problem solving were rated as having a low language proficiency level by their teachers, compared to many high language performance students who scored low on both the number sense and problem solving tests. Ninety percent of the students, with low problem solving performance scores who were interviewed through the TASRI, failed to note the key points, although they could read the problem perfectly well. When probed further, these students showed that they understood what the problem was asking them to do. What was observed was that they do certain things which caused them to go off track, such as:

- comparing data which should not or cannot be compared:
- failure to identify and distinguish between relevant, extraneous and landmark information:
- displaying a tendency to confuse relevant, extraneous and landmark information: and
- being unsure of whether the problem solving item contained adequate, inadequate, or redundant information with regard to the problem solution.

Although this research did not venture into specific focus upon the influence of language upon students' problem solving, there emerged one particularly obvious situation where it could be said that language was definitely hindering the student from getting a correct answer; when these students were confronted by certain specific mathematical terminologies such as 'congruent' in the fourth item. Hence, it seems that this is in tandem with Barton's (1995) comments, where the issue is mainly one in which mathematical concepts that involve specific vocabulary related to shape, size, volume, measurement and comparisons are particularly difficult for some students to understand, thus making it difficult for them to solve word problems. However, the difficulty that such students face with respect to word problems might be due mostly to lack of mathematical language proficiency rather than normal English language per se. As pointed out by Barton (1995):

> mathematics discourse has distinct features not found in normal English. For example, it is particularly dense, it is very precise, it is read in multiple directions (not just from left to right), and it contains familiar words with precise meanings which are different from their normal meanings. (p. 160)

Impact of Student Preference upon NS and PS

The students were asked questions designed to ascertain: whether they were distinguishing between number and number sense; and the degree of preference they had for problems requiring reasoning about number and a numerical solution (NSIP) compared to those requiring no reasoning about number and no numerical solution (DNSP) — they also had the option to indicate that they preferred both or neither of them. The eventual analysis would reveal the extent to which NSIP preference was related to NS and PS (through correlation). Subsequent analysis will focus preference for number sense inherent problems (NSIP) as opposed to those which are devoid of number sense (DNSP). The students' preference for number sense inherent problems or devoid of number sense problems was compared to their number sense scores in Figure 4.8; and to their problem solving scores in Figure 4.9, according to which performance proficiency band they belonged. The only instance where there was a marked difference between number of students favouring NSIP or DNSP was for students in the upper and lower proficiency bands. More than 70 percent (58% of the 24 students preferring NSIP) of Hns students preferred NSIP as opposed to none of them showing any preference for DNSP (Figure 4.8).

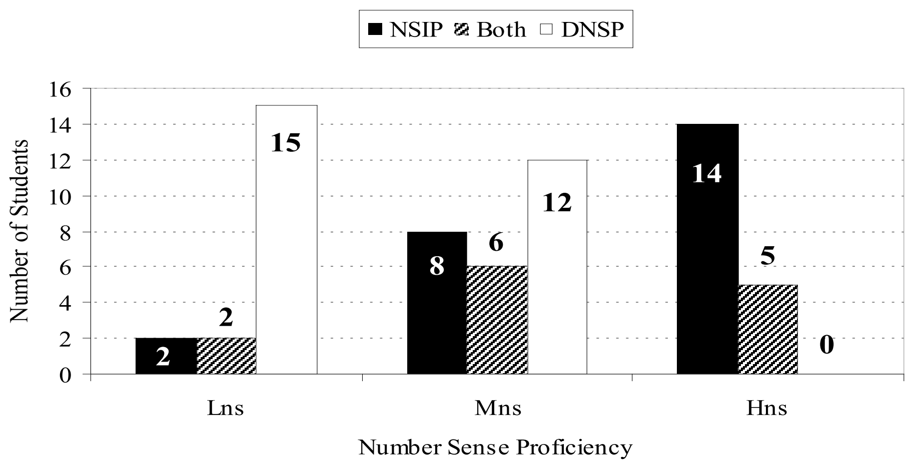


Figure 4.8 Students' problem solving type preference by number sense proficiency

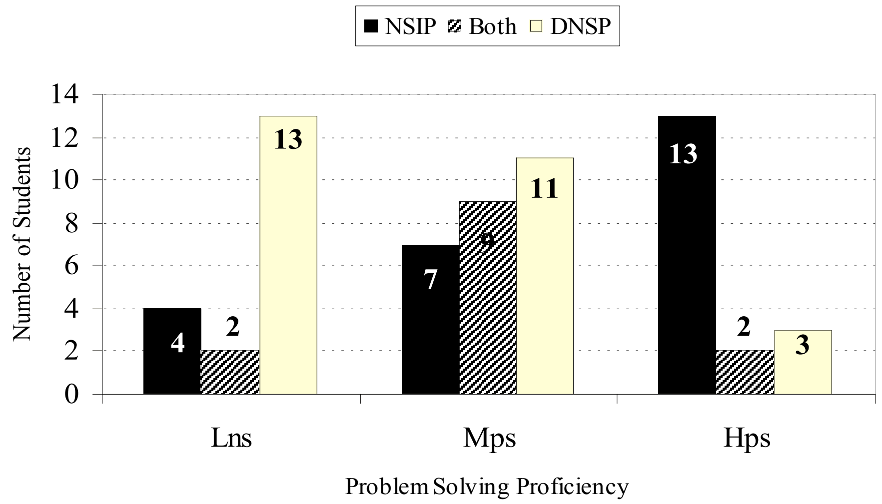


Figure 4.9 Students' problem solving type preference by problem solving proficiency

In contrast almost 80 percent (56% of all students preferring DNSP) of the low number sense students preferred DNSP as opposed to only 11 percent (8% of all students preferring NSIP) of the low number sense students who expressed a preference for solving NSIP items. This trend is also reflected, although to a lesser degree, for students' problem solving performance, where 68 percent of the high problem solvers, compared to only 21 percent of the low problem solvers, preferred NSIP. This suggests that students' preference for problems involving or not involving number sense could be a factor influencing both their number sense and problem solving performance, which is a fact supported by the correlation analysis presented in Table 4.23.

It is worth noting that as a student's number sense performance increases it appears that there is also a relative increase in the proportion of students preferring to solve NISP items (Figure 4.8). Since a similar frequency distribution was obtained when problem type preference was compared to problem solving performance (Figure 4.9) a correlation analysis was carried out to ascertain the association of preference for problem type, and NS and PS performance.

Students were asked to express their preference for NISP, DNSP, both or neither by circling the letter next to the one they preferred most. To give direction to the preference scale for this question, it was decided that since the majority of high performing students preferred NSIP, the preference for NSIP should be coded highest (0 = neither preferred; 1 = prefer DNSP; 2 = like both; 3 = prefer NSIP) so that the greater the correlation coefficient the closer the relationship between the preference for NSIP and the respective test score. Table 4.23 shows comparative correlation coefficients for students' preference for problem type (NSIP, DNSP, both, or neither) versus PST and NST performance scores.

Table 4.23 Correlation of students' preference and specific academic performance

		NSIP Score	DNSP Score	Number Sense Total Score	Problem Solving Total Score
Preference for NSIP, DNSP or Both	Pearson Correlation	0.561(**)	0.290(*)	0.687(**)	0.515(**)
	Sig. (2-tailed)	0.000	0.02	0.000	0.000

Note: **Correlation is significant at the 0.01 level (2-tailed); *Correlation is significant at the 0.05 level (2-tailed). N = 64

Preference for NSIP was more highly correlated to number sense ($R = 0.69$) performance than it was to problem solving ($R = 0.52$). Although preference to solve NSIP was related to both high problem solving and high number sense performance scores, it is interesting to note that when this preference was compared to the two major types of problem solving items the correlation favoured NSIP ($R = 0.56$, $p = 0.005$) while having a very low correlation with DNSP ($R = 0.29$, $p = 0.02$). Moreover, the correlation between NSIP preference and NSIP performance was greater than NSIP preference and overall problem solving performance, suggesting that preference for solving NSIP tended to result in higher performance scores in the NSIP component of problem solving, which according to the teachers are also inherent in most problems. This could be an indication that students' problem type preference is related more to number sense performance than it is to problem solving performance. In the interview with students, 64 percent were of the view that since they had been in Year 7 their number sense performance had improved a lot and consequently as expressed by one of them, "I used to prefer drawing problems and logic problems more. But now I prefer those with numbers in them". This could imply that one way of enhancing a student's problem solving performance could be to upgrade his or her number sense performance and preference for solving NSI problems.

Assertion 8

A preference to solve number sense inherent problems is mostly associated to number sense and solving number sense inherent problems.

Assertion 10

A preference to work mentally was popular among students who preferred to solve number sense inherent problems, and also among students who preferred solving devoid of number sense problems.

Assertion 9

Enhancing a student's number sense could in turn enhance that student's problem solving performance

Assertion 11

A large majority (82%) of students who preferred NSIP and working mentally were able to score full or partial marks for at least three of the four TASRI problems, while only 31% of students who preferred DNSP managed to do the same.

Other Aspects Noted Through the TASRI

Results of the TASRI interview has indicated that the written test might not be giving teachers a true picture of the mathematical strength and content preference of a child. In the list which follows the most pertinent results and respective students' comments or examples are presented. Through the TASRI interview some important discoveries were made and the results suggest several points.

1. Many students, mainly those with medium and low number sense performance scores, lack understanding of fractions and division in context;

Comments 1

- I don't like fractions because they are not like normal numbers.
- It is too difficult for me to work with fractions because I don't understand them.
- I can multiply fairly well, but when it comes to division it does not work like the others [operations]
- I can divide some whole numbers, but when they are too big or have remainders it's really hard.

2. For lower ability NS performers, it is not that they cannot read and understand the problem; it is more a case of the problem being placed in a more realistic setting in which they have to figure out the role of any number given. Below are four typical examples of such students in terms of their PS type preference, comments and performance scores. Such students tended to mostly prefer DNSP only or both DNSP and NSIP. Their number sense scores were in the low proficiency band (Below 60%). In most cases these students expressed that

they would either panic or want to give up the moment they were presented with an NSIP; whereas they expressed a great desire to try and solve any DNSP.

Examples 1

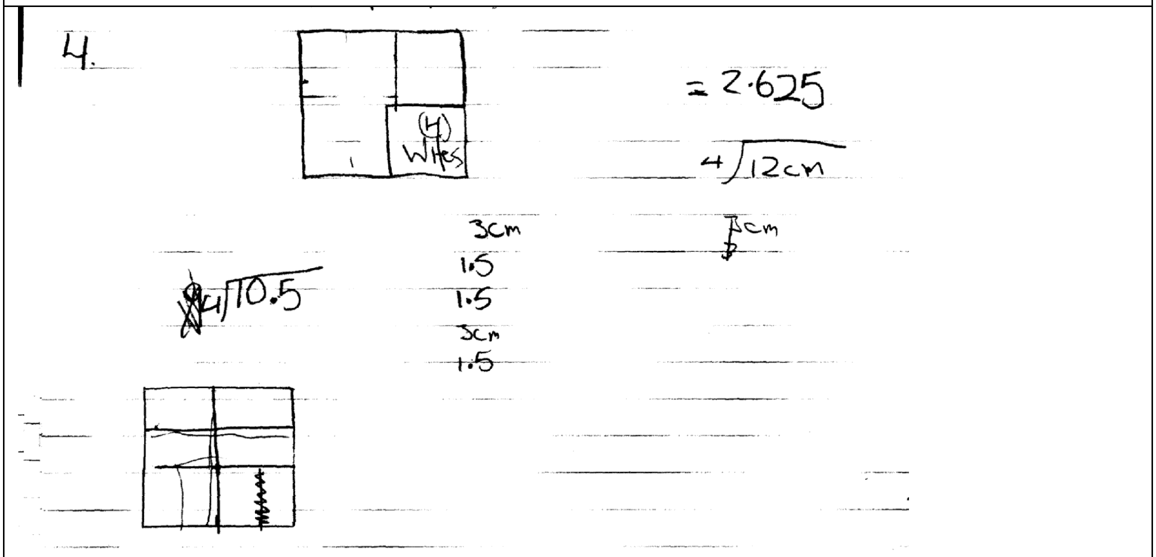
		Perceived Reaction when having to solve		TASRI Score (%)		Main Test Score	
Student	Preference	NSIP	DNSP	NSIP	DNSP	NS	PS
Wix[$S_{(2,36,1)}$]	Both	Wants to give up	Tries to solve it	0	75	56.7	64.6
I did understand what I was reading but I could not figure out what the problem was. So, I found it hard to find an answer.							
Belinda[$S_{(3,52,2)}$]	DNSP	Panic	Likes it	1	1	44.4	74
After I read this problem I understood the words, then I had to solve it. I did not know exactly what the numbers were telling me. There's more work with numbers than drawing or logic.							
Mary[$S_{(1,23,2)}$]	DNSP	Wants to give up	Tries to solve it	37.5	100	58.9	77.1
I like reading and so it was not difficult for me to read the problem. What was hard for was there's so many figures, the age of the kids, McPenny's age and so on. These age and numbers make it hard to solve. Sometimes I can, but not always.							
Annetta[$S_{(3,61,2)}$]	Both	Wants to give up	Tries to solve it	0	50	44.4	67.7
I don't think that I did not understand what I read. I understand all these words, but there's calculations to be made, but I don't know which calculation to do. I don't always understand what the numbers, what to do with the numbers.							

3. Having a preference for number problems does not mean that the person with such a preference can solve the problem, but it does highlight the fact that the person enjoys solving such problems and will come back to it very often until it is solved;
4. Number sense is a pre-requisite for problem solving, although students who prefer DNS problems could solve the latter in certain cases. Nevertheless, students with above average number sense proficiency level tended to be more all-round problem solvers than those with low number sense and a high DNSP performance index;
5. There is quite a strong relationship between a student's number sense and problem solving performance;
6. Most students (90%) stated that they have a preference for logic problems. This was the case regardless of whether they could solve NSIP or DNSP;

7. Assessment of student performance in mathematics could be improved through the use of the TASRI in conjunction with other assessment methods.

8. Some students with low number sense and medium problem solving performance who expressed a preference for solving devoid of number sense problems, but stated that they did not mind dealing with the numerical aspects of a problem, tended to attribute a numerical dimension to problems which necessarily required a non-numerical answer. For instance in the following work sample Belinda [$S_{(3,52,2)}$] tried to solve the drawing problem (Appendix VII) through calculating a numerical area. Although one student was successful in that regard, all the others who attempted to use such a method failed to get beyond drawing a few inappropriate lines.

Work Sample 1



9. The higher a student's number sense the more they seemed to prefer to work mentally, and if they did use pen and paper none of the high number sense students used the standard written algorithm. The lower a student's number sense the more they seemed reliant upon using standard written algorithms. Nevertheless, as highlighted in Excerpt 15, it seemed that the teacher's emphasis on encouraging students to work mentally and to use alternate or self-invented algorithms succeeded in getting most students, even those with low number sense to use non-standard written algorithms.

Excerpt 15

Erin [$S_{(1,14,2)}$]:

Mrs (Amanda) always asks us why we have used a certain way [algorithm] of calculating and she likes it when we use our own way [non-standard algorithm]. I used to work in rows and columns [standard written algorithms], like we were taught [in the lower grades], but now I feel free to work in my own way as well. If I can explain my method and it is alright then the teacher [Amanda] says it is good.

Joseph [$S_{(2,31,1)}$]:

I always try to find another way of doing the calculation because Mr [Bob] always asks us to come up with another way of doing it. Often he wants to know if we have a better way or our own way. Some ways are better tan others, he [Bob] says, and I've found that it is often best that I do it my own way.

Gitanne [$S_{(3,62,2)}$]:

In class we are all encouraged to compare our own calculation [with] those of our friends and then we have to also compare with those very organised calculations, like those in where the numbers are in straight lines [Standard algorithms]. Mrs [Chantal] asks us which one is faster or easier to use to get the answer. Usually it's best to use your own method because you understand it better.

4.5.4 Summary

Data analysis indicated that there was a strong relationship between number sense and problem solving. Triangulation of data revealed that this relationship manifested itself through various factors:

- Students' preference for solving either NSIP, DNSP or both was closely related to both their number sense and problem solving performance;
- Mathematics problem solving performance could be enhanced through solving number sense inherent problems;
- Students with high number sense were more likely to successfully solve mathematics problems; and
- The higher a student's number sense the greater the chance of that student detecting and rectifying errors in their working solution to a problem.

4.6 Analysis and Results of Research Question 2

How does teaching style impact upon students' number sense and problem solving performance?

Since the teachers selected to participate in this study were identified as effective teachers of mathematics it was expected that the teaching style they employed would have considerable impact upon the students' number sense and problem solving performance. Hence, to answer the second subsidiary research question, the discussion of results is first presented through the analysis of students' problem solving and number sense performance scores, as a means of ascertaining whether there was significant growth in regard to the respective scores between the administration of the pre-tests and the post-tests. This initial analysis will be followed by discussion of results pertaining to which facets of these teachers' teaching style could have had marked influence on the students' improved performance.

4.6.1 Growth in problem solving and number sense performance

The frequencies presented earlier in this chapter indicate instances of improvement from the pre-test administration time up to administration of the post-test. The pre-test scores range from 12.5 percent to 100 percent, while there is a marked increment when it comes to the post-test scores, with the latter having three fewer score categories than that of the pre-test. The scores are also becoming more concentrated towards the middle of the distribution by the time the students sat the post-tests; a range of 25 percent to 100 percent. With the exception of two students, who scored below 25 percent for the pre-test, all students scored at or above that, and this is even more evident when it comes to analysing the post-test scores.

With regard to change in performance and the impact of teaching and learning style upon NS and PS, the study specifically evaluated progress of mathematics learners through the teaching and learning period spanning the beginning of Term one to the end of Term four in 2004, and compared their performance at the beginning and end of this period on tests measuring Number Sense (NS), Number Sense Inherent Problem Solving (NSIP), and Devoid of Number Sense Problem Solving (DNSP).

To analyse growth, only students who had completed both the Pre-tests and Post-tests were used. Hence, the sample comprised a group of 64 students coming from three Year 7 classes, who took the pre-test which was administered at the start of the second school term, and the same group of students taking the parallel, equivalent post-test at the end of Term four in 2004. In the intervening months students were observed during mathematics lessons taught by the teachers.

Across the three classes, a majority of students increased their number sense (32) and problem solving (48) proficiency level and showed significant growth in NS, solving NSIP and solving DNSP, and overall PS. This growth occurred during this 34-week period, of which there were 30 teaching weeks, between the pre-test and post-test administrations. This trend supports the effectiveness of the mathematics learning experiences implemented by each of these three teachers.

For every class, a comparison between the mean pre-test score and the mean post-test score shows highly significant growth at the 0.000 statistic level, indicating that the increases in means are real, and not simply due to chance. Hence, when all the three classes' scores are pooled there is evidence of a real increase in performance.

Although this research is not aimed at making predictions or generalisations, a focus on the PS pre- and post-test scores would be beneficial in suggesting that an effective mathematics teacher's intervention could have an important role in enhancing students' performance in PS. For the sake of evidence in support of the aforementioned suggestion, the following line of thought is proposed, and this would require that one is not concerned with flaws in this design, such as lack of a control group. Here the focus is more on the 'change score' value. First an analysis of the Problem Solving Test (PST) scores was carried out to ascertain whether there was significant improvement from PS pre-test to PS post-test. From Table 4.24 it can be seen that the mean of the PS pre-test data is 65.4 percent and that of the PS post-test is 75.5 percent giving a mean difference of 10.1.

Table 4.24 Basic statistics of pre- and post-PST percentage scores

	Mean	N	Std. Deviation	Std. Error Mean
PS Pre-test	65.4	64	16.9	2.1
PS Post-test	75.5	64	18.3	2.3

This is very encouraging since according to Table 4.25, the true population mean lies between −14 and −6.3 with a confidence interval of 95%, which implies that the hypothesised mean of zero does not fall within this range. Evaluation of this result reveals that there is significant difference between the pre-test and the post-test scores [$t(63) = -5.2$, $p = 0.000$].

Table 4.25 Paired samples T-test statistics of pre- and post-PST percentage scores

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
PRPS − PSPS	−10.1	15.5	1.9	− 14.0	− 6.3	− 5.2	63	0.000

A similar analysis pertaining to the Number Sense Test (NST) scores was also carried out to ascertain whether there was significant improvement from NS pre-test to NS post-test. The NS pre-test and post-test mean percentage scores presented in Table 4.26 indicate that there was an increase in the students' overall number sense performance from the time they were pre-tested to the time the were posted.

Table 4.26 Basic Statistics of Pre- and Post- NST percentage scores

	Mean	N	Std. Deviation	Std. Error Mean
PS Pre-test	58.8	64	16.7	2.1
PS Post-test	72.2	64	16.7	2.1

A paired samples t-test analysis was also applied to the students' NST pre- and post- tests percentage scores, and the data presented in Table 4.27 indicates that the mean difference of 13.3 percent was highly significant [t(63) = −9.2, p = 0.000]. Since these students were taught by the teachers who participated in this research, the improvement in the former's performance could be attributed to the influence of the teacher on his or her students. Although these results seemed strong enough to support such a claim it was deemed more appropriate to conduct further detailed analysis of the students' scores to ascertain the extent to which they did or did not improve. To analyse students' improvement in students' number sense and problem solving performance the respective scores were ranked according to the quartile proficiency bands, which resulted in four proficiency groups.

Table 4.27 Paired samples t-test statistics of pre- and post- NST percentage scores

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
PRPS − PSPS	−13.3	11.6	1.5	− 16.3	− 10.4	-9.2	63	0.000

For this analysis it was deemed more effective to group the students' scores, by quartile rank, into four equal proficiency bands, instead of the three (top 30%, middle 40% and bottom 30%) which is the proficiency grouping used throughout most of this thesis. The purpose of grouping students' cores by quartile instead of three groups was that having four groups would enable the researcher to see more clearly how students progressed or regressed in relation to the proficiency bands. A substantial number of students (77%) advanced their level of Number Sense proficiency. In order to calculate the percentage of students who progressed or regressed one or more proficiency levels, or who stayed at the same level, the pre-test and post-test results for each student from each of the three schools were compared. A summary of this growth is shown in Table 4.28. On the whole students made significant improvement in both Number Sense and Problem Solving, with 25 percent more students advancing one or more quartile proficiency levels in Number Sense as opposed to Problem Solving. Around 88 percent of the 17 students who stayed at the same level were those who at the start of the first term showed signs of mathematical anxiety when they were informally interviewed. However, of those who decreased in their level of performance, approximately 96 percent had been missing school on a more regular basis than the rest of the class. A comparative analysis of the percentage of those who graduated to a higher performance level, those who stayed within the same level and those who decreased one or more levels, results in a ratio of 64:13:23. This indicates that the proportion of students whose combined NS and PS performance scores placed them in a higher proficiency level was nearly twice that of those students whose scores did not place them in a higher proficiency band after the post test. This is more evidence that the learning experience provided through these teachers' guidance could be highly influential in enhancing the students' NS and PS performance.

Table 4.28 Growth summary of students NS and PS performance

	Increased one or more levels		Stayed within the same level		Decreased one or more levels	
	Number	%	Number	%	Number	%
PS	33	52	15	23	16	25
NS	49	77	2	3	13	20
Total	82	64	17	13	29	23

Note: N = 64; Levels = Four groups of scores ranked by quartile.

Hence, the evidence pertaining to an increase in number sense and problem solving performance, as obtained from the students' PST and NST scores, suggests that the influence of the teaching provided during the period between pre-test and post-test could be one of the factors which helped the students upgrade their NS and PS performance.

Assertion 12

The improvement in both the students’ number sense and problem solving performance scores were statistically significant.

Assertion 13

When students’ problem solving and number sense performance scores are banded into four proficiency groups, based on quartile ranking, 64 % of the students improved from one proficiency band to another.

Hence, the presentation of data and discussion of results which follow focuses first and foremost upon identification of common elements of these teachers’ practices obtained through observation data. These were validated through triangulation with interview data.

4.6.2 Analysis of teaching style

Teaching Emphasis and Teaching Style

In presenting some recent research findings, Grasha (2002) observed that “style is reflected in how [teachers] present themselves to students, convey information, interact with learners, manage tasks, supervise work in process, and socialize learners to the field” (p. 140). Thus, to gain substantial insight into these three teachers’ teaching styles, they were observed on a weekly basis from term one 2004 to term four of the same year, and the observation data were validated through formal and informal interviews. In addition, this data was triangulated with data obtained from the teaching style inventory. It is worth noting that although these teachers showed certain preferences and differences in their teaching, the aim of this research was mainly to gauge how best these effective teachers taught for the development of number sense and problem solving ability. Hence, focus was placed upon teasing out the common points of practice as opposed to issues which were too different from one teacher to another. In this way it is hoped that other professionals will have access to data pertaining to how effective teachers teach for NS and PS. What follows is an account of the data analysis results presented as per the most prominent factors in the teaching repertoire of these teachers.

To gain comparative data about each teacher’s teaching preference, interviews, classroom observations and a 40-item Teaching Style Inventory (TSI) designed by Grasha (1994) were used. In regard to the TSI, the teachers were required to respond to a seven-point scale for each item. Other explanations regarding the scales and other relevant information about the TSI are given in Chapter 3 and Appendix V. Once the questionnaire was completed it was assessed on-line. Table 4.29 shows that these three teachers were well matched on only one preference; namely Delegator. A preference for this modality was also confirmed through observation and interview data analysis. All three teachers were very concerned with enhancing their students' ability to be self confident and autonomous, which is the main descriptor of a delegator teaching preference. The inventory results have also indicated that both Amanda and Bob are moderate facilitators, while Chantal was assessed as being high on that modality. Triangulation of other data indicated that all three teachers encouraged much student-student interaction with Bob and Amanda being slightly less teacher-student interactive than Chantal. Through discussions with the teachers and data gathered about their backgrounds it would seem that all three were high in expertise. In fact all three teachers acted very much like an expert/facilitator/delegator in the way they showed that they were knowledgeable, understanding and profoundly analytical in their discussions about the teaching of mathematics, the students and education in general, as evidenced through subsequent discussions of the results presented in this thesis. Some of the observation and interview data from this present study, which are presented below, are very closely related to the findings of Grasha (1997).

In presenting his Teaching Style Inventory's research results, Grasha (1997) claims that there were mainly five teaching styles — expert, facilitator, delegator, formal authority, and personal model. He also observed that these styles converge into four clusters which comprise the characteristic ways that teachers design instructional settings. The delegator/facilitator/expert cluster is the one which is of interest in this study since this was the cluster for which all three teachers were commonly classified as having moderate to high preferences. According to Grasha (1997) this cluster places much of the onus for learning on the students. Grasha (1997) further pointed out that the tasks provided by teachers who belonged to this cluster were most often complex, and required students to take the initiative. A common feature of this style cluster was how the teachers made much use of getting students to work in collaborative groups, which was also evident in this present study. Table 4.29 presents the teaching style preference scores of the three teachers, obtained from Grasha's teaching style inventory.

Table 4.29 Combined teaching styles preference scores and range

Teacher / Preference	Amanda		Bob		Chantal	
	Score	Range	Score	Range	Score	Range
Facilitator	5.2	Moderate	5.1	Moderate	5.5	High
Delegator	5.1	High	4.3	High	4.6	High
Expert	4.7	Moderate	5.2	High	5.8	High
Personal Model	4.6	Moderate	4.3	Low	5.8	High
Formal Authority	4.5	Moderate	4.0	Low	5.2	Moderate

Note: = Most similar teaching style preferences.

These teachers seemed to have what Ma (1999) termed as Profound Understanding of Fundamental Mathematics (PUFM), which they even tried to instil in their students as well. Moreover, each of the three teachers used virtually the same teaching 'ingredients' but they modified their approaches according to their lesson objectives and how the class reacted to the teaching and learning experience. As will be discussed later on, these teachers thought that one reason why their teaching-learning experience were most often effective was because they took into consideration contemporary issues which affected the 'learning reality' that each student was confronted with. One major aspect of this practice seemed to relate closely to the belief that the students live in a society influenced by information technology, of which the television and computer are most influential. Hence, the teachers tended to go to great lengths to ensure that there was variation in how the number sense and other mathematics experiences, that the students were engaged in, were presented to the students. In this regard the presentation of the lessons observed seemed to vary from lesson to lesson on quite a regular basis. Quantification of the observation data revealed that these teachers employed a variety of presentation formats in the introductory part of the lesson, and that some of these were commonly used by all three teachers. Table 4.30 presents the elements which occupied more than one third of the duration of the introduction phase of an observed lesson.

Table 4.30 Categorisation of types of lesson introductions by most common elements used

Term		Role Play	Real Life Situation	Artificial Situation	Dramatisation	Hypothesis	Total Number of Observed lessons Tot	%	
		Teaching Elements							
1	Amanda	1	3	1	0	2	7	35	
	Bob	0	3	2	0	2	7	35	
	Chantal	1	3	0	1	1	6	30	
		2	9	3	1	5	20		Tot
		10	45	15	5	25		100	%
2	Amanda	0	5	2	0	2	9	34.6	
	Bob	0	4	3	0	1	8	30.8	
	Chantal	2	3	1	2	1	9	34.6	
		2	12	6	2	4	26		Tot
		7.7	46.2	23.1	7.7	15.4		100	%
3	Amanda	0	5	2	0	1	8	33.3	
	Bob	0	5	2	0	1	8	33.3	
	Chantal	0	4	1	1	2	8	33.3	
		0	14	5	1	4	24		Tot
		0.0	58.3	20.8	4.2	16.7		100	%
4	Amanda	0	3	3	0	1	7	33.3	
	Bob	0	2	3	0	2	7	33.3	
	Chantal	0	3	3	0	1	7	33.3	
		0	8	9	0	4	21		Tot
		0	38.1	42.9	0.0	19.0		100	%
		4	43	23	4	17	91		GT
		4.4	47.3	25.3	4.4	18.7		100	%

Note: Tot = Number of lessons observed; GT = Grand total number of lessons observed.

Hence, although there were many other elements used, Table 4.30 presents only those which were most prominent in terms of duration, which means that there were other instances where either any of these four elements or others were used in the introduction, but due to them lasting no more than a third of the lesson, they are not represented in Table 4.30. As shown in Table 4.30, there were five common types of elements that all three teachers tended to use in the introductory phase of the teaching-learning experiences. It was very difficult to detect any new category of introduction format and focus during the third and fourth terms. Nevertheless, it should be noted that although four types of introductions are listed, none of the introductions seemed to be a repetition of one observed previously. It can be seen that the majority (47%) of the introductions involved some aspect of real life situations while the least employed of the four common presentation elements were 'dramatisation' and role play (only 4%). The instances which have been coded as dramatisation were ones in which the teacher would come in with a dramatic story or just do something a bit unusual which caught the students by surprise. For example, at the beginning of one lesson, Chantal was partially dressed up like a sinister type of person and she was talking in a sort of gangster-like voice which got the students amused and wanting to know what all this was about; she was introducing a lesson on probability. It is also worth noting that an introduction would usually incorporate more than one teaching element. In 19 percent of cases the teacher would start with a hypothesis; for example, all square numbers are odd, even or prime. In such cases the next phase of the introduction would involve a discussion which most often took the form of an open debate. This might then be followed by individual and group work where students try to prove their assertions. An interesting pattern is that as the year progressed there seemed to be more focus on employing 'artificial situations'. These were usually teacher-or student-made, and further analysis revealed that increased use of such an element could be related to increased focus on getting students to work in the abstract, as will be discussed later. It is also interesting to note that during the first term the teachers tended to employ the hypothesis element more than they did in the second and third term, but then tended to use more of it in the fourth term. Just like the introduction was varied through the use of a variety of elements, the other phases of the lessons were also varied in terms of format and presentation. Unfortunately it proved too demanding a task for these to be quantified.

All three teachers incorporated an element of flexibility, to various degrees of emphasis, in their teaching. Although all three were observed adapting the flow of the lesson to suit the students' reactions, Bob was the one who was always on the lookout for any signs of frustration so that he could completely change the teaching emphasis, the topic and mathematics content. Compared to Bob, the other two teachers preferred to adapt their teaching to the situation of the moment rather than change to a completely new topic. Chantal usually stuck to her plan more than the other two teachers, but just like them she was never observed referring to any notebook or learning experience plan. Amanda and Bob were the ones who used mostly a few sketches on paper as a guide, although similar to Chantal they kept to the term's scheme of work. Amanda supported this method by stating that "teaching is a flexible activity. For it to be fruitful it has to be planned mostly in the head. You have to review it over and over again in your head so that at the time of execution you will not be shackled to your plan". Bob's comment, "it's no use having very detailed teaching plan written down on paper if you don't mentally know this plan inside out", was in tandem with Amanda's. Although Chantal insisted that "it is always better to have your written plan a bit more detailed and handy", she still taught mostly from memory, instead of reading from her plan. When asked why she did not refer to her detailed lesson plan she remarked "I always think [mental preparation] about what, why and when to teach a certain topic, then by engaging in writing all these details I end up memorising the whole lesson". Hence, it seemed that all three teachers relied much on mental preparation, mental previewing of the delivery and also on memory recall of their teaching plan during delivery. Nevertheless, it should be noted that whatever was taught always revolved around the policies and curriculum adopted by the school. Hence, when the teacher met with the curriculum coordinator or deputy head and other teachers, before the first term, they discussed possible intended objectives and then selected those which were most appropriate and relevant to the educational level of the students, the national curriculum framework principles and goals, and the goals of the school. The individual teacher then transformed these into performance objectives according to the particular group of students to be taught. The realised objectives were usually shaped by the individual preferences and educational disposition of students in the class. Hence, the achieved objectives were based mainly on what students actually learnt, which, as explained above, would not necessarily be exactly the same as the original objectives.

Assertion 14

The learning experience plan must be flexible enough to accommodate students' moods, unanticipated interests, motivation and preferences.

Assertion 15

Most of the lesson preparation was done in the head, key ideas were mostly of the jotted- down form, and lesson delivery was done mainly from memory.

Why Students were more Balanced on the Understanding Scale

Understanding Information Globally and Sequentially

Since the *Understanding Information* learning style dimension was the one on which students were more balanced, after the ILS had been administered the researcher went through the students' worked assignments to re-check their methods of working. This exercise revealed that students were working in a more sequential manner during the first term, and then tended to be less sequential as the term progressed. Hence, it was deemed important to find out how these teachers helped students to become more global-oriented in their method of understanding. Although none of the teachers referred to this as developing a global learning preference, it was evident that they felt the students were "too linear in their approach to solving problems", as explained by Bob. Chantal explained that she taught in this way because "students need to learn and understand mathematics both step by step [Sequential] and also through seeing the big picture [Global]".

In Chantal's class, once every fortnight, after completing work on a variety of topics, students were given teacher-made exercises and activities [See Example 2, below] which tested their understanding of how the elements learnt are related. Such work was then corrected orally with the whole class as a means of getting students to highlight how various topics are linked. The links were highlighted on the board as a result of answers to questions such as "What is the probability of removing a red ball from the bag", "Can anyone express this as a fraction", "What is this fraction as a percentage", "How about converting it to a decimal". Afterwards students were asked to summarise what they had discovered, with the teacher seizing on this opportunity to get them to see and 'accept' the links among fractions, decimals, percentages and probability.

Example 2

Mary has 10 red marbles, 5 green marbles, 15 blue marbles, and 30 yellow ones.

(a) What fraction of the marbles are yellow?

(b) Jim says that one quarter of the marbles are blue, while Jack insists that only 0.25 of the marbles are blue. Who is right?

(c) What percentage of the marbles are blue?

(d) Rita places all the marbles in a black bag. Then, without looking into the bag, she removes one marble. What chance (probability) does she have of removing a green marble?

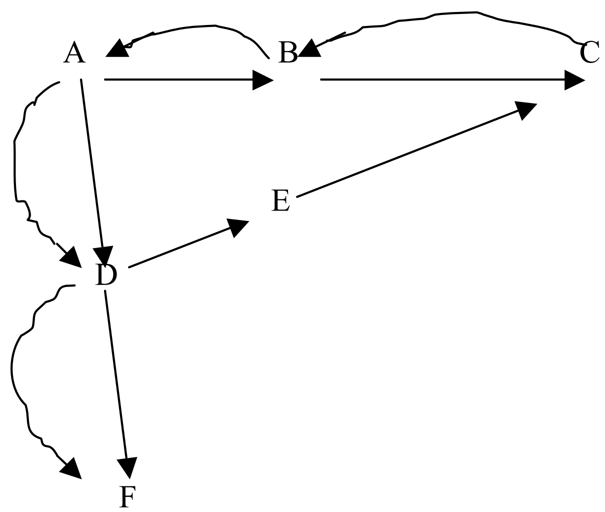
In regard to Bob, he referred to inductive and deductive reasoning instead of using the terms global and sequential. He encouraged his students to use various forms of methods to help them understand the information and also retain and recall these.

At least once a week Bob's students were involved in a whole class discussion and blackboard interaction with the end product being a sort of mind map diagram showing how the topics learnt are linked. Vignette 1 shows an example of how Bob moves from a simple activity to getting students to make some form of mathematical generalisations, which they were then encouraged to represent as a visual *aide-memoire*.

Vignette 1

Problem: Ashley's brother gave him a puzzle to solve. First Ashley was asked to add the following pairs of numbers together: (a) 2 + 6 (b) 3 + 5 (c) 8 + 4 (d) 10 + 7 (e) 5 + 2 (f) 6 + 14 (g) 7 + 9 (h) 11 + 9 (i) 16 + 6 (j) 13 + 15 (k) 17 + 12

Students are asked whether there is a way of grouping these pairs of numbers according to a pattern. From this a discussion ensued until they discovered that there could be three main groups; ODD + ODD, ODD + EVEN, and EVEN +EVEN. Through the discussion they learnt that the sum of two odd numbers is an even number, that of two even numbers is even and that only when one of the two numbers is even, and the other is odd that the sum is odd. Students were then asked to draw their own diagrams to show these links. The diagram below was then drawn on the board and the teacher then asked students to replace A, B, C, D, E and F by the word ODD or EVEN, after which they had to explain their answers to the rest of the class.



For homework the students were asked to do the following, first mentally, then through other means, if they were not able to do it mentally. Extended Problem: Ashley's brother then wanted to know the solution to the following:

(1) ODD – ODD = ……. (2) ODD – EVEN = ……. (3) EVEN – ODD = ……

(4) EVEN – EVEN = ……. (5) ODD x EVEN = ……. (6) Do your own

In the next lesson homework is corrected and solutions discussed. Suggestions are provided for extension; multiplication and division. This was revisited, at a higher level, two weeks later, at which time students were then made to construct a more advanced diagram relating all four operations upon pairs of odd and/or even numbers.

Although each of the three teachers tended to employ some different teaching strategies and methods one common observation across all three classes was that as the term progressed, more and more topics were linked together. In the case of Amanda, at the beginning of the week she employed a question-and-answer technique to get students to come up with a 'schema' of what they covered the previous week. They were then encouraged to think about the elements of the major topics to be covered during the week and how they are related.

Teaching for Understanding the Language of Mathematics

These teachers used specialised language adapted to whoever they were interacting with. For example, when talking to:

a. the researcher about mathematics they used such terms as 'multiplicative reasoning' 'proportional reasoning';
b. some colleagues about maths they selected their words carefully depending upon what they thought about the mathematics knowledge of the person. For instance, to a maths teacher whose mathematics ability they considered to be high they would use the same words they used with the researcher. With other teachers they tended to use simpler terminologies that they would employ with a child.
c. the students they taught, quite often they tended to use certain advanced words (e.g. "Jim, how come four raised to an exponent of 3 gives you 64") with the more able 'mathematicians'. If the student did not understand the term used he or she was asked to do a search for its meaning and to explain it to the teacher next time. The interview with the teachers indicated that they deliberately adapted the language they used depending on their perception of the student's ability to understand what they were saying. As highlighted by Amanda, "I do not use such language with a child unless I am sure that it won't put her off. The more mathematically minded students tend to like such challenges".

Each teacher prepared and engaged students in activities which helped them in gaining a better understanding of the mathematics vocabulary, syntax and structure of word problems. Although this was done in very different ways from one teacher to the other, there were certain common elements across each of these practices:

- Problems were most often given in a worded format.
- In each lesson observed there was always an element of problem solving language which students had to overcome.
- Questions were always being asked to check students' understanding of the mathematical vocabulary used in the problem.
- Parallel problems were given in both in-context and non-context formats.

The three vignettes presented below give an idea of some common recurring practices which were observed on a regular basis throughout the whole year. These were more like problem solving teaching strategies and they were done in a very subtle way so that they were seen as the natural thing to do when solving a problem.

Vignette 2

Bob:

There is this tendency to always get students to dissect a problem into its component parts. For example, a problem was given and students were asked to prepare a set of questions about the problem which would help them understand what the problem was about. There was open discussion about what clues exist, how to detect them and what could be done about them which would assist in finding a solution to the problem.

Bob's focus was more upon analysing the problem by understanding how it was constructed. This involved learning how to detect clues and how to use these effectively in the solution process.

Vignette 3

Amanda:

Students were asked to rewrite a problem in a different way, using terms similar but not exactly the same as those in the original problem, in such a way that the requirements of the problem were the same. They then tried to solve each other's reworded problem. After that they engaged in a discussion about the language structure of the problem.

Amanda was more into reconstructing the wording of a problem as a means of owning the problem first and then solving it. Chantal's focus was more on getting students to construct their own problems from scratch so that students got a feel for how to make the words say exactly what they wanted the problem to convey. Although each of the three teachers tended to lay more emphasis on a particular mode of engaging students in understanding the language of the problem, the three examples chosen were commonly practised to various extents by each teacher.

Vignette 4

Chantal:

Given a few Ordinary English words, numbers and mathematical terms, students were asked to create their own word problems. They were then asked to compare problems. Discussion ensued about the wording of these problems. A lot of emphasis was placed upon understanding how the mathematical terms and language combined to communicate clues to the problem solver.

Use of Peripherals: Enhancing Students' Visual Modality Preference

The results of the students' learning style also revealed that more than 90 percent of the students preferred to learn through the visual modality. Hence, it was deemed necessary to look into how the teaching-learning experience could have influenced the students' preference for receiving information via the visual modality. A striking feature of these three classes was the way in which the teachers used nearly everything in class in an attempt to engage the students in the teaching and learning interaction process. Every little bit of wall space was covered with some type of information in the form pictures, diagrams, charts, graphs, posters and notices. The majority of these were students' work, followed by slogans, sayings and reminders. When asked about the rationale for having these displays the main theme which came out of these teachers' comments was that they were left on display as a means of improving memorisation and recall of facts. The most common affective principles which were being encouraged were perseverance and self esteem. This was disguised through posters, slogans and students' work. The excerpts which follow highlight the purpose for having these displays, with emphasis placed upon the two principles of self esteem and perseverance.

Excerpt 15

Amanda:

They serve different purposes. You have these posters at the back [Pointing towards the soft board at the back of the class]. I use them to remind students of certain things that they have learnt. The displays over there [pointing towards the walls] are mostly what students have done. It gives them a sense of pride in what they do. I sometimes have those words of wisdom displayed opposite the entrance, where students can see them as they come in. I use these to remind them of certain important principles such as these [Pointing to some rectangular strips of paper on which slogans or sayings such as "To achieve your best you must believe in yourself", "Never give up even when it seems impossible to achieve"].

Excerpt 16

Bob:

Many of these displays are left there to help the students in many ways. I want them to learn and understand many things in life. But it is not easy for them to remember all that I would like them to learn. So, I have these posters, sayings, drawings showing some mathematical relationship and so on. In this way they help the students to memorise certain important facts. The students have to read these because every now and then I make reference to them.

As pointed out in both Excerpt 15 and Excerpt 16, both Amanda and Bob used these peripherals mainly as a 'silent' aid which served to remind students of important mathematical and essential life facts. In Chantal's class, on the left of the writing boards, there is a picture of a bird trying to gobble down a small animal, but the latter is strangling this huge bird in an attempt not to allow itself to be swallowed. The moral of it is summed up as "Never Give Up" at the bottom of this poster. Since no authorisation was sought for taking pictures, a similar poster, presented in Figure 4.10, was obtained from the website http://www.cafepress.com/cp/browse/store/marshbunny.21011894 .

Figure 4.10 Poster in Chantal's class which reminded students to always persevere

When quizzed about why such a poster was on display Chantal had this to say:

> The principles that we want to instil in our students are many, but since they are extremely important we have to find ways to get them to memorise these principles. In my case, it has been very difficult to decide which of these principles is most important. You see…I do not have enough space…There's not enough space to write all the principles, but I did find this simple drawing…it's beautiful and it summarises a very important principle.

Chantal explained that this poster is used to "encourage my children to persevere even when there seems to be no immediate way of solving a problem".

Praise and Reinforcement

Another common aspect of these teachers' teaching which could have had much influence on the students' number sense and problem solving performance is the use of praise as a means of creating rapport and fostering self-belief. All three teachers employed various ways of praising their students, and this was generally undertaken in three ways as follows.

Instantaneous Spontaneity

The dialogue presented in Example 3 shows that as the student was explaining something the teacher would come in with a word of acknowledgement and encouragement about what the child had just said. This form of praise was used mainly with less able students as a means of spurring them on. Yet as they moved from one term to the other instead of just praising students for their effort the teachers engaged students in reflective activities about their needs and what they envisaged doing in life later on. Chantal pointed out that "these kids are on the verge of going to high school. Therefore they have to be prepared to challenge and motivate themselves to learn". In addition to that, Bob felt that, "Year seven being at the crossroad between primary and secondary education means that students have to be a lot more mature. They cannot rely on external motivation to spur them on to higher levels anymore". Amanda explained that she was trying to "hand over the reign of motivation to the girls so that they will be the ones in control", while Bob added to that when he said that "to be successful in life one has to be intrinsically motivated". In Amanda's thinking "these girls had to be treated as babies when they first came in, but they are growing up now, and they must also rely on their own self-evaluation about the quality and level of their work". This theme of empowering the students by gradually moving away from teacher-dependent and teacher-directed teaching-learning experiences, towards a more self-dependent and student-centred environment seemed to be central to these teachers' beliefs and practice.

Example 3

Iridis [$S_{(1,22,2)}$]:	There are three balls in a plastic bag….
Amanda:	That's true; there are three ping pong balls in a plastic bag.
Iridis [$S_{(1,22,2)}$]:	And since there are five plastic bags…yep, each box contains five plastic bags.
Amanda:	That's it!
Iridis [$S_{(1,22,2)}$]:	Then,…then you get fifteen balls in a box.
Amanda:	That's great Iridis. Yeah, that's very good…what did you do to get fifteen…go on, tell the class, how you got fifteen Iridis.
Iridis [$S_{(1,22,2)}$]:	There are three balls in a bag, and…um, and five bags will go in a box.
Amanda:	Do you see what Iridis is doing class? Do see that she is doing something to three and five?...What did you do to the three balls and the five bags Iridis?
Iridis [$S_{(1,22,2)}$]:	I multiply three by five….
Amanda:	Fantastic. Isn't Iridis correct class? Three times five equals…?
Class:	Fifteen
Amamda:	That's great Iridis. Iridis will now tell us what that fifteen means…what does fifteen represent here?
Iridis [$S_{(1,22,2)}$]:	[Iridis is silent for a while. She seems to be thinking, and suddenly she starts talking] Yep, there are fifteen balls [Then talking to herself she says:] because each bag has three balls, yep, and there are five bags in a box.
Amanda:	That's it Iridis! Fifteen represents…[The teacher waits for Iridis to complete her sentence]
Iridis [$S_{(1,22,2)}$]:	There are fifteen balls.
Amanda:	That's great Iridis. Well done.

Felicitation

These three teachers do not lavish cheap praise on students left, right and centre, just for the sake of praising students. They praised their students through genuine mention of what the student has done to deserve the praise (Excerpt 17). When some students were asked about how they felt when praised in this way, a common answer was "I felt good. It makes you feel good".

Excerpt 17

Chantal:	That's really good Robin. You have noticed that three quarters of $120 is the same as dividing by four and multiplying by three. So you divided $120 by…
Robin [$S_{(3,59,1)}$]:	...four…
Chantal:	…and this gave you…
Robin [$S_{(3,59,1)}$]:	…30
Chantal:	That's great Robin…and then you…
Robin [$S_{(3,59,1)}$]:	multiplied 30 by 4
Chantal:	Well done Robin.

Peer Acknowledgement

The three teachers often encouraged peer acknowledgement and support. For example, at times when the teacher knew that a student's answer or solution was correct he or she often asked rhetorical questions such as:

- "Can you see what he has done children…how he has solved this problem?";
- "Is she correct?…Has she solved the problem correctly?...What do you say Alice about Alana's answer?";
- "Boys, do we need any more lines to complete the drawing? [Students unanimously say "No"]. Well done Peter, the class agreed that you were right!

Relevant Recurring Themes Directly Related to Teaching Style Cluster 4

Throughout the discussion about common teaching behaviours and beliefs shared by all three of these teachers the following five themes were reiterated quite often:

- reading to stay up-to-date with both academic and pedagogical information;
- having a positive attitude towards mathematics and teaching it as a subject;
- presenting factually accurate information so as not to confuse students;
- focusing one's belief on being determined to build students' confidence; and
- being prepared to accept that as a teacher one is not infallible.

The themes presented in Table 4.31 are summaries of the field notes and observation data. Although there were other recurring themes, those presented in Table 4.31 were the only ones which seemed to permeate all aspects of these teachers' teaching. The most common reason given by these teachers for placing much emphasis on reading was that it allowed them to keep up-to-date with new developments, not only in regard to mathematics education, but also with respect to the current events "which do have a lot of influence on these children's day-to-day life". According to Bob "it very important that the teacher knows as much as possible about what is happening, both in his own world and in the world around him".

Table 4.31 Five common themes which were most frequently reiterated by the teachers

Code	Observation and field note comments	Teaching focus
Reading	This indicates that they read a lot to stay up to date with research and relevant developments in mathematics education.	Reading to stay abreast of new developments
Attitude	All three teachers expressed that they "loved mathematics" and that they were very good at it. This is evident in the way they teach, the enthusiasm they show and the type of questions they ask their students.	Teaching Proficiency
Belief	They believe that building confidence is the main key necessary for helping students learn mathematics and that there are certain key elements which are instrumental in developing and maintaining this confidence: 1) Loving and caring teaching environment 2) Free play 3) Concentration 4) Proportionate challenge 5) Reasoning prowess 6) Freedom of expression 7) Using own method of solution 8) Opportunity to question and challenge 9) Opportunity to present method and thinking 10) Mistake allowance 11) Posing own problem 12) Customised Teaching	Building confidence
Factual accuracy	When explaining mathematics to students these teachers seem to be monitoring their speech instantaneously, and they are so conversant with what they are communicating that the phrasing of the explanation is always done in a factually correct way.	Instant monitoring of speech
Mistakes	If they do make a mistake, which is extremely rare, they will always apologise once this is discovered. Since their students have been trained to exercise 'controlled freedom of expression' it is often the latter who discovers and politely bring such mistakes to the attention of the teacher.	Acceptance and remediation of mistakes

4.6.3 Common learning experience preparation and engagement themes

During the first three weeks the researcher went around with the teachers as they taught various subjects to the students participating in this research and sometimes to students other than their usual class. During these visits notes were taken as to what characterised each teacher's lesson delivery. Once these were analysed some recurring themes common to all three teachers emerged. These themes were coded first at the end of the first week, then at the end of the second week and the third week. By the fourth week a checklist was made based on what were considered to be practices which were commonly indulged in across these three teachers. This helped the researcher to gather quantitative data in the form of the number of occurrences of the different aspects of lesson organisation and delivery. The checklist was easy to fill in since all that was done was to tick an appropriate box whenever the teacher engaged in any of the components listed. During the course of the observations the list was modified six times as a result of the teachers tending to use other methods and approaches as their acquaintance with the children matured. Hence, the following list contains only the most popular components which will hereby be called essential elements of the lesson.

Each of the three teachers observed used a problem-solving approach together with extensive use of modelling and simulation exercises, aimed at connecting the students' experiences with the physical world through simulation modelling, and with the mathematical world through algebra, graphical representations, tables and mastery of basic mathematics facts. Of the 91 lessons observed only one was taught without any number sense or problem solving required or employed, but even then the lesson involved some corrections of number work which had been given for homework. The data collected was often quantified through counts of occurrences of certain observed behaviour and interview data, and factors derived from them.

Types of Tasks Students were Engaged in

The teaching and learning tasks employed were usually focussed on a combination of real world problems, abstract mathematics, and a mixture of the two. Although the teachers focussed a lot on problem solving their teaching tended more towards getting students to work mathematically. All tasks were carefully prepared beforehand; at least one week in advance. An interesting feature of these teachers' planning of the teaching-learning experiences is that most of this planning was done mentally as will be explained below. In the case of Bob and Amanda these tasks were revised so often that by the time the actual lesson was delivered they had taken on a totally new dimension. Although she did allow for a lot of flexibility in her teaching, of these three teachers, Chantal was the only one who tended to stick more to whatever was planned; she would make major changes only if there was a disruption caused by school activities. A possible reason for this could be that she was the only one of the three teachers to prepare quite detailed lesson plans, and as explained by the teachers, it would be more challenging and difficult to change, adapt or discard altogether a detailed written plan upon which considerable amount of time had been spent. Nevertheless, compared to the strict teaching-to-the-plan that would be observed in most, if not all, average mathematics teacher's class, Chantal adapted her plan *in situ* to fit in with the students' responses and participation level.

From Table 4.32 it is apparent that while 36 percent of activity occurrence was devoted to games, puzzles and mini projects, the majority (64%) of the total occurrences were devoted to investigations and the teaching and learning of various mathematics strategies such as pattern recognition and application.

Table 4.32 Most common teaching and learning experience tasks per lesson observed

	Amanda (31)		Bob (30)		Chantal (30)		Total	
	Count	%	Count	%	Count	%	Count	%
Investigations	20	65	23	77	22	73	65	71
Games	8	26	7	23	6	20	21	23
Puzzles	10	32	16	53	11	37	37	41
Strategies	21	68	26	87	21	70	68	75
Mini projects	5	16	4	13	7	23	16	18

Note: The number of lessons observed per teacher is shown in parentheses.

During the early part of the first term these strategies were most often suggested by the teacher, but from the middle of the first term onwards the focus was placed mostly on the students' suggested or invented strategies. These strategies consisted of standard, non-standard and alternative problem solving and computation strategies. Another interesting feature pertaining to the exploration, invention and use of strategies was that from the first encounter the students were encouraged by the teacher to "feel free to use mental" strategies.

Customised Teaching

One-on-one Interactions

Although these teachers believed that in practice it was virtually impossible to cater for each student's learning style, in all their teaching they tried to reach each student, in spite of this being seen by all three of them as a very challenging task. During the observations every time that a teacher spent two minutes or more working with a particular student (from now on this will be referred to as one-on-one interaction) the details pertaining to the student and the duration of the interaction were recorded. This data was previously presented in Table 4.18. Table 4.33, which is derived from Table 4.18, shows that all three teachers provided more one-on-one interaction with the less able students than they did with the high ability students. The interview data revealed that the teachers believed that the more able students were better equipped to work independently, although they were constantly monitored by the teacher. When asked whether it was "possible to attempt to satisfy each student's individuality", each teacher's answer was very striking in that they all explained that the effective teacher must believe that although it seems impossible to cater for every student in practice, the teacher must believe in finding better ways to satisfy each student's need, especially in large classes being taught for short periods. Bob answered that "if I were to think of catering only for the majority of students then I would definitely be excluding this little minority". Chantal stated that "unless a teacher attempts to cater for each individual student there is no way that that teacher will be able to take into consideration each student's ability and performance during preparation and teaching". Amanda expressed her concerns that "as a teacher you have two choices; to conscientiously try your best to reach each and every student or to reach only a group of students. The only way to reach everyone is to believe that it is possible to find new ways to engage each student to his or her satisfaction". She went on to explain, "…that's why I talk to the students about what they appreciated in the lesson and what could have been done better". This was seen and used as a method of getting student feedback which was taken into consideration in subsequent lessons.

Table 4.33 Frequency of one-on-one teacher-student interactions

		Number of One-on-one Interactions				
	Number of students	Once	Twice	Thrice	Total	Mean per student
Hns	9	26	0	0	26	3
	7	94	5	0	104	15
	3	5	0	0	5	2
Total	19	125	5	0	135	
Mns	12	162	2	0	166	14
	6	93	36	3	174	29
	8	28	0	0	28	4
Total	26	283	38	3	368	
Lns	3	55	11	2	83	28

Student-Friendly Teaching-Learning Experiences

Data analysis revealed that all three teachers attempted to cater for students' learning differences in various ways, although there were only three very common areas in their practice: (i) prepare and teach according to the ability; (ii) students' preferences;

and (iii) experiences of the students. Triangulation of the observation and interview data provided an indication of how these three teachers tended to focus on individual ability and readiness of students before engaging in a new task. Nevertheless, this focus on readiness was more about how the teaching-learning experience should proceed, in order to facilitate learning, rather than deciding not to teach a topic until students were deemed to be ready for it. As explained by Amanda, “If I was to wait for every single student to be ready before we could move to a new topic then the whole class and the particular students in question would be held behind”. What was done was to use the student-readiness information in planning the new teaching-learning experience so that students would be able to go to the next level with as much ease as possible. For example, the teacher would discuss with students about a topic to be introduced in a subsequent lesson and from this gain insight into what might be a better course of action. Hence, as exemplified through Bob’s statement, the teachers “stay[ed] abreast of what could make future [teaching-learning] experiences more student-friendly”.

The observation and interview data revealed that the emphasis on making the teaching-learning experience student-friendly could be the major factor behind teachers checking for student-readiness. Hence, this was quite different to the traditional emphasis on readiness for the purpose of deciding whether or not to engage students in a new learning experience pr on a new topic. In this study the teachers used this student-readiness check so that they were very much aware of each student’s weaknesses and strengths and what students might be better prepared to respond to in subsequent lessons. Therefore, although the teachers were the ones mainly responsible for preparing the plan for administration of the teaching-learning experience, the students’ input was considered to be crucial to the whole teaching-learning process. The consequences of such a practice — of trying to make the teaching-learning experience as student-friendly as possible — were observed to be of various types and influences with the most influential being the constant variation in the way the teaching-learning experiences were presented and administered. For instance, the teachers attempted to ensure that no two lessons were started off or delivered in exactly the same way, although certain characteristics of the lessons seemed to stay intact due to the teacher’s personality, and both subliminal and overt feedback from the students. In this way the teacher ensured that as much as possible they gauged students’ prior knowledge, skills and ability, and then attempted to build on that. It was apparent through the observations and discussions with the teachers that no mention of either ‘teaching style’ or ‘learning style’ was ever made, unless these terms were used by the researcher, although as is discussed through the evidence presented below, these teachers did cater for diversity in their pupils. They all preferred to refer to this as individual differences.

The observation and interview data indicated that three of the most common components of what these teachers termed as 'individual differences' were: individual ability; special interests and students' experiences. Quantitative analysis of the data, stemming from the observations, questions posed by the teacher and teacher-student interactions, which were aimed at gauging students-readiness and preferences, resulted in a frequency distribution, presented in Table 4.34, of the three most common factors that these teachers focused upon.

Table 4.34 Catering for students' learning styles (individual differences) before engaging in a new task

Catering for:	Amanda (31)		Bob (30)		Chantal (30)		Total	
	Count	%	Count	%	Count	%	Count	%
Individual ability	22	71	20	67	19	63	61	67
Special Interests	5	16	5	17	7	23	17	19
Building on student's experiences	24	77	22	73	24	80	70	77

Note: The number of lessons observed per teacher is shown in parentheses.

These teachers were very subtle in the way they subliminally involved students in decision-making. Hence, it could be very difficult for an uninitiated observer to discern that the teacher might be soliciting suggestions for lesson preparation from the students. Upon closer examination it was revealed that about 75 percent of the lessons observed from term two to term three incorporated elements which were specifically dedicated to teaching students according to what they felt happy with (Special Interests), although this was not done in the same way in each school; although this practice could have been present in term one it did not become evident until the third week of the second term. Amanda preferred to engage students in discussion about various topics of interest and asked the students what they knew or did not know about these topics. Analysis of the classroom observation audio recordings and field notes revealed that Amanda subliminally got students to also suggest what they would like to do; although these were asked as general questions it was observed that many of the students' suggestions were present in the teaching-learning experiences. Although not done in an overt conspicuous manner, it could be ascertained that the students were involved in the preparation and delivery of this lesson.

On the other hand, Bob preferred posing a problem and then getting students to come up with different ways that it could be solved. Once this was done they were free to extend this problem in any way they wanted. Bob then collected the work, corrected it and then prepared a worksheet containing a mixture of items, some of which incorporated the students' contribution in the form of a problem for the class to solve. In the case of Chantal students were asked to discuss with those in their group what mathematics was involved in some current events. Subsequent lessons would revolve around some mathematical aspects stemming from the students' suggestions. It was revealed through discussion with the students that although the teacher did not tell them overtly, they felt that she used their ideas in some activities, and hence, many of them felt good about this. As one student explained "when I feel like the activity seems to mean something to me or that it is like something we had discussed in class before then I feel more enthusiastic in presenting my solution to the class". It should be noted that the choice made by the students was confined to the curriculum content which the teachers had already decided to focus upon.

Two very interesting findings about this practice need to be noted:

1. The interview with the students revealed that they felt that their contributions were appreciated and used by the teacher. For example, when asked to list the reasons why they enjoyed the lessons in Amanda's class, the students expressed appreciation for being made to feel part of what happened in class. A typical example is Erin's [$S_{(1,14,2)}$] comment "I feel part of what is going on because on many occasions we [did] some activit[ies] which I feel we ha[d] suggested before"; and
2. All three teachers initially tried to play down their role in getting students to participate in planning certain aspects of the teaching-learning experiences. One possible reason could be that this was so ingrained in these teachers' practice that they felt it was natural. As expressed by Chantal, "I do not see anything strange about this, except that I am still the one who does all of the planning". Bob admitted that he "got a lot of ideas about his teaching from the students", but according to him he was not getting students to help in planning and delivery, but rather encouraging them to participate in their own learning". Amanda expressed a similar view to Bob. Maybe the researcher should have asked these teachers whether they were playing down what had been observed due to the traditional belief that teachers are the ones who are solely responsible for preparing lessons.

Appreciating the Problem Solving Journey instead of Focusing on the Final Solution

As the focus of the study was more on gaining insight into the common practices of these three teachers, instead of their differences, it was deemed necessary to see whether a major common teaching and learning goal pertaining to all three teachers could be identified. It was agreed that if this was possible then it would provide a more unified theoretical platform from which to view all the other common aspects of their beliefs and practices. Hence, a search through their beliefs revealed that although the philosophies of these teachers had some qualitative differences it was possible to extrapolate at least one major common goal: to ensure that all students experienced some success at doing mathematics.

According to Amanda "in order to achieve this" she engaged in a form of "teaching which relies heavily upon making the students feel good". Bob expressed this as a sense of "wanting to create in my students not only a sense of being able to manipulate numbers, but also to cultivate this yearning for problem solving". This is something that is shared by all three teachers as evidenced in Chantal's statement that "I aim to develop in my students this need to want to do mathematics. They might not always be successful in getting the right answer, but if they thirst for problems to solve, I believe that I would have achieved my goal". Another common practice of all three teachers is this seeming preoccupation with getting the students to talk a lot about what they have done, with focus on 'why they did it this way', and constantly asking the question 'is there another way of doing it'. Hence, they were each asked "Why do you engage the students in so much talk?". Amanda's reply, presented here because it is most representative of what they all expressed, highlights their belief that "one important duty of a teacher is to get the child to be ready to solve problems, and to do this they have to be encouraged to communicate what they are thinking to their peers". In all three classes students were made to engage in activities where they had to discuss a lot with their peers, so much so that this was witnessed in nearly every lesson observed, even before and after an assessment exercise; although the use of visual aids was more prominent. In answer to the question, "What is the main reason behind you getting students to discuss their work with their peers nearly every time I've seen you teaching?", many propositions were advanced, but the most common was "to help the kids develop a mathematician's attitude". Since this could mean anything they were asked to clarify what was meant by "developing a mathematician's attitude?" Their answers contained diverse topics and philosophies, but they all included developing the learners' confidence in their ability to explore mathematics, solve problems and be willing to communicate their findings to others.

Assertion 16

Although these teachers believed that it was not practically possible to cater for individual learning styles, they used feedback gained from their students as a means of satisfying each student through their interests and experiences.

Assessing Students' Thinking as a means of Deterring Rote Learning

During the observations the three teachers exhibited different personal behaviours which are too numerous to report here, hence, only a few most prominent ones will be discussed. Although it was expected that effective teachers of mathematics would necessarily encourage relational as opposed to instrumental learning, it was an overwhelming experience to observe the high level of emphasis that these teachers placed on discouraging rote learning. They were always on the lookout for students who might be relying on memorised algorithms to solve problems. Although they did encourage students to use any appropriate method to successfully solve a problem, these teachers were always testing the thinking of students who tended to do well on standard written algorithms. Since on most occasions these teachers were witnessed challenging students to use other forms of algorithms instead of the standard one, it was deemed strange when Amanda gave students some calculations to do and then asked them to strictly use the standard written algorithm. Her explanation of why she allowed this to happen is quite illuminating. She believed that every now and then such tasks must be given "to check the students' performance; especially those who are unable to solve most word problems or real-life problem situations, and to encourage them to compare [the use of standard written algorithms] with other free [non-standard and alternative algorithms] methods invented by themselves or their friends". Although Bob did not do likewise to his students he did take time on various occasions to dwell a bit more on workings done through the use of the standard written algorithm. He said that "in this way I get a chance to challenge their thinking and the way they do operations". Chantal, on the other hand overtly encouraged them to "use this method [standard written algorithm] only if you know what you are doing, how it functions and why you are using it". In the Think Aloud and Stimulated Recall Interview (TASRI), except for one student, all of Chantal's students preferred not to use the standard written algorithm on its own. In fact, except for students with low number sense, all other students from all three classes preferred to use a combination of algorithms.

Then there are those students who whether they were very good or poor at mathematics had a lot of difficulty using the standard written algorithms. These students were not directly discouraged from using memorised rules and procedures when attempting to solve a problem, but every opportunity was taken to remind these students to explain what was hindering them. This was surprising, since all three teachers seemed to prefer their students to use non-standard and alternative algorithms instead of relying on the standard written algorithm. It was even more surprising to observe that in all three classes the teachers tended to encourage such students to use the standard written algorithm. When asked to explain why they seemed to be encouraging these students to use the standard written algorithm the most common answer given could be summarised as *the standard written algorithm is as important as any other algorithm*. To Chantal "the more tools the students have the easier it would be for them to solve various problems". It was acknowledged by Amanda that "students who master only one form of algorithm are at a disadvantage compared to those who are able to use both standard and self-created algorithms". In Bob's view "these students are also being prepared for secondary and tertiary education, and since at such higher level a student's solution steps need to be logical in progression, it is important to also master the standard algorithm". When pressed to explain why, Bob replied that "the standard [written] algorithm requires a sort of logical and systematic presentation of a students working". Nevertheless, all three teachers agreed that those students who rely solely on the standard written algorithm should be encouraged to also use non-standard and alternative algorithms. The reason for that as explained by Amanda is that "if students are not careful it is easy to fall into the trap of learning and working through rote memorisation, without understanding the underlying principles, which is what could happen if they use only the standard written algorithm". Hence, the teaching emphasis was always on relational learning and encouraging students to work through various algorithms instead of only a preferred one. As expressed by Bob this was very important because "for them to be successful in dealing with any challenging mathematical concepts and to be able to solve various types of problems, they must be reminded and encouraged to understand that it is not enough to memorise rules and procedures". This reminding was done through dialogue with individual students and was supported by activities and questions which, in Amanda's words "force such students to demonstrate an understanding of number processes". To this end it was observed that the students were continuously being bombarded with sense making questions as a means of encouraging them to make sense of the mathematics they were dealing with.

The Importance of Problem Solving Strategies

The strategies were taught mainly as possible problem solving procedures. Hence, the students developed a very good working knowledge of the different basic strategies such as 'educated guess and check', 'working backwards,' or "drawing a table'. According to Chantal these strategies were seen as "stepping stones towards more advanced problem solving" which would require students to be "creative and flexible" (Bob) and "analytical and synthetic" (Chantal) and "systematic, organised and evaluative" (Amanda) in their approach. Hence, most of the emphasis was placed upon learning how to analyse, think and solve as many different categories of problems as possible. Amanda stressed the fact that "the most important thing is to constantly put them in situations where they have to apply the strategy in unfamiliar contexts". The unfamiliarity of the context, as explained by Bob, is achieved through "setting problems which require strategies other than the basic ones taught in class". In Chantal's view the teacher must not focus on teaching certain strategies as if they are absolute per se, since "some of these problems are such that students might need to apply a combination of strategies…some invented and some learnt in class".

Since these teachers got students to solve various types of problems to develop the latter's problem solving ability, it was deemed appropriate and effective to use checklists during the observation period as a means of allowing the researcher more freedom to write other field note details — the checklists just needed a tick. Table 4.35 presents a checklist which was adapted from the St Louis University's website at http://euler.slu.edu/Dept/SuccessinMath.html, and which was used to count how often teachers got students to work on these different types of problems. Each time that the teacher gave an exercise or asked students to solve a problem the researcher noted which type of problem was being emphasised or appeared to be most prominent in the exercise. It should be noted that the frequencies in Table 4.35 indicate how often the students were asked to focus on a particular problem type and hence, do not represent a record of the number of problems students had to solve per session.

Table 4.35 Number of problems, per type, observed

Problem type	Frequency per School		
	Amanda (31)	Bob (30)	Chantal (30)
Problems testing memorisation ("drill")	18	21	26
Problems testing skills ("drill")	19	25	22
Problems requiring application of skills to familiar situations ("template" problems)	26	32	22
Problems requiring application of skills to unfamiliar situations (you develop a strategy for a new problem type)	34	40	28
Problems requiring that you extend the skills or theory you know before applying them to an unfamiliar situation.	17	23	8

Note: The number of lessons observed is shown in parentheses

Thinking Beyond the Basic Facts

All 64 students thought that knowledge and skills pertaining to the basic number facts was essential not only for number sense but also in problem solving. It seemed that the students' beliefs had been partially influenced by their teacher constantly reminding them of the importance of the basic facts. Moreover, in each of the lessons observed the teacher managed to find an opportunity to check on students' mastery and recall of the basic number facts and other fundamental mathematical facts. One common belief among the teachers, which also matched the observation data, was that the teacher had to find "ways and means to encourage students to think beyond the basic number facts" (Amanda). An interesting observation, which could be a consequence of such a belief, is that it was translated into actual practice in which each of the three teachers was observed presenting students with at least one new problem everyday. Furthermore, there was a marked difference in the way the problems were designed to cater for the students' ability levels as the year progressed. During the first two terms the problems were designed so that students of different ability levels could achieve success at different stages of solving the problem. Whereas towards the end of the school year not only were all students given the same problems to solve, but also less emphasis was placed on incorporating special aspects into the problems so that the less able could be partially successful. As discussed in other sections of this thesis, these teachers prepared the students to become more self-dependent and self-confident, which appeared to be the main reason why as the year progressed all students were made to work on the same task level. This practice could be classified into two types: (i) partitioning a problem through the use of leading questions; and (ii) altering a problem to cater for different ability levels.

Bob thought that the partitioning of a problem in this way, in the early part of the year, was "extremely important as a means of encouraging these students [the less able] to feel motivated by at least [having] solved parts of a problem". Although, as pointed out by Chantal "this [providing students with problems which they could at least solve partially] was not always possible; it has to be used quite often at the start [during the early part of the problem set]". Although popular belief would be that it is common sense to start off a mathematics task with the easiest problems presented first, it is still interesting to note that all of the exercises and worksheets presented in the lessons observed in the first term and the first half of the second term, started off with problems of which the question was structured in such a way that it would be easy for most, if not all, students to obtain some success. During the first term it was quite common to set out problems as presented in Example 4, which was observed in Bob's class.

Example 4

At Funnymore School they have a fruit lesson day once a year. For one mathematics lesson the teacher asked the children to bring apples to school, and place their bags of apples in a box on the teacher's desk. In the only shop which was near the school, apples were packed in either bags of three red apples or bags of four green apples. A bag of green apples cost $1.20 and a bag of red apples cost 60 cents. The teacher distributed the bags of apples so that in Maureen's group they got three bags of green apples and four bags of red apples.

1. How many red apples were there in Maureen's group? [repeated addition as multiplication]
2. How many green apples were there in Maureen's group? [[repeated addition as multiplication]
3. Without doing any calculation can you state:
   (i) whether there were equal number of green and red apples in Maureen's group; [mental estimation through use of commutativity of multiplication]
   (ii) how did you figure out this answer? [Check for reasoning and understanding]
4. What was the cost of: (i) one red apple; (ii) two green apples; [division and multiplication, unitary proportion]
5. What is the total cost of the apples in Maureen's group? [Totalling money]
6. What would be the total cost of the apples in Maureen's group if the number of green and red apples in a bag were reversed? [Open question – implications of interpretation]

The interview with Bob revealed that in this example the teacher was trying to gauge students' understanding and application of the following fundamental mathematics concepts and principles:

- Repeated addition can be speeded up through multiplication;
- Using known basic facts such as the commutativity of multiplication to aid mental estimation and computation;
- Practice using the concept of unitary method (multiplication and division) to solve problem involving direct proportion;

- Simple money computation are carried out in the same way as ordinary basic calculations involving decimal numbers; and
- Open questions could have various interpretations. Students need to know how to make and justify the most appropriate conclusion.

The practice of partitioning the problem through the use of leading questions, as a means of encouraging all students to at least be partially successful, was not confined only to addition, division, multiplication and division problems, but also to extended application of these four basic operations, as shown through the fraction problem presented in Example 5, which is from Amanda's class. Although, due to space limitations, only one example each from Bob's and Amanda's class have been presented, they are both typical examples of what all three teachers tended to embed in the problems they set the students during the early part of the year or as introductory problems in a set of exercises. Nevertheless, as time progressed the use of leading questions in the problems were reduced and eventually disappeared altogether by the latter half of the fourth term. Therefore, problems which contained leading questions, such as questions 1 and 2, and 4(i) from Example 1, were seldom used in terms three and four.

Example 5

Brenda had to share a pizza with her three sisters so that each would get an equal share. Unfortunately the dog ate the original share of Mary's pizza. And when Mary started crying each of her three sisters gave her a half of their share.

(i) How many sisters were there altogether?
(ii) How many parts would Brenda have cut the pizza into?
(iii) What fraction of the pizza did each girl receive before the dog ate Mary's share?
(iv) What fraction of the original pizza did each sister give to Mary?
(v) Jack says that finally Mary had three eighths of the original pizza, while Phil said Mary must have received a half of the original pizza; who was telling the truth, Jack or Phil?
(vi) Merna and Frank did not agree with either Jack's or Phil's answers. Merna thought that Mary received $^{12}/_{32}$ while Frank though Mary received $^{12}/_{24}$ of the original pizza. Who was right between Merna and Frank?

Another important revelation in relation to the students' learning of the basic facts was the emphasis placed by the teachers on repetition through various modes. The most common aspect of this pertained to both the teachers' and students' perceptions that recall of multiplication facts was very important. It was observed that since the mathematics curriculum is quite extensive, the students were not encouraged to memorise their multiplication tables in the traditional way. Instead, these teachers tended to have a basic fact memoriser component built into their teaching repertoire and exercises, as indicated in Example 4 and Example 5. Some typical examples were getting students to apply their understanding of multiplication facts through games, investigations, puzzles or word problem solving. Of particular interest here was the use of problem solving for the purpose of enhancing factual recall. This was done in very well organised phases, based on the 'principle of readiness'. The analysis of the observation and interview data indicated that there were five common aspects employed by these teachers when planning and implementing problem solving experiences which were used to enhance students' memorisation and subsequent recall of both basic and advanced facts. In the majority of cases it seemed that the teacher would:

1. Prepare a major problem;
2. Extrapolate simpler problems from it;
3. Distribute these problems as per her perception of a student's group ability;
4. Try his/her best for students from different groups to not be aware of what others were doing; and
5. Increase the difficulty of the problem as the student's understanding improved

During this process the teacher would be looking for signs that indicated improvement in self-confidence. He or she would test the students by asking them to volunteer to go to the board to explain something. For those who would not volunteer the teacher would get them to 'privately' explain their solution to him or her. Then the teacher might make the following encouraging statement to the class "[student's name] has solved this problem in a very interesting way. Would you like to share your method with us [student's name]?" In the case of Chantal, as the students would gain more confidence she would move them to another group to work with others who she judged to be at a similar level or higher. Bob and Amanda rarely moved students around in this way, although a student would sometimes be encouraged to work with a teacher-designated partner so that they could help each other.

Another method used frequently by these teachers during the early part of the year, to enhance students' mastery of the basic facts, was providing students with different versions of the same problem, which had been altered to suit the observed performance level of the students. For example, students might have been asked to solve the following pattern problem:

> Joel's father has started a small tennis ball distribution business, called Tenidis. He first prepares small plastic bags of six balls each. Five of these bags are then placed in a small paper tray. Eight paper trays are then packed in a big carton. These cartons are then placed in wooden cases, which are finally loaded into metal containers. Mister Briggs runs a sports shop in Perth. Once every six months he receives a container of 12 large wooden cases of tennis balls from Tenidis. How many tennis balls does Mr Briggs receive in a year?

Usually the teacher would give this original problem to all the students in the class, and then go round to monitor their progress. The teacher would spend more time with anyone encountering major difficulties in solving the problem as it is, and if he or she deemed it necessary might then do some *in situ* alterations and present an altered form of the same problem to the respective student. Hence, the problem just presented above would now be presented in a simplified form as follows:

> Joel's father has started a small tennis ball distribution business, called Tenidis. He first prepares small plastic bags of two balls each. Three of these bags are then placed in a small paper tray. If you buy four full tennis ball trays from Tenidis, how many tennis balls would you have altogether?

It is worth noting that whenever the teacher felt that certain students could be struggling with a problem, the latter was either altered or leading questions inserted into it *in situ*. This point needs to be re-emphasised since the teachers stated that in most cases they would "rarely prepare alternative questions for supposedly weaker students", as expressed by Bob. The use of the term 'supposedly weaker students' warranted some clarification, since usually the teachers were explicit in their use of 'weak' or 'high' ability. Interestingly Bob explained that "a student might be weak for certain problems but not others", and he also explained that this was one reason why he preferred to alter the problem *in situ*, "because you are not always sure how they would respond". It was a common belief among these teachers, as explained by Amanda, that "students could be very surprising in their performance", which was why she thought "I use the term weak, better, good at, very sparingly, for when these girls are motivated they find it easier to recall what they've learnt [such as the basic facts] and how to use them". According to Amanda that's why she prepared activities which would motivate them to learn. Chantal mentioned that "as a teacher I have to be ready to make appropriate changes as the need arise [*in situ*], for these children need immediate attention. There is not much time for me to go home and think about it when I could have solved it on the spot". Chantal went on to explain that this was one reason why she thought that "mathematics teachers, and any teacher for that matter must have very good knowledge and understanding of what they are getting the students to learn". Amanda's explanation was also in tandem with Bob's and Chantal's, as she stated that "a teacher has to be able to think on her feet [*in situ*]. If she cannot do that then she might lose some very important opportunity to engage the students in meaningful learning situations". It was also a common belief among these teachers, as expressed by Amanda, that "I often have to think very fast on my feet, to help the students resolve some issues which I could never have foreseen". Hence, one main reason given for the belief that it was "important that the teacher can assess the various queries and challenges posed by the students on the spot" (Bob), was that "once you allow students to participate freely and actively in the lesson you have to be prepared to deal with the unknown on many occasions" (Chantal). Nonetheless, the teachers did accept that it was not always possible to remedy all situations *in situ*, although they still insisted that in most cases it should be possible.

Assertion 17

Teaching-learning experiences were designed in such a way that the basic facts would be encountered in almost every lesson through games, puzzles and problem solving activities.

Assertion 18

Alterations of problems were done *in situ* whenever there was a need to adapt the problem being solved to the student's present understanding and performance.

Procept Formation: Linking the Symbol to the Process and Concept

A common practice of these three teachers was the emphasis they placed on getting students to be well grounded in how a symbol was just another representation of a concept, which in turn was very closely linked to a particular process. It was noted that none of these three teachers hesitated in using the exact terminologies (symbol, concepts, processes) although this was done in a gradual manner. Throughout the observations they insisted that students must understand the meaning of any mathematical symbol that they had to use, as shown in Excerpt 18, which presents extracts of typical answers to the question "why do students have to understand what the processes, concepts and symbols mean?"

Excerpt 19

Amanda:
A symbol is a very complicated thing to understand, and unless the children understand what it means there's no way they can appreciate how it is linked to the operations and computations that they do. The task of learning is made a lot easier for these girls once they know what they are dealing with, why they are like that and how to apply them. Of course I don't think that the teacher must get them to understand everything, but at least they must be at ease with understanding and using the symbols and concepts with confidence. How will they be self-confident if they do not know or have not had the opportunity to appreciate their [symbols, concepts, processes] meaning and use?

Chantal:
Those students who know what the symbols stand for and can explain their understanding find it easier not only to retain the facts and strategies they have learnt but also apply these in new situations. They can do this because they can manipulate these ideas [concepts] which gave rise to the respective symbols. I don't mean that I have to spend my time getting students to learn these facts [about symbols, concepts and processes] by heart, but I try to make it an enjoyable experience for them. They learn the concepts and processes through games, solving problems. What I do is to get them to see that in fact without the symbols mathematics would be extremely difficult to communicate.

All three teachers thought that mathematics sessions were not to be only about teaching students how to solve problems in a "typical lesson delivery fashion", as expressed by Chantal. To them it was important to make allowance for students to engage in discussion about mathematics itself. In some cases students were asked to carry out a research on a mathematical issue, such as symbols, and then to present that in class. Students were encouraged to discuss relevant mathematical issues such as why use symbols, and to create their own symbols for the concepts and processes they learnt, as exemplified in Vignette 5.

Vignette 5

In Bob's class the students had been discussing whether they thought they had the right to invent their own mathematical symbols and why the same mathematical symbol was interpreted in the same way in most places around the world. This was followed by Bob writing '2 x 3 + 2 → 2 ⌂ 3' on the board and asking students what the operation symbol '⌂' meant. After much discussion students were asked to work with their peers to come up with an appropriate word statement. Students proposed various statements such as:

1. Multiply a certain number by 3 and then add 2 to the result';
2. Do twice a number. Then plus two
3. Twice the first number added to two'
4. Multiply the first number by 3 and then add the result to the first number.

Bob then asked the class to see whether their definition fitted the expression '3b + 2' '3b + b' or '4b'. Some students (Mainly those with high number sense) argued that it cannot be 3b + b since "this could easily have been written as 4b". Bob did not comment but asked the others what they thought. Another heated debate ensued for about four minutes. Since there were now two distinct camps — one favouring '3b + 2' and another favouring '3b + b' — Bob wrote another expression; 4 x 3 + 4 → 4 ⌂ 3. Students went into debate mode again, but after three minutes about 64 percent of the students supported the expression '3b + b'. Bob got students to work in groups of three or four to discuss why '3b + b' was different from '3b + 2'. This eventually led to all 14 students accepting '3b + b' as the most relevant expression, upon which they were asked to explain why — which they did to Bob's satisfaction. Some students suggested that the first term of the numerical expression be written as '3 x 4' and not '4 x 3'. They were asked to convince the class, which they did. Students then went through virtually the same pattern of discussion for the expression 6 + 12 – 3 upon which they eventually settled for Δ6 as the condensed version for the explanation 'Add a number to twice itself and then subtract half of the original'.

It is interesting to note that the students were surprised when Bob told them that he had invented this symbol himself, and that it was not a conventional symbol. Although they had already agreed in the previous discussion that anyone could invent a mathematical symbol most of the students seemed taken aback by Bob's revelation. As one student, Peter [$S_{(2,28,1)}$], put it "I still thought that we were not allowed to make our own symbols in this way, [although] it's true…we [did] discuss that we can invent them".

Some interesting points to note from Vignette 5 are:

- The teacher directed the flow of the lesson according to the topic under scrutiny and how students responded [Teacher acting as delegator]
- The teacher did very little talking and allowed students to do most of the discussion [Teacher was more like a facilitator].
- The teacher engaged students in meaningful discussions and acted as an intermediary who simultaneously mitigates by channelling all students' contributions, advises students as to possible consequences and solutions, and proposes new insights through the 'what-if' approach [Teacher acts as an expert]. For example, after some contributions from the students they were asked to discuss why we sometimes need to have new symbols and whether they thought they were going to meet new mathematical symbols when they go to Year 8. Since students could come up with virtually anything, the teacher had to be prepared to deal with both the expected and the unexpected.
- The cyclic transaction, from process to concept to symbol, did not therefore have a definite starting point since students were simultaneously acting on something they were doing [applying the process of the operation]; and the result of the action of applying this process as a new idea [concept]. For example, by carrying out the operation 2 x 3 the student is doing the process of multiplying, while the activity is such that it forces the student to hold the result of this process as a product [concept] somewhere, as he or she attempts to add two to this product. While all of this is going on the learner is also engaged in using known symbols which are then transformed into a new symbol. This evokes the concept of a *procept* as explained by Skemp (1971).

It is also interesting to note that in all lessons observed the discussions involved students in breaking the concepts and processes into components which can be explored individually so that students understand how the whole function [Analysis]. While the majority of the follow-up exercises — of which a typical example is presented in Example 6 — got students to apply what they had learnt in a new context [application]; be creative [work at the synthesis level]; show that they understood the respective concepts and processes [comprehension]; and solve at least one word problem. The students were then asked to do an exercise which followed from Vignette 5, as shown in Example 6.

Example 6

- Make up your own symbol to represent the following mathematical operations (e.g. (a) 6 + 2 + 3 (b) 10 – 5 x 2 (c) 18 ÷ 9 x 2)
- You are given the following English language statements: (a) Half a number added to one third of that number; (b) divide by four then add six.
  (i) Write a numerical expression using the conventional symbols for each of these two statements.
  (ii) Condense each of your expressions into one which has only one invented symbolic expression.
- Anna has invented a new kind of thermometer called an 'Annametre'. To convert from 'degrees Celsius' to 'degrees Anna' you have to multiply the temperature by 5 and then add 3 to that. Anna used the expression 'C * 3' to convert from Celsius to Anna.
  (a) What does the symbol * mean?
  (b) Which of the following is equivalent to Anna's expression?
  (i) 3 + 5C (ii) 5C + 3C (iii) 5(C + 3)
  (iv) 5C + C + 3 (v) 5C + 3

Another interesting observation pertained to how the students were engaged in exercises where they were required to solve problems in which the traditional mathematical symbol had been given a meaning that was totally different from its basic conventional meaning. Example 7 provides a microcosmic view of an exercise in Amanda's class which highlights how the teachers would invent an interesting story which appealed to the contemporary environmental experiences of space travel of most students. Amanda stated that such an exercise was used "to help them [students] develop a sense of flexibility in the way they interact with operations and number symbols".

Example 7

Jane was staying on planet Naz when she was asked to attend a maths class. There she learnt that two apples add five apples make ten apples. One afternoon she bought a hat for 6 Naz dollars. She gave the shop assistant a 10 Naz dollar note, and then heard the assistant say "10 dollars divided by 6 dollars is four dollars". Jane was very surprised, but did not say anything.

When she got back to her hotel she discovered some inscription in a wall which read like this:

(i) 2 + 2 = 4	(ii) 5 + 3 = 15	(iii) 2 + 3 + 4 = 24
(iv) 72 – 6 = 12	(v) 8 – 2 = 4	(vi) 39 – 3 = 13
(vii) 18 ÷ 3 = 15	(viii) 7 x 3 = 10	

The next day she found some more inscriptions, but this time a number was missing from each equation.

1. Write the missing number in the space.

(a) 3 + 3 = ---	(b) 8 x --- = 1	(c) 16 ÷ 8 = ---
(d) --- + 9 = 36	(e) 2 – 1 = ---	(f) 26 ÷ 2 = ---

2. Why was Jane surprised when she heard what the shop assistant said?

Hence, although each teacher taught in a different way, it seemed that when it came to linking the process, concept and symbol there was a common pattern, although this was not always in the order suggested. In fact the intention here is not to present a definite sequence but rather the aspects of this whole cycle, since when it came to process and concept formation both were presented simultaneously, with the symbol coming most often after these two:

1. Present a problem;
2. Try to solve a single aspect of the problem;
3. Describe the process involved;
4. Describe the concept which is evoked by this process; and
5. Suggest a symbol to represent both the process and the concept

The Relationship between Teaching, Assessment, and Students' Learning

Since, the interview and observation data had revealed that teachers thought that number sense (NS) and problem solving (PS) were also related through the way they were assessed, it was deemed important to find out how these teachers went about assessing particularly for number sense and problem solving. The observation data was analysed for frequency of questions asked by both teachers and pupils, and although this was in itself a very time-consuming exercise, the results were quite interesting. It was noted that all three teachers employed both higher-order and lower-order questions to engage students in discussion and problem solving activities. In a typical lesson the ratio of higher-order to lower-order questions was about two to one. The analysis revealed that students were assessed mainly on their comprehension, application, analysis and synthesis of the respective number sense concepts, processes and contextual implications. Figure 4.11 shows how the three aspects of number sense concepts, processes and contextual implications were relatively assessed.

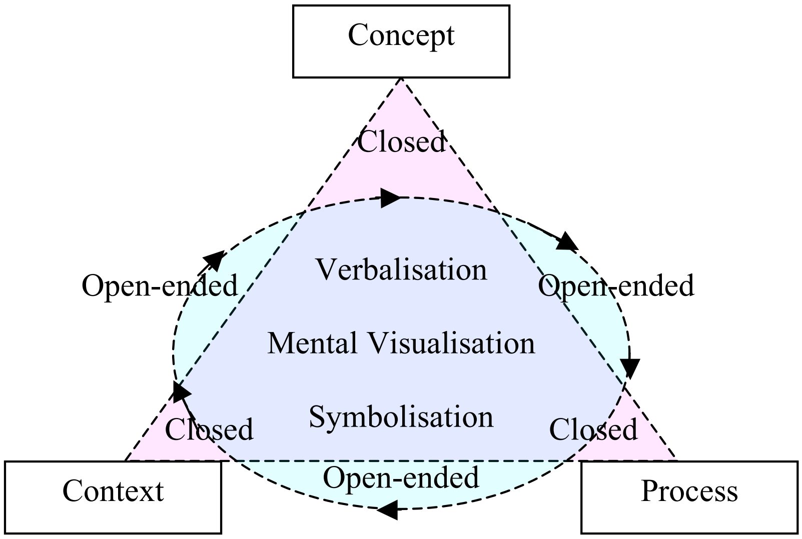


Figure 4.11 The relationship of types of problems, medium of expression and number sense aspects

A combination of open-ended and closed tasks was used to challenge students to think beyond the confines of the three aspects; the concepts, processes and contextual representation of the numerical problems they had dealt with. The problems were mixed in such a way that some of them were more closed or open on one or more of the three aspects than on others. According to Bob:

> this also provided the less able [lower number sense] students with a way of gaining confidence as they engaged in the problem solving activities, since such students would not always cope as well with open-ended items as would the more able [higher number sense] students.

The most common medium of expression through which students were assessed were in the form of questions which elicited:

- Students' understanding of facts, procedures and context of the problem through verbal interactions between teacher and student and student with other students;
- what students were thinking about the different number sense aspects under discussion, and how they might visualise these [Mental visualisation]; and
- how students interpreted, manipulated and applied the symbols when solving the problems given [Symbolisation].

When interviewed about the forms of assessment they employed all three teachers claimed that a variety of assessment methods and instruments would be a lot more effective in providing a "clearer picture" of a student's ability. Such a belief was in tandem with observation data which indicated that these teachers assessed students through various modes as shown in Table 4.36.

Table 4.36 The most common assessment modes used by the teachers

Assessment Mode	Teacher Amanda	Bob	Chantal
Portfolio	4	3	4
Self-Reflection	3	3	2
Problem Solving Exercise	3	3	3
Peer Evaluation	2	3	3
Journal	4	4	4
Written Test	3	3	2
Interview	1	2	1
Investigation	3	3	3
Project Work	2	2	2
Cooperative Group Work	3	2	4
Student Board Explanations and Demonstrations	3	3	2
Debates and Discussions	3	3	3

Note: 0 = Never; 1 = Seldom (Above 0 but less than 40% of the time); 2 = Occasionally (Above 40% but less than 70% of the time); 3 = Often (70% to 90% of the time); 4 = Always (Above 90% of the time).

In response to the question, "Which assessment strategy could be the most effective and accurate in providing the teacher with a more authentic picture of a student's number sense ability?" teachers expressed a preference for interviewing students. Nevertheless, they expressed their concerns that time constraints would not permit them to interview students as much as they would have preferred to. Both teachers and students felt that using various forms of assessment tools and methods helped not only in catering for individual differences in the students, but also to monitor both teaching and learning progress, and as expressed by Bob, "to create opportunities for more quality teaching and learning interactions". In view of the teachers' expressed willingness to diversify the assessment strategies and instruments as much as possible, they were asked to explain why they thought this was important. Chantal thought that "using a single form of assessment [would] give me only one perspective about my students' performance" and she thought that her judgement would thus not be as accurate as if she was obtaining information in various ways from various sources. Bob expressed a similar view and added that "the traditional [assessment] instruments used focus more on how students perform on written tests" and according to him this fails to explain "why a student might be doing very well, or why another student might not be doing so well". Amanda's response, which reflected the same belief, highlighted the importance of "knowing each student's personality", which she felt could be gauged through “interviews, listening to students as they discuss a point, and observing their reactions and body language”. Students were asked about their experience of being assessed before they came to Year 7 and to compare that to now. The interview with the students revealed that they did not feel the pressure of assessment, which could be due to the fact that they were not formally assessed through written tests, as highlighted in the typical responses presented in Excerpt 19, Excerpt 20 and Excerpt 21.

Excerpt 19

Terry [$S_{2,26,1)}$]: I used to fear having exams. I even feared home work and the written work [assignments] in class. Here [in this class] we do a lot of exercises [and] home work. We take part in mathematics competitions as well. But this is most often fun. The way Mr [Bob] does it doesn’t put me under pressure. I did very well for the latest [mathematics] competition, although I was a bit surprised because I was not afraid [during the exam].

Excerpt 20

Lena [$S_{(3,54,2)}$]: Mrs [Chantal] told us that we are being assessed everyday. So, I have got used to it because that’s what she includes in our portfolio; the things I have done. I think this is more realistic because it does not show only what I did in the exam [but it] also shows what I’ve done during the lessons [term]. Now I do not bother about exams because I have to be prepared all the time. Sometimes she comes in and asks us questions which makes you think a lot. I usually do well on these problems, but I think if it was like [what I was used to] before [joining Chantal’s class] I would not have really enjoyed doing it.

Excerpt 21

Nanette [$S_{(1,18,2)}$]: I’ve never feared exams, but I enjoy it a lot more nowadays. We do not do exams as such in class, but I am part of the team which participates in mathematics competitions, and we have to sit those tests which are like exams. We have a lot of home work exercises as well, but these are really fun to do because the teacher explains everything when we do the corrections in class. She never gets angry. She asks us questions about what we have been doing and also how we solved a problem. So, she knows more about us, which is good because sometimes the teacher does not know what you are thinking or why you got something wrong. There’s many positive things. Yep, I enjoy it [the way they are assessed].

The observation and interview data revealed that these teachers used an amalgamation of assessment methods to assess for number sense through a problem-based teaching-learning experience. Figure 4.12 presents an illustration of what the teachers meant when they said that NS and PS were also related through an assessment method where teachers tried to gauge the extent to which students were able to make connections in their learning of number sense and other mathematical aspects.

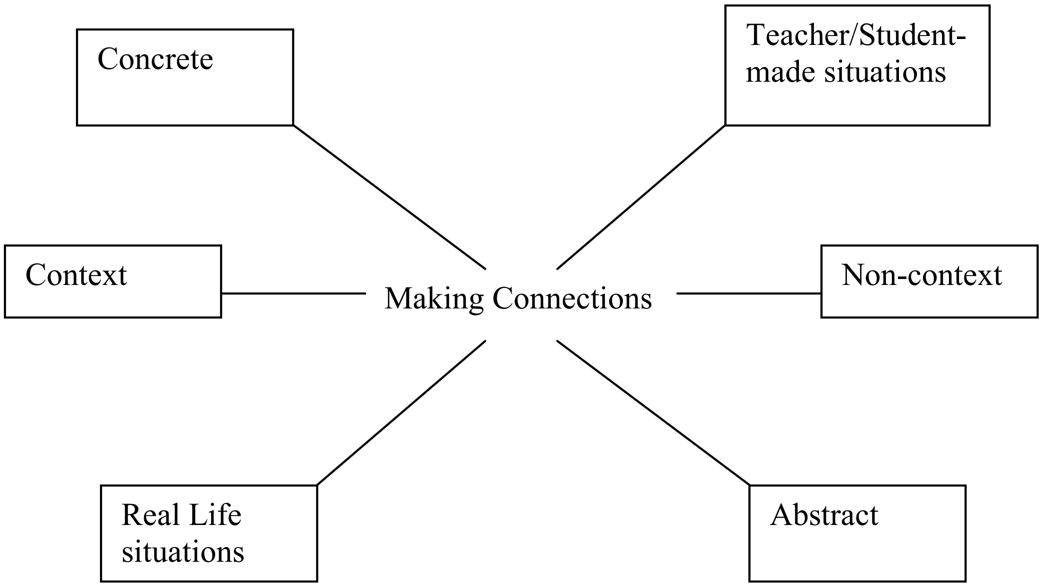


Figure 4.12 Six major assessment connectors commonly used by the teachers

Content and observation analysis of the tasks given revealed that although there were many factors that were considered and employed, in the problems given, which served as assessment cues, only six of these factors were commonly used by all three teachers in at least 85 percent of the activities and assignments given. In this assessment method there were three pairs of related connections which were used to assess students' number sense and problem solving: (i) concrete and abstract mathematical situations; (ii) real life and artificial situations; and (iii) context and non-contextual problems. As an example of how this assessment process works, an observed lesson in Chantal's class is used in Vignette 6, Vignette 7 and Vignette 8. It should be noted that throughout this assessment exercise all three teachers would be noting down things either mentally or in writing about the students, just as exemplified through the vignettes from Chantal's lesson. The most striking observation is that in a large majority of cases, assessment and teaching-learning experiences were not separated. In fact they were treated nearly as one and the same thing; the teacher seized on the teaching-learning experience as an opportunity to assess the students through interacting verbally with them and also monitoring their written work and how they engaged in mathematical thinking.

Vignette 6

In a previous lesson Chantal had provided students with various small objects, such as counters [concrete], and asked each student to place half of the counters on one side and the other half on a sheet of A4 paper [Teacher-made situation - Context]. She then asked each student to compare how many objects there were in his/her half with the number in someone else's half. They were asked to write their half as a fraction [Abstraction-symbolisation – Non-context]. She then asked them questions such as "What did you do?", "Why did you write four over eight and not eight over four?" [Linking concept and process – Sense making – non-context]. Students were then asked to compare their fraction with those of others close to where they were sitting. Issues were discussed as to whether there were similarities and differences between their friends' halves and their half [making connections & abstraction]. The same activity was repeated for other fractions such as thirds, quarters, ninths, thirteenths and so on [generalising]. Students explored many fractions through symbolically writing the numerals [forming 'procepts' – Non-context]. Students were asked to visualise fractions which were equivalent and how these related to each other, and then explain what they saw to the class [Mental Visualisation]. Other activities took place which will not be elaborated upon here.

In Vignette 6 Chantal moved from the concrete to the abstract through a teacher-made situation, in which she also got students to interact with fractions in context and also non-contextual situations. She got students to work in the abstract at the same time that they were working with concrete materials, hence encouraging them to connect the two as soon as possible. Students were also involved in comparison tasks which helped them to make the connections between equivalent fractions and to eventually make generalisations. A common practice was for the teachers to get students to make use of mental visualisation as a means of engaging mathematically with the process, concepts and context of the problem.

Vignette 7

In the subsequent lesson Chantal asked students "What would be an equivalent fraction to ¾?" [Making connections – Working in the abstract – Non-context]. Once the answer was given she asked the respective person to explain what he/she did to get the equivalent fraction [sense making] how secret agents would send each other encoded messages to be deciphered or decoded [Real life context]. Chantal got students to think through questions such as "What are you actually doing to three quarters to get thirty fortieths?" [Sense making liked to shortcuts]. "Try to see this in your mind's eye" [Mental visualisation]. After having made sure that students knew what was done to change a fraction into an equivalent form, and why, Chantal used a number line and got students to position fractions of the same denominator on it in relation to 0 and 1 [Transfer of knowledge - Context], and then asked them what whole number represents eight eighths [Making connections and sense making – Non-context]. Other activities took place, such as:

- students being made to count properly in eighths, tenths, seventeenths, etcetera, up to a whole [Relevance of terminology, concept and process];
- comparing the denominators of equivalent fractions, where students revised the concept of denominator, multiples and factors, least common multiple, highest common factor [Relevance of terminology, concept and process]; and
- how to convert a fraction to another equivalent fraction, which brings into play the concept and process of cancellation, and the distinction between odd and prime numbers [Linking application and short cut methods – Non-context].

In vignette 7 Chantal gets students to make connections between equivalent fractions by encouraging them to think about this concept in the abstract. She then used a real life situation as a means of concretising the concepts, processes and contexts she wanted to assess. Number sense-making formed a central part of the whole teaching-learning and assessment exercise, and this sense-making was used as a stepping stone to getting students to discover and understand what short cuts to use, when and why. This observation is in tandem with Amanda's belief that "once students can make sense of the mathematics they have learnt it becomes easy for them to understand the short cut". Another interesting issue from Vignette 7 is how Chantal engages students in discussions about relevant terminology and how these are linked to the concepts and processes applied to solving the problem. Most of the work revolves around non-contextual situations. The teachers always seized on opportunities to get students to clarify their understanding of related concepts. For example, when getting students to explain how they went about finding the lowest common multiple, Chantal met with students whose notion of the concept of an odd number needed to be refined so that they would see that what they were talking about was more specifically called prime numbers.

Vignette 8

Towards the end of the lesson Chantal used a worksheet [Teacher-made situation] to engage students in the LINE UP activity, which required the students to draw lines to link a fraction and its reduced form [Abstract symbol]. As a straight line is drawn from one fraction to its equivalent a letter and number are covered, and these are then entered in the appropriate box so that when all equivalent fractions have been linked a message will be decoded [Visual exercise].

Vignette 8 reveals that Chantal was using teacher-made situations to engage the students in solving the problems she was posing. The activities were always very carefully selected and as evidenced through Vignette 8, they usually channelled the students' learning focus towards applying what they have learnt through abstracting the mathematics. This was achieved through some enrichment activities which according to Amanda "removed the students focus from how difficult the problem could be, to how much fun they can have in solving this problem". The use of teacher-made situations, as explained by Bob, was seen as being "very appropriate and effective in helping students move towards the abstract". A closer look at Chantal's teaching reveals that she was using visual aids as a way of accessing what students could be thinking or what they were mentally seeing in their mind.

The teachers were always getting students to make connections, and through observations and on-the-spot interviewing, they assessed the students' competence at making such connections. The activities and interactions presented in Vignettes 6, 7 and 8 also indicates that the teachers prepared tasks which got students to navigate between context and non-context situations. This was also assessed through how they managed to make the connections between context and non-context number sense and other mathematical problems. According to Bob “it is important to get students to link every little piece of mathematics they learn” since he saw this as “one powerful way of assessing not only how much they are learning but also the quality of their learning. Hence, as pointed out by Amanda, making “connections is employed as a major component of developing students’ sense-making ability”. The use of visual imagery and presentations was seen as a very powerful assessment tool since according to Chantal “it opens a window into the child’s mind which I would normally be unable to access through conventional pen and paper tests”. Hence, number sense and problem solving were simultaneously assessed in various non-conventional ways so that students would often not feel the pressure of being assessed. When the teachers questioned provided various explanations as to why they used such an assessment method, but the most common answers indicated that the aim was most often three-pronged and focussed on:

- getting as much authentic information about the students’ performance and ability as possible;
- assessing number sense and other mathematical aspects through problem-based assessment;
- and linking the teaching-learning experience and assessment in such a way that students would enjoy both doing the mathematics and being assessed.

The interviews with the students tended to support these results in that students felt that most often it was not easy to discern between teaching, learning and assessment, since all three seemed to be happening all at once.

Moving from Pedagogy to Andragogy

Another factor which could have contributed to the improvement in student’s number sense and problem solving performance could be the way in which students were increasingly given more control over their own learning. The observation data indicated that as the terms progressed the teacher tended to talk less, provided fewer instructions, and praised students in a softer manner than previously. It was simultaneously observed that students were gradually given a lot more opportunities to suggest ideas for subsequent lessons, provide feedback to their peers, engage in self-correction of assignments given and encouraged to ask more “what if” questions. This discovery seemed to suggest that the teacher was slowly relinquishing most of the power and vesting students with more control over their own learning. Hence, the teachers were questioned about what was being observed. Bob pointed out that "when students first come in you don't know them enough. Therefore, to get a starting point I have no other option but to teach them according to their age level". Bob went on to explain that through the various tests, conversations with students and other teachers, and observation of his students he then started understanding "how developed and confident they are to take responsibility of their own learning". Chantal's view was based mainly on "an efficient teacher must search for avenues to extend her students; be it in their performance, their thinking or their competence". She reiterated Bob's point that "although the curriculum framework document stresses outcomes based education, it is difficult to start off from a student's level or standard of performance when you first get them". When asked why she thought this was difficult she explained that "I tend to treat the class as a whole when I first meet up with them, and the only way to do that is to know the year level [age] they are in". Amanda's explanation was even more striking in that she maintained that "it's good to have standards, but the first time that I get a new group of students I think it would be wrong to assume their standard of performance from tests alone". She felt that the teacher needs to get acquainted to the students first and the best way to do this is to "treat them at the same level according to how old most of them are". Further analysis of these teachers' answers indicated that they seemed to reason around whether to treat students according to their ages or according to their level of mathematical maturity, and their final verdict was to start off by considering their ages. When asked, "Which student characteristic do you think is most important to the success of your teaching; focusing on:

1. learning maturity; or
2. chronological maturity?"

The teachers tended to favour a combination of both, with focus on chronological maturity as the initial factor and then gauging and working according to the student's learning maturity. This practice could be compared to starting off by teaching the students as dependent individuals with little experience, in a semi-subject-centred system where they were told much of what to do, and most of the motivation came from the external where rewards in the form of verbal praise were lavished on the students for the slightest contribution. As the teacher and students got to know and trust each other the students were empowered through having more say in their own learning, treated as if they were very experienced (based on genuine information gathered by the teacher), lessons prepared according to their perceived needs, encouraged to ask and answer higher order questions, taught more through a problem-based system, and encouraged to motivate themselves intrinsically and engage in more problem posing. Hence, it seemed that the teaching and learning experiences moved from a pedagogically oriented one to a more andragogically oriented one, in which students had more ownership of their own learning.

Assertion 19 All three teachers' teaching style tended to move away from being pedagogic towards being more andragogic as the year progressed.

Assertion 20 Students were empowered by having more input in how the lesson was prepared, delivered and enhanced, and also by taking more responsibility for their own learning through problem posing, self correction and intrinsic motivation.

Assertion 21 The teachers' teaching styles were simultaneously proactive and reactionary; when they programmed the learning experience according to what they knew about their students' readiness and when they instantly adapted their teaching to fit in with students' immediate response, respectively.

Grouping Students

Since Bob and Amanda employed a less sophisticated and more familiar group work approach, it was deemed more insightful to report on Chantal's grouping methods because it was the only one of the three which was unique while simultaneously producing a similar atmosphere of wellbeing which was present at the other two schools. In Chantal's class there were two main groups. Within these groups students would normally be paired with a similar ability partner, and such pairs would form part of a smaller group of four students. Although the class was split into two major groups, there were in fact three groups which could be identified; a very strong group on the far right, a fairly-good-at-maths group in the middle right hand side, and a not-so-good-at-maths group on the far left. Although the only overt groups were the main two, the teacher inconspicuously worked with and treated the class as four distinct groups.

Students were mostly promoted to the 'better' group and very rarely would there be a 'demotion', and even when there would be one it would be done in such a way that it was not seen as such, as evidenced through an informal interview with a student from Chantal's class, who was 'demoted' to another group. Although the information presented in Excerpts 22, 23, 24, 25 and 26 is from one child only, it is used here as a typical example of how students felt when made to work with others of lesser mathematical ability than their's. It is important to note that such a situation was not observed in the other classes, although it had many similarities with students being made to work with those of a lower mathematical ability in both Bob's and Amanda's class.

Excerpt 22

Res:	Why are you working in this group today?
Jono [$S_{(2,3,1)}$]:	I have not been working, erm, not doing well, as expected.
Res:	What do you mean?
Jono [$S_{(2,3,1)}$]:	I was getting too many things wrong.

There seemed to be an acceptance here, in both his body language and voice intonation, that the teacher did the right thing. The discussion was pursued for more clarification, and what came out tended to show that, according to the student, the teacher was justified in her actions.

Excerpt 23

Res:	What things?
Jono [$S_{(2,3,1)}$]:	Like, I'm good with fractions,…it's easy, yeah most of the time I work well, I...
Res:	You like working with fractions?
Jono [$S_{(2,3,1)}$]:	Yes, but I did not do my home work properly; I did not have time to do it.
Res:	Why?
Jono [$S_{(2,3,1)}$]:	I played basketball. I played and rushed…just…I did not have time for my homework.
Res:	So, what happened?
Jono [$S_{(2,3,1)}$]:	Mrs (Chantal) has asked me to sort my fractions out and then I can go back to my place.

It seems that this action was taken as a means of:

- getting Jono [$S_{(2,3,1)}$] to see that he was not performing up to his potential;
- helping him rediscover his ability to work with fractions;
- reflect upon his misbehaviour.

What follows further reveals that the teacher's action seemed to have more positive connotations than negative ones. In Excerpt 24 Jono [$S_{(2,3,1)}$] pointed out that he was gaining something in terms of not only helping seemingly less able students but also getting help from them on aspects such as solving drawing problems, which was not his forte.

Excerpt 24

Res:	How do you feel about being in this group?
Jono [$S_{(2,3,1)}$]:	I can help. And they help me as well.
Res:	How do they help you?
Jono [$S_{(2,3,1)}$]:	Like. Like…they are good with those drawing problems.
Res:	You do not like drawing problems?
Jono [$S_{(2,3,1)}$]:	I like them, but…I prefer to work with, adding, multiplying things and…
Res:	Do you prefer working with numbers?
Jono [$S_{(2,3,1)}$]:	Yes

An interesting point to note from Excerpt 25 is that higher ability students seemed to prefer not only to work faster than the lower ability students, but also to solve number sense inherent problems. Moreover, in similar fashion to Jono [$S_{(2,3,1)}$], many of the high ability students who had worked with those of lesser ability, were very explicit in the explanation of their preference for solving number sense inherent problems, although "working faster" was not a common theme. In fact through subsequent interviews it became apparent that wanting to work faster was more like a perception which was mismatched with the student's belief, as exposed through the interview with Jono [$S_{(3,3,1)}$] presented in Excerpt 25.

Excerpt 25

Res: Would you like to go back to where you were before…in the other group?
Jono [$S_{(2,3,1)}$]: Yes, but I also like it here.
Res: But why would you like to go back?
Jono [$S_{(2,3,1)}$]: Yes, because…we work faster over there, and we help each other with numbers.
Res: You mean number problems?
Jono [$S_{(2,3,1)}$]: Yes

Jono's [$S_{(2,3,1)}$] response to the question "What makes you like it here" is presented in Excerpt 26. The data reveals the importance of the interview as a tool for clarifying many issues about students' preferences, which cannot be gathered through any preference questionnaire on its own. For example, in Excerpt 25 Jono [$S_{(2,3,1)}$] had suggested that he wanted to work faster, but upon further probing it was revealed that this could be due to subliminal pressure due to having to work in a high performance group.

Another unexpected form of pressure was revealed as having to "work a lot without a calculator". Although Jono [$S_{(2,3,1)}$] cited peer influence as a possible contributor an interview with eight (all of them boys) of the other high number sense students, during the latter part of the first term showed that peer pressure was the main reason why some of them did not use a calculator. This phenomenon also occurred in Bob's class, although the girls' from both Amanda's and Chantal's class did not seem to have such an experience. Hence, this could be something which is gender related, and needs to be confirmed through further research, as proposed in the recommendations of this study. It should be noted that when the same students were interviewed during the second week of the fourth term, they indicated that the teacher's "comments and constantly encouraging us to use any [computational] method or object [instrument] which we thought was better [most appropriate]" had virtually eradicated the tendency for them to feel pressured not to use a calculator.

Excerpt 26

Jono [$S_{(2,3,1)}$]: You have more time to think. In my group they work very fast and…yes…everyone works fast…and I have to be quick…to get the answer. Here [in this other group] I can work with a calculator. Over there [in his usual group] we work a lot without a calculator.

Res: Who does not allow you to work with a calculator?

Jono [$S_{(2,3,1)}$]: We…we…try to solve the problem in our heads or with pencil.

Res: Yes, I understand this, but is there anyone who asks you not to work with a calculator?

Jono [$S_{(2,3,1)}$]: No…Yes…the others. We do not, we…we all don't like it…like when one of us…yes, tries to use a calculator for something…a cal…a problem which can be done…without one…without a calculator.

Res: Would you like to work with a calculator?

Jono [$S_{(2,3,1)}$]: Yes…yep, but… not all the time. Not when it's [the problem] simple. For difficult problems.

Another striking revelation from this interview, which supports what was observed, is that students from the 'high performance' group were more at ease with number problems than those who were in the other groups. This was confirmed by Chantal when she responded that "it's strange, my best students are very good at numbers…solving number problems, but when it comes to logic problems, some of my less able students usually do as well or sometimes better". The results of the Think Aloud Stimulated Recall Interview (TASRI) were in tandem with this observation. When attempting to solve Number Sense Inherent (NSI) problems, over half of the 19 students with lower number sense seemed to identify and solve a different problem, and eventually got no marks for both NSI problems. It is worth noting that such students verbally expressed that they did not like working with number. On the other hand, 10 of these same students got a combined score of 100 percent for one or both DNS problems.

Focus on Number Sense through Problem-based Teaching-Learning Experiences Quantification of the number of lessons observed (Figures 4.37 & 4.38) revealed that teachers placed a lot of emphasis on number sense and problem solving, although many other mathematical contents were presented to the students. The 10 lessons which focused only on problem solving were all observed in the first term. From the second term onwards all teaching-learning experiences observed either pertained wholly to number sense and problem solving or contained elements of both. A more specific analysis revealed that a large majority (81%) of the lessons observed were specifically designed to emphasise number sense development, while focus on problem solving was devoted to 91 percent of the 91 lessons observed. This in effect indicates how much emphasis was placed on teaching for the development of number sense through a problem solving approach. This finding is made even more striking when it is considered that only one of the lessons seemed to have had no focus on number sense at all as shown in Table 4.37and Table 4.38. Another important observation was made pertaining to how focus on problem solving, number sense and other mathematics contents were distributed per class (Table 4.38).

Table 4.37 Percentage of observed learning experience focus

Focus of learning experience	PS only	NS only	Others only	PS & NS only	PS & Others only	NS & others only	PS, NS & others
Percentage	11	5.5	1.1	39.6	6.6	2.2	34.1

In this regard there was no marked difference in distribution among the three classes. Given such a situation it was deemed very important to understand why the teachers taught through problem solving with much of the focus on number sense. As pointed out earlier and in answering question four, one main reason why these teachers had to focus on number sense was because it permeated all the other strands, but the question remained as to whether the focus on number sense was influenced by other factors. The analysis and discussion which follows suggest that there were other possible influential factors.

Table 4.38 Learning experience emphasis per class

School	PS only	NS only	Others only	PS & NS only	PS & Others only	NS & others only	PS, NS & others	Total
S_1	2	2	1	13	2	1	10	31
S_2	4	1	0	12	2	0	11	30
S_3	4	2	0	11	2	1	10	30

The observation data revealed a common pattern, with some slight variations, about the way the problems were designed and presented in all three classes. During the first term learning experience activities were designed to demonstrate various methods that could be used to solve the same problem. Each week the teacher concentrated on the application of a particular strategy which was new. This teaching sequence was cumulative in nature in that problem solving strategies learnt previously were revisited in subsequent lessons. For example, in Bob's class tt was only during the last lesson of the week that the teacher directed students' attention to specific strategies they had been doing. This lesson involved a lot of discussions about problems, but students were not asked to solve these problems in class; usually they were encouraged to try and apply the strategies they had identified in solving the problems discussed during this last lesson.

The problems presented provided the students with many authentic applications for a particular strategy. These lessons revolved around the students being encouraged to communicate their solutions and their reasoning, whether or not the latter were logical or not. Initially students discussed their solutions with those in their group. The teacher went round to monitor their progress and in doing so he or she pounced on opportunities to get the less able students to present their solutions first. Even when such students had not reached an acceptable answer, they were made to present how they started solving the problem. From there other students' inputs were solicited in an attempt to guide the class towards discussing which additional steps could be taken to reach a reasonable conclusion. It is worth noting that no new problem solving strategies were introduced from the second term onwards. As remarked by Chantal, as the teacher, her focus would be more on getting students to work through a "problem solving environment" instead of just focusing on problem solving strategies. When questioned during the fifth week of the second semester, her answers gave the impression that she was employing a systematic method of teaching students "problem sense".

Excerpt 27

Researcher: What is your reason for not teaching any new problem solving strategy?

Chantal: By now all of these children…all of them…they would have at least an idea that certain problems can be solved using certain strategies. That's all I wanted them to know…to understand.

Researcher: But, don't you think that some of them might forget the strategies they've been taught!

Chantal: I am not too concerned about that. My aim is not to teach them problem solving strategies. I want my children to become good problem solvers…above everything else. You see, in solving these problems they have to use certain strategies.

Researcher: What if you had not taught them these strategies?

Chantal: Some of them, the brighter ones, those in the top group, already knew some strategies when they first came to me. Some children just went about trying to solve the problems I gave them. A few mixed their strategies…some could solve the problem, others could not…but they were using some form of strategy. I just took them a step further…a little bit higher. I was trying to…to get them to see something…to realise that some problems could be solved in a certain way, using a method. You see, we are working in a problem solving environment.

When the teachers were asked about how getting students to work in a 'problem solving environment' instead of continuously teaching them problem solving strategies had helped these students a common theme from their responses was that the initial exercise was to help students understand that they could meet up with problems which could be solved through the same strategy. As explained by Bob “once they know the names of certain common strategy then they understand that they can name their own strategies”, which according to him made it “easier to recall having solved similar problems in the past through a certain specific strategy”. Chantal also felt that naming a strategy was important because:

> when you have a name for something you feel less afraid of it. As you have seen, sometimes I ask them to name the strategy they have used…but this is not very important. I do this just to get them to recall what they did in the first term. Like, I insist that they do not say just guess and check. So… you see that they stress on this ‘educated’ when they say “educated guess and check”.

When the researcher suggested that it seemed the teacher was saying that “the strategies are that important”, it emerged that engaging in problem solving activities should take priority over learning of strategies. As expressed by Amanda:

> I am not saying that they are not important…otherwise I would not have spent time getting them to … know them. All I am saying is that children need to spend more time on solving problems and not learning problem solving strategies.

The next question required the teachers to explain whether they would “encourage another teacher to get students to learn some problem solving strategies during the first term that they have a new class?” The response was more about what would be best for the students. As explained by Amanda, the teacher should move in a certain direction if students responded positively to a situation, and find other ways of proceeding if students’ respond negatively. To Amanda:

> It depends…you have to consider the students first…what do they have? What they know? [Readiness] Then there is the question of focus. If your focus is on teaching problem solving strategies then your lessons will be boring…there won’t be any mathematics in there [the teacher must widen the teaching-learning focus and make it interesting]. You’ll be encouraging these children to muck around and hate doing maths. If the class is ready to learn about these strategies…then…yes, only then would I say “go ahead…teach them some strategies”, but if they are not ready…if they are not this type of class…you know…[then] it’s best not to teach them about these strategies. Teach them only a few and…I do not think…there is no need to teach them the names of the strategies, for them to learn these by heart. The names should be learnt only as a reminder that we can identify strategies which occur again and again, and that it is a good idea to have a name tag for them.

Since it seemed that while these teachers were observed getting students to learn certain problem solving strategies while simultaneously insisting that the focus should be on problem solving they were asked “How would you advise someone to teach these strategies?”. The teachers’ responses proposed many diverse situations and focal points, four of which were commonly referred to as the teacher’s need to:

1. have a very good knowledge of the students' ability and preferences;
2. assess students' preferences and understanding;
3. present students with problems at various levels of difficulty; and
4. ascertain the extent to which the students are able to make mathematical sense

In Excerpt 28 Bob's response is used to give an idea of how these four issues were expressed by these teachers. Although the other teachers mentioned that it was important to gauge whether students could make sense of the mathematics they were learning, Bob was the only one at this stage who explicitly focused on number sense.

Excerpt 28

Like I've said…you must have a feel for what they can and cannot do [Knowledge of students' ability and preferences]. I give them some questions…like a test… to test them. I ask them questions to learn about them, what they like, what they don't like, what they know, what they do not know [Assess students' preferences and understanding]. I also give them easy and challenging questions…problems to solve [Mix the problems level of difficulty]. Then…once you know…you have a feel for what they can do and what they are not capable of doing…you make a decision.

Bob: For example, your research gives me an idea…it is about number sense?

Researcher: Yes.

Bob: I think the teacher should look first at the students' problem solving level…their ability…Do they have a sense of what the problem is about? Do they have a sense of how to start solving this problem? Do they have a way…a sense of what they will do to solve the problem? So you have number sense, and in this case I think this could be the case with problems as well.

The teachers were asked to respond to the question, "What do you mean by 'students making sense of the mathematics"?. This resulted in teachers expressing their views about students being able to explain what the problem was asking them to do, how they solved the problem and whether their solution was reasonable. Excerpt 29 shows some aspects of how Chantal responded to this question, which highlights common issues raised by all three teachers.

Excerpt 29

Chantal: You have to be able to make sense of what the problem is about, what it is telling you. You must also be able to make sense of your solution…the answer you get…Does it make sense? You have to ask yourself this question. I do not know what I would call that but it has to do with making sense…some form of sense making.

Researcher: Have you read or heard about problem sense?

Chantal: No, but I would think that if there is number sense then there is geometric…al sense, statistic sense. Yes, there could be something called problem sense. Is there such a thing as problem sense already?

Researcher: I have not heard of it, but if it does exist I presume it would be in line with what you were saying.

Since then the researcher searched the internet and discovered that there is such a concept as problem sense which is described by Montana (2005) as:

> To develop competency in problem solving requires the teacher to view problem solving from the perspective of presenting mathematical concepts for the purpose of developing "problem sense".
> (http://www.yale.edu/ynhti/curriculum/units/1980/7/80.07.10.x.html)

Montana's (2005) view of problem sense captures the essence of what these teachers described as making sense of the mathematics that the students were learning.

Focus on the Learner's Learning Reality

Since according to these teachers it is not possible to give a hundred percent one-on-one attention to each student, it was important to understand how they dealt with individual differences, especially in the area of learning style. A most common theme, expressed by Amanda as "Learning Reality" seemed to be central to all the other factors considered in preparation for the learning experience, although the other two teachers did not exactly used this term. What follows highlight that these teachers took the following into consideration:

- the learner's learning reality and experience;
- how prepared the learner was to embark on participating in the learning experience;
- weaknesses and strengths of the learner;
- how to be caring towards and empathise with learners;
- a learner's emotional state;
- a learner's knowledge baggage;
- each learner's level of self-confidence; and
- how to monitor the learner's progress.

The following excerpts provide an illustration of how the above concepts, which seemed to guide the teacher's preparation and delivery of the lesson were prevalent in these same teachers' beliefs. Although other issues were also raised, it is interesting to note that in response to the question "How do you plan a learning experience for the students in your class?" the teachers tended to be in accord on the points summarised in parentheses, in Excerpts 30, 31 and 32. Although Amanda was the only one to use the term 'learning reality' the idea evoked by this concept was also present in both Bob's and Chantal's deliberations. Amanda suggested that she used a caring attitude to gauge the students' readiness through the student's experience, weaknesses and strengths, and mental preparedness. According to Amanda she looked at all of these components and by asking certain questions she obtained information about the student's learning reality. Amanda also brought out the issue of mentally preparing teaching-learning experiences. This issue is further explored when answering question four.

Excerpt 30

Amanda: You have to focus on the child's own learning reality [Learning reality]. What does she know already [Prior knowledge]? What is her experience [Child's experience]? Is she ready to add to her experience what I intend to teach her today [Principle of readiness]? Where might she fail [Weaknesses and strengths]? What do I need to be careful of [Caring attitude]?

R: Do you mean that you ask these questions first before deciding on the topic to teach?

Amanda: Not necessarily. I do not think that I plan in any definite sort of order. I suppose it all depends on how I am feeling, I think. But I never prepare anything without thinking [Mental preparation] a lot about each child…their strong points and their weaknesses [Strengths and weaknesses]. Like I've said, the child has her own learning experience to deal with [Learning reality].

To gain more understanding about what Amanda really meant she was asked to explain what she meant by 'learning reality', which she expressed as:

> It's more like a reality check [Learning reality]. We all know certain things, and we all don't know certain things [Knowledge baggage]. There is also the mood that the girls are in at the moment of teaching and learning [Emotional state of students]. Each girl will react differently to my teaching according to her ability, her way of learning and what she is confident of doing [Self efficacy]. That's her learning reality and I have to cater for this [Empathise with learners].

Although Bob expressed similar issues to Amanda's he was more explicit in explaining how he cared for students by empathising with them, which he did through mental preparation. In fact Bob provides another clue about how these teachers use the information they get to prepare students. As explained previously, these teachers did not intend to use student-readiness information for them to decide whether to teach a topic or not, but rather as a means of preparing them for what they were going to experience. In fact Bob stated that what would happen in the teaching-learning experience was unpredictable in terms of how the students would respond. Bob argued that it would be futile to teach students anything for which they have not been prepared. Hence, he always came to the class prepared with various activities, and employed both group work and individualised learning situations to maximise the students' readiness to cope with the material presented through the teaching-learning experience. His method also involved trying to replace whatever uncertainties and misconceptions that students had in their "jar" by enjoyable experiences. As presented in this study, Bob was always on the look-out for signs of fatigue, boredom or irritation from the students, and if he felt that they needed to be motivated he would change the activity for a while until he felt they were ready to continue with what they were doing previously. It seemed that

Bob's analogy of replacing what was in the jar by something else was more related to keeping students interested and motivated to learn rather than seeing them as empty vessels which needed to be filled.

Excerpt 31

Bob: I do a lot of thinking about the situation first [Mental preparation]. I try to see myself as a student [Empathise with learners]. How would I react if I were asked to do a certain thing? You see, if I am not ready for learning [Principle of readiness] then there is very little use in forcing me to learn anything. So, I tend to go over how I can get the boys ready [Principle of readiness]. You can never predict what is going to happen [Prepare for the unpredictable].

R: So you think it is important that they are ready.

Bob: I have about fourteen to twenty students per class, which means that class size is not really a big issue here. Hence, it is easier to work with the students as if you are working both with them as a group [Group work] and simultaneously as individuals [Individualised learning]. But this won't be possible…it will be very difficult for me to… if each student is not ready for what I want to teach them [Principle of readiness]. That's why I prepare a lot of various exercises…worksheets and various activities…students can get tired, and if I were to keep going…teaching them something that they are not taking in…it is like pouring water in a jar which is already full. You must first empty the jar or remove some of its content [Caring attitude]. I mean obviously it is only then that,…that…erm the…that anything can be poured into the jar.

When asked to explain the analogy of the jar in relation to what he taught, Bob explained that:

> Once I notice that I am not getting the response [Monitoring of student response]…I am not getting anywhere with them, I change the direction of the whole lesson [Change of setting], as you have seen many times. I give them something new to focus on…a problem which has nothing to do with what they were doing. This helps in many ways.

It is worth noting that when Bob was asked how his practice of engaging students in a totally new activity helped them in many ways his response evoked the same sense of accommodating the learners' learning reality. An interesting and very insightful revelation from Bob's answer is how he thought that his teaching should be in accord with the influence of the information culture that the students live in, as he stated:

> I mean, we live in the present. Therefore we have to deal with the present. That's real life…these kids watch a lot of television where the…the pictures are constantly changing [Learning reality]. I have to train their concentration…but then I have to be careful not to stretch them too far. By switching to a totally new topic I prevent them showing more…becoming more frustrated. In this way I also don't waste their time, they will still be learning something.

Chantal's view of what she focused upon when planning a teaching-learning experience is presented in Excerpt 32. In the case of Chantal, she gave a new perspective to this notion of 'learning reality' in that teaching and learning are linked, since both teachers and students teach and learn from each other. Just like the other two teachers, her notion incorporated a sense of caring for the students through ensuring that they were ready to cope with the next stage of the teaching-learning experience.

Excerpt 32

Chantal: Teaching and learning are inseparable. Therefore it is very important for the teacher to learn about and understand each student's idiosyncrasies in relation to how these affect their present learning situation [Learning Reality]. The teacher has to develop an effective mechanism for tracking [Monitoring] her own teaching and learning, the development of her students in terms of what they are capable of achieving and what they are prepared to achieve [Principle of Readiness], since it is not appropriate to force them to learn what they do not want to learn [Caring Attitude].

4.6.4 Summary

From the discussion which has just ensued, some of the most common themes coming out of these three teachers' comments revolved around the issue of catering for students' learning as individuals within a group situation. In comparing the observation data and the teachers' beliefs about what they viewed as the most influential elements in how they planned the teaching-learning experiences for the students, the following themes seemed to be very important:

- Customised teaching

  The planning and implementation of learning experiences should revolve around the learning ability, style, capacity, attention span and prior knowledge of the students.
- Learning reality

  Teaching must take place in the present. Hence, it must be realistic in its presentation. It should not be forceful. It should take into account the present culture that students find themselves living in. Teaching must be contemporary both in approach and expediency.
- Amalgamating theories and practices

  Contemporary teaching does not mean using only modern theories and discarding older ones. For contemporary teaching to be effective it must incorporate what is good from traditional teaching practices and theories with what is good from the more recent ones.
- Content guides learning style

  It is the content which influences the students' learning preference. All three teachers thought that although the association between learning style preference and academic performance is usually considered to be one in which learning style is expected to affect academic performance, the relationship could be in the reversed direction. Hence, they always encouraged students to explore a problem in various ways, using various methods of solution and applying different learning preferences on the situation. Through alteration of the context, teachers felt they could change the students' adopted learning approaches.

- Flexibility of curriculum

  Emphasis must be placed more on expediency than on principle, although the latter is very important. The principles which govern execution of the curriculum must be easily adaptable and should be used as a means to an end and not an end *per se*.
- Monitoring for signs of frustration

  This is a theme which recurs under various guises throughout the interviews with both students and teachers. The aim is to prevent boredom from setting in. If this is achieved then the teacher has less disciplinary problems to deal with.
- Teaching for estimation

  All three teachers encouraged their students to estimate, and they were given exercises specifically geared at developing their estimation skills. Hence, every now and then they were given exercises in which they had to decide whether the problem was requiring an actual measure or an estimate. They always had to explain the rationale for their choice. As pointed out by Amanda the students "are encouraged to always estimate the answer to a problem as a means of checking whether the final answer they got was in the ball park".
- Mental emphasis

  This involved a lot of emphasis on working mentally as much as possible, with the more able students expected to work mentally all the time before using pen and paper. On the other hand the less able were given more leeway by the teacher to use written computation. As the year progressed the teacher allowed students more autonomy to use whatever method they preferred, although they often had to justify why they used a particular method. Although Swan (2002) observed that "self-generated mental strategies harmonise extremely well with constructivist thinking, whereas a transmission approach does not", it is interesting to note that all three teachers used a combination of both the constructivist and the transmission approach to get students to use estimation to solve number sense problems, and any other problem for that matter.

- Teaching for selection of appropriate calculation method

  Students were encouraged to use any method they wanted, but at the end of the exercise there was always a discussion about strategies used; calculation methods employed; which is best to use first; and why such decisions were made.

4.7 Analysis and Results of Research Question 3

How does learning style impact upon students' number sense and problem solving performance?

As the research data was being analysed and new situations were encountered it started becoming apparent that although the Index of Learning Style (ILS) inventory was providing some important information which was consistent with the pilot study results, it was not necessarily designed to provide the most accurate answer as to "*The impact of learning styles upon students' number sense and problem solving performance*". It seemed that the results were suggesting that, in regard to the present study, the issue of personal and individual learning style differences would be better informed through number sense and problem solving style. Hence, the analysis is presented through three sets of data pertaining to: (i) the learning style inventory; (ii) teacher interviews and (ii) student interviews, especially the Think Aloud Stimulated Recall Interview (TASRI). It should be noted that statistical tests of significance were applied to the data only after more simple analysis had been carried out.

4.7.1 Number sense and problem solving performance and students' learning style results

To facilitate comparison analysis the students were ranked by their academic performance in number sense and problem solving so that three proficiency groups were obtained for each academic component; the cut off points were set at the 30th and 70th percentiles (Table 4.6). First the learning modalities were analysed as per high and low performance in number sense and problem solving. Then the focus of the analysis shifted to combined NS-PS proficiency levels as High number sense High problem solving (HnsHps) and Low number sense Low problem solving (LnsLps). Although separating student scores into percentile proficiency bands inevitably led to the high problem solving proficiency band containing one less student than the other bands, the distribution was compatible and consistent with those obtained when students were banded with the same number of students in each group; quarters (quartiles), thirds and halves (separated through the median number of students). Comparison of the high and low performers presented in Table 4.39 shows that thirty-two categories of number sense and problem solving by number of students preferring each modality was obtained.

Table 4.39 Number of students by learning modality preference per number sense and problem solving proficiency

	Number Sense			Problem Solving		
	Low	High	Total	Low	High	Total
Active	11	8	19	13	9	22
Reflective	8	11	19	6	9	15
Sensing	12	11	23	9	11	20
Intuitive	7	8	15	10	7	17
Visual	19	15	34	19	16	35
Verbal	0	4	4	0	2	2
Sequential	10	10	20	7	9	16
Global	9	9	18	12	9	21

Note: High Number Sense score ≥ 76.2%; Low Number Sense score < 58.4%; High Problem Solving score ≥ 80.2%; Low Problem Solving score < 61%.

A diagonally symmetrical frequency pattern was observed for preference to learn through the active or reflective singular modality, with 58 percent of low number sense students preferring the active modality compared to 42 percent of high number sense students. The reverse was observed for the reflective modality, indicating that there were more high number sense reflective students (58%) than low number sense ones (42%). The proportion of low PS students who preferred the active modality (68%) was more than twice that of the low PS students who preferred to process information reflectively (32%). The frequency of high problem solving students who preferred to process information actively (50%) was equivalent to the reflectors (50%), although the proportion of high problem solving reflective students was greater than that of the low problem solving reflectors (32%).

Assertion 22

For both NS and PS there were proportionately more reflective students scoring in the high proficiency bands than those scoring in the low proficiency bands, while there were proportionately more active students in the lower proficiency bands than in the high proficiency bands.

No marked differences were observed in terms of the proportion of students showing a preference for either of the two polar learning modalities pertaining to the perceiving information or understanding information dimensions. The result in regard to intuition was particularly unexpected, since number sense can be seen as "an intuition about numbers that is drawn from all varied meanings of number" (NCTM, 1989, p. 39), Moreover, this result was not in tandem with the teachers' beliefs (Excerpt 33) that number sense develops students' intuitive perception which they saw as an important element necessary for effective problem solving. A possible reason for such a contrast between these results and these teachers' and NCTM's (1989) perceptions could be that the items used in the ILS inventory (Appendix IV) were gauging general intuition instead of more mathematics-related intuition. At least six (items 2, 6, 14, 18, 26 and 34) of the 11 items pertaining to the perceiving information dimension had very little to do with number sense or mathematics and of the five remaining items only one (item 42) directly addressed a mathematical preference. Hence, a more directly relevant inventory might have revealed a closer relationship between students' number sense and their mathematical intuition. This is not to say that the ILS inventory employed was useless, but the analysis of this study suggests that the type of items of the inventory could possibly be responsible for the unexpected preference results obtained.

Excerpt 33

Bob:	It is more through number sense than any other mathematical process that students develop a greater sense of reliance upon their gut feeling [Intuition] when solving a problem.
Chantal:	Students with higher number sense tend to be a lot more intuitive in the way they deal with any sort of problem solving situation.
Amanda:	To be good at problem solving students need to be creative, more innovative and [adept] at discovering patterns and relationships when confronted with new situations [Intuitive qualities]. It is mostly those students who are not afraid to rely on a 'hunch' [Intuition] rather than logical reasoning who have such qualities. But these qualities are easier to develop through number sense activities than through other problem solving activities.

On the other hand, the balance expressed between preference for understanding information sequentially or globally could be explained by the teachers' insistence on encouraging students to focus first on seeing the problem and its solution as a whole, and to estimate the solution from this perspective before working systematically to check their holistic assumptions. Moreover, the teacher tended to start off by encouraging students to work more deductively during the early part of the school year, and then got them to work more inductively as the year progresses. It was only during the first term that the students were sometimes given some prefabricated information with the teacher’s intention being to get students to master and apply them to certain problems which could be solved through particular known strategies. During the last three quarters of the school year students were exposed to conceptual problems which required a lot of lateral thinking. As students became more proficient at problem solving they had to find appropriate solutions to the problems by themselves with very little help from the teacher. The teacher’s input was mainly in the form of encouraging students to identify their weaknesses and errors in their work, to understand these and use such understanding to rectify the mistakes in their solution strategies and processes. According to McKeachie (1980) and Silverman (1988) compared to the deductive approach, induction fosters deeper learning and prolongs retention of information. Moreover, research indicates that through the inductive approach the teacher provides students with more opportunities to develop greater confidence in their problem-solving abilities (Mckeachie, 1980; Silverman, 1988). Hence, since students are more inclined to use the deductive approach, focus on the inductive approach could have also played a major role in getting them to be more or less balanced in the way they preferred to understand information.

Assertion 23

The teachers believed that number sense develops students’ intuitive perception which is an important element necessary for effective problem solving.

Since there were only five students (out of 64) who expressed a preference for the verbal modality as opposed to 59 who preferred the visual modality the focus for this section of the analysis pertained mainly to the number of students preferring the visual modality. It is nevertheless worth noting that since all of the five verbal students’ number sense and problem solving performances were classified as high, there was no verbal preference indicated by the students scoring low for either number sense or problem solving. This directly implies that all students scoring low on both number sense and problem solving preferred to receive information visually.

Assertion 24

With respect to both number sense and problem solving all low scoring students preferred the visual modality, while some of the high scoring students preferred the verbal modality.

> **Assertion 25**
> There is no marked difference between the proportions of high problem solving students preferring to: (i) perceive information either through the sensing or intuitive modality; and (ii) understand information through either the sequential or global modality.

The next stage in the analysis was one in which students were grouped according to simultaneously scoring high for both number sense and problem solving (HnsHps, N = 12) or simultaneously scoring low for both NS and PS (LnsLps, N = 12). The graph presented in Figure 4.13 was used to bring together the data just presented so that now these were then seen with combined high performance and combined low performance statistics, which could be readily compared. As a means of facilitating visual presentation and interpretation of the frequency data per pairs of corresponding learning modalities the data presented in Figure 4.13 was then mapped so that both count and percentage frequencies could be compared both across modalities and across combined proficiency levels.

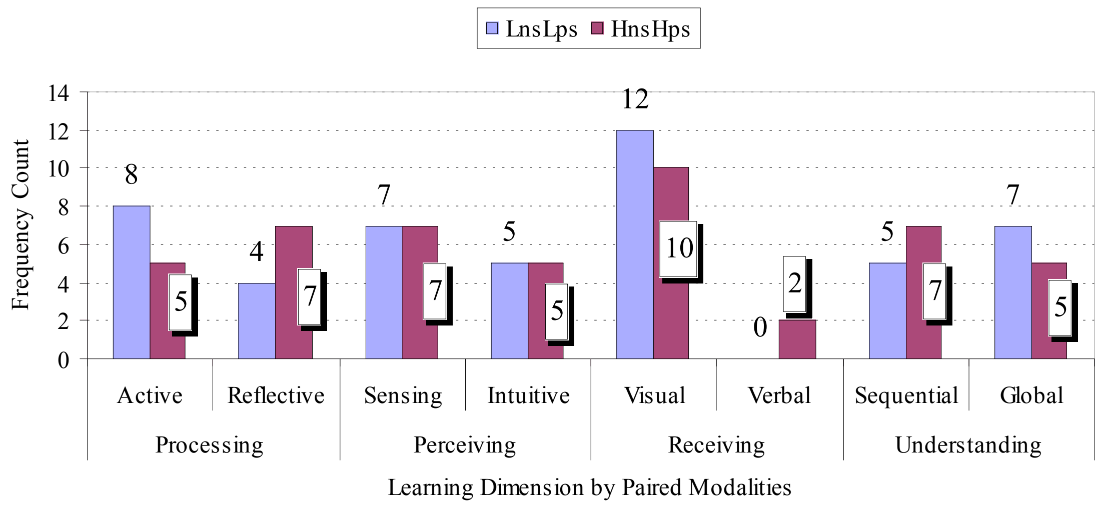


Figure 4.13 Learning modality preference by number of students per combined NS-PS high and low proficiency

Since a difference of three or more students was considered to be indicative of a marked difference, except for the active and reflective modalities, all of the others showed no marked difference between the proportions of high and low combined number sense and problem solving performances. A *coco-de-mer* mapping diagram was used to summarise the frequency data as per combined NS and PS proficiency levels. In the *coco-de-mer* map presented in Figure 4.14 the arrows leading immediately from the inside of the proficiency box compare with each other, whereas the arrows leading from a proficiency box to a learning modality box compares to the other arrow leading from the other proficiency box to the same learning modality box. Hence, as shown in Figure 4.14, there was a greater proportion of HnsHps students who preferred the reflective modality (58%) as opposed to the active modality (42%), while there was a greater proportion of active LnsLps (67%) than active HnsHps students. Therefore, it was concluded that with regard to processing information, a greater proportion of students with simultaneously high number sense and problem solving ability tended to be more reflective than active, while the reverse was observed for LnsLps students. More specifically, the HnsHps frequencies tended to follow the pattern for Hns students' performance, while that of the LnsLps tended to follow the frequency pattern of both the Lns and Lps students' performance (Figure 4.14).

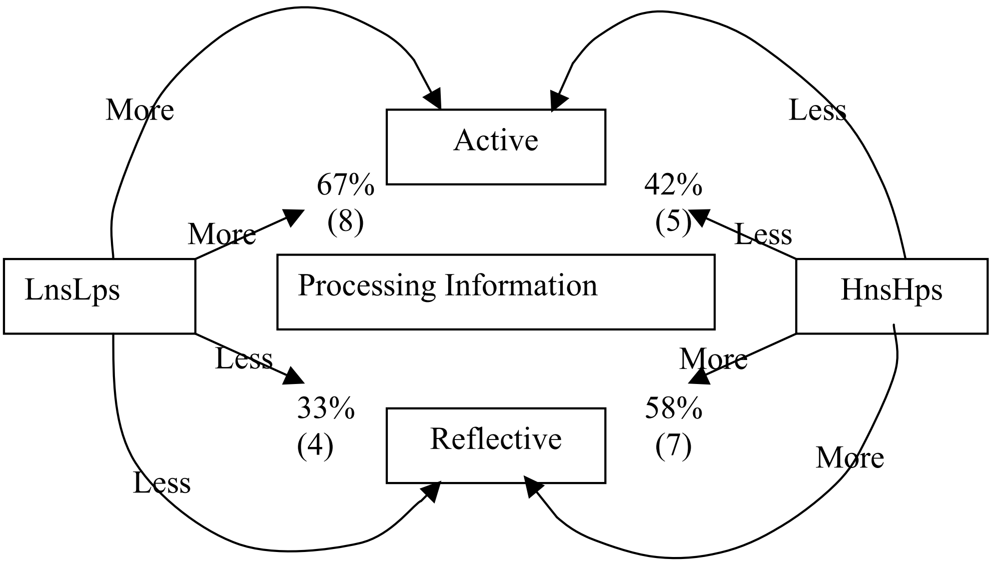


Figure 4.14 Comparative frequencies of students per NS-PS proficiency by the processing information dimension

> **Assertion 26**
>
> The active and reflective modalities were the only instances where a marked difference was observed between the proportions of high and low combined number sense and problem solving performances.

> **Assertion 27**
>
> High number sense and high problem solving students preferred to process information reflectively rather than actively, while both low number sense and low problem solving are linked more to the active preference.

Since the only instances where a marked difference was observed pertained to the processing of information, it was deemed inappropriate to extend this analysis to the other three learning preference dimensions of perceiving, receiving and understanding information.

Table 4.40 presents comparative students' mean percentage for separate number sense and problem solving scores per related twin-modality. With respect to number sense, independent t-test results for dichotomous learning dimensions also revealed no significant difference between mean number sense performance scores of polar (twin) modalities. Nevertheless, the mean differences, although very small in variation, revealed that students with reflective style preference had higher NS and PS scores than students with active style preference. Likewise students with verbal style preference outscored those with visual style preference for both NS and PS. For the two other dimensions of Perceiving and Understanding this trend was interrupted. In the first case the intuitive students outscored their sensing counterparts for number sense with the reverse occurring for problem solving; maybe this is due to intuition being more closely associated with number sense.

Table 4.40 Difference in test scores for different preferred learning modality

Learning Style Polar Modalities (a) vs. (b)	N = 64	Number Sense Mean $(M_a - M_b)$	Number Sense Mean Difference	Problem solving Mean $(M_a - M_b)$	Problem solving Mean Difference
Active vs . Reflective	37 vs. 27	63.7 – 68.1	-4.4	69.3 – 72.0	-2.7
Sensing vs. Intuitive	41 vs. 23	65.1 – 66.1	-1	71.3 – 68.8	2.5
Visual vs. Verbal	59 vs. 5	64.5 – 78.0	-13.5	69.4 – 82.5	-13.1
Sequential vs. Global	33 vs. 31	64.8 – 66.3	-1.5	71.7 – 69.1	2.6

As will be shown later, except for the processing information dimension, no statistical tests revealed any significant mean variations for number sense performance and the learning style modalities pertaining to the perceiving, receiving and understanding information dimensions. Thus, the mean number sense and problem solving performance scores for each dimension were compared to see whether there were any marked differences between performance scores per learning style preference.

Analysis of Paired LS Dimensions against NS-PS Mean Percentage Scores

The students' number sense and problem solving mean percentage scores were analysed with their preference for either of two related learning dimensions. The results indicated that there was no marked difference between any of the related paired learning style modalities in terms of the students' mean number sense or problem solving scores. Hence, Figure 4.15 is presented only to illustrate a typical example of no marked differences as explained previously. Figure 4.15, presents students' number sense (NS) and problem solving (PS) mean percentage scores by the paired active and reflective learning style modalities. For the following analysis any difference in mean lower than 10 percent, was considered to be negligible enough for the two means to be categorised as being similar; implying no marked difference between means. Hence, any difference of 10 percent or more between a pair of means was considered to be 'marked'. Figure 4.15 has to be read one section at a time because there are more than one type of variable on the independent axis. For example, when interpreting the last two bars on the far right hand side of Figure 4.15, starting from the left (Active preference bar), the percentages shown pertain to the mean number sense score. This is categorised per the students' number sense proficiency grade (i.e. Hps). Hence, the reading of 77 indicates that those students who scored in the high problem solving proficiency band scored an average of 77 percent on the Number Sense Test (NST). Similarly the 83 on the other bar indicates that the reflective students who scored in the high problem solving proficiency band scored 83 percent on the NST.

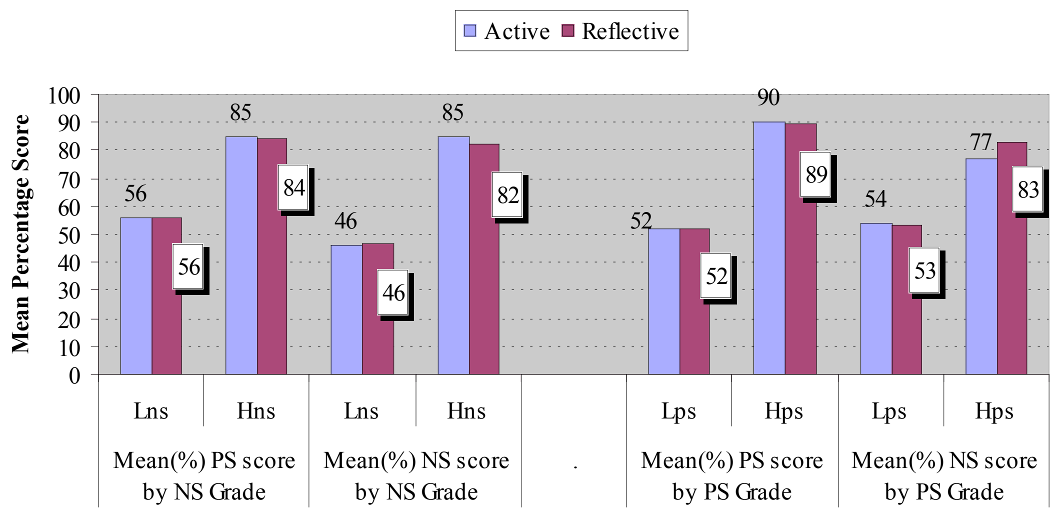


Figure 4.15 Students' Mean PS and NS scores per proficiency band by preference for active or reflective modality

Assertion 28

There was no marked difference between any of the related paired learning style modalities in terms of the students' mean number sense or problem solving scores.

Comparing the Students' Mean Number Sense and Problem Solving Performance Scores across Learning Style Preference

During the classroom observations the researcher identified only six students who could be described as being always more reflective and quiet than they were active. The other students seemed to practise a mixture of the two, with the more able students being as active as or even more so than the less able. This observation, coupled with the fact that before data collection the researcher believed that high number sense students' preference would be overwhelmingly reflective as opposed to being active, prompted the researcher to analyse the students' mean number sense quartile scores. The analysis revealed that a shift from Very Low towards Very High number sense showed virtually no difference in mean scores between students preferring one dichotomous modality and those preferring the respective modality in that pair. Hence, Figure 4.16 is presented only as an example to highlight the result just mentioned, while Table 4.41 provides an overall illustration of the data.

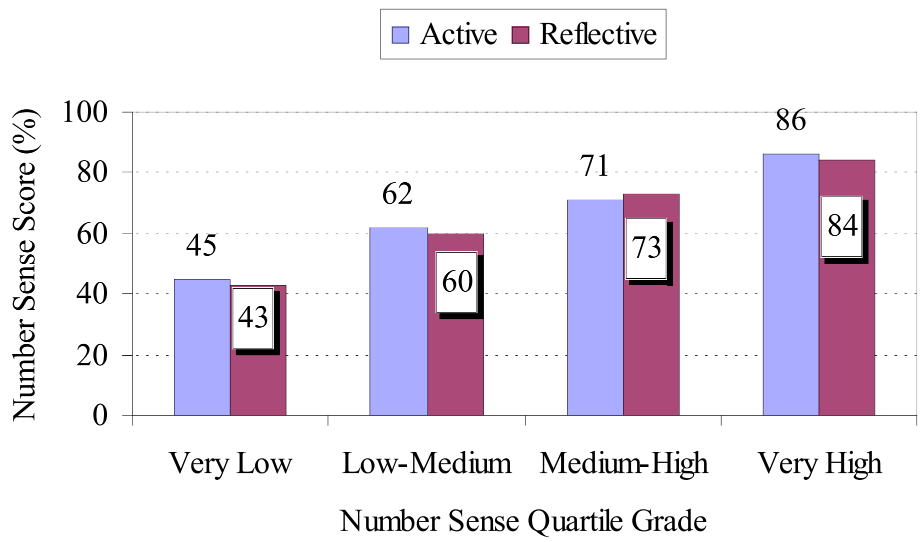


Figure 4.16 Mean score comparison for High and Low number sense by the Processing dimension (N = 64)

Due to intuition being one of the requirements for good number sense, particular attention was also given to the perception dimension. The perception scale used in this research is Felder and Silverman's adaptation of the sensing-intuition scale of the Myers-Brigs Type Indicator for personality types.

Table 4.41 Number of students per number sense score by quartile and learning modality preference

	ACT	REF	SEN	INT	VIS	VRB	SEQ	GLO	N
VL	10	6	9	7	16	0	8	8	16
L-M	12	4	12	4	15	1	11	5	16
M-H	8	8	10	6	14	2	6	10	16
VH	7	9	10	6	14	2	8	8	16
N	37	27	41	23	59	5	33	31	64

Note: VL = Very Low; L-M = Low to Medium; M-H = Medium to High; VH = Very high

When number sense results of students scoring in the top 25 percent were compared to those of students scoring in the bottom 25 percent approximately 60 percent of $H_{NS}H_{PS}$ students preferred the sensing mode of perceiving compared to 40 percent who preferred the intuitive mode. Yet those who preferred an intuitive modality had a slightly higher mean percentage score than sensors on the number sense test, but scored lower than sensors on problem solving. As suggested earlier, the intuition implied through the ILS might be too general in nature and not necessarily relevant to number sense and mathematics problem solving intuition. Hence, it could be that an inventory designed to gauge students' number sense and problem solving related intuition would produce more incisive data.

As a means of validating the results presented hitherto, a multiple regression analysis was performed to ascertain whether there was any significant differences in the means of the students' learning style scores and their number sense and problem solving scores. This exercise resulted in only one significant difference being found, which is presented below. A regression analysis was performed using number sense percentage score as the outcome variable and the variables reflective, intuitive, verbal and global as predictors, to see whether the eight learning style predictors produced any statistically significant differences and, if so, the direction of the relationship. The results are presented in Table 4.42. The reflective preference score ($b = 2.265$) is significant ($p = 0.009$), and the coefficient is positive which would indicate that stronger preference for the reflective modality is related to higher number sense performance — which also implies the reverse for the active modality. It should be noted that this thesis is not claiming that the reflective preference is causing higher number sense performance.

Table 4.42 Regression coefficients for number sense performance by learning style modality

Model		Unstandardised Coefficients B	Unstandardised Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	45.628	8.791		5.190	0.000
	Reflective	2.265	0.835	0.335	2.714	0.009
	Intuitive	0.597	0.791	0.094	0.755	0.454
	Verbal	-0.642	0.986	-0.081	-0.651	0.517
	Global	1.376	1.104	0.155	1.246	0.218

Note: Dependent Variable = Combined pre- and post- number sense percentage score

The reflective variable is related to working mentally and functions more as a proxy for working with abstract constructs and theory building. Thus, higher levels of abstract construction and theory building are associated with higher number sense performance. This result makes sense. Next, the effect of the intuitive modality ($b = 0.597$, $p = 0.454$) is not significant, although its coefficient is positive, indicating that the greater the proportion of students preferring the intuitive modality, the higher the number sense performance. Similarly preference for the verbal modality ($b = -0.642$, $p = 0.517$) did not result in significant effect upon number sense performance, and it has a negative coefficient which suggests that the greater the number of students who preferred learning through the verbal mode the lower their number sense performance. Finally, the percentage of students showing a preference for the global modality ($b = 1.376$, $p = 0.218$) seems to be unrelated to number sense performance. This would seem to indicate that the percentage of students preferring to work in the global mode is not an important factor in predicting number sense performance — this result was somewhat unexpected. Nevertheless, these results are in tandem with the simpler statistical analysis presented earlier, which situates the processing information dimension as the only one of the four ILS dimensions which could have a significant impact on Year 7 student's learning.

Assertion 29

Stronger preference for the reflective modality is related to higher number sense performance.

4.7.2. Students' Learning Style as Portrayed through Interviews and Observations

The student-interview data was analysed and the results used to identify preference attributes of students in relation to their number sense and problem solving proficiency levels. These preference attributes were then grouped into number sense-problem solving style factors.

How Students coped with the Teacher's Teaching Style

Although the ILS inventory results produced no statistically significant mean differences between students' learning style modalities and mathematics performance the observation data revealed some interesting patterns. For instance, during the classroom observations it was observed that 53 percent of the students repeatedly (in two or more teaching sessions) requested for them or the teacher to present information through an alternative mode and 50 percent of these students wanted information presented in a mode identified as their strongest by the ILS inventory. Of the 53 percent of the cases which could be identified it is worth noting that eleven students were observed requesting for information to be presented in another mode on more than eight occasions. Upon comparing these students' learning styles against their requests it was observed that eight of these eleven students had asked for information to be presented in a mode which the ILS results suggested as their preferred one (strong preference). This suggested that ILS results could be reflective of a student's learning style preference in cases where such preference was strong. Table 4.43 presents the number of students observed requesting for information to be presented in a mode other than the ones the teacher had employed.

Table 4.43 Number of students requesting for information to be presented in an alternative mode per number of sessions

Modality	Frequency of sessions		
	2	3	More than 8
Visual	9 (4)	6 (2)	8 (6)
Active	3 (2)	1	1 (1)
Sequential	1	2 (1)	2 (1)
Sensing	0	1	0

Note: The figures in parentheses indicate the number of students requesting information in their preferred learning modality. There were no cases observed for 4 to 7.

Although these teachers varied their mode of instruction in an attempt to satisfy every student's preferred mode of learning, it was impossible for the teacher to always teach through a mode which would readily fit in with all students' appreciation and understanding. To make up for this lack of cover the students were encouraged, and in fact trained, to request for the mode of presentation to be changed to one which suited them at the time. Hence, it was quite common that whenever a student felt uncomfortable with an explanation, or how he or she was asked to respond, for that student to ask for clarification or be allowed to respond via a different mode. For instance, in recognising that the teacher had given a visual explanation with which he or she was not comfortable, a verbal learner could ask the teacher to provide explanations incorporating some diagrams, drawings or written algorithm. A typical example (Appendix VIII) of such a situation occurred prior to administration of the ILS, when Amanda had set students the permutation problem, "A businessman from Melbourne has to visit three cities: Sydney, Adelaide and Canberra. How many different ways can he fly around the cities?", which she and most of the students were solving through drawings and tree diagrams. When she asked Leah what was her solution, the latter gave a verbal explanation. The teacher wanted to see her work, but Leah had only written down the explanation, to which the teacher asked, "Why didn't you use a diagram or some drawings?" and Leah's answer seemed to surprise the teacher "I got confused by the drawing. I got it [the solution] faster when I wrote it [written explanation] down". In the following interview extract, Leah [$S_{(2,9,1)}$] who had the second highest verbal preference score (7 out of 11), expressed a certain 'subliminal' preference for explanations.

Excerpt 34

R: Why did you sort of implore your teacher to allow you to explain this verbally only?

Leah: I sometimes get confused when there are diagrams involved. In this one there were so many lines and circles.

R: How did you get the answer? What went on in your mind?

Leah: I said to myself that there is a pattern…only one way of flying from one city to another. If I fly from one city to two other cities I can go to the second city, then the third…that's one way. Or I can go to the third and then the second. That's two ways [She writes down '2 ways']. So, there's one way for two cities, two ways for three cities…but if I now fly to one more city I can go to second, third and fourth…that's one way. I can also go to second, fourth and third…that's two ways. For each city after the first one I can reach the last city in two ways. Since there's three cities [without counting the first city] that's 3 times 2…that's 6 ways.

Some students were compared as to their behaviour and performance both during classroom observations and the TASRI. In that regard it was interesting to discover that Leah was one of seven students whose behaviour during the TASRI was detected as exhibiting certain characteristics which were mostly similar to those that they exhibited in class. These students worked a lot mentally and seemed to be more of the abstract-reflective type. It could be that since they were not strongly active they preferred not to draw that much. This was further elucidated through their TASRI results; they got a basic score of zero for the 'drawing-item' number four, and only 17 to 25 percent when the item was process scored. Interestingly enough, all of these seven students scored 100 percent for each of the first three TASRI items.

Assertion 30 The Index of Learning Style results could be reflective of a student's learning style preference in cases where such preference is strong.

Assertion 31 About 50 % of students, who requested for information to be presented in another mode, asked for information to be presented in a mode which the ILS results suggested as their preferred one.

It was discovered that questions asking students about their preferences must be carefully structured so that a more accurate picture would be obtained. For instance, when asked "Do you prefer logic or other types of problems?" a large majority of students (89%) stated that they had a preference for logic problems. This was the case regardless of whether they could or preferred to solve NSIP or DNSP. The most popular explanations given by the students are presented in Table 4.44.

Table 4.44 Students' most popular explanations for preferring logic problems

Factor	Sample quotes	Coded element
Ease of planning	they are easy to plan	[easy planning of solution steps]
	It'd not difficult to see where to start and how to solve it.	[easy to start and chart a solution route
Readily obvious clue	there is always a clue…it is there looking at you	[the clue is very evident]
	it's as if the answer is given in the question	[the relationship between the entities are given in the problem]
Readily obvious reasonableness of solution	very easy to know if the answer is correct	[reasonableness of answer is more readily obvious]
	you don't have to check whether the answer is correct	[reasonableness is self-evident]
Numbers facilitate making connections	in logic problems which have numbers, the numbers guide you	[numbers act as signposts towards solution]
	some logic problems have numbers in them. the numbers are linked to each other	[built-in relationship between number components]
	…it is easy to see the link between the numbers	[easy to identify relationship between number components]

However, when asked whether they preferred solving logic problems, drawing problems or those involving number, the proportions of students, according to their preference, changed markedly. Table 4.45 shows that although most students still preferred logic problems, the inclusion of other possible preferences had reduced the frequency of students expressing a preference for solving logic problems.

Table 4.45: Preference for solving logic, drawing shape or number problems

	All Students		HnsHps		LnsLps	
	f	%	f	%	f	%
Logic Problems	35	55	6	50	10	83
Drawing of Shapes	9	14	1	8	2	17
Involve Number	20	31	5	42	0	0
Total	64		12		12	

Table 4.46 shows the number of students per explanation category (factor) in regard to why they preferred logic problems. Although equal number of HnsHps and LnsLps students are categorised as believing that they preferred logic problems because it is easy to see whether the solution is reasonable or not, it should be noted that on the

whole the LnsLps students clarified that in the case of logic problems “you do not have to bother about checking for the reasonableness of the answer”. This indicated that the LnsLps students were also aware that they needed to check the reasonableness of their solutions, but did not do so in the case of NSIP because, as one of them said “It [numbers] makes it even more difficult to check whether your result is appropriate [reasonable] or not”. Upon further investigation these same students revealed that the difficulty was more to do with knowing what to do with the numbers, although 70 percent of them stated that they had “improved a lot” since they have been in Year 7. Hence, one possible item, that a learning style inventory aimed specifically at discovering a student’s learning style, could be “I find it: (a) difficult (b) easy to know what to do with the numbers in a problem? This could be counter checked through another item of the form: I find it: (a) easy (b) difficult to check for the reasonableness of the answer if the problem involved making sense of numbers.

Table 4.46 Students’ response in regard to why they preferred logic problems

Factor	All Students	HnsHps	LnsLps
Ease of planning	12	1	2
Readily obvious clue	17	3	5
Readily obvious reasonableness of solution	19	4	4
Numbers facilitate making connections	9	4	1
Total	57*	12	12

Note: *Only 89 % of all the students suggested reasons which could be classified under the four factors presented.

Preference for Solving NSIP or DNSP

The student interview data, presented in Figure 4.8 and Figure 4.9, indicated that the higher a student’s number sense and problem solving performance the greater the proportion of students preferring to solve Number Sense Inherent Problems (NSIP), while the reverse occurred where preference for solving Devoid of Number Sense Problems (DNSP) increased as students’ performance decreased. Hence, preference for solving either NSIP items or DNSP items was identified as one possible factor which could help in determining a student’s number sense-problem solving learning style. To further validate this assertion the data presented in Figure 4.8 and Figure 4.9 have been aggregated and presented in Figure 4.17.

Figure 4.17 Number of students per combined NS-PS proficiency showing a preference for solving a particular type of problem

Assertion 32
Preference for NSIP or DNSP was identified as one possible factor which could help in determining a student's number sense-problem solving learning style

Analysis of Visual Students

Since the results of the ILS indicated that a large majority (92%) of the students expressed a preference for receiving information via the visual modality the students were asked questions about the necessity of having visual aids in class. Table 4.47 gives an indication of the number of students per level of preference to receive information in visually.

Table 4.47 Number of students according to level of preference for receiving information visually

Preference Level		Frequency	Percent
Very Low	(0-2)	0	0
Low to Medium	(3-5)	5	8
Medium to High	(6-8)	30	47
Very high	(9-11)	29	45

Except for those students who were strongly visual, there was no marked consensus among the less visual students about whether visual aids should be used in class, and if so, how effective they were in helping them develop their number sense and problem solving ability. Of the students identified as strongly visual, 90 percent stated that the teacher should employ visual aids in the teaching of number sense and problem solving and about 52 percent of these same students indicated a strong preference for solving NSIP, compared to 34 percent who preferred DNSP. The most typical responses from these 29 strongly visual students are presented in Excerpt 35. In most cases they are interpreting anything, whether written or drawn, as visual aids; as long as they can be seen as visually helping them in their learning.

Excerpt 35

Lolita[$S_{(1,1,2)}$]:	The drawings help me memorise what I have learnt [Helps memorisation]
Nanette[$S_{(1,18,,2)}$]:	I read the problem then I draw what I can see in my mind and this helps me to understand what the question is about [Mental visualisation]
Joel[$S_{(3,45,1)}$]:	The pictures on the wall are usually what we have done in class. So they remind me of my tables and also of other facts [Aid factual recall]
Aline[$S_{(3,41,2)}$]:	Sometimes it's hard to see how the numbers are connected. I use drawings and diagrams and whatever I can draw to see how they are connected [Making Connections]
Antoine[$S_{(2,30,1)}$]:	It's easier for me to visualise the actual objects and then I understand what the numbers are really showing [Mental visualisation]
Wix[S(2,36,1)]:	Once I have seen the drawing I can visualise it over and over again in my head, and this helps me understand the problem [Mental visualisation]

From the interviews and observations, it seemed that many students viewed visual aids and visualisation as important mainly for helping them:

- memorise and recall both basic and more advanced facts;
- make connections between the various concepts;
- retain mental pictures of events, facts and concepts which are then transposed into mathematical algorithms, equations and formulae; and
- understand and solve the problem.

Furthermore, 15 of these students (52%) stressed the importance of visual aids in helping them understand, master, memorise and recall number-related facts, and to solve numerical problems, which is akin to solving number sense inherent problems.

Assertion 33

The only substantial evidence to show that students might want to be taught through their preferred learning modality exists for students expressing a strong preference for the visual modality.

Assertion 34

Having a strong preference for receiving information visually is more closely related to preferring to solve Number Sense Inherent Problems as opposed to solving Devoid of Number Sense Problems.

Observation of Learning Style from TASRI

Analysis of the Think Aloud and Stimulated Recall Interview (TASRI) data, which was obtained through observation of students at work and interviewing them, revealed that to discover students' learning styles it might not be sufficient to base such analysis solely on data gathered through a questionnaire. What follows are suggestions as to possible factors which could be more accurate predictors of learning style pertaining to number sense and mathematical problem solving performance.

By the time that the TASRI was administered, information gained from previously collected data had already indicated that both the teachers and about 78 percent of all students were of the view that number sense ability influenced problem solving ability. Hence, at the start of the TASRI the high number sense students were interviewed first so that their behaviours could then be compared with those of other students. During the TASRI it was noticed that the eight students who were classified as simultaneously high in number sense and problem solving performance (HnsHps) solved problems in at least four main noticeable ways. Table 4.48 provides the nature of the strategies and the respective descriptors. It should be noted that these were first observed through the behaviours of the simultaneously high number sense high problem solving students. Once these common strategies had been identified and described the observation was extended to other students in other categories. Although some students in other proficiency bands also used one or more of these strategies, the Catalytic-Clarifying strategy was more obviously used by all high number sense students as evidenced through Table 4.49.

Table 4.48 The most common type of strategy use by the HnsHps students

Type of strategy	Action	Most Common Reason(s)
Single effective strategy	Employ only one strategy throughout, if to them the problem was 'straight forward'	• Could recall a similar problem; • Straight forward problem, straight forward strategy.
Combination of 'helpful' strategies	Use a combination of various strategies to solve a problem, depending on the natural requirement of the respective aspect of the problem. Usually a previously used strategy is not re-used.	• This strategy has served its purpose. A new one is needed now; • It would take too long with only one strategy; • These strategies help each other out.
Trial and error strategy leading to more effective strategy	Start off with a strategy of which they are not 100% sure, and then switch to a completely new strategy if they found that the one they were using was hindering their progress.	• Had to start somewhere, otherwise would lose time; • By attempting to solve with the first I can discover the 'correct' strategy.
Use of catalytic-clarifying strategy	Start off with one strategy, then switch to a new one if the first one seems 'difficult' to work with, then go back to using the first one and solve the problem	• Need another strategy to confirm what has been discovered so far; • Use whichever one would speed up the work of the first one.

It should be noted that except for students belonging to the categories HnsHps, HnsMps, HnsLps and MnsHps, other students employed less common strategies outside the four presented in Table 4.48. Some interesting discoveries were made with respect to the most common problem solving strategies used by students, and some of the most striking are presented in Table 4.49. It seemed that as the students' level of number sense shifted from high to low, their reliance on a single problem solving strategy increased from 25 percent to 79 percent of cases for HnsHps and LnsLps respectively.

Table 4.49 Number of problems for which students applied a particular strategy

Type of strategy		HnHp (8)	HnMp (5)	HnLp (1)	MnHp (2)	MnMp (13)	MnLp (4)	LnHp (1)	LnMp (5)	LnLp (6)
Single effective strategy		8	6	2	5	38	10	3	14	19
Combination of 'helpful' strategies		16	8	1	2	11	3	0	0	1
Trial and error strategy leading to more effective strategy		3	3	1	1	1	0	0	0	0
Use of catalytic-clarifying strategy		5	3	0	0	0	0	0	0	0
Number of problems	Strategy Applied	32	20	4	8	50	13	3	14	20
	Max	32	20	4	8	52	16	4	20	24

Note: The number of students per category is presented in parentheses. The total number of problems was calculated as the number of students in a category multiplied by four (as there were four problems). In order to fit the codes into the table the 's' has been dropped from both 'ns' and 'ps'; e.g., instead of Hns the code used is Hn.

This discovery was surprising in that high problem solving performance did not seem to make a difference as to whether students stuck to a single strategy or not. The reverse trend was observed for the strategy "combination of 'helpful' strategies" where it was observed that the lower the number sense proficiency level of the students the lower the probability of them using a combination of effective strategies. It should also be noted that students with higher number sense tended to use "educated guess and check" more than those with lower number sense, who tended to use a more random trial and error strategy which was mostly not based on an inappropriate estimate. Another interesting result was that in only eight instances, pertaining to the HnsHps and HnsMps categories, did students tend to assess the effectiveness of the strategy being used and decide on whether to use another strategy to act as a catalyst. On the other hand students with simultaneously low number sense and low problem solving (LnsLps) tended to stick to a single strategy about 79 percent of the time. Only one of the LnsLps students tried to switch to a new strategy if the first one was proving unsuccessful. Some of these LnsLps students (21%) would rather give up than attempt to use a new strategy. Based on what has been discussed so far the following assertions are in order.

Assertion 35

The lower the number sense ability of a student the greater his/her reliance on using a single problem solving strategy, regardless of whether this strategy is effective or not.

Assertion 36

High problem solving performance, irrespective of number sense ability, does not seem to make a difference as to whether students stick to a single strategy or not.

Assertion 37

Only HnsHps and HnsMps students tended to use an additional strategy to confirm what had been achieved so far as a means of speeding up the solution process.

Another observation which resulted in the provision of pertinent information as to the factors which could be most influential in the number sense and problem solving habits of the students was how they behaved in situations where they were 'stuck'. When students stated that they were 'stuck', their behaviour was observed closely for any repetitive or prolonged behaviour patterns. Data analysis revealed that while students belonging to the intermediate proficiency bands exhibited conflicting behaviour patterns, there were some notable clusters of behavioural differences between the HnsHps and LnsLps students as highlighted in Table 4.50.

Table 4.50 The most common behaviour of students in the two extreme proficiency bands who got stuck while solving a problem (N = 45)

Factor	HnsHps	LnsLps	Total	
			L & H	All
Inadvertently starts all over again	0	21	21	32
Doodles, scratches a body part and/or mumbles	12	13	25	29
Reads problem once again, goes over main points and estimate(s)	15	2	17	22
Gives up instantly and goes to another problem	1	9	10	15
Maximum Possible Total	8 (x 4 problems) = 32	9 (x 4 problems) = 36	68	180

Note: Since for the TASRI there were 45 students who had to solve 4 problems each, the total number of solution attempts was 180.

Of the 10 students who decided to start all over again without even trying to do anything to see why they were stuck, seven were LnsLps students. Some of their most common reasons for acting this way are recorded in Excerpt 36.

Excerpt 36

Sonia[$S_{(1,48,2)}$]:	I don't like wasting time because usually it's difficult to know why you are stuck.
Tarrick[$S_{(3,50,1)}$]:	When I start all over again sometimes I manage to solve the problem.
Lolita[$S_{(1,1,2)}$]:	If I go back to the beginning I find it hard to concentrate and I can't find a way, and then I lose a lot of time.
Charlotte[$S_{(1,15,2)}$]	To get the answer you cannot be too slow because you won't finish all the questions.
Terrence[$S_{(1,1,2)}$]:	When I go back and start all over again I don't feel the pressure. It's just like a fresh start. You forget about the mistakes.
Mena[$S_{(1,1,2)}$]:	I hate going round in circles and if I don't start again that's what happens.

From Excerpt 36 it was apparent that compared to the HnsHps students, the LnsLps students had a learning and problem solving style where they seemed to be overly concerned with loss of time. Another common theme in their preference was that they thought they could hide from the mistakes and pressure of solving the problem in this way. These low performing students tended to provide the same answer for why they suddenly stopped and wanted to go to another problem without giving the present one another try. On the other hand the HnsHps students tended to share a totally different belief where they thought that "it is better to go over the problem and see what could be stopping you from doing it". As explained by Joseph [$S_{(2,32,1)}$], "if I am unable to keep going there must be something wrong. I have to search for it before I can continue working". Hence, the HnsHps students seemed not to focus as much on time as the LnsLps students. Moreover, the HnsHps students were more ready to discover and rectify any mistake before moving on.

Although each student worked in certain personal ways the HnsHps students exhibited certain patterns of behaviour which incorporated some common elements as presented in Figure 4.18. Compared to students performing lower proficiency levels, the HnsHps students seemed to be the only ones who consistently employed a learning and problem solving style in which they would jot down the key points after reading the problem. From there they usually tried to come up with a reasonable estimate.

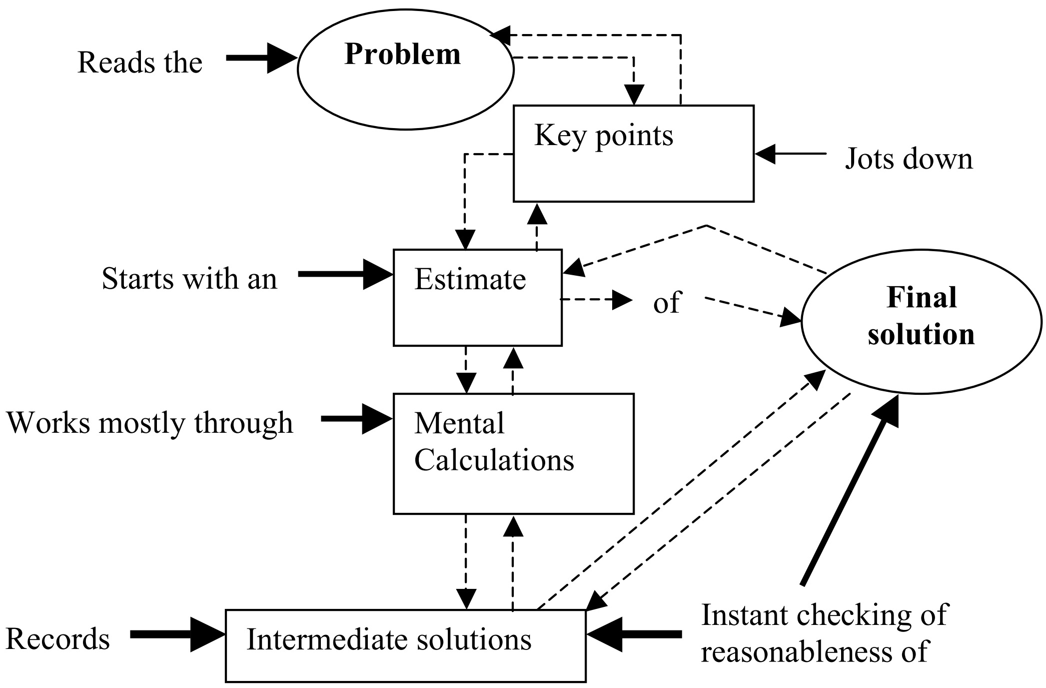


Figure 4.18 Most common problem solving pattern used by HnsHps students

Eighty-nine percent of the HnsHps students expressed similar views to Antoine [$S_{(1,1,2)}$], who attributed this to "…I am used to solving a lot of number problems, and this has given me the habit of trying to figure out [estimate] the answer before even solving the question". It seemed that the HnsHps students worked mostly mentally, and unlike many other students who preferred to use written algorithms, they used pen and paper mainly to record solutions that they felt cumbersome to keep in the head. As stated by Elnada [$S_{(1,16,2)}$] "…usually I prefer to work in the head, but when my head is like filling up with information I jot down the important points [intermediate solutions]". As previously explained, the HnsHps students were engaged in constant instantaneous monitoring of the reasonableness of any partial solution they obtained which, as explained by Alana [$S_{(1,3,2)}$] and Serge [$S_{(2,27,1)}$] in Excerpt 37, helped them keep track of errors and even sped up the solution process. It should be noted that it was typical of the HnsHps students to express a belief that when solving number sense inherent problems it was advisable to check for reasonableness of any partial solutions the moment these were obtained. Hence, it seemed that this process in successfully solving NSIP items was instrumental in HnsHps students showing a preference for instantly checking the reasonableness of any intermediate solution in any type of problem being solved. From Figure 4.19 it can be seen that this checking for the reasonableness of intermediate solutions was facilitated by constant reference back to the other aspects; from the immediate solution back to the mental calculations, which led back to the initial and intermediate estimates, going back to the key point and finally the problem itself. In this way checking for the reasonableness of the final solution was facilitated, since according to Sonia [$S_{(1,8,2)}$] it "…is easy to check how accurate your answer is if you have been checking all along".

Excerpt 37

Alana[$S_{(1,3,2)}$]:	I don't think it is a good idea to wait until the final answer for me to check whether I'm right or wrong. What if I have the wrong answer! Then I have to go back and try to find out where I went wrong. This is a waste of time, especially if you have numbers like fractions to deal with.
Serge[$S_{(2,27,1)}$]:	Every time that I look at what I have done so far it helps me keep track of what I have been doing. If you are doing a problem which has no numbers, like drawing or some puzzle or logic problem, sometimes you can get away with it. But I want to gain maximum correct [solutions] all the time, so it is best that I discover any mistake as soon as they happen.

Assertion 38

For the HnsHps students, being successful at solving Number Sense Inherent Problems provided them with a ready-made set of tools to solve other types of problems.

Other learning style factors which could be transformed into questions for a learning style inventory will be illustrated while presenting the amendments to the original theoretical framework at the end of this chapter, when a causes and effect diagram of attributes of HnsHps students will be proposed in Figure 4.29. As it will be discussed when presenting the amendments to the theoretical framework, the six major factors derived from the TASRI were:

- PS Tool bag — Familiarity, facility and success in using standard and non-standard PS strategies;
- Versatility — Not being afraid to use, discard, and re-use various strategies. Being ready to invent, combine and adapt strategies;
- Self-Correction — Constant monitoring of progress. Look out for and rectify errors. Assess reasonableness of conclusion;
- Gut Feeling — Don't be afraid to start. Trust inner feeling if not sure where to start. And start anyway;
- Preference — Develop a Practical and intrinsic preference for solving both NSIP and DNSP; and

- Persistence — Develop and maintain a persevering attitude. Always believe it is possible to solve the problem.

As suggested earlier, the behaviours of the students as observed during the TASRI could be used as indicators of their learning styles and that these could be compiled into a learning style inventory specific to gaining information about a student's number sense and problem solving learning style. For instance, the six major factors just presented and the strategies that students used could be reworded into questions which would elicit certain number sense and problem solving attitudes and habits in the students' memorised recollections. Since these questions would be based on specific factors derived from the TASRI, the students' responses would be more relevant to how they preferred to respond to the mathematics problems they had to solve. In this regard the factors presented at the end and earlier in this chapter would be useful.

4.7.3 Summary

Although, except for the processing information dimension, none of the statistical tests used on the ILS inventory data produced any significant mean differences in regard to the other learning dimensions, some interesting results were obtained indicating that HnsHps students are mostly reflective compared to the LnLps students who are mostly active. Through the ILS it was also discovered that students with high number sense were more intuitive than they were sensing, and that a large majority (92%) of students preferred to receive information visually. Except for a marked difference between students processing information actively and reflectively, and receiving information visually and verbally, all other differences, pertaining to either sensing and intuitive, or sequential and global, were deemed to be negligible.

With regard to classroom observations it was discovered that there were some instances where students requested for information to be presented in a mode which was compatible with their learning style preference, as obtained through the ILS. About 50 percent of students, who requested for information to be presented in another mode, asked for information to be presented in a mode which the ILS results suggested as their preferred one. Preference for NSIP or DNSP was identified as one possible factor which could help in determining a student's number sense-problem solving learning style. The only substantial evidence to show that students might want to be taught through their preferred learning modality was observed for students expressing a strong preference for the visual modality. Moreover, having a strong preference for receiving information visually is more strongly related to preferring to solve NSIP as opposed to solving DNSP. This could be related to the fact that the teachers employed a lot of physical visual aids and encouraged students to use visual imagery to concretise the abstract numerical elements they were dealing with. Although this practice was extended to other mathematics curriculum strands it was most prominent in regard to solving number sense inherent problems since most of the teaching-learning experiences observed involved number sense.

Results of the TASRI revealed that the lower the number sense ability of a student the greater the reliance on using a single problem solving strategy, regardless of whether this strategy was effective or not. High problem solving performance did not seem to make a difference as to whether students stuck to a single strategy or not. Only HnsHps and HnsMps students tended to use another strategy to confirm what had been discovered so far as a means of speeding up the solution process. According to the HnsHps students, having good number sense problem solving habits provided students with a ready-made set of tools to solve other types of problems.

Hence, it is suggested that when considering implications for research, using a learning style inventory on its own might not be sufficient for gauging the preferred learning style of a student unless another form of data collection, such as the TASRI, and classroom observations are employed. The data collected could then be triangulated to give a more accurate picture of a student's learning style preference. Furthermore, the proposition made about using various types of data collection methods and instruments seemed to be the most appropriate approach to use for obtaining and analysing accurate data about the impact of learning style on a student's number sense and problem solving ability.

4.8 Analysis and Results of Research Question 4

How do the teachers' beliefs concerning the link between number sense and problem solving impact on their teaching of number sense?

Since the three teachers participating in this study were specially selected as effective teachers of mathematics the focus was more on their common beliefs and practices rather than their differences. Hence, in the following section the teacher's beliefs will be explored through analysis of common issues emerging from the four formal interviews and the twenty-five short informal interviews. These common issues will be presented through themes extrapolated from the data. To confirm how the teachers' beliefs seemed to impact on their teaching of number sense, the interview data was triangulated with all the other data collected.

4.8.1 Gaining information about the teachers' beliefs

Formal Interviews

Each term the teacher was engaged in one formal interview (Appendix VI). The questions asked throughout the formal interviews were geared mainly at discovering what could be the implication(s) of the teachers' beliefs upon getting learners to develop good number sense and problem solving ability. The students' views were also solicited on some parallel question themes as a means of validating the authenticity of the teacher's views in terms of their students' perceptions. Once these were coded and analysed any common themes which emerged were then explored through short discussions, from here forth referred to as informal interviews. The themes explored in these informal interviews were also informed by other data gathered through observation.

Informal Interviews

Before the collection of data each teacher was briefed about participating in informal interviews. Hence, the researcher had a short five- to ten-minute discussion with each teacher usually once per week. An informal interview format was designed so as to maximise the collection of relevant data around pre-discovered themes; to this end the first interviewee was used more or less like a guinea pig. Although the researcher purposefully engaged the teacher in a sort of interview-like discussion, the interaction was conducted in such a way that it seemed like just a normal little chat with the teacher. This ensured that both teacher and researcher interacted in a very relaxed 'normal' atmosphere, unlike the usual scheduled and anticipated interview format. Due to the nature of the design of this sort of interview, as much as possible, visit days to each school were rotated so as to ensure that each teacher got a chance to be interviewed first. The informal interviews usually revolved around no more than three questions. In most cases only one main question was used to start off the discussion with other subsequent questions being formulated as per the direction of the discussion. Hence, it was normal for only one major question theme to be prepared and put to the first teacher interviewed on a particular week, with all other questions being formulated according to the flow of the answers, explanations and other statements made by this first teacher, as the discussion progressed. When the first participant was interviewed, it was quite usual for a particular pre-formulated question to be followed by other spontaneous questions ensuing from this first discussion. Once the first interview for a particular pre-formulated question was over a simple trend analysis was done to ascertain certain relevant directions that the subsequent questions had followed throughout this first discussion. Consequently the next two teachers were interviewed in such a way that question themes explored in the first interview would be more or less preserved. So, although most subsequent questions were not pre-formulated for the first informal interview, some form of pre-coded question themes were used in the next two interviews. In this way it was ensured that:

- the discussion would flow as freely as possible;
- these teachers were made to reflect and discuss nearly the same issues;
- common themes could be explored in a semi-flexible interaction format;
- the element of irrelevancy of information would be reduced considerably; and
- coding of data would be more controlled and focused on relevant issues.

The Teachers

All three teachers taught at Year 7 level only and they were each very active in their school's professional development programmes. Amanda had taught at this year level for 20 years. She taught Mathematics, English, Society and Environment, Science, Health, Technology and Enterprise. Bob had been teaching at this year level for 23 years. He taught Mathematics, Religion and Values Education. Chantal had been teaching at this Year level for 30 years. She taught all subjects except LOTE and Music. What follows are these teachers' common beliefs as obtained from samples of the interviews, from which information was extracted for use as direct or indirect quotations in the thesis. These beliefs are presented in the form of themes and extrapolated factors.

4.8.2 Beliefs about catering for students' learning styles

When the teachers were asked whether they thought a teacher should cater for individual students' learning style, it was discovered that they employed certain common approaches — one of which was providing individual attention — in order to help their students improve their number sense proficiency. Moreover, the observation data pertaining to the number of times that each teacher was engaged in a one-on-one interaction (which lasted two minutes or more) with a student, revealed that on average these teachers interacted in this way with about nine students per lesson.

Table 4.51 Number of teacher-student interactions, per lesson observed, lasting two or more minutes

	Amanda	Bob	Chantal
Mean	9	11	8
SD	5	7	6

Since 81 percent of the 91 lessons observed were either totally focused on or incorporated some aspects of number sense, it was no surprise to find that much of this one-on-one interaction revolved around enhancing the students' number sense proficiency. As explained by Chantal, teachers were of the view that "students need to master the fundamentals…[which comprised] the basic number facts". Bob explained that he thought it was "…very important to be one-on-one with the students… [since this]…provided the teacher with an opportunity to cater for the specific needs of the student". In regard to number sense this was seen as being "…necessary to engage students on an individual basis so that I can understand where they might be encountering certain difficulties" (Chantal). Amanda also expressed the importance of engaging students in one-on-one discussion so that "…I can relate to them according to their specific needs", and further stated that "…although I don't go out to place special emphasis on number at the expense of the other strands I must admit that number is one area where the weaker students sometimes need more time with the teacher".

Further questioning revealed that this was part of these teachers' method of trying to teach according to students' learning styles, although none of the three teachers were of the idea that they were catering to learning style as such. One common reason given for this belief was that in regard to number sense a student's usual way of working was not really what they preferred but rather a habit which most of them were not aware of. Moreover, according to the three teachers involved in this study, it is not easy to change one's learning style, although all three agreed that components of learning style can be enhanced "…so that students can gain a better understanding of the underlying principles which one must master in order to develop good number sense" (Bob). The major components mentioned were: (1) method and level of concentration; (2) how individual learners process and retain information; (4) intrinsic motivation; (5) past experience; (6) self confidence vis-à-vis the subject under study; and (7) learning strategy. They all agreed that instead of focussing on teaching to the learning style of individual students, it is best to vary the teaching strategy and approach as much as possible. The observations confirmed that these teachers incorporated the following components in 80 to 90 percent of the delivered number sense or number related learning experiences: (1) graphical visual aids; (2) 'mental' manipulatives; (3) verbal-aural participation; (4) use of allegories; (5) prompts for analysis; (6) encouraged synthesis and sense-making; and (7) integration of information. Chantal explained that:

> Some of these students, the less able girls, come to my class feeling that they cannot solve number problems. If I do not sit down with them and go through various ways of understanding what they are doing, what the numbers mean, why they are like that…they find it hard to cope in class.

This belief was also shared by most students, as explained by Miranda [$S_{(3,64,2)}$], who had the lowest NST performance score (30%), "…when I first came to Mrs [Chantal] class I did not understand the calculations [algorithms] I did. Mrs [Chantal] worked with me in class ... I feel I have improved a lot". Figure 4.19 shows the mean duration of the lessons observed and how much time was spent on average on number sense related experience activities. Except for Bob, as summed up by Amanda, the other two teachers agreed that lack of time prevented them " from providing students with much needed one-on-one help". The fact that Bob had fewer students and longer average teaching time per session could be the reason why he was the only teacher who managed to spend two-minutes or more with each student in some of the lessons observed in his class. None of the other two teachers could achieve this in any of the lessons observed. Hence, the teachers were probed further as to how they catered for individual differences, especially in relation to their students' number sense problem solving.

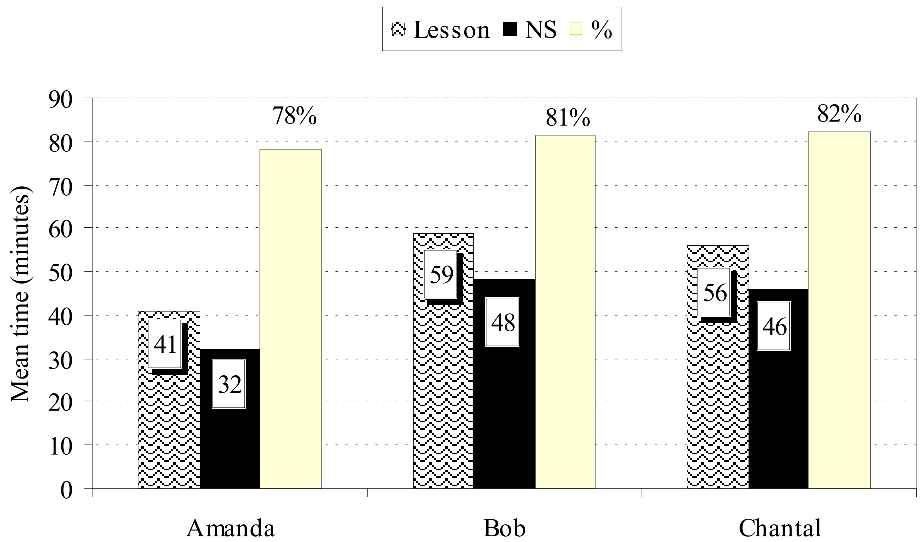


Figure 4.19 Comparative time spent by the teacher on whole lesson and number sense

Catering for Individual Differences

Although all three teachers used verbal exposition in most of their deliveries, none of the number sense problem solving lessons observed involved whole class teaching throughout. When questioned about whether they would encourage anyone to use whole class teaching throughout a teaching period to teach for number sense, the teachers' answers pointed to the need to satisfy the differences in their students. Just like the other two teachers Amanda believed that:

> if the teacher had a class where all students were of the same ability, same personality, and so on, then the easiest option would have been to use the whole-class teaching approach most, if not all, of the time.

Bob summarised it all when he stated that students should be taught "according to their ability and performance", and according to Chantal "sticking to one teaching method throughout one lesson fails to cater for individual differences". With regard to number sense the teachers thought that the main reason for diversifying the teaching approach and style was that students were different in the way they preferred to learn about and solve number problems. As explained by Amanda:

> When it comes to number some students, like the less able ones, learn best through visual aids and manipulatives. While the more able might want to work more with equations and formulae

As highlighted earlier when answering question 2, these teachers thought that an effective teacher must always believe that it is possible to reach each student. Nevertheless, the data further revealed that these teachers did not believe it was possible to cater for each individual student's learning style, as highlighted through Chantal's statement that "[although] there'll be people who'll tell you it is possible,…I don't believe it's possible…to cater for thirty different levels". Hence, the crucial question revolved around how these teachers believed they catered for individual differences in their students, upon which it was discovered that there were five major common aspects which were referred to repeatedly by all three teachers (Figure 4.20): (i) evaluation of students' ability; (2) grouping of students; (3) problem-based learning experiences (4) variation and differentiation of learning experiences (5) monitoring and on-going evaluation. Further investigation and analysis of data eventually resulted in the emergence of a picture in which individual differences were catered for through a grouping system whereby students' number sense level was used (at initial evaluation stage) as an indicator of their problem solving ability, which in turn was used to group the students. As illustrated in Figure 4.20 the two major considerations pertaining to individual preferences and the most common preferences shared by the whole class were also taken into consideration.

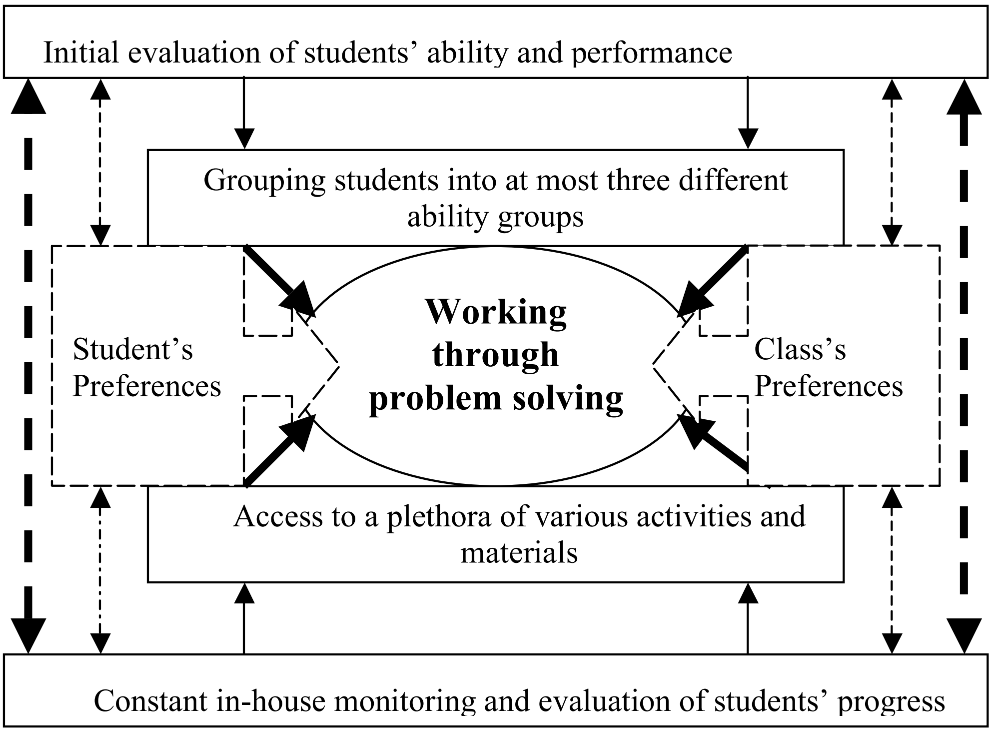


Figure 4.20 The process through which teachers cater for individual differences

The initial evaluation of students was a mixture of short problem solving activities which mainly 'tested' a student's number sense, impromptu and informed interviews. In the case of Amanda, she stated, "I don't formally test to find this information. My approach is more informal through discussion when we begin a new topic". Similarly, Bob's view was that "by talking to the students and watching their behaviours I gain a lot of information, which I then use to gain an idea about their personality", and as he later explained, the students' "number sense [proficiency] was also gauged during this initial evaluation phase". Chantal did nearly the same thing as Bob, but maintained that this was "due to lack of time…[since] it takes time to get to talk individually to all the kids to understand how they feel…and what they understand", which was a concern expressed by all three teachers since they felt that "talking to the students individually…you gain more information about them as individuals…", as expressed by Amanda. Just like Bob, the other teachers agreed that the information gained from tests, observation and interviews helped them understand the students better. Amanda believed that "to understand these students better you have to allow them to express themselves through discussion with their peers and asking questions to the teacher". This was a prevalent belief of all three teachers which was supported through observation data, and it could be the main reason why most of these students did not show a great preference for receiving information via the verbal modality. Bob maintained that although they did group the students he coped:

> …with them by making sure that I understand and know my students and their capabilities fairly well and which ones need some extra help. We're trying to put in place an educational program whereby we've got a lot of evaluating of our students and their learning, and where they're at. And by keeping that in mind when I'm actually teaching and also making sure that those students are getting extra help, when they need it, in particular areas.

In all three classes this evaluation was done through observation, talking to the students and comparing these with the respective academic performance (in this case NS and PS) of the students. Although there were some differences in the way students were grouped in each of the three schools, a common practice was that the students were placed in groups only after the initial evaluation data had been analysed. The interesting aspect of this practice was that some grouping was overtly done so that students were directly aware of them while other sub-groupings — within the major groups — were done subliminally in most cases. Just like with the other teachers, to Chantal it was important to "protect the students' self-esteem", and as pointed out by Amanda "by allowing them to work with whoever they want as well, helps in making them feel more secure and part of the whole class". In addition Bob stressed that "there must never be some thick boundary lines around the groups so that students would start feeling alienated". Hence, evaluating the students according to how proficient they were at solving number sense problems was seen, by all three teachers, and as explained by Bob, as "…a way of ensuring that they were being judged more fairly…" since, as pointed out by Amanda "although I do not formally test my students…judging them on any aspect of mathematics would surely involve number sense, which is more common [than other mathematics curriculum] content and processes". In the same vein Chantal explained, "…if I were to group them based on their level of chance and data, or shape and space I would place most of my students at a disadvantage [since] these [non-number strands] do not cover as much a large chunk of the curriculum …", whereas "…number permeates all of the other strands [of the mathematics curriculum]".

Assertion 39

Group work was seen by teachers as a way of bridging the gap between the belief that it was important to attempt to reach each and every student, and catering for individual differences in practice.

Therefore, it seemed that these teachers were very concerned about the students' confidence and anything which could affect that, which caused them to initially evaluate the students more on their number sense ability than any other mathematics content or process due to the students being more familiar with the number strand as opposed to the other mathematics strands. These teachers' tendency to act fairly towards the students also played an important role in the way the students were grouped and degrouped according to their number sense, problem solving ability and personal preferences.

Assertion 40

Since the number strand is the only one which permeates all other mathematics curriculum strands, the informal evaluation used for grouping the students involved a lot more focus on number sense, as opposed to the other mathematical aspects, mainly because students were more familiar with the number strand.

Given that lack of time was perceived as preventing these teachers from engaging all students in one-on-one learning experiences, all three teachers thought that group work was therefore the most effective solution in terms of catering for the various differences in their students. When asked to explain how they went about grouping students it was discovered that proficiency in solving problems was a very important common factor. Bob explained that students were grouped "not only according to individual personality differences but also their problem solving ability". Since Chantal's initial view also encompassed "number proficiency" as a requirement for grouping students, the other two teachers were asked: How influential was the students' number proficiency in placing them in ability groups. Amanda thought that "it all depends how you look at it. I won't say it is directly influential in getting students to work together, but it does play a role since most problems contain a numerical aspect", whereas Bob thought that "…number provides an easy way of gauging the students' problem solving ability because they [number] permeate most problems". In that respect it became apparent that when the teachers referred to getting students to work in problem solving groups, the criteria used were more focused on student's number sense proficiency than on any other mathematical aspect.

Assertion 41

A common way of catering for individual differences was through problem solving ability grouping, which in turn was gauged through students' number sense proficiency.

The link between catering for individual differences, problem solving and number sense was, as extrapolated from Bob's statement, "…maintained through creating an environment where the more able students would often get on with their work while the teacher spent more one-on-one time with the weaker students". Hence, all three teachers tended to share a belief that "the above average students needed very little coaching", as expressed by Bob, while Chantal insisted that "…to get to [the level of] the high problem solvers, they [weaker students] need to learn the strategies by direct teaching methods". Amanda indicated that she set up "…problem solving groups…so that if they finish another task that we are all working on together then they always have those speeding [items] so that they can go on with their problem solving". At no point during the study did any of these teachers waver about this belief that students who are below average in number sense and problem solving need to be taught the strategies through a more or less direct teaching approach while the above average work a lot more independently. As explained by Amanda "by teacher-directed I mean as a teacher you are giving satisfaction where the teacher is setting up a situation and explaining what it entails, how it is done and why". A note of caution is appropriate here, since if such a belief was taken into consideration on its own, without observation, many important and relevant applications could be missed. For instance, although these teachers stated that they also used direct teaching, the duration of such instances, especially the ratio of teacher-talk to other more student-directed aspects was on average about one to five. Even when these teachers engaged in what they called direct teaching, their belief was that "the students must be presented with many opportunities, through a huge variety of problem solving activities, to be actively involved in their own learning of mathematics". This was confirmed through the observed teaching which revealed that on average the teachers employed a mixed method comprised 75 to 80 percent of student-involved time. This was seen as being "…of great importance in getting students to help each other make sense of the mathematics they are learning" (Bob). Chantal also thought that "…when the kids explain what they have done, to each other, they learn to make sense of the numbers and other mathematics from another perspective…other than relying on the teacher". When asked whether she thought that getting students to work in collaborative groups developed their number sense, Amanda stated "it's not only number sense — although it does occupy a big chunk of what they discuss. They are always being prompted to make sense of any mathematics they learn, and…yes, working in groups provide them with this sort of platform". To Chantal "…a lot of number work is very important because once they can make sense of number it is easier for them to apply this to other areas [mathematics content]. Amanda's comment was also in line with Chantal's when she stated that "it is obvious that the girls with better number sense can solve many more…and various types of problems on their own", which is why, just like the other two teachers, she tried "…to get them [both high and low ability students] to work together…to help each other out".

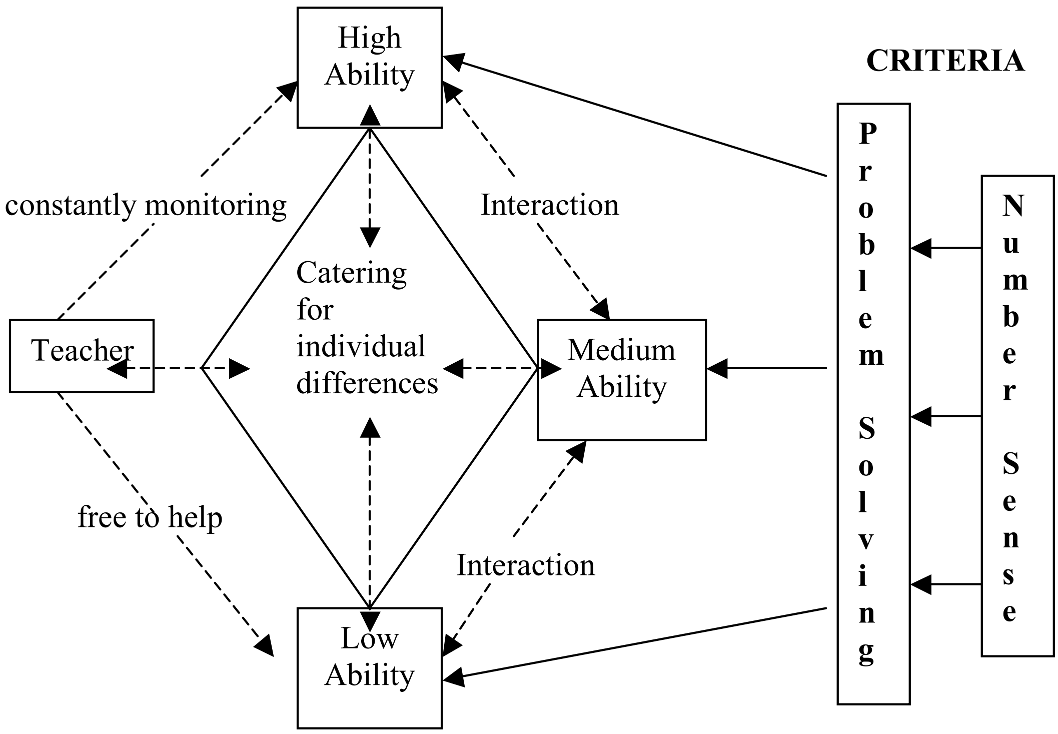


Figure 4.21 Catering for individual differences by grouping through NS and PS ability.

It was a common belief among these three teachers that in this way, as expressed by Bob, "the less able gain a better understanding of how numbers and other mathematics contents are related, since they are learning from their peers and not from the teacher". Figure 4.21 illustrates the teachers' beliefs that grouping students through their number sense and problem solving ability facilitates teacher interaction with, monitoring and helping students according to their needs. As number sense is believed to be a major contributor to problem solving ability, it serves as an indicator of a student's problem solving ability, which in turn is used to place students in problem solving ability groups. There are usually two main groups — low ability and high ability — with the middle ability students either subliminally shared between the two extreme groups or sometimes formed into a group of its own. As explained by Chantal it was a common belief that the teacher should"…go around the class to monitor the students' progress", and according to Bob, "…spend more time with those in need of help", which are most often "…the less able students [who] are the ones who need more individual attention". Table 4.52, which is a summary of data previously presented in Table 4.33 and Table 4.34, gives an indication of the number of interactions — lasting two minutes or more — in which the teachers were observed working one-on-one with individual students. Since these teachers seemed to evaluate and group students more on their number sense problem solving proficiency than on any other mathematical aspect, after the data had been collected the researcher decided to group the students according to their performance on the Number Sense Test (NST). Hence, it was discovered that the teachers were giving most attention to the low number sense students (an average of 18 visits per student) and comparatively very few to the high performers (7 visits on average). This was in tandem with the teachers' belief that the less able students needed the teacher's guidance much more than the more able ones. Nevertheless, any request for assistance from the high performers was always promptly attended to. A common reason for the teacher giving unsolicited one-on-one assistance to the less able was that "the high flying students prefer to overcome a challenge without the teacher's assistance" (Bob), whereas the less able students "are often introverts, who could be very shy or prefer not to be seen as needing help all the time" (Chantal). As explained by Amanda "I have to be on the lookout for those who might need more help but for some reasons are holding back. Hence, I go round and check [what each student is doing] all the time, and in this way I can help the less able without intimidating them".

Table 4.52 Number of teacher-student one-on-one interaction lasting at least two minutes

		Number of one-on-one interactions				
Number Sense Proficiency	Number of students	Once	Twice	Thrice	Total	Mean per student
Hns	19	125	5	0	135	7
Mns	26	283	38	3	368	14
Lns	19	242	38	8	342	18
Total	64	650	81	11	845	13

Note: The mean number of visits per student pertains to a total of 91 lessons observed.

Chantal echoed the other teachers' sentiments when she claimed that "…interaction with the students is a two-way process…they talk to me and I talk to them…They propose ideas and we discuss these". As pointed out by Amanda, the students are "…encouraged to work collaboratively with each other…The more able help the weaker ones", and it is also believed that "…the weaker students also contribute ideas which the more able students might not have thought about" (Chantal). In that respect the teachers employed a hierarchical consultation system in which students were encouraged to develop confidence by solving a problem on their own first [Initial phase]. If they encountered any situation which prevented them from solving the problem their first point of contact would be other students in their group [Intermediate phase]. The teacher's responses presented in Excerpt 38 show that consulting the teacher was believed to be used only as a last resort [Emergency phase] if the student concerned still could not resolve the issue. Even then the teacher might still decide to ask other students to help answer the query of that particular student, and the teacher would intervene only when he or she was satisfied that his or her help was really needed [Emergency phase].

Excerpt 38

Bob:	I insist that students try to solve the problem first on their own, then they need to consult each other about their solution. If their friends cannot provide them with a good answer then they can come to me.
Amanda:	The girls are now used to getting immediate help from their peers before consulting me. This reduces the pressure on me to deal with each little issue…most of which they can resolve among themselves.
Chantal:	…the [more able students] rarely call me unless those in their groups have been unable to help them. I encourage the [less able students] to also consult those in their group first, but their case is a bit special. Sometimes they prefer that the teacher helps them.

Figure 4.22 illustrates how these three phases relate to each other.

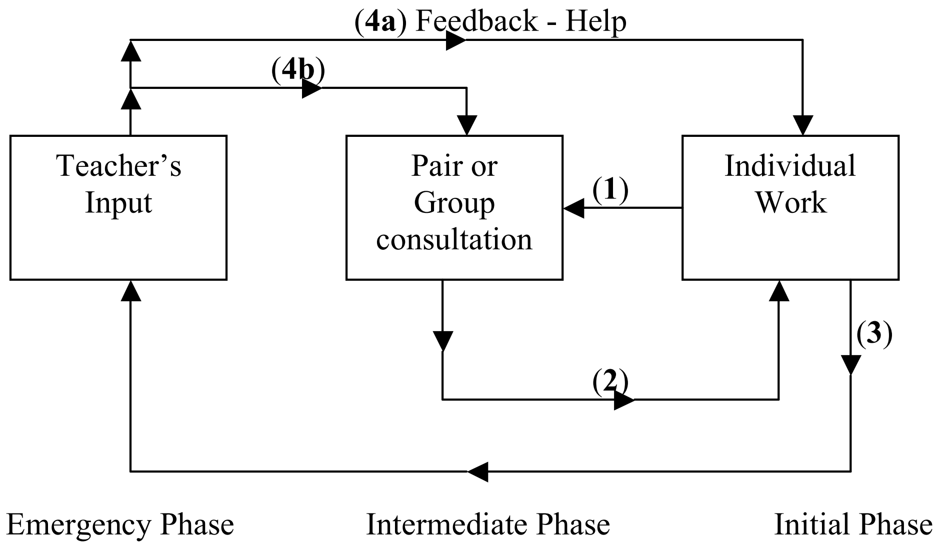


Figure 4.22 Getting students to be more autonomous and reducing pressure on the teacher

> **Assertion 42**
>
> Grouping students according to number sense and problem solving ability allowed teachers to work with the less able students, while those who needed less teacher-attention got on with their work.

> **Assertion 43**
>
> Students were encouraged to: (i) first work on their own first; (ii) consult others in their group if they still had an issue which they could not resolve; and (iii) seek the teacher's help only if both of the two previous avenues did not satisfy them.

It is worth noting that as the school year progressed the individual attention (one-on-one teacher-student interaction) rate was reduced considerably as shown in Table 4.53 and graphed in Figure 4.23.

Table 4.53 Frequency of one-on-one teacher-student interaction throughout the year.

Teacher	Term 1		Term 2		Term 3		Term 4		Total	
	Count	%	Count	%	Count	%	Count	%	Count	%
Amanda	77	(28)	104	(38)	75	(27)	19	(7)	275	(100)
Bob	94	(29)	123	(38)	92	(29)	11	(3)	320	(100)
Chantal	72	(29)	111	(44)	50	(20)	17	(7)	250	(100)
Total	243	(29)	338	(40)	217	(26)	47	(6)	845	(100)

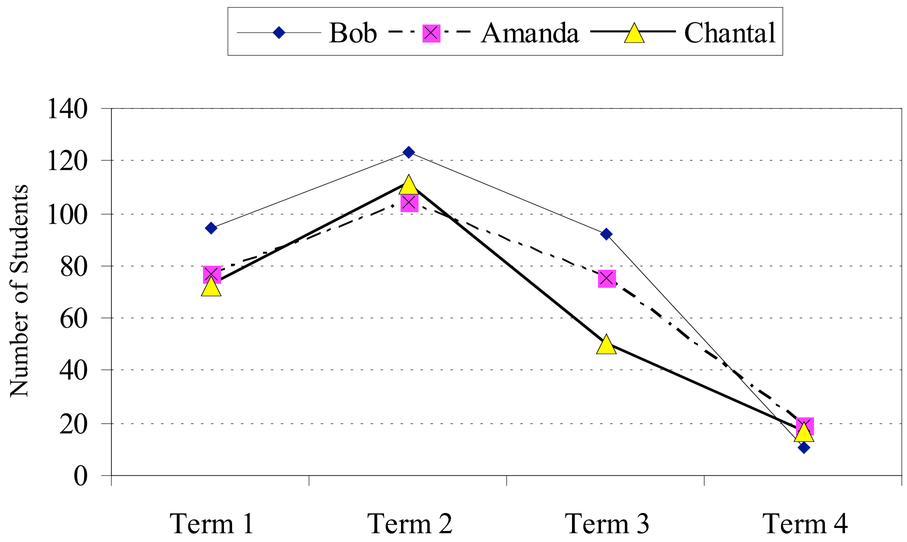


Figure 4.23 Frequency trend of one-on-one teacher-student interaction per term per teacher

Although Chantal seemed to have interacted with less students than the other two teachers she was the one who inevitably had to spend longer periods with certain weaker students,since she had more (15) low number sense students in her class than both Bob (1) and Amanda (3). All three teachers thought that at the start of the first term the teacher "…needs to learn about the personality of the students" (Amanda), try to "…gain their [students] confidence and gauge their knowledge base" (Bob) and "evaluate their performance, especially in terms of their basic facts" (Chantal). This could explain why during the first two weeks of the first term there were very few instances of the teacher working one-on-one, for two minutes or more, with individual students. The observation data revealed that the teachers spent this time getting to know their students, creating a good rapport with them and evaluating their number sense and problem solving ability. Bob's statement that, "I tend to use number sense as a barometer, to gauge the students' mathematical ability", was somewhat more similar to Chantal's belief, whereas Amanda was more concerned with their personality, although she did admit that she "…also made mental notes of how they coped with number problems".

Assertion 44

Since number permeates all the other content strands the teachers deemed it appropriate that when they get a student for the first time that the teacher gauges the student's mathematical ability based on the latter's proficiency at solving number problems.

From Figure 4.24 it is apparent that the rate of one-on-one teacher-student working together for two or more minutes reached its peak during the second term and then started dropping very quickly as the school year moved towards the third and fourth terms. Since all three teachers shared a similar belief, as expressed by Amanda, that the "…teacher should always aim to develop in the students a sense of wanting to work independently" they were asked whether they were aware of the decline in one-on-one teacher-student problem solving together. Their answers indicated that although they were aware of this decline they believed that they did not necessarily plan to reduce their one-on-one contributions. Nevertheless, further analysis of their responses, samples of which are presented in Excerpt 39, revealed that the teacher's belief about getting students to become more autonomous was very closely connected to:

- weaker students' improvement in mathematical performance and ability;
- allowing them to work mostly on their own;
- encouraging them to engage in cooperative pair and group work; and
- reduced interference from the teacher.

Excerpt 39

Bob:	I don't go out to stop providing help to the students. I still try my best to answer their questions as much as I can…but I also need to encourage them to become more autonomous.
Chantal:	By then [fourth term] many of the weaker [students] are more willing to work on their own or with their peers…So, I encourage them to do more on their own, and also to work together in pairs [and] in groups.
Amanda:	I think it is still important for me to monitor their progress and ask the odd question here and there to check upon their reasoning…[but] I do not believe that I should interfere if they are coping very well on their own.

This in turn could be the main influential factors in reducing the teacher's working one-on-one with individual students. When the teachers were asked about what aspects of the curriculum helped them in knowing that particular students had become more autonomous there was a diverse response, such as:

- improvement in problem solving performance;
- how they are able to make sense of the mathematics;
- being able to argue about and defend their solution method;
- mastery and application of the basic facts in more complex problems;
- increased use of mathematical language, equations, formulas;
- being able to explain and make use of mathematical relationships; and
- increased number sense.

As reported in Excerpt 40, in response to the question "Which of these factors [in the above list] was most easy to notice and face-evaluated", a student's number sense problem solving performance was deemed to be a very good indicator.

Excerpt 45

Amanda:	It is easy to notice the students' progress through how much they know about the basic facts, especially how they have improved in their use of number operations, how they can make sense of the numerical parts of a problem,…how they reason about these…and how they solve problems. If they can do this then there is a greater chance of them improving in solving spatial problems, chance and data, and so on…"
Bob:	…I watch a student for signs of improvement in their number sense. Can they explain what they have done, what the numbers mean? If yes, then that's a good sign…because most of the problems need good number sense.
Chantal:	The easiest way is to look at the extent to which they are able to put to use what they have learnt about the relationship between the number facts in a problem solving context. For instance, a child who could not see the connection between odd, even and multiplication might now be able to state that 'since there is an even number of house, number of people in all the houses together would be (a) 268, and not 267, 265 or 269.

Assertion 46

As students became more mathematically proficient, especially in their number sense, the teachers gave them more freedom to work independently.

Sense Making and the Importance of Number Sense

After it was noticed that in most of the interviews the teachers were stressing the importance of mathematical sense-making, they were asked to fill in a table to show which content sense was most important and to what degree (Table 4.54).

Table 4.54 Importance of mathematics content sense according to teachers' beliefs

	Rank by importance				Degree of importance			
	Teacher			Mean	Teacher			Mean
	Amanda	Bob	Chantal		Amanda	Bob	Chantal	
Algebraic Sense	N/A	3	3	2	7	7	7	7
Statistical Sense	N/A	2	3	1.7	7	7	7	7
Spatial Sense	N/A	2	2	1.3	7	7	7	7
Number Sense	N/A	4	4	2.7	7	7	7	7

Note: For Rank, 4 = Most important and 1 = Least important; for Degree, 7 = Extremely important; 1 = Not important at all; and N/A = Not Answered.

Figure 4.24 shows the mean of the aggregate of the rank and degree of importance for each content. It should be noted that getting the teachers to state the degree of importance for each aspect proved to be a dilemma for them, and hence it was deemed a poor indicator of their respective beliefs. Nevertheless it helped to confirm that they thought all mathematical strands and content components were very important.

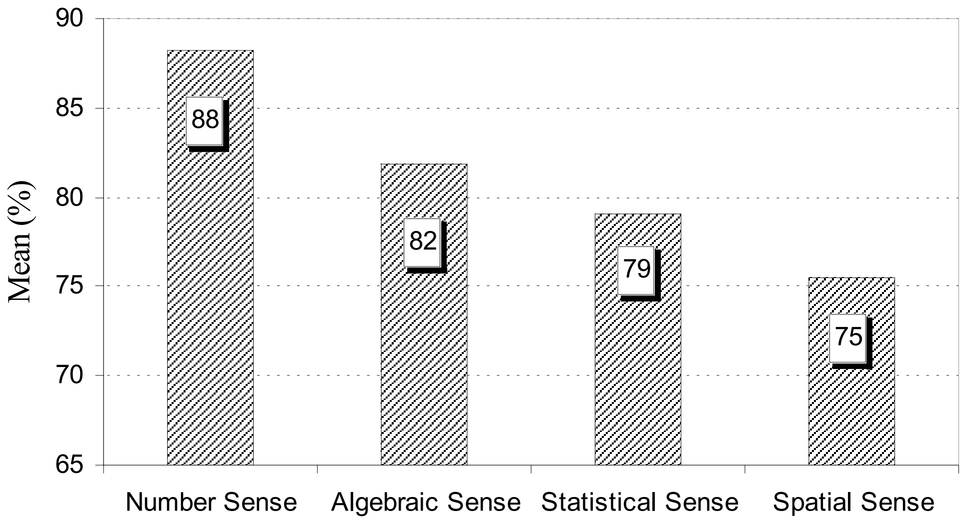


Figure 4.24 Aggregate of rank and degree of importance of four mathematics sense-making areas

The striking thing about this result is that two of the three teachers unanimously indicated that number sense was the most important sense making aspect of mathematics. Amanda was the only one who tried to argue that all mathematics strands and contents were equally important, although she eventually conceded that:

> it's true that number work is essential in most maths problems. So, that's why we encounter them in nearly every lesson…[although she stated that she did]…not plan to deliberately put more emphasis on number sense. It just happens that it [number sense] is present in most problems.

To gather comparative data, both teachers and students were asked to comment on what was required for a student to be successful at problem solving. Some typical comments given by the students and teachers are presented in Excerpt 41 and Excerpt 42 respectively.

Excerpt 41

Hanna [$S_{(1,12,2)}$]:	I think it takes practice and an understanding of what they are doing with the numbers…A student should be able to check the answer and if they have miscalculated and have a wrong answer the child should be able to see where they went wrong. [Attributes of number sense]
Elenda [$S_{(1,4,2)}$]:	You have to understand number and basic mathematics such as multiplication, division and tables. [Attributes of number sense]
Hyacynth [$S_{(1,10,2)}$]:	…to be successful it is usually understanding the wording of the question and the student must be good in number [Number sense].
Indigo [$S_{(1,13,2)}$]:	You have to be able to read the question and understand it well …. You then need to know the basic facts, like multiplication, subtraction, division and product [Number sense].
Kristofe [$S_{(2,38,1)}$]:	They should understand numbers and adding subtracting and multiplying and dividing [Number sense]
Joseph [$S_{(2,32,1)}$]:	A very good knowledge of equations. They must have good number sense.
Mona [S(3,44,2)]:	They should be good at number patterns and understand numbers and how they are used.
Lena [$S_{(3,54,2)}$]:	Be good with number…understanding about number.
Jono [$S_{(3,53,1)}$]:	You must know your basic number facts like addition, multiplication, division, and understand what to do with the numbers.
Anne [$S_{(3,55,2)}$]:	Know how and when to use various problem solving strategies. Have good number sense [Number sense]. Be able to think logically.

Analysis of the students' responses revealed that, similar to the teachers, there was substantial agreement among the students that number sense was essential for successful problem solving as evidenced through data presented in Table 4.55.

Table 4.55 Students' responses to the question: What does it take for a student to be successful in mathematical problem solving? (N = 64)

School	Factor				
	Number Sense	Reason Logically	Problem Solving Strategies	Understanding the Language	Think it through
Arlenta	18	12	7	15	6
Baden	8	0	2	3	3
Cotton	14	12	12	1	4
Total	40 (63%)	24 (38%)	21 (33%)	19 (30%)	13 (20%)

The results in Table 4.55 showed that nearly two thirds of the students considered number sense to be the most important factor responsible for a student's success in problem solving. It is interesting to note that although Amanda was the only teacher who seemed reluctant to set number sense as the most important aspect of her teaching and her students' learning, the majority (69% of Amanda's class) of those 40 students who perceived number sense as being a requirement for successful problem solving came from her class. The teachers' responses presented in Excerpt 42 shows that except for Amanda, both Bob and Chantal thought that number sense was an important pre-requisite for successful problem solving. Moreover, it is interesting to note that the Think Aloud Stimulated Recall Interview (TASRI) revealed that competences such as 'confidence' and 'persistence',which were mentioned by Amanda as requirements for success in problem solving, are hallmarks of good number sense. Hence, the results of this analysis confirmed the teachers' interview reports which situated number sense as the most important aspect of mathematical sense making which was linked to problem solving.

Excerpt 42

Amanda:	Confidence, persistence, ability to understand what the problem requires of them to solve it, accumulated mathematical knowledge
Bob:	They must learn to think critically, to be prepared to explain what they are doing through a thorough analysis. They must learn to explain their answers and solutions with clear reasoning. They need to feel comfortable in their manipulation of number [Number sense]; to be prepared to experiment, to be willing to try a variety of strategies until they find one that works. They must learn to become actively involved- it is through being an active learner that they learn to construct their own knowledge. They need to learn collaborative skills- through critical thinking, lateral thinking and group skills there will be a positive impact on their cognitive development. The children also need to develop a responsibility for their own learning and most of all they should experience some enjoyment!
Chantal:	Positive approach. Teacher to teach in a manner which allows students to work in a problem solving way. They must master the basics well so children have the tools they need to have a better number sense [Number sense]. Learn strategies and when to apply them. Practice. Challenges, e.g. maths competitions.

Assertion 47

Number sense was perceived by the teachers as the most important aspect of mathematical sense making that was linked to success in problem solving.

Eclectic Teaching Style: A Powerful Means of Adapting to Students' Needs

Since the question being answered is looking at the teachers' beliefs vis-à-vis their teaching of problem solving and number sense, the teachers were asked about how they taught for the development of their students' number sense. It was revealed that this was achieved through a mixed teaching style. The triangulation of data from the Index of Learning Style (ILS) inventory, the interviews and the tests results indicated that:

- teachers changed their teaching styles to adapt to the students' preferences and academic ability;
- eleven of the 19 students with low number sense unexpectedly tended to welcome the opportunity to work in the abstract, although to a lesser degree than those with medium and high number sense;
- teaching for number sense involved a lot of visual aids, which could be one reason why a large majority (92%) of all students preferred to receive information via the visual modality instead of verbally; and
- visual aids were not restricted only to the tangible, visible and manipulative materials, but also to what could be internally visualised and memorised.

The analysis seemed to show that much of this tendency could be due to the degree of emphasis placed on teaching through a problem solving approach which made use of the relationship between visual and abstract constructs, encouraging students to move from the concrete and work in the abstract, and a low ratio of teacher talk to student talk. Although this research was not designed to investigate explicit causes and effects interaction, it was discovered that these teachers' teaching styles tended to favour visual instead of verbal representations of numerical entities, and placed a lot of emphasis on relating the abstract to the concrete. Therefore, it was deemed necessary to gain insight into the reason behind this, as per the teachers' beliefs regarding how they taught for the development of number sense.

Mixed Teaching Style

The observations revealed that the teaching style of these three teachers varied according to the subject being taught, the level of the students, and the nature of the topic under discussion. Bob commented that he had *a mixed teaching style*, which he termed as being *eclectic*. Of the three teachers Chantal was the only one who thought that she was a bit more of a prescriptive type, and went on to say:

> So I have to be careful that I allow the students to do activities and that they're fully involved and immersed in what they're doing — I have to be sure of that, because I know it's the way I should go, but I do tell rather than let them find out. Probably because I like to move quickly. So, that's something I always have to be careful of. But in general I focus on skills, 'cause I'm a sort of a skills focused teacher, and when I'm happy that their skills are fine then everything else gets going.

The observation data and interview with the students tended to contradict Chantal's belief. Furthermore, all of the 16 students, from Chantal's' class, who were asked whether they would like the teacher to allow them to have more time to express themselves, indicated that Chantal allowed students to do most of the talking. Chantal's case was typical of these three teachers in the sense that they were always on the look out for improvement in their own practice, and were sometimes over critical of their own performance. Hence, on certain occasions when the interview data did not match the respective observation data it was found that these teachers seemed to be quite demanding of themselves, as they set a very high personal standard of performance. With regard to Chantal's belief about her being too prescriptive the observation data revealed that in her teaching sessions the students were engaged in practical, discussing with their peers and talking to the teacher on average about 75 percent of the time per lesson. In the case of Amanda, she expressed the belief that she worked according to the group of students she had, which was also part of the other two teachers' sets of beliefs.

The way these teachers employed various methods to teach for number sense development in their students was more evident in the way they organised the classroom and how they taught the students. In practice these teachers seemed very versatile and would adapt their teaching style to suit the occasion rather than stick to one which seemed not to be working, which made it virtually impossible to anticipate what they might come up with in the next stage of a lesson. As shown through Vignette 9, Vignette 10 and Vignette 11, this was a common practice, which was in tandem with the teachers' belief of diversifying the teaching approach in regard to the various number sense learning experiences that the teachers engaged their students in.

Vignette 9

Chantal would sometimes begin the lesson with a question and then diversify students' activities according to their mathematics performance level. Hence, she would challenge the more able students to pose their own number sense problems, encourage the medium group to investigate the relationship between some number concepts, and take the opportunity to work one-on-one with the less able students. The whole class would finally be involved in a class discussion.

In another lesson she might start off by asking students to write a list of daily life situations in which mathematics is used. She would then involve them in a whole class discussion about what sort of mathematics is used in each context, and pull out the numerical aspects. Then students might be given group activities where they have to prepare a short report on how number was or could be used in a particular context. This report would then be presented to the class.

Vignette 10

Amanda would start all students off through one real life problem, such as "a bridge can support no more than 50 people at a time. There are 922 people waiting to cross from one end of the bridge. If it takes one person between 10 and 13 minutes to cross the bridge, how estimate the shortest time it might take for everyone to cross the bridge. She would then diversify according to how students responded, so that those who preferred a certain mode of solving a problem would be asked to present their work to the whole class.

On another occasion she would read a story to the class, get students to take notes about any numerical mathematics involved. Each group would then be given a theme. She would then ask each student in a group to use this theme to write a totally different story to what they've heard, which also employs various numerical aspects. They would then get into groups to try and make a bigger story using what they have written. The groups would then be asked to swap stories, read another group's story, and then each group would explain the relationship between the numerical aspects in the story they have read.

Vignette 11

Bob might start the lesson by writing a number pattern on the board. Students would then be asked to work in pairs to continue the pattern. This might result in seven different pattern continuations. The class would then engage in a discussion about each pair's suggested continuation. Students would then work on their own to write the next three or so terms of a number pattern. If Bob notices any drop in the students enthusiasm he might decide to stop because he has detected some sign of 'fatigue' on the part of the students, and move on to a completely different activity. Later on during the lesson he would get the students to go back to the activity they were doing and work in groups of four to compare their results. Finally there would be whole class discussion.

For another teaching session Bob might show the students the plan of a house. He would then ask them some general questions about the numerical dimensions of the house. They would then be asked to work in groups of three or four where each group would have to estimate: how long it might take to walk from one room to another; how many people are needed to fill up the house; what area of land it would cover. These groups would then form two bigger groups where students would compare their results and then students could be involved in drawing the house to a specific scale and finally providing whole class explanations as to how they estimated the answer.

Vignettes 9, 10 and 11 also provide a microcosm of the most common teaching patterns of these teachers. The only aspect of their belief and practice about teaching of number sense and problem solving which seemed to have an element of constancy was how they got students to work individually, in pairs or in groups. Bob and Amanda believed in a sort of pyramid group work system in which students worked mostly in pairs or individually and then formed into larger groups to compare and discuss their results. Whereas Chantal believed in sitting her students into two major groups of high and low performers, within which students would work individually, in pairs or in larger groups, and sometimes move from one ability group to another.

Analysis of the teacher-interview and observation data also revealed that any differences between how these teachers taught for the development of number sense and any other mathematical aspect was so subtle that none of the teachers could identify these differences in their beliefs. Hence, in most cases what applied for the teaching of number sense and problem solving necessarily applied to the other mathematics strands.

Nevertheless, when it came to developing number sense the teachers' beliefs and practices indicated that diversifying the teaching methods, approaches and activities was important. To Amanda diversification:

> …of the teaching approaches was important [if the teacher wanted] to develop in students the ability to make sense of the numbers they encounter in [mathematics] problems.

Hence, although all three teachers accepted that their grouping of students was quite constant, they still maintained, as Bob explained, that:

> …it is the way that the number sense is taught which is important. You cannot expect students to be enthusiastic about developing a number sense if they do not enjoy your teaching of it… [and] to enjoy what they are learning the students must be taught through a variety of different activities, [through] various teaching [and] learning approaches.

As pointed out by Chantal "…students would easily become bored if I were to always employ the same teaching method for getting them to develop their number sense".

Encouraging Students to Work in the Abstract

In each lesson observed where the emphasis was on developing number sense, the teacher always attempted to get students to contrast the existence of a numerical concept with its attributes. For instance, in a lesson where students obtained a certain percentage as a solution to a given problem, they would be asked to explain what difference it would make if instead of '50%' they wrote the answer as '50'. The students were also required to explain their reasons and how '50%' could be written in another way.

Since all three teachers employed an approach which involved students in solving number sense problems through discovery and inquiry, their teaching had a greater element of induction than deduction. This suggestion is supported by the fact that no lesson observed was delivered in a purely traditional manner. Moreover, none of the 91 lessons observed was introduced through teaching the fundamental number concepts and then moving to a phase where what was learnt was put into practice. Usually the teacher would start with a problem or a question — about 84 percent (76 times) from everyday life — and from there engage the students in finding a solution or discussing a possible solution strategy. Nevertheless, it was observed that the teachers immediately engaged students in abstraction activities which involved situations such as discovering a pattern, presenting it in symbolic form, and asking 'what if' questions.

The reasons given for such a trend in their practice indicated that there could be four major contributory factors as shown in Figure 4.25.

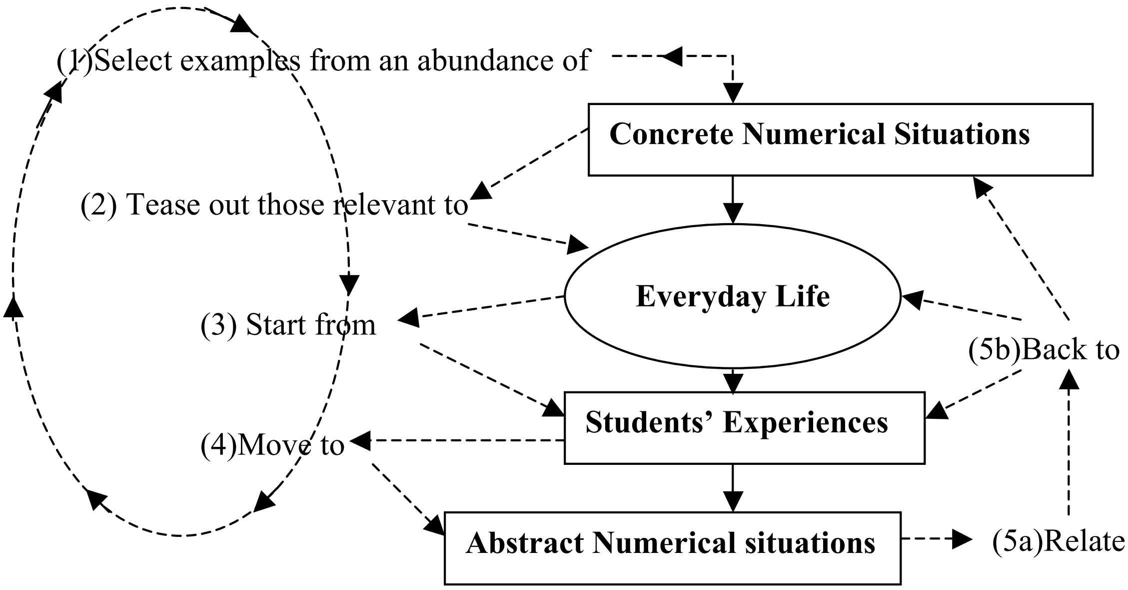


Figure 4.25 Teaching through linking the concrete and the abstract

In tandem with a large majority of lessons observed the teachers thought that they taught through a cycle of selecting concrete numerical situation examples from everyday life and selected those which were relevant to most, if not all students' experiences. Once students had engaged in the activities and "are showing very good understanding of the underlying [numerical] concepts" the teachers challenged them to "express this in pure mathematical language" as Bob would call it. This latter phase was mostly done through oral discussion first between students and then between the teacher and the students. It should be noted that the observation data revealed that once the lesson had started it was most often impossible to rank these four aspects in terms of a timeline of occurrence, since they all seemed to sometimes happen almost simultaneously, and at other times in sequence. Hence, although these aspects have been numbered from one to four, they should not be strictly interpreted as phases but rather as part of a continuum of interacting experiences, which sometimes happened one after the other, in pairs, in threes or simultaneously, and started at any point on this continuum. Thus, the *raison-d'être* for both the cyclic and the linear components of Figure 4.25.

There were two interesting aspects which came to the fore when comparing these teachers' beliefs and the observed practice. The first one pertains to the teachers' belief that they started from the concrete and moved to the abstract, and the observed data which indicated that the teachers and students spent more quality time abstracting the mathematics and referring it back to the concrete than they spent on getting students to focus on the concrete. Since none of the three teachers expressed any surprised reaction when the researcher pointed this observation out to them, they were asked about their belief concerning getting students to start working from the concrete and then moving to the abstract. Hence, the second interesting aspect was how in hindsight the teachers acknowledged that they did not always intend to start off through the concrete. A common point which was raised indicated that the teachers were very much aware of what they were doing and that they attempted to vary the introductory activities so that sometimes they started off with the abstract and at other times through the concrete. Amanda felt that it was important to vary the approach. She stated that she did not "believe that you can always teach in this way [From concrete to abstract]". She maintained that she did "not always start from the concrete. It depends on what you want students to learn and how best they can learn it". When she was asked for further clarification it became apparent that Amanda was attempting to cater for students' preferences and how well they could cope with both the concrete and the abstract, as shown in Excerpt 43.

Excerpt 43

R:	So, you do not believe that a teacher needs to start from the concrete, move on to language aspect, and then bring in pictures and diagrams before going on to using abstract symbols?
Amanda:	This is a sort of text-book prescription which can never be followed in that exact sequence for all students. What about those students who do not like starting from the concrete? My lessons would become boring if I always do that. Some of the girls prefer to start off from an abstract problem, like the problem about the business man from Melbourne who had to visit three cities; I got them started through the worksheet which was mainly a real life situation presented through abstract symbols. As you would have observed some students really liked that introduction, and they were ready to jump into answering the question about the number of different ways that the man could fly around cities, but I quickly moved on to asking one girl to go to the board and physically simulate a way of flying from one city to another.
R:	What was the purpose of moving quickly to having a child perform such a simulation?
Amanda:	I started off with the worksheet which was sort of abstract and then I had to move to a sort of concrete example. This got the abstract students thinking and working out the solution mentally, but I could see that some girls would still need the concrete simulation; some children, they need to see it happening in one way or another. Others can work in the abstract.

This belief was quite universal among all three teachers. Bob brought out the comparative notion of teachers who are confident in their academic and pedagogical background, while he simultaneously brought out the same issue of catering for the students when he stated that:

> It all boils down to flexibility of approach. The teacher who is not sure of himself, who does not know his students and who maybe lack content expertise, might follow a prescribed routine of teaching from the concrete to the abstract all the time. But I have fourteen kids in this class and each of them seems to be at a different level in their way of thinking and tackling a problem. I prefer to vary my introductory activities so that I would cater for as many differences in my students as possible.

When Bob was asked the question "don't you think that moving from the concrete to the abstract is more logical and supported through research findings?" his response emphasised the link between concrete and abstract mathematics, which he felt were important in real life situations. According to Bob:

> I have had classes in the past who had to be taught a lot through the concrete first and then move on towards the abstract, but there again I had to prepare them for life. In real life I don't think you can separate the concrete and the abstract in most cases. I think that the abstract is already there in your mind. I help my students to see how both compliment each other.

Further discussion with Bob (Excerpt 44) revealed that just like Amanda he was concerned with the needs of the students. He proposed that the latter had to be prepared for "enjoying more advanced mathematics" as well. It seemed that the important issue was finding a balance between teaching and learning through abstract and concrete representations.

Excerpt 44

R: In catering for the different abilities, as you mentioned previously, would you say that most often you start with the concrete and then move to the abstract or vice versa?

Bob: It depends. Sometimes I start off from the concrete because of the nature of the problem and my experience of the mood of the students when it comes to solving such problems. At other times I start from the abstract. With younger kids I would tend to select problems which would lend themselves well to being presented in a concrete form. But at year 7 level I am always looking for this window of opportunity to stimulate students to think in the abstract, because after all that's what maths is about. But I am careful to cater for those students who still need heavy doses of the concrete stuff. Mind you, most of my students seem to be abstract thinkers. Therefore I have to find a balance.

R: What sort of balance is that?

Bob: I mean, I see it as something very important, that I have to find a way to satisfy the individual needs of the students while at the same time helping them cover the mathematics they need and also push them towards enjoying more advanced mathematics. They all have different needs and approaches when it comes to mathematics and I need to respect that and cater for it.

The theme of catering for students' personal differences, in terms of being able and preferring to work in either the abstract or concrete mode, kept recurring throughout this part of the interview, as evidenced through Chantal's notion that:

> It is very important to help students understand the concepts and to master the skills of mathematics, and if teaching through the concrete will help them achieve these then I have no problem with starting from the concrete all the time. Unfortunately if I do so I won't reach all of my students' needs.

When asked what caused her to think that the students' needs might not be met, Chantal emphasised even more on the importance of reaching all students, as she expressed that:

> Each student has a personal feel for doing something. Each has a sort of culture that they bring to the class. It's all those little contributions and personal cultures which make up the class. If I teach only to satisfy one or two personal cultures, then I won't be reaching the other students. I won't be accommodating their culture.

Hence, choosing to teach through the concrete or the abstract was not seen as a theory that the teacher had learnt somewhere and which they felt they had to apply, but rather as a means to getting students to learn mathematics according to their level of readiness to work through the mode being suggested. As shown in Excerpt 45, Chantal thought it was important to consider the students' disposition while attempting to engage them into any sort of abstract or concrete experiences.

Excerpt 45

R: Hence, what advice would you have for a newly qualified mathematics teacher? Would you encourage her to start from the concrete and going to the abstract?

Chantal: Yes, I would, but it all boils down to what sort of level you're teaching at, how many students you have, whether the class is mixed genderwise or not. And more importantly the teacher must learn to know each student's likes, preferences, dislikes, things they are comfortable with and things that might put them off. Not all students are keen on learning maths through concrete experience all the time. Some students, for instance, are number crunchers. They enjoy working with the bare bones of mathematics. They enjoy working with the non-concrete aspect of mathematics.

R: Would you say that starting with the concrete is as important as starting with the abstract?

Chantal: Like I've said before, it all depends on the circumstances that you are teaching in. The teacher must always be gauging the students' ability and what they enjoy doing [Principle of readiness]. Otherwise many of them won't respond to your teaching. If you listen to them. If you take note of how they do their written work and so on. Then you get an idea about them. How much you can push them. Whether they prefer to start from the concrete or from the abstract.

Further discussions led into an exploration of how these teachers thought they managed to get students to move from concrete to abstract. In this regard the observation data revealed that these teachers constantly got the students to refer back to concrete every day life number sense situations while simultaneously trying to relate this experience to its abstract counterpart. Since these teachers did not specifically prepare special lessons for the various ability groups and preferred to get them to set high standards, the teachers were asked about how they managed to get all students to work in the abstract. The data presented in Excerpt 46, Excerpt 47 and Excerpt 48 suggest that this was done to various extents with different ability students while the teaching-learning experience was being engaged in. The reasons provided for such a practice were as follows:

Excerpt 46

Bob:
When it comes to the use of numerical algebraic formulae, equations and the like I prefer that students do most of the discovering for themselves, while I act as partial interpreter, although it depends on the ability of the students. With the weaker students I sit down and reason with them from a simple starting point. With the better problem solvers I just throw some number sense questions at them. In fact they already know that I expect them to go into the abstract. In fact I expect all of them to be able to work in the abstract, whether they are of low or high ability or whatever stage they are in their learning.

Excerpt 47

Amanda:
With my lower ability students we sometimes discuss what they've discovered and how we could write this down in the English language first. Then I get them to try and come up with the equivalent in mathematical number symbols. The brighter girls …do not need much prompting from me. I always require them to work in the abstract as soon as possible and not to rely too much on the concrete unless they really need to do so.

Excerpt 48

Chantal:
I always challenge the more able [students] to go one step beyond their present number sense level…beyond what they have done, and usually they don't like working with manipulative materials, although I try to encourage them to do so. But they [less able] benefit more and enjoy working with manipulatives. Hence, it is a little bit more difficult to get them to move [from concrete numerical situations] to the abstract [numerical and algebraic symbols]. So what I do is I challenge them to try to resolve this now without the manipulatives, without concrete materials. The critical part is when they have to write it [concrete numerical situations] in maths symbols. I usually get them to use drawings, tables and diagrams to help them, which I also encourage the brighter [students]…to do as well.

Hence, the higher the number sense ability of the student, the more the expectation of both teacher and pupil, for the latter to work in the abstract. In fact some high number sense students were observed always wanting to bypass the concrete manipulation activities and go straight to the abstract, as pointed out by the teachers.

Moreover, as discussed while answering the second research question, all three teachers involved their students in very well organised discussions and enrichment exercises which helped students in understanding the link among respective processes, concepts and symbols. As explained earlier by both Amanda and Chantal, it was seen as being very important for students to know what processes give rise to any number concept that they deal with and why these are represented as symbols, which relate to the idea of a 'procept' as first presented by Skemp (1971). This was seen as a very striking finding for three reasons:

- the notion of a 'procept' is not common yet in the literature;
- teachers are not encouraged to focus on getting students to understand terminologies such as what is a concept, process and symbol and how these are linked; and
- the way the teachers tended to incorporate that into the fabric of their lessons so that students were learning about these through very well prepared, challenging and motivating activities.

Hence, the teachers were asked about why they tended to repeatedly get students to link the number processes and concepts, to which Bob stated that he:

> …personally think that it is not possible to deal with the process without immediately forming a relative concept. Therefore, it is important to help students understand why they must develop a sense of awareness about how each process has a related concept attached to it.

When queried about how important it was to developing students' number sense, to get them to understand the meanings of mathematical symbols Amanda was quick to emphasise that "symbols are present everywhere and more so when recording or quantifying objects. How can students understand what these number symbols and the addition sign, and subtraction sign, and so on mean?" She then went on to explain that:

> …for instance, division and fractions [are] very closely related. Some students might not [see this] link unless they understand what the division symbol mean and that it is related to the horizontal bar which separates the numerator and denominator [in a fraction].

This belief in getting students to understand how the respective processes, concepts and symbols were linked together was also quite prevalent among the students. For example, when asked whether it was important to know about processes, concepts and symbols, Ashdela [$S_{(1,20,2)}$] explained that:

> before that, when I did not understand what a process was, or what a concept was, or how the symbols were linked to the number concept, I could do mathematics to a certain level. But since Mrs [Amanda] has helped us to understand that we always have to know what we are dealing with, we have learnt how to link the symbol to the idea, to the number, to the concept of what it means. It is easier now to work with the abstract.

Therefore, it seemed that the teachers succeeded not only in passing the message across regarding how important it was to know these terms, but also how to use such a knowledge to be better able to work in the abstract, as pointed out by Ashdela. Nevertheless, the whole exercise of encouraging students to understand these different aspects was always done in a subtle manner so that it was seen as a 'natural' part of the lesson instead of a focus on learning the meanings of abstract mathematical terminology.

Using the Visual and Verbal Modalities as an Aid to Working in the Abstract

The ILS inventory results indicated that a large majority (92%) of students preferred to learn through the visual modality and as already pointed out, these teachers believed in getting students to work in the abstract according to their number sense, problem solving and mathematics ability. Since their belief was that:

- number sense is very closely related to making sense of the abstract nature of mathematics (Chantal);

- in order to make sense of the numerical aspects of a problem one has to be able to relate the abstract numbers to the concrete quantities (Amanda);
- students should work in the abstract as soon as possible (Bob);
- a teacher must not rush a student into work for which they are not ready. She must provide opportunities to prepare them first (Amanda);
- different students learn at different rates, at different levels of understanding and according to certain personal preferences (Chantal); and
- students prefer to learn about number through visual presentation (Amanda).

Questions were asked to ascertain how this was done in relation to students' learning styles. The data revealed that these teachers believed in and taught through a system where:

1. Visual methods are usually used as the first point of introduction for number sense inherent geometrical problems;
2. When it comes to non-geometrical number sense problems these are usually introduced through a written story from real life;
3. The aim is always to lead students towards abstracting some form of relationship.

The following Excerpts provide a glimpse of the teachers' thoughts as to why they believed that getting students to work in the abstract is important and related not only to learning through concrete manipulatives but also through the visual and verbal modalities.

Excerpt 49

Amanda:
I see number sense as a vehicle, or I'll say a translator from real life context to the abstract. Hence, I am always leading my students towards representing the number sense problem in an abstract form. These girls are taught from day one to extract the mathematics [number] from the problem, and I think the only way in which we can do that in less space, time and word is through making it abstract. But it's good if they are first given the chance to see [visual] and discuss [verbal] where it [the abstract] comes from. That's why real life contexts are very important…as important as the abstract.

Excerpt 50

Bob:
It is easier for the boys when the number sense problem is presented in a way that they can relate to. But it would be a big mistake on my part if I were to, sort of get them to learn only through drawings and written words instead of encouraging them to use [numerical] symbols. The symbols make it easier to show the relationship between the different parts [numerical components]. Nevertheless, I use a lot of visual aids to capture their interest, and also encourage them to see where the abstract comes from.

Excerpt 51

Chantal:
Number mathematics is not just an abstract subject. To me it is more of a way of explaining what happens in real life. That's why I place a lot of emphasis on encouraging them to use drawings [visual], to speak [verbal] about what their understanding and solution ways…of solving a given question…mathematics problem. I believe that [numerical] symbols have been invented to simplify things. Like, instead of making long sentences just to explain that some objects [concrete numerical representations] plus some of the same objects equals so many. I mean, with symbols we can say things that we would not be able to say in a few words.

Hence, the belief among these teachers is that the abstract is the most important aspect of making sense of mathematics — in this case number sense — and that the visual is a very powerful means of enticing students to be motivated to 'see' the abstract in relation to the concrete. In that sense it is a bit like what is portrayed by Figure 4.26, where the learning interaction process is made to move to and fro between the concrete and the abstract, via verbal and visual presentation. It is worth noting that the visual input comes mainly in the form of diagrams, drawings, graphical representations and written or drawn number sense related mathematical symbols which are often displayed as peripherals inside and outside the classroom. Whereas the verbal input comes mainly from the students who the teachers believe should be encouraged to get involved in a lot of peer discussion and in challenging the teacher through questions. Hence, these students were used to a teaching approach which employed much visual input from the teacher and less verbal input from him or her.

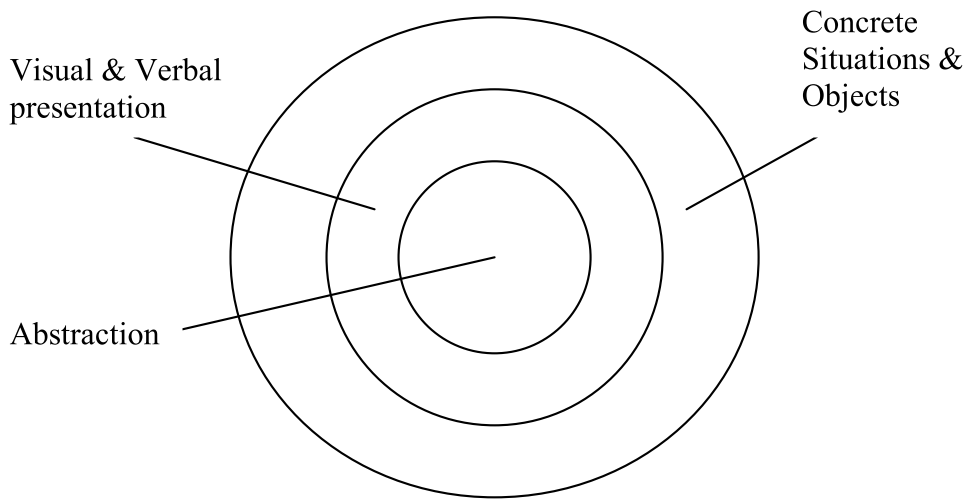


Figure 4.26 Using visual and verbal media to interact between concrete and abstract number sense problem solving situations

Teachers' Planning Beliefs

Excerpt 52, Excerpt 53 and Excerpt 54 present glimpses of the three teachers' responses when asked to describe: (a) how they generally prepared a typical lesson; and (b) how much of this preparation was devoted to catering for the development of number sense and problem solving. In terms of mathematics lesson preparation Amanda thought that since both number sense and problem solving formed part of all the other mathematics strands, preparation for number sense problem solving learning experiences formed an inherent part of most of her preparation as stated in Excerpt E.

Excerpt 52

Amanda: (a) I choose my material from my resources. What happens in my lessons is dependant on what happened in the previous lesson. I have an understanding of what I want the students to learn but it[‘s] the knowledge and progress of the students that determines the pace and content of each lesson.

(b) Since number sense is present in most mathematics topics a substantial amount of the preparation will involve number sense related learning, but I do not focus too much on that aspect. Once I know what my students' need are, from the previous lesson, I choose my materials, which definitely would contain mostly things having to do with number sense. Then there is the aspect of problem solving which involves a lot of mathematical sense making. Therefore, whether I plan for it or not, both the number sense and problem solving aspect appear voluntarily.

Similarly Bob emphasised that a "good portion" of each lesson involved number sense, since it is intricately linked to problem solving. Moreover, his use of a "free-from" teaching format, where students' input played a greater role in the direction that the lesson took, usually resulted in students requesting for more number sense problem solving assistance and opportunities. In Chantal's case she believed in purposefully engaging her students in a lot of number sense and problem solving learning experiences most of the time, since she felt that teaching through a problem-based approach necessarily involved a lot of number sense experiences.

Excerpt 53

Bob: (a) I prepare by identifying what I hope to achieve. I clarify in my mind what the steps might be to get me to where I want to go. I prepare any examples, work sheets, etc that I need.

(b)) I think that a good portion of the lesson will be built around developing some aspects of number sense. I'm not sure about quantifying the amount, as it will depend on how the lesson develops. Many of my lessons tend to be somewhat freeform, as they will depend on student response, success of ideas, the depth of understanding that becomes evident etc, and all of them [lessons] are problem-based. That's another reason why number sense forms part of the greater portion of my lessons; the students' usually ask for help in solving number sense problems. [Hence] In terms of preparation, I do try to be clear about what aspects of number sense I need to prepare for, but I wouldn't want to waste an opportunity to develop some thing not foreseen, should it arise.

Excerpt 54

Chantal: (a) I do 5 maths lessons of one hour each week. For some areas I will prepare specific lessons. Often I rely on good work already prepared. I use a text which is outcomes focused. [It is] impossible in the time frame available to do all my prep[aration] from the ground up. [I am]Very experienced, so for most things [I] use tried and successful methods and lessons, then adding anything new which I think will be useful.

(b) [I] Always look to use approaches which develop number sense and problem solving. In fact this aspect permeates most, if not all of my lesson preparation and teaching. Making sense of number is the key to developing ones problem solving prowess, and since all my lessons are problem-based there is always an opportunity to prepare for students to develop their number sense.

From the interviews with the three teachers it was possible to identify 21 major pedagogical decisions taken during the planning stage of the learning experience. It was seemingly apparent that all of these decisions took into consideration mathematical sense making and problem solving, both of which were thought to incorporate a substantial proportion of number sense. After coding of the interview data, eight most common pedagogical decisions relating to number sense and problem solving were identified, then each teacher was given a list of the eight decisions and asked to rank them according to the level of importance they would generally attach to each factor prior to and during the planning stage; assigning a ranking from 1 (most important to be considered) to 8 (least important). Table 4.56 presents the eight most common issues considered as most important by teachers prior to and during lesson preparation.

Table 4.56 The most important issues considered prior to and during lesson preparation vis-à-vis number sense and problem solving

Rank	Factor	Description	Teacher Rankings T_1	T_2	T_3	Mean	SD
1	Needs of the class:	The mathematics educational needs of the class	1	1	2	1.3	0.6
2	Needs of each student:	Individual mathematics educational needs of each student	3	2	1	2.0	1.0
3	Ability of students:	Mathematics strengths and weaknesses of the students	2	3	3	2.7	0.6
4	Curriculum obligations:	Possible content areas that need covering	4	5	4	4.3	0.6
5	Students' content preference:	Which content areas students might prefer and which they might not be comfortable with	5	4	5	4.7	0.6
6	Students' everyday life interests:	What everyday, fictional or real, issues and situations students are interested in	6	7	6	6.3	0.6
7	Teacher's capabilities:	What are the possible learning experience approaches available to the teacher which are within that teacher's capabilities	7	6	8	7.0	1.0
8	Selection of effective approaches:	Which of these are most appropriate for effective teaching and learning of the identified content	8	8	7	7.7	0.6

Note: T_1 = Amanda; T_2 = Bob; and T_3 = Chantal.

Although none of the teachers was willing to explicitly state which of these issues was taken into consideration first, the data indicated that they believed that students' needs and ability should take priority over all other aspect. According to Amanda it seemed that "all these processes could be happening all at once,…and they could all pertain to solving number sense problems to a great extent", which fits in with Bob's perspective that "how to amalgamate all of these [factors] to fit in with the rest of the students' educational needs is an ongoing processs…[since] preparing to teach for number sense and problem solving development has no fixed format". It should be noted that although the needs of students' was considered as most important, all three teachers selected their content according to the curriculum framework document, of which the number strand and the working mathematically strand — to which problem solving was central — were the ones mostly referred to.

Ideal versus Actual Teaching Approach

When asked: What would be the ideal way they would like to teach number sense and problem solving? Bob was the only one who felt that since he was teaching only mathematics most of the time then he could do most of what he would have liked to do as expressed in his statement where he said “I think that I have it pretty good at present! I have about eighty percent of my teaching time teaching mathematics”. Bob felt that this allowed him “… ample time to develop students’ number sense through a lot of problem solving experiences”. Amanda stressed that she would prefer to teach in a very informal environment where she would allow “…the students to set the pace and direction to a large extent”. Although the observation data suggested that she was already doing this to a level that surpasses what would be observed in most mathematics classrooms she supported her perception by referring to her work with a class she takes after school hours. According to Amanda, “my best work is done with my after school group which is very informal and where I work one-on-one with the students”. This is a wish which was expressed by both of the other teachers in the first interview, with Chantal in particular repeatedly expressing a wish to “teach the top students separately” since their mathematics, and consequently their number sense problem solving “…level demanded that they would be in a class with mainly students who would challenge their sense-making ability”. Whereas all three teachers thought that the less able students sometimes needed to feel more adequate and powerful in the way they solved problems, and being in a same-ability class would help them satisfy this need. Nevertheless, all three teachers thought that such a system should be flexible enough to allow for students to also interact in mixed ability settings.

All three teachers taught the brighter students, the average students and the below average ones differently, although as explained by Bob this was “done in a subtle manner so that students’ self esteem is not adversely affected”. As explained previously, a common method employed, according to these teachers and confirmed through the observation data, was to teach the whole class as a group while simultaneously taking time to go to the ability groups and teach them at their level. According to Chantal:

> two major ability groups of the more able and the less able, which were less visibly split into three other ability groups. [This was done because] there’s less of a stigma within the class, ‘cause quite often I teach them as one group and I don’t move them to any particular maths seat.

Amanda stated that she attempted to “…use the same grouping system whether it was for number sense development, problem solving or any other mathematical aspect”. Amanda believed in employing a less rigid grouping system which she said was possible due to her classes having only girls. In this case she encouraged students to:

> work in pairs according to where I have purposefully placed them. Then they are free to form larger groups with those sitting near them, without them knowing that they are seated in such a way that they can work in ability groups. At other times they are free to move about and work with anyone they like.

In Bob's school some classes are grouped differently to the extent that it resembled a new way of streaming students. This was very much in tandem with Bob's idea of an ideal situation for enhancing students' number sense and problem solving performance. As he explained:

> We have got some of them grouped differently. For example my class, that I've just taken off to music are the group of boys that we identify in year seven as probably being the weakest academically. So I set a different sort of program for them. I do a lot more… basic mathematics with them … [I place] a lot more emphasis on [the] four [number operations] processes and … improving their table skills…[I employ] a combination of those kinds of things. The other two groups are pretty much homogenously mixed. But most of the weaknesses are out of there so that they have a reasonably good background in terms of their tables and combinations of number sense and problem solving performance.

To clarify some of the beliefs expressed through the question about ideal teaching, the teachers were asked to identify changes they would like to see in their current mathematics programme. Although this question failed to result in any common point being raised, it was interesting to note the four concerns expressed, which were:

- I'd like to have more time for maths…(Amanda)
- …I'd prefer to teach students of similar ability together.(Amanda)
- I would like to establish a mathematics laboratory with plenty of manipulative equipment, problem solving activities etc.(Bob)
- Refine what I teach to match with learning outcomes. (Chantal)

Of these four issues lack of time to engage in problem solving activities and teaching students of similar ability together were two major concerns expressed through the other interviews. When asked to identify areas in which improvements could be made to lift their current programme a major concern expressed was that there was too much paper work which involved:

- written programming;
- preparation of detailed written lesson plans;
- rubric preparation;
- detailed reports; and
- examples for portfolios.

Although these teachers saw some benefits of documenting students' performance and progress and reporting these to parents, just like the other two teachers Bob stated that:

> I am doubtful that many parents really understand much of the jargon that schools are getting bogged down with in terms of evaluation and I think that there is a danger of encouraging teachers to avoid the truth by using language to hide it. I would prefer to see clear reporting.

Furthermore, Chantal thought that "such time could be spent on more effective teaching and learning experiences", while Amanda highlighted that "…if the teacher is not careful you might end up with lots of students' work to correct, which is extremely important, plus losing sleep on all this paper work". All three felt that a reduction in the amount of paper work and less teaching load would "free the teacher to prepare more student-friendly activities" according to Bob, "provide more time for teachers to provide students with more one-on-one interaction time between the teacher and the kids" according to Amanda, and "provide me with more time to refine my teaching so that it really matches the learning outcomes". Nevertheless, these teachers made it clear that an effective teacher is one who "…always attempts to work for the benefit of the students within the limits imposed by the policy makers" (Bob), but such teachers also "make time to prepare, execute, monitor and evaluate everything which has to do with my students' learning" (Chantal). In other words, as stated by Amanda, "No teacher should deprive the children of an excellent opportunity to engage in fruitful and effective learning just because there is too much paper work and less time available for preparation and teaching". Hence, the bottom line seemed to be that the teacher's "…primary responsibility was towards the students, as expressed by Chantal, and as reiterated by Amanda "…all other issues which act as obstacles to what I would really like to do should not deter me from giving my best for these girls". Further investigations and subsequent analysis revealed that in spite of their concerns about too much paper work and the workload, these teachers tried to strike a balance between the various factors and concepts in their teaching, so that in the end the students would benefit. This eventually led to discussions about balance in the curriculum itself which Bob expressed as follows:

> whether I like it or not…I have to strike a balance between working mathematically, appreciating mathematics, conceptual understanding, procedural knowledge, problem solving, and sense making, so that my students would benefit.

As discovered from the observation data this belief tended to permeate these teachers' practice on a daily basis. They believed that all these aspects, similar to what Bob highlighted, and more are so important that neglecting one of them could be fatal to a student's mathematical sense-making and problem solving development. When asked to explain why they strived to maintain such a balance, although they felt that the work load and paper work was too much, it was revealed that there was one main reason for this; the close link among the process and content strands of the mathematics curriculum, as highlighted in Excerpt 55, Excerpt 56 and Excerpt 57.

Excerpt 55

Bob: Because these cannot be separated one from the other, although we tend to think that some, like working mathematically and appreciating mathematics, are greater in the sense that they are made up of the other mathematical abilities.

Excerpt 56

Amanda: It takes time and effort to get some students to really work mathematically. Hence, everyday that I have these children I need to get them to solve a lot of problems and through these I manage to get them to appreciate mathematics, work mathematically and at the end of the day enjoy mathematics.

Excerpt 57

Chantal: If I were to separate these then I would not have time to develop the students' problem solving. I think that problem solving is extremely important because it unifies all these concepts such as working mathematically and appreciating mathematics. And the most important part of problem solving is number sense, because without it most students would find it extremely difficult to solve any problem.

Attributes of Students as to their Mathematics Proficiency Level

When asked to describe the most noticeable attributes in terms of skills, attitudes and behaviour of above and below average ability students in their class the teachers brought out the elements listed in Table 4.57. Many of the attributes listed in Table 4.57 are very closely related to the requirements of good number sense.

Table 4.57 Teachers' perceived attributes of students according to ability

Below average	Above average
Shy away from mathematical challenges	Happy to meet mathematical challenges
Anxious about mathematics	Motivated about mathematics
Lack self-confidence	Self-confident
Enjoy DNS problems	Enjoy most, if not all, types of problems
Fail to check for errors	Monitor work for errors
Try to do the bare minimum	Proactive and readily extend given activity and problem
Lack number sense	Very good number sense
Lack of perseverance	Have persistence
No comment made	Intuitive about their work
No comment made	Very good parental support
Fear failure in mathematics. Worried about being wrong.	Revel in being able to do well and are not fazed by mistakes or not getting correct answer
Find it extremely hard to communicate understanding	Willing to discuss, explain and defend a position or idea
Less accurate with number facts and not as good at seeing or understanding of number ideas	Very good recall of basic number facts
Don't know their tables	Know their tables
No comment made	Most often check the reasonableness of their answers
No comment Made	Estimate the final solution before starting
No comment made	Very systematic and logical even when written work seems untidy to others

All three teachers expressed the belief that having "excellent number sense as giving such students an advantage over others when it comes to problem solving" (Amanda), although Amanda was the only one who believed that all mathematics strands and content are equally important. The other two teachers were very explicit about number sense being the most important pre-requisite for good problem solving in mathematics, as evidenced through Chantal's statement that:

> …, the high problem solvers have excellent number sense…, well developed problem solving strategies, and they know when to apply the problem solving strategies. Therefore, they are able to cope with a mixed bag of problems. No matter how simple or how difficult they might be. They can cope with a page of examples, mixed up addition, subtraction, addition mixed with subtraction; they can cope with that different ways of showing the problem because they have excellent number sense. The weaker students need more drill.

The following comments from Bob add even more weight to what Chantal had expressed. In Bob's view, although problem solving is slightly more important than number sense, the latter is also very important since:

> … I encourage students, for example, to look for pattern in number because you can see the patterns in things. Then it reduces the stress of being able to work things out. So we always spend a fair [bit of] time looking at those kinds of things: looking for patterns in number; understanding the shortcuts in things; looking for shortcuts; discussing how do you do this; how do you do that is better than that; and is this best for you? Those kinds of things.

Nevertheless, just like Bob and Chantal pointed out when asked about the place of number sense in the students' success in problem solving, Amanda indicated that number sense was necessary in the sense that it permeated most other strands:

> It [number sense] comes into the other strands. Because if you're going to deal with most mathematics strands you will most often meet up with number…like in measurement for example, once you get past doing concrete measurement there becomes a number component of that. So if you know what you're doing in measurement… usually [it] involves some of those things [number sense] and … although it is not to the same extent in all problems. So well, I don't think that number is more important than the other strands… in that…although it usually requires you to have some facility with number to solve problems, and in order to work within the other strand as well. So, [number sense is needed to read] charts and data also.

Interestingly, 70 percent of students thought that a good problem solver needed to know the 'basic facts and tables', be able to 'estimate the answer', and 'check how correct his answer was'; which are all attributes of having good number sense. Excerpt 58 provides some snippets of typical answers by students in regard to the attributes of good problem solvers, which indicates a similar belief to the teachers'.

Excerpt 58

Arnold[$S_{(2,34,1)}$]	To be good at problem solving you must be able to estimate the final solution before [embarking on the actual] solution work is done. You must also be able to tell if your answer is acceptable or not. Number facts is also very important. [Have] problem solving strategies, which you can use to solve the problem.
Sheila[$_{(1,19,2)}$]	You must understand which [problem solving] strategy to use.…The students who are very good problem solvers are constantly checking [the reasonableness of] their [intermittent and partial] solutions as they are working, as well as checking [the reasonableness of] the final answer. They must know how to estimate, and know their basic facts and tables.
Savinia[$_{(3,60,2)}$]	You have to know how to invent a new strategy if [a ready-made] one is not available …and if you are unable to make connections between the [different numerical concepts] numbers in a problem, then you won't be able to solve most problems. If you can estimate the answer it helps you to compare your final answer [check reasonableness].
Rico[$_{(3,46,1)}$]	…master the basic facts with the aim of reducing the pressure on [recalling] getting facts from memory or having to spend time working out basic facts which could easily have been stored in [memory] the head and then easy to remember [recalled] when needed.

On the other hand, when asked to describe what could be the main reasons behind students being poor at solving mathematics problems 70 percent of the students identified lack of number sense as a major culprit. Figure 4.27 shows the three most common factors identified by the students as being influential in a student's poor problem solving performance. Hence, the students' perceptions were in tandem with those of the teachers, in regard to having good number sense being important for higher problem solving performance.

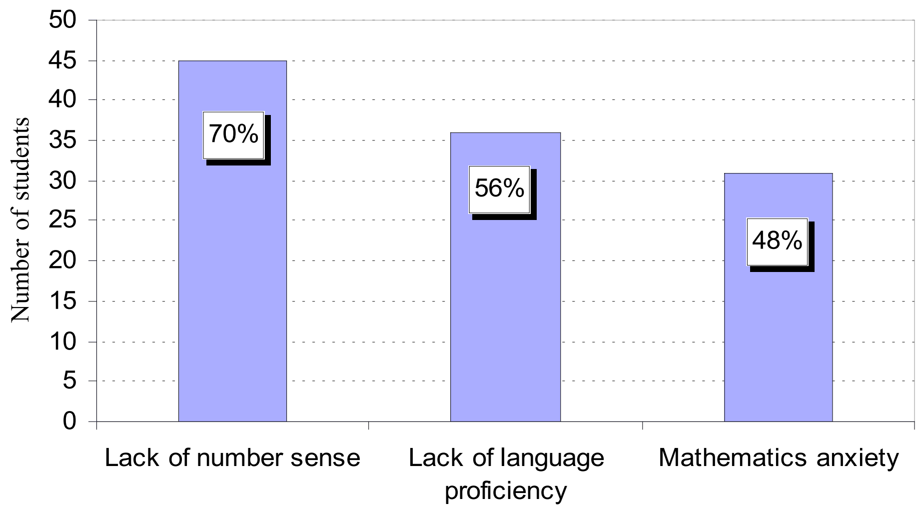


Figure 4.27 Main factors identified by students as responsible for poor problem solving performance (N = 64)

4.8.3 Themes

The results of data pertaining to the teachers' beliefs were categorised into seven major themes which are presented in turn below. It should be noted that although there were other themes which were identified, the seven themes presented below were the only ones which were common among all three teachers.

Importance of the Teacher's Mathematics Knowledge, Ability and Confidence

The confidence that these teachers seemed to exude, as they guided their students through activities which were partly curriculum driven and partly influenced by the teacher's perceived student interest and experience, seemed to be partly due to their belief that an effective teacher must also posses a very good mathematical background. Moreover, they believed that the teacher must always be up-to-date with new developments in number sense discoveries and applications.

The teachers believed that a mathematics teacher:

- must have a very solid mathematics background (Bob);
- should be very conversant with the content of mathematics curriculum relevant to the level at which [he or she] is teaching (Amanda); and
- needs to be good at maths and know how the fundamental parts relate to each other (Chantal).

They believed in active participation in professional development which they attended regularly. They also expressed the notion that the teacher must keep abreast of new developments about number sense, problem solving and mathematics in general, through reading of relevant research and literature. Such a belief was in accord with Ma's (1999) who discovered that effective teachers of mathematics had Profound Understanding of Fundamental Mathematics (PUFM).

Teaching Mathematics Through a Problem-based Approach

All three teachers shared the belief that number sense is best taught through problem solving. This belief is in line with the classroom observation data which revealed that number sense in combination with problem solving were involved in more lessons than any other combination. Figure 4.28, which is related to data presented earlier in Table 4.37 and Table 4.38, shows the number of observed lessons according to the main perceived mathematics contents and processes focused upon. Nearly 70 of the 91 lessons observed seemed to be dedicated to focus on number sense and problem solving. Only 6 of the lessons observed involved problem solving with no focus on number sense.

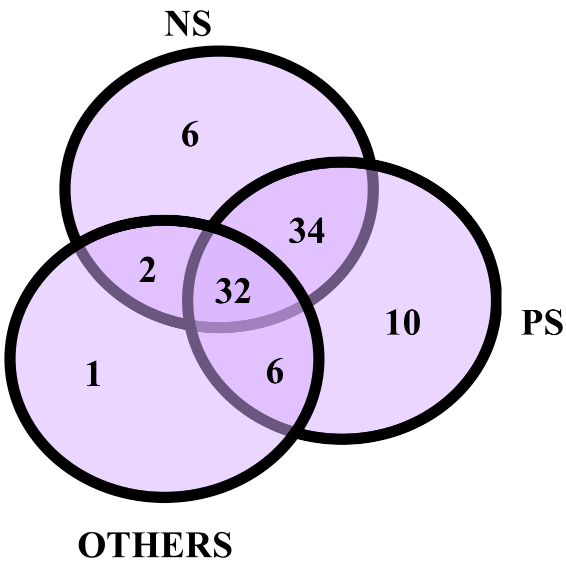


Figure 4.28 Distribution of learning experience sessions per mathematical aspect

Getting Students to Think and Work Mathematically

Problem solving was seen as a necessary approach within which students learnt how to think laterally and work as mathematicians would do. In this sense the teachers believed in giving students the opportunity to get involved in investigations which comprised mostly of number sense work. It was believed that number sense had certain inherent qualities not found in other aspects of mathematics which lent themselves well to facilitating the development of a culture of good problem solving.

All Mathematics can be Taught Through Problem Solving

The belief that all mathematics can be taught through problem solving is reflected through the amount of learning experience time devoted to problem solving where 82 (90%) of the lessons observed were purposefully designed around problem solving activities (Figure 4.29).

Number Sense is the Key to Problem Solving

Students with good number sense found it easier to solve most mathematics problems since both number sense and problem solving share similar requirements. The teachers expressed a belief that success in solving number sense problems bred confidence into students, which in turn enabled them to confidently want to solve other types of problems.

Students as Detectives

This belief permeated all of the teaching sessions observed. Students were always challenged to investigate the situation presented through a problem and then asked to explain what clues they used to arrive at the solution. Although the correct answer was desirable it was never treated as more important than understanding the process. As stated by Chantal "…the process is most important, not the calculation", which is one reason why according to Bob, "students are most often given the freedom to choose which solution method, algorithm or calculating instrument they need to use".

Combining Teacher-centred Teaching and Student-centred Learning

The system employed was one where on the one hand the teacher decided on most of the curriculum, when it would be taught and how [teacher-centred], and on the other hand the teacher became a learner and students were often taught by their peers. Although these teachers believed that they were the ones responsible for preparing most of the lessons and worksheets and to deliver them according to a pre-planned scheme [teacher-centred], their lessons were also very student centred in that they believed that students should most often be free to discuss with their peers and use any method of calculation and/or instrument, and make propositions about the work given.

Encouraging Student-Student Interaction

Students were made to work in various situations where they had the opportunity to work on their own, with someone else, in a group or as a whole class. Table 4.58 gives an indication of the proportion of student-involved activity time spent working in each particular setting. It should be noted that since students sat in close proximity to each other and that they were most often free to interact with others the percentages given pertain to instances where the teacher would specifically instruct students to work in these particular settings. To obtain the data in Table 4.58 each time the teacher explicitly asked students to work in a particular setting the duration of such work was timed.

Table 4.58 Percentages of student-involved activity time spent in each particular learning setting

Setting	Percentages
Individual	27
Pair	25
Group	30
Whole Class	18

The teachers believed that allowing more time for students to work with their peers helped them to: (i) share their solution strategies; (ii) develop confidence in explaining their work; and (iii) and most importantly provided them with an opportunity to feel like an important part of the class. Teachers also felt that students needed to be grouped according to ability, but not always required to work in such groups.

Striking a Balance

According to these teachers it was very important that a balance was struck at:

- the working level among lesson planning, delivery, marking of students' work, professional development and family life;
- the cognitive level among concepts, procedures and applications;
- the problem solving level among working mathematically, problem solving, number sense, appreciating mathematics and all the other strands; and
- the affective level among developing students' self confidence, self esteem, interaction with others and valuing of achievements in relation to goals set.

The teachers believed that due to the constraints imposed by a heavy workload and a lot of paper work, to achieve such a balance it was important to empower the students by slowly giving them more autonomy and jurisdiction over their own learning. Some measures used to facilitate this empowerment process were:

- Prepare enrichment activities as a means of encouraging students to master the basics of arithmetic, upon which it was expected that mastery of the basics in other areas such as algebra, geometry and statistics would be built;
- Develop students' number sense ability so that they will be better able to solve problems pertaining to other strands of the mathematics curriculum;
- Encourage students to develop good cooperative learning habits;
- Encourage students to actively participate in all discussions and teach them how to do so successfully so that their communication skills would be enhanced;

- Devise ways of discovering students' weaknesses and construct appropriate activities to help them overcome such weaknesses. For instance, students who are thought to lack concentration are given extra home work which would help them improve upon their concentration, those who have a particular problem in mathematics are given extra home work in that area; and
- Make allowance for students to participate in contributing ideas for the design or more effective and student-friendly learning experiences.

It is Possible to get Students to Improve upon their Performance

Combining Cooperative Group Work and Individual Attention

The practice related to this belief employed a system of the less able learning from the more able and vice versa, with the teacher also learning from the students. It was believed that in such an environment the teacher would learn more about each student's preferences, weaknesses and strengths, and devise appropriate and relevant situations tailored to the individual student's specific needs. Nevertheless, none of the teachers believed that it was possible to cater for the individual learning style of each student. Instead they got students to work in ability groups, which in turn gave them time to attend to those students who needed their attention most at a particular time. As Amanda would say it would "remove the stress from teaching".

Operating through the affective domain

Caring for the students as important human beings was a major priority of all three teachers. Hence, the emphasis on teaching through the affective domain was a theme which permeated all the discussions with the teachers and most of the discussions with the students. When asked to state what they thought were the characteristics of a good mathematics teacher Amanda stated that the teacher "…must be non-judgmental, accepting of children having a go, willing to explain many times and in many different ways". When asked the same question, just like the other teachers Bob was very passionate about being fair to the students, as shown through Excerpt 59

Excerpt 59

Bob: …compassion for the students. An understanding that different students learn in different ways…[The teacher must] be happy to learn from the students… [The teacher must be] able to create a risk free environment for students, so that they can feel free to have a go without fear of being ridiculed by others or by worrying about getting it "wrong" for whatever reason.

Have an awareness of what is happening in the classroom- being flexible, so that you can be prepared to stop, or move or change direction if something is not working. It is pointless in flogging a dead horse. Better to accept that it didn't work, let's think about it and come back another day with a different approach.

Chantal emphasised that everything boiled down to having "patience…[and providing a] Supportive environment, [and allowing students to] work in groups. Encourage contributions and value them. Try to teach to suit different learning styles". The main points coming out of these teachers' beliefs about caring for the students could be summarized as:

- Having patience and being willing to revamp the explanation until students are at ease with whatever they are learning;
- Respecting individual differences in learning preference of students;
- Allowing them to get support from their peers through group work;
- Being flexible enough so as to make allowance for the unexpected behaviours of the students
- Not tiring them down through lengthy activities if they are not showing any interest;
- Zero tolerance for ridiculing the student in any way, shape or form; and
- Both teachers and students must be non-judgmental towards each other.

It was further revealed by the students, as exemplified through Excerpt 60, that they felt "at ease", "loved" and "respected" when they knew that they could interact freely with anyone in the class without fearing being "put down [by their peers] or the teacher".

Excerpt 60

Germaine[$S_{(3,56,1)}$]:
I like being in Mrs [Chantal's] class because she is never mean at you. I can ask her any questions I have…At first I felt [embarrassed] because when she said we could always ask questions I was the one asking a lot of questions because I did not understand a [lot] of things in maths. But she never complained. She always tries to make us feel at ease.

Mona[$S_{(3,44,2)}$]:
Mrs [Chantal's] is the best maths teacher I've ever had. I do not like to talk a lot and sometimes I have questions, but I am afraid to ask them. Not in Mrs [Chantal's] class. Like today she changed her explanation three times to try and get us to understand, and when I said [asked] if she could draw it, she asked me to help her [draw it] on the board. Many of us feel that she loves us because of [the way] she is kind to us. Of course you don't fool around in her class, but she is a very good teacher…very understanding and very caring.

Asunta[$S_{(1,24,2)}$]:
I cannot remember a day when I did not feel like coming to school. Mrs [Amanda] always have a joke, but she is a very serious teacher, but at the same time she is like one of us. She would say "girls what do you say about this". Or I like it when she calls us by our short name or our nick name. She never gets tired of finding ways to make us understand. And she does not boss us around…she respects us and gives us things to do. I never feel put down by the teacher or the other students in this class.

Francoise[$S_{(1,21,2)}$]:
Some teachers try to hide their anger. Yep, they get angry for the slightest thing. That's why I would love it if Mrs [Amanda] was our class teacher in High school. She is firm but fair. [She] Always encourages you to have a go, and always finds something good to say about your answer. She is also very disciplined, but she never raises her voice or make you feel afraid to try something out. She'll say "O.K. [Francoise] go to the board and show us how you did that".

Donald[$S_{(2,31,1)}$]:
In Mr [Bob]s] class we all work very hard because he always wants us to give all we have. But when you get tired you can let him know [about it] and he will do something to… [get you to] enjoy the lesson. He is always asking how we are feeling and…wants our opinion on what goes on in the lesson. The only time that he really gets angry is if we tease someone over and over again.

Terry[$S_{(2,26,1)}$]:
When I came to Mr [Bob's] class I thought he will do like other teachers…when I cannot concentrate some teachers used to get angry. With Mr [Bob] I can crack a joke, and when I am too tired to work or just don't want to work I can tell him…I am not afraid. Of course he never says "good [Terry] you can stop working". No, but he talks to me very calmly and tries to give me something else to do. He is like my friend, and I understand maths in his class. He does not allow the others to laugh at me when I make a mistake.

As shown in Excerpt 60, the students felt empowered to "…participate freely as a respected and valued member of the class…" (Bob). According to Chantal, when students "…do not feel threatened to voice their opinion or to suggest their own way of solving a problem, they start improving beyond your wildest dreams". Furthermore, as explained by Amanda "caring for students as important beings who have something to contribute in the class makes them fell more confident and:

> the confidence gained is like money that you deposit in a bank account. It gathers interest, and you can see this as every one of the students make some progress [in their mathematics] according to their ability. That's why I think [that] the most important thing is to provide them with the reassurance that you care about them. Then everything else falls into place very nicely. They become more motivated to learn, to participate, [and] to give a lot of their time and energy.

When asked whether they thought it was important to consider the affective aspect when getting students to develop their number sense (Excerpt 61), the teachers' responses suggested that it could be even more important because, as explained by Chantal "number permeates a large chunk of the mathematics they [the students] learn at school". The teachers' sample responses presented in Excerpt 61 shows that they thought that catering for the students' affective needs help to reduce the fear that some students could have previously experienced vis-à-vis their learning of mathematics and consequently number sense. In this way the teachers thought that students' self-belief grew and they became more willing to try new ways of working and employed new strategies to solve number problems.

Excerpt 61

Amanda:
Mathematics itself could be threatening if not learnt in a nurturing, safe and enjoyable environment. Since most of the work we do involves dealing with number sense it is very important that this [Affective aspect] is carried over into how number concepts and processes are dealt with. They have to enjoy working with number, and the teacher can help them do that by removing most of the threat through a loving and caring attitude towards them [students].

Bob:
The teacher has to be very sensitive to these students' feelings and needs because some of them haven't got a high self-respect for their own competence and understanding of the number system and how it works. Once this type of rapport is built there is some form of trust from the students, that they have an understanding [of] these number concepts and can solve not only number problems but also those which have none [no number in them].

Chantal:
Many of the students I get come in [the class] feeling inadequate and incapable of solving most number problems… [They] Don't know [their] tables, don't work in logical steps. Unable to determine the key elements. Have experienced failure in maths and don't have a positive attitude when they arrive in my class… [but] With the right encouragement the students, especially the weaker ones, become a lot more willing to give it a go and learn their number facts, and try to make sense of these facts.

Combining Contemporary and Traditional Teaching

Extrinsic and Intrinsic Motivation

It was felt that the best teaching approach was one in which the "good aspects of traditional teaching are combined with the more contemporary ones", as pointed out by Chantal. Hence, during the first term students were motivated more through the extrinsic channel than through the intrinsic one, and as the year progressed the emphasis was slowly and deliberately shifted towards getting students to be intrinsically motivated. The extrinsic motivation pattern employed in the first term did not seem to be a very genuine one in the sense that students were being praised for nearly every little piece of contribution, but just when the teacher's instant praise seemed to be becoming a bit too obvious it also started becoming less abundant than before. As explained by Bob “this is more like behaviourist thinking…when someone is motivated through an external impulse”, and as he further explained, this was seen as being “…a very important aspect of my [Bob’s] teaching”. The interesting thing about this is that only Bob seemed to have planned it to be this way. Amanda and Chantal were not necessarily aware of the fact that by the third term they were employing a lot less verbal praise and external rewards than they would have been using in the first term. Maybe this is something which is practised by most effective teachers.

Learning Mathematics Through Making Connections

The teachers thought that in the past not much emphasis was placed on getting students to make connections. Chantal referred to this as “…in many instances students were made to learn by rote, reciting the multiplication table without understanding what it was about, or how these were connected to each other”. Bob insisted that it was possible for a good teacher of mathematics “ …to see the connection between different aspects of maths…” and “guide students towards seeing these links”.

One aspect of these teachers’ beliefs, which might be a direct result of them having a very good mathematics knowledge base, was the applicability of what the students have learnt through getting them to make connections between classroom mathematics— in this case number sense — and everyday life. This was also reflected very strongly in their teaching, and as highlighted through Excerpt 62 it seemed that these teachers believed that the way to develop number sense was to:

- give students a sound background in the number system;
- develop their sense-making ability;
- empower students to be able to apply and relate number sense to real life contexts; and
- hand over control over their own learning to the students

Exerpt 62

Bob:	Generally I think that I try to provide students with the opportunity of developing number sense skills or problem solving skills [Sense-making] by setting up a series of activities that will either strengthen or introduce a particular concept [Developing a sound number system background] — then try to lead them to a series of opportunities to apply [Application] these concepts in a variety of ways or to develop them to a greater depth. Thus I try to move from a controlled situation towards a more open ended, challenging situation [From teacher-controlled to student-owned].
Amanda:	Number sense can be developed by giving the students a sound background in the number system [Developing a sound number system background]. They need help in understanding our number system and practice in working with it. They can benefit from explanations of how and why things work that they might not have seen for themselves [Teacher-controlled]. They need lots of exposure to problem solving and permission to go about solving it in a way that makes sense to them [From teacher-controlled to student-owned]. They also need to be able to explain what they did and why [Sense-making], but there is never just one way to solve a problem. Working with other students is important I think. They gain more ideas about how to apply number sense in everyday life [Application].
Chantal:	I don't believe that a child's number sense will improve much, or even improve at all, if he or she does not know the basic facts, how the place value system we use functions, cannot recall these when needed [Developing a sound number system background] and more importantly cannot make sense of the numbers present in his environment [Sense-making]. They [students] must be trained to seek for numbers everywhere and to find the relationship between the numerical maths they learn at school and what they meet elsewhere [Application and relationship].

A very important aspect of the four points just highlighted revealed that these teachers believed in empowering the students to take control of their own learning. Thus they tended to make it a priority to encourage students to develop a classroom culture of using their own methods of accepting whether their discovery was mathematically reasonable. Hence, as reflected in Excerpt 62, a major purpose of providing students with the opportunity to develop number sense was expressed through three connected perspectives of enabling students to:

- move… [them] towards a more open ended challenging situation [Moving from teacher-controlled to student-owned]
- seek for numbers everywhere and to discover numerical relationships [Search for numerical relationships]
- gain more ideas about how to apply number sense in everyday life situations [Application of number sense in everyday life]

These three closely connected perspectives intersected through the way these teachers strived to get students to make connections among the mathematics learnt, the various numerical aspects, and the real world. It was very common that towards the end of a learning experience session, the teacher would usually engage the students in discussion about the applicability of the problems they had solved. In this way, as pointed out by Chantal, it was thought that the students were "…enabled to see how they could relate number sense problems to other subjects [curriculum subject-matter areas]". For instance, when the researcher chose four topics and asked the teachers to prepare and teach four lessons from these, towards the end of the lesson on number bases Bob challenged the students with some related real-life problems for home work. In a subsequent lesson he went over their results and engaged the whole class in discussions about what they had learnt. Amanda had the habit of presenting students with a real life situation and then getting them to see how they could discover a pattern, come up with a formula or construct an equation, which they were then asked to fit into a different real life context. The students were often given tasks which required them to search for the numerical aspects in other subject areas. The students in Amanda's class seemed aware of what she wanted to develop in them through such exercises, as expressed by Alana [$S_{(1,3,2)}$]:

> Mrs [Amanda] tries to get us to see the relevance of what we learn in class…She sometimes gives us those activities where we have to find about the maths done by others,…ancient people, like the Egyptians, the Greeks, and others.

Other students also gave specific accounts of their recollection of how the teacher got them to engage in 'research' about how other civilisations dealt with the mathematics we are learning. For instance, Indigo's [$S_{(1,13,2)}$] account which corroborated what Alana and others had recounted:

> Like when we had this lesson on time we had to find out: how the Babylonians recorded time; how the Egyptians managed to calculate when the river would flood; [and] how the Mayan calendar works. We then did that project about cogwheels in the clock. During the English class we wrote a paragraph about time, clocks and calendars, and then we stuck them on the walls…It was fun because I learnt about what the numbers meant, why the time was read in a different way.

Chantal was no exception to the rule; she got her students involved in projects at least once every six weeks, and some of these were "…extensions of what students had learnt in class and how they could relate these to everyday real life happenings…". For instance, after students had been doing substantial amount of work on fractions she got them to discuss about how they could express these fractions in other numerical terms such as decimals and percentages. Then she organised for them to engage in searching for how fractions are used in music, science, in the bank, in sports, geography and some other areas. Students then drew a chart to show the different links of the same numerical aspects in the various curriculum learning areas.

Teaching for Logical Reasoning

These teachers saw computation, the basic facts and instant memory recall as mere basic number sense tools, which are needed for but not as important as problem solving, understanding numerical structures and patterns, and constructing ones own problems, structures and patterns. Thus the teachers' believed that "…it [problem posing] has an important place in [their] teaching [and] students' number sense problem solving development and, much emphasis was also placed on getting students to discover the logic in a mathematical argument. This was seen as an important exercise which also prepared students to "…appreciate what seems illogical at first sight", according to Bob. Excerpt 63 presents a brief sentence about each teacher's belief concerning why they spent so much time getting students to explain the logic behind their answers.

Excerpt 63

R:	Can you briefly state, in one sentence, your philosophy about logical reasoning in your teaching and the child's learning?
Bob:	Logical reasoning is the basis of problem solving in mathematics.
Chantal:	If a child cannot find the thread of reasoning in the problem then it could be very difficult for him to solve that problem.
Amanda:	The children need to learn how to think logically…how to make a logical argument. This will definitely help them discover logical patterns in the words,… the structure of the problem.

The teachers pointed out that in "…most mathematics classes of long ago to understand the logic behind the solution to a problem one had to be able to understand axioms and so on, and use a lot of algebraic or geometrical proofs". It should be noted that what students called logic problems and their perception of logical reasoning were given two very different meanings. Logical problems were those of the type in which some descriptions are given and the problem solver is asked to match these with the most appropriate item such as a person, object or occupation. On the other hand, logical reasoning was seen as being able to think through and solve a problem in a systematic way. Surprisingly the students with lower number sense tended to perform as well, if not better, in solving those types of logic problems than some students with higher number sense, while the latter were better at using logical reasoning to solve problems which required lateral thinking.

Short-cut, Instrumental Learning and Relational Learning

All three teachers expressed their belief that in the old traditional system too much emphasis was placed upon teaching and learning short cuts. When asked whether he was not reverting back to the old system when he taught students algorithmic rules Bob explained that compared to traditional teaching this was more about "…[understanding] the why, the what and the how". Although all three teachers maintained that they did not lay any emphasis on teaching short cuts, on certain occasions students were asked to explain how they worked through an algorithm. Unlike the method employed in traditional teaching, Amanda explained that what the researcher was calling 'the short cut' "… is taught parallel to the relational understanding", which was confirmed through the observations, and highlighted in Excerpt 64 of Amanda and one of her students who had obtained 0.12 as the product for 0.6 times 2. Eventually Chantal went on to get Tarick [$S_{(350,1)}$] to understand that "decimal points don't have to go in line". Although the student was taught how he could use a short cut to get the decimal point in the correct location, most of the interaction time was spent in enhancing relational understanding of the concept. Later on Tarick said to the researcher that he "had not thought of relating 0.6 to a half", and that he now finds it easier to understand "where it [the decimal point] should go".

Excerpt 64

Chantal:	[Pointing to 0.6] Is that more than a half?
Tarick:	Yes.
Chantal:	What do two halves make, Tarick?
Tarick:	A whole
Chantal:	But you told me that two numbers bigger than a half make[s] a very small number less than a half.
Chantal:	'Cause if it was a half, two halves, it would be one, isn't it, one point something?
Tarick:	Yes.
Chantal:	Which of these could be the correct answer, 1.2 or 0.12?
Tarick:	1.2
Chantal:	Why?
Tarick:	Because I have more than two halves.

According to Ernest (1989) there are three main philosophies about the views of mathematics: the instrumentalist; the Platonist; and the problem solving. The data suggested that these teachers tended to share some aspects of the first two while showing a tendency to fully embrace the tenets of the third philosophy. The belief of both teachers and students that to be good at mathematics one has to be very conversant with the basic facts, rules and skills tend to be partially instrumentalistic, but as pointed out by Amanda "rote learning is when the teacher just teaches students to learn rules…such as 'multiply the top by the top and the numerator by the numerator', without understanding where they come from, how they function and why". Hence, Bob maintained that "I don't believe in rote learning because it does not help the students become independent thinkers…[which] affects their progress in developing good number sense…[and] problem solving]", and Chantal stressed that "it is important to teach students the problem solving strategies and basic facts, but what use is that to them if they don't understand them…[and] can't make sense of the numbers involved or the other mathematics involved?"

4.8.3 Summary

Analysis of the interview data revealed that the teacher's beliefs about how the link between number sense and problem solving impacted upon their teaching of number sense revolved around a complex set of interactions between the teacher's expertise, the student's ability and preference, and the curriculum. The importance of the teacher's mathematics knowledge, ability and confidence was considered to be a major necessity for effective teaching of number sense and problem solving. It was believed that a teacher who is not very competent in mathematics could find it difficult to deliver effective number sense problem solving lessons. Hence, the teachers involved in this research kept abreast of new developments pertaining to number sense, problem solving, mathematics and mathematics education, especially through their engagement in professional development sessions. Teaching mathematics through a problem-based approach was seen as being central to any curriculum which aspired to develop mathematical sense-making in students. This belief was manifested through the way the teachers got the students to think and work mathematically, which was facilitated through the belief that all mathematics can be taught through problem solving. Number sense was seen as the key to problem solving, and the students were provided with opportunities to engage in productive investigations. Furthermore, these three teachers belief that combining teacher-centred teaching and student-centred learning was a key component of their teaching repertoire. In this teaching system it was thought that student-student interaction should be encouraged and monitored appropriately, as a means of enhancing the development of students' number sense ability. Moreover, it was deemed important to facilitate this development by taking into consideration the students' affective needs and to create a learning environment through which these were met. According to the three teachers observed, instead of preferring one over the other the teacher needs to find a way to combine contemporary and traditional teaching. This involved learning mathematics through making connections; teaching students to develop a sense of logical reasoning; and encouraging them to learn about the relational aspects of number before moving on to using short cut methods. There was also the issue of the need to strike a balance, which, according to these teachers, was a very important aspect of their teaching. It was believed that this involved finding a balance among the mathematics curriculum content, process and principles. Crucial to the planning of learning experiences and the teaching approaches used was the belief that it is possible to get all students to improve upon their performance.

Hence, number sense was seen as being a very important component in a student's problem solving tool box since students with good number sense had a greater chance of being confident enough to tackle problems from other mathematics strands. This is possible due to number sense being required for successful solution of most problems which incorporated a numerical element. The teaching being advocated is one where the teacher and students interacted through affective considerations of each other, so that they willingly participate in cooperative as well as individual learning activities. Hence, the three teachers observed believed in and practised a system in which the teacher sought to make advantageous use of an amalgamation of effective components of the different philosophies of teaching and learning advanced through both inter-personal and ultra-personal research. Since the three teachers who participated in this study were categorised as 'effective teachers of mathematics', it is worth noting that these beliefs and practices might be exclusive to such teachers. Hence, the average teacher could gain much both pedagogically and academically by taking into consideration how these effective teachers put into application their knowledge and expertise and how their belief impact on that.

4.8.4. Theoretical model from the study

During the study it was observed that both of the frameworks presented previously in Figure 1.1 and Figure 1.2 seemed to be in use, but the second one was predominant throughout while the former was used mainly at the start of the first term. The discussion of the results presented in this chapter has shown that as the study progressed, grounded analysis of the data indicated that there was another more complex model at work. When all the various components of the framework are brought together the model obtained resembles the one presented in Figure 4.31 below, and this model will serve as a succinct graphical answer to the main question, which was:

What is the relationship between teaching and learning styles, and the number sense and problem solving abilities of Year 7 students?

An amalgamation of the themes from the answers to the four secondary research questions resulted in the construction of a theoretical model, which depicts the interaction of the following major factors, as the main ones involved in the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students.

- Teacher's beliefs;
- Flexibility of the curriculum;
- Students' beliefs;
- Thinking skills;
- Teaching style;
- Learning style;
- Teaching and learning through the affective domain;
- Number sense;
- Problem solving ability; and
- Mathematical sense-making

Classroom observations revealed that most of the teaching time was spent on teaching problem solving in combination with number sense, while only six percent of the time was devoted to teaching problem solving in combination with other topics not necessarily involving number sense. In more than 35 percent of lessons observed number sense was specifically involved in combination with Devoid of Number Sense (DNS) topics and problem solving, while more than 80 percent of teaching time was devoted to topics involving Number Sense Inherent Problems (NSIP). Yet there was no marked difference between the students' DNSP NSIP performance. Hence, the results of the NST and PST, in combination with on-going observations of the classroom learning experiences, suggested that it is possible that increase in students' performance on a particular type of problem was not necessarily based on how much emphasis was laid on solving such problems in class. As discussed previously, the teachers explained that since most mathematics problems involve number sense an emphasis on solving NSI problems helps students improve in solving DNS problems; a view which was shared by a large majority of students as well.

In terms of the importance of number sense and problem solving the teachers seemed to agree on the notions that:

1. Problem solving is more important than number sense because it is needed to solve problems not only from all the strands of mathematics but also areas other than mathematics.
2. Number Sense is more important than any of the other mathematics curriculum content.
3. Good number sense is central to having good problem solving ability.

The model being proposed revolves around a teaching style which emphasises higher-order thinking and conceptual understanding in relation to developing number sense and problem solving. This teaching also encourages students to engage in systematic mastery of the fundamental number sense skills and concepts such as recall of basic facts, estimation, and use of self-made algorithms and computation methods. Hence, this teaching incorporates use of technology like calculators, manipulatives, educated guess and check, collaborative group activities, standard and non-standard algorithms, repeated practice, problem posing and problem solving. According to the teachers the aim of employing such a system was to produce in students a "mathematical self-belief" which would help them become autonomous learners. When asked about what qualities are present in an autonomous learner the most common attributes stated by these teachers which were supported by their students were:

- Self-belief;
- Perseverance;
- Intuition;
- Knowledge and recall of the basic facts and fundamentals; and
- An ability to adapt to new situations.

Although this research was not designed specifically to look at causes and effects the Think Aloud Stimulated Recall Interviews (TASRI) revealed that there were mainly six factors which seemed to play a role in the performance of students who were classified as high performers in both number sense and problem solving (HnsHps). Figure 4.29 provides an illustration of these six factors. These six factors seemed to be in line with the five attributes suggested by these teachers. In all aspects of their teaching these teachers tended to encourage students to: (i) explore the mathematics they were learning as deeply as possible; (ii) always ensure that they understand and master the fundamentals; and (iii) employ the most efficient method of solving the problem at hand. For this to occur the learning experience seemed to rely heavily upon the affective interaction between teacher and students. The teachers believed that for students to learn mathematics effectively they must be empowered so that they can make sense of the mathematics they are dealing with. Hence, Instead of problem solving the teachers were talking about 'problem sense'. The emphasis in these lessons was always on getting students to work mathematically since to these teachers problem solving forms part of working mathematically. Both teachers and students felt that to be good at mathematical problem solving and also have good number sense students must be able to make sense of what they learn.

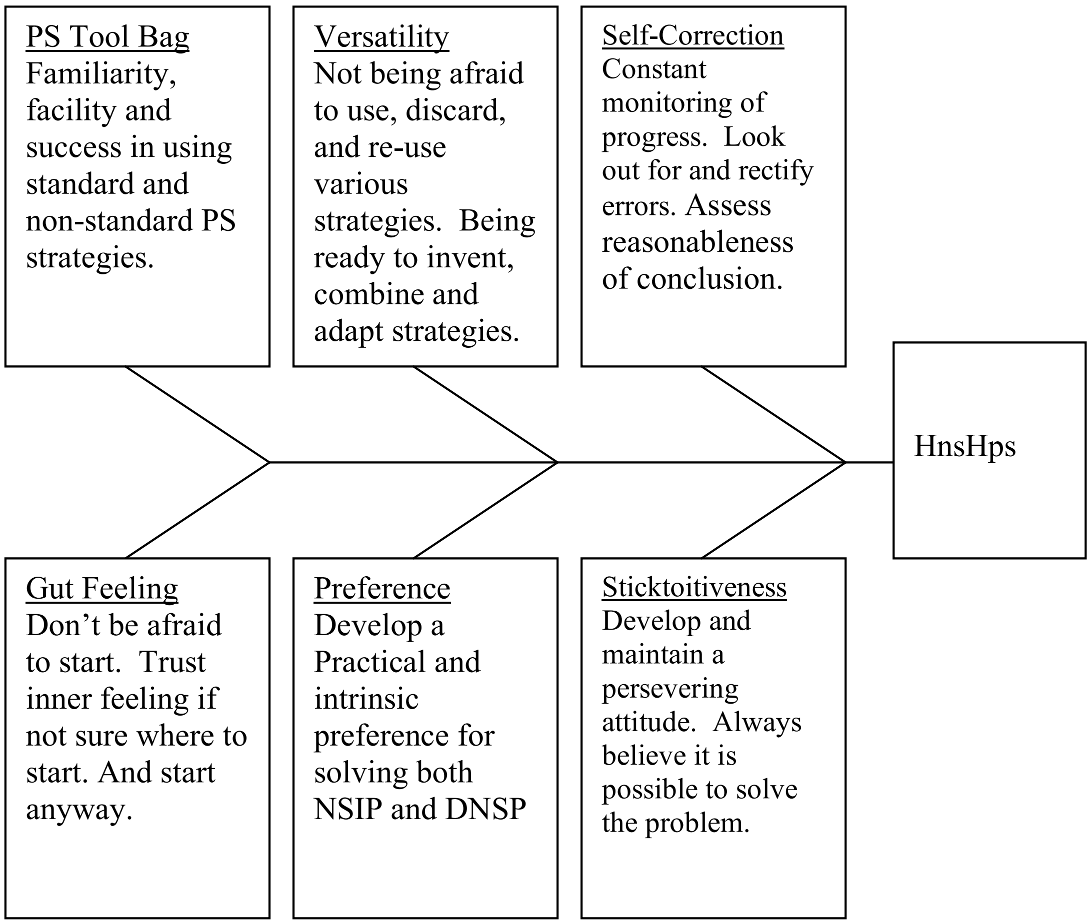


Figure 4.29 Causes and Effect Diagram of Attributes of HnsHps Students

Figure 4.30 is an attempt to capture how the teachers and students tended to see the relationship between sense-making, problem solving and the numerical aspect of learning mathematics. The working mathematically component forms part of the first level of the framework, since according to the teachers this is the component that guides their selection of mathematics content, concepts and processes.

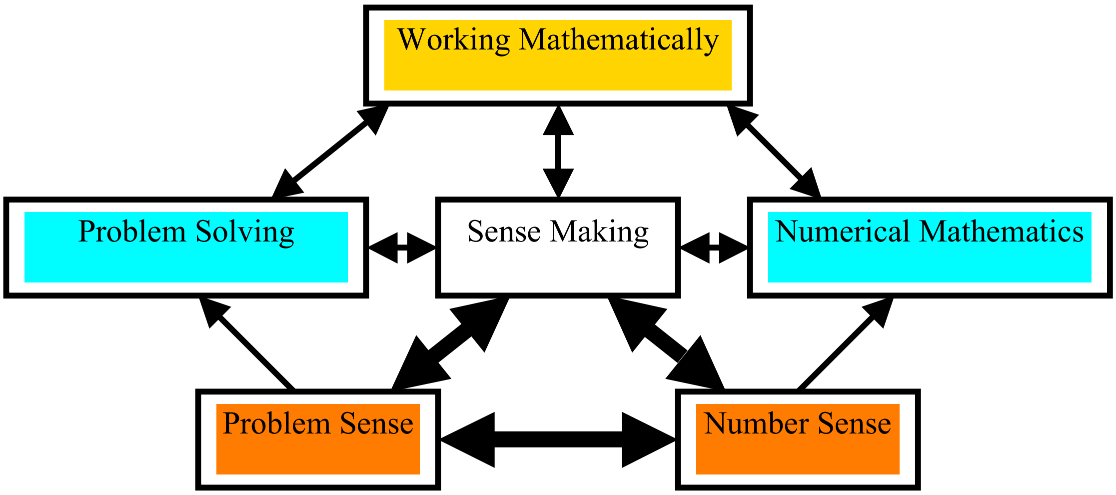


Figure 4.30 Sense-making is central to learning number sense and problem solving

By 'Numerical Mathematics' is meant exposure to rigorous, content-rich mathematics which focus on mastery of the fundamental components of number as a branch of mathematics. The focus here is not on higher-order thinking, but on various algorithms — mainly invented ones — repeated practice through play (puzzles, various numerical operations), recall of basic number facts and applying these through certain basic mathematical processes.

Through problem solving the students are placed in new situations where they have to find ways to apply what they have learnt as Numerical Mathematics to describe the outcomes of these new situations. The common elements stemming from these teachers' and their students' views of problem solving could be summarised in a definition of problem solving as "Whenever one encounters a new mathematical challenge one is faced with a problem". This implies that although a student might have been involved in solving a particular situation before, if upon meeting that situation again everything seems new to that person, then the situation is considered to be a problem for that person; it will cease to be a problem until it can be solved permanently and recalled easily whenever it is met. This notion, of a mathematical challenge being identified as a problem whenever one cannot recognise how the problem was solved, is different to the traditional definitions which tend to classify problems as routine or non-routine.

The teaching strategies employed create a situation in which 'Sense-making' instantly permeates both numerical mathematics and problem solving. Every time students explored the solution to a given situation they were prompted to make sense of the situation, their solution plan and the final answer they got; they were then taken through various problem solving processes either as suggestions from the teacher or students, or their peers' personal methods, strategies and algorithms. The same thing happened the moment the student embarked on dealing with numerical mathematics; the teacher introduced that student to making sense of the operations, contents and processes through questioning, enriched repeated practice situations and recall through linking the different components together.

When all the results are compared the common themes which emerge could be summarised into the theoretical framework presented in Figure 4.31. As expressed by these teachers and students, and as observed by the researcher, while the sense making aspect is definitely acting somewhere in the intersection of problem solving and number sense, it also permeates all the other components of the model as it exerts a sort of hovering influence over them, as shown by the dotted circular path.

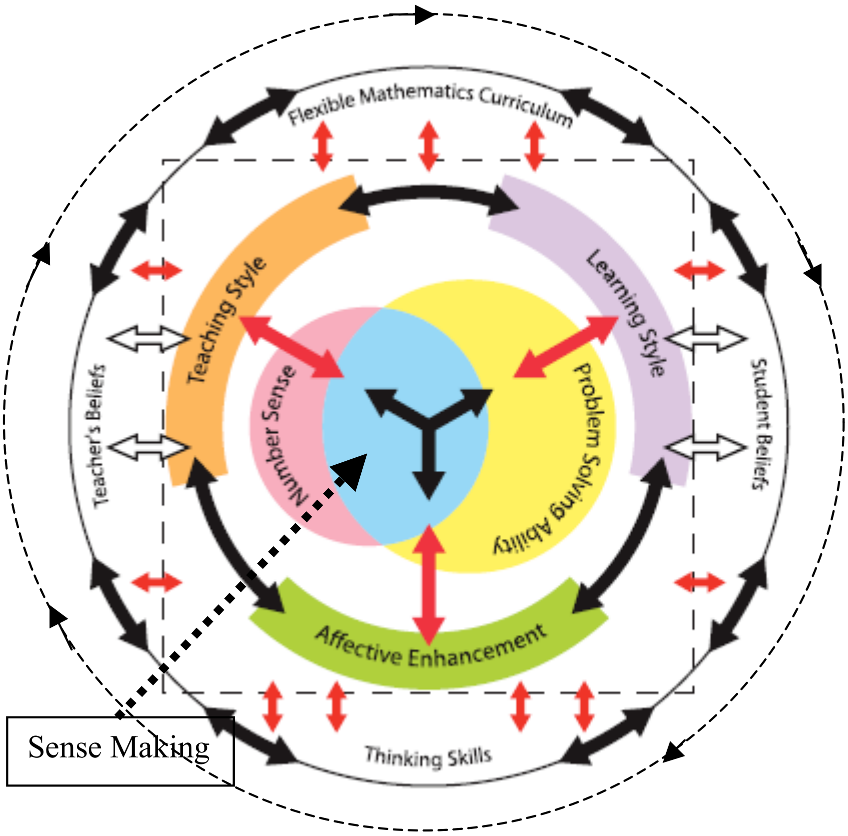


Figure 4.31 Theoretical framework of the interaction among ten essential factors

The whole framework can be viewed as a vehicle being directed through two main pilots; teaching preference and learning preference. Hence, the driving force is comprised of the sense making aspect of the learning experience, which acts as the engine of the whole framework; it empowers the vehicle of problem solving and numerical mathematics. These in turn are conveyed by the respective wheels of problem sense and number sense, which intersect whenever a student has to make mathematical sense of a numerical problem and its solution.

On the whole, the comments from the three teachers and those of their students suggested that problem solving and number sense are neither synonymous nor always part of each other. Similar to what was anticipated, prior to data collection, through the theoretical framework, the observation and interview data indicated that many problems were solved without recourse to number sense and not all number sense was applied within a problem solving context. This was further supported through the distribution of teaching sessions focussing on number sense, problem solving or both.

The flexible curriculum implies both the official curriculum and the hidden one. Although the curriculum is very important, the weighting given to it by these teachers and what was gathered from the observations suggest that students' beliefs, teachers' beliefs and quality of thinking skills each tend to have, or are given, equal bearing on the aspects occupying the inner components of number sense and problem solving ability.

In the next chapter the study will be summarised and a conclusion presented, together with some implications and recommendations.

Chapter 5: Summary, Conclusions, Implications and Recommendations

5.1 Summary

This study investigated the relationship between teaching and learning styles, and the number sense and problem solving abilities of Year 7 students, for a duration of one school year comprised of four terms. The investigation started off with the identification of effective teachers of mathematics, from which three were approached to participate in this study. Once the three effective teachers of mathematics had been identified and the researcher had gained the consent from all the participants, there was preliminary observation of all the lessons taught by the three effective mathematics teachers. The three teachers and their 64 students were all from three Perth Metropolitan schools. They were interviewed and asked to fill in questionnaires, with the students also participating in pre- and post- written number sense and problem solving tests. Data was also collected through classroom observations and documentation. The number sense and problem solving pre-tests were first administered to the students during the first term. This was followed by classroom observations and one formal teacher interview per term, with the students being formally interviewed as per availability of time and personnel. Short teacher and student informal interviews were conducted in between the formal interviews. During the fourth term the teachers and their students complete a learning style inventory with the former also completing a teaching style inventory. The number sense and problem solving post-tests were administered during the latter part of the fourth term, after which 45 students were selected to participate in a Think Aloud and Stimulated Recall Interview while completing four set problems.

The results from this study found that there was a close connection between students' number sense (NS) and problem solving (PS) ability. In fact the higher a student's number sense the greater the student's performance in solving both Number Sense Inherent Problems (NSIP) or Devoid of Number Sense Problems (DNSP). This result was further confirmed through analysis of data pertaining to students' preference for solving either NSIP or DNSP. In that regard it was revealed that students with above average number sense, who preferred to solve number sense inherent problems were better at solving most types of problems, than students who preferred to solve devoid of number sense problems. Moreover, the high problem solvers seemed to transfer and make successful use of the number sense performance attributes of: (i) estimating the final solution; (ii) checking for reasonableness of answer; and (iii) detecting and using the relational aspects of the mathematical components, coupled with other personal qualities such as perseverance and not being afraid of committing mistakes.

In regard to the impact of teaching style upon students' NS and PS ability, various factors were identified, which could be responsible for enhancing students' number sense performance. From this the observation data analysis revealed that teaching for number sense development through a problem based approach seemed to be one major factor which could be responsible for students' improvement in number sense performance. This result was supported through analysis of these teachers' teaching style through Grasha's Teaching Style Inventory (ILS), which showed that the teaching style of the three effective teachers of mathematics who participated in this study emphasised the Delegator/Facilitator/Expert blend of Grasha's (1996) Cluster 4. According to Grasha (1996) such a learning style cluster:

> …sends message to students that "I'm here to consult with you and to act as a resource person." A warmer emotional climate is created and students and teachers work together, share information, and the boundaries between teacher and student are not as formal. (p.144)

Since comparison of the pre-tests and post-test data revealed that there was a marked improvement in the number sense and problem solving performance of students from all three classes, it could be suggested that the teaching style cluster of these teachers were partly responsible for the students' marked improvement. This teaching style cluster preference could also explain why both students and teachers felt that the emphasis placed upon catering for the students' individual differences through consideration of their affective and cognitive needs was very important in helping students' number sense development.

Although, except for the Understanding Information learning style dimension, none of the statistical tests of inference performed on the students' Index of Learning Style (ILS) scores were statistically significant, there were some interesting results. In the first instance it was revealed that students with high number sense performance were more reflective than they were active, whereas those with a low number sense profile preferred to process information actively instead of reflectively. Furthermore, all students with a low number sense and low problem solving performance preferred to receive information through the visual modality instead of verbally. The only students who expressed a preference for receiving information verbally were some of those with simultaneously high number sense and high problem solving performance. In most cases where students were observed requesting for information to be presented in a mode which the ILS results suggested as their preferred one, the learning preference was at the strong end of the scale. This was most apparent for students with a strong visual preference. In regard to students' preferred mode of perceiving information it was expected that the greater a student's number sense performance the more would be their preference for the intuitive modality. The ILS results indicated no such tendency, although the teachers expressed the belief that number sense develops students' intuitive perception, which they felt is an important element necessary for effective problem solving. Moreover, except for modalities pertaining to how students preferred to process and receive information, there were no marked differences between the proportions of students who preferred either of the two understanding information learning modalities or the two perceiving learning modalities. When the analysis was shifted to which type of problem students favoured, preference for NSIP or DNSP was identified as one possible factor which could help in determining a student's number sense-problem solving learning style. Moreover, according to those students with a simultaneously High Number Sense and High Problem Solving (HnsHps), being successful at solving Number Sense Inherent Problems provides students with a ready-made set of tools to solve other types of problems. It was also revealed that the lower the number sense ability of a student the greater his/her reliance on using a single problem solving strategy, regardless of whether this strategy was effective or not. While HnsHps and HnsMps students were the only ones who tended to use one or more additional strategies to confirm what had been achieved so far as a means of speeding up the solution process, it seemed that high problem solving performance, irrespective of number sense ability, does not make a difference as to whether students stick to a single strategy or not.

Analysis of the interview data revealed that the teacher's beliefs about how the link between number sense and problem solving impacted upon their teaching of number sense revolved around a complex set of interactions between the teacher's expertise, the student's ability and preference, and the curriculum. The importance of the teacher's mathematics knowledge, ability and confidence was considered to be a major necessity for effective teaching of number sense and problem solving. It was believed that a teacher who is not very competent in mathematics could find it difficult to deliver effective number sense problem solving lessons. Hence, the teachers involved in this research kept abreast of new developments pertaining to number sense, problem solving, mathematics and mathematics education, especially through their engagement in professional development sessions. The teachers believed that teaching mathematics through a problem-based approach was central to any curriculum which aspired to develop mathematical sense-making in students. This belief was manifested through the way the teachers got the students to think and work mathematically, which was in turn facilitated through the belief that all mathematics can be taught through problem solving. Number sense was seen as the key to problem solving, and the students were provided with opportunities to engage in productive investigations. Furthermore, these three teachers stressed that combining teacher-centred teaching and student-centred learning was a key component of their teaching repertoire. In this teaching system it was thought that student-student interaction should be encouraged and monitored appropriately, as a means of enhancing the development of students' number sense ability. Moreover, it was deemed important to facilitate this development by taking into consideration the students' affective needs and to create a learning environment through which these were met. According to the three teachers observed, instead of preferring one over the other the teacher needs to find a way to combine contemporary and traditional teaching. This involved: learning mathematics through making connections; teaching students to develop a sense of logical reasoning; and encouraging them to learn about the relational aspects of number before moving on to using short cut methods. There was also the issue of the need to strike a balance, which, according to these teachers, was a very important aspect of their teaching. It was believed that this involved finding a balance among the mathematics curriculum content, process and principles. Crucial to the planning of learning experiences and the teaching approaches used was the belief that it is possible to get all students to improve upon their performance.

Hence, number sense was seen as being a very important component in a student's problem solving tool box since students with good number sense had a greater chance of being confident enough to tackle problems from other mathematics strands. This was possible due to number sense being required for successful solution of most problems which incorporated a numerical element. The teaching being advocated was one where the teacher and students interacted through affective considerations of each other and willingly participated in cooperative as well as in individual learning activities. Hence, the three teachers observed believed in and practised a system in which the teacher sought to make advantageous use of an amalgamation of effective components of the different philosophies of teaching and learning advanced through both inter-personal and ultra-personal research.

5.2 Conclusions

At the outset it was believed that there was a connection between students' number sense and problem solving ability — which was eventually confirmed through a high correlation coefficient — and that matching teaching and learning style would facilitate and enhance such a connection. The existence of statistically significant differences between pre- and post-tests for both the NST and PST shows that students from all three classes improved considerably. Students in all three classes advanced in their use of both number sense and problem solving strategies as evidenced through comparison of the pre- and post- tests workings and performance results, and the TASRI data. Furthermore, these results do not only indicate high retention rates but also an improvement in the efficiency and novelty of the strategies used. This indicates that among the possible causes for such an increase in performance level, being taught by an effective teacher of mathematics could be very important.

5.2.1 Empowering students instead of catering for individual learning styles

It was believed that the improvement in the students' number sense and problem solving performance could also be attributed to how the teachers catered for individual differences in their students. However, the literature is divided on the issue of compatibility of teaching style and learning style (McLoughlin, 1999; Klein, 2003; Cassidy, 2004; Denzine, 2005). For instance, some researchers assert that matching the teacher's learning style with the students' learning will produce higher academic performance (Raines, 1978; Hunter, 1979; Zippert, 1985; Van Vuren, 1992; Carthey, 1993) while others argue that there is no significant relationship resulting from matching of teaching style and learning style (Hunter, 1979; Scerba, 1979; Charkins, O'Toole, & Wetzel, 1985; Battle, 1982; Campbell, 1989; & Lyon, 1991). Moreover, all three teachers participating in this study felt that it was more important to focus on learning strategies instead of learning style. The position taken by all three teachers were more in tandem with the evidence and arguments presented by Messick (1984), Barris, Kielhofner and Bauer (1985), Talbot (1985), and Streufert and Nogami (1989). Messick (1984), and Streufert and Nogami (1989) showed that learners adapt their learning style based on perceptions of the requirements of a learning task. This contention was also supported by Talbot (1985) who maintained that students were eclectic in their learning styles depending on the learning task being undertaken. Furthermore, Barris, Kielhofner and Bauer (1985) proposed the possibility for learning to vary throughout a course of study. Further support for such a stance came from the high negative correlation between the three teachers' learning style and that of their students which confirmed that for most learning modalities teachers and students had opposite preferences. This was a very striking result for three main reasons: (i) the students still made very significant progress in both number sense and problem solving performance; (ii) all students indicated that they thought the teacher adequately catered for their individual learning needs; and (iii) research which states that a teacher's learning style is indicative of that teacher's teaching style. In the latter case Stitt-Ghodes (2001) stated that "research supports the concept that most teachers teach the way they learn" (p. 136). On the other hand, the present study tends to suggest the opposite; that all three teachers deliberately discarded any preference they could have so that they could enhance their students' learning.

Furthermore, both the teachers' and students' beliefs indicated that it is virtually impossible to cater for the individual learning styles of all students in a normal class of up to 30 students, although it is possible to engage in teaching-learning experiences which compensate each other according to the 'mood of the moment'. This is in tandem with Stanley's (2006) suggestion that "a competent deliverer of learning won't necessarily enable every sort of learning in every learning event" (p. 17). Although, on the surface this would seem like the teacher having no say in how the lesson would progress, the observation and interview data revealed that teachers still had control over content selection and delivery of the lesson, but students were gradually empowered to dictate the pace of the lesson and how they wished to engage in the learning experience. This type of teaching style seemed to have been successful due to certain purposeful teacher-designed situations.

First of all this type of teaching style calls for the teacher to be highly skilled in monitoring students' on-task progress in relation to what extent they are still enthusiastic about the activities they have been given. The teachers in this study employed various methods to enhance the quality and voluntary participation of students in the lesson. One teacher always prepared more than one learning experience plan so that he could direct students to a totally different learning experience if he detected any signs of tiredness or boredom. Another teacher relied on allowing students to set personal targets for the day and to take as many occasional short breaks as they wished. This seemed to work due to the students having to fulfil their promise to reach their own self-set targets. The third teacher relied more on having students suggest what sort of activities would motivate them more. All three teachers made use of all three teaching-learning strategies, although in different degrees as pointed out above.

Secondly the teacher had to be well equipped and ready to use a combination of activities and situations which incorporated a high dose of the affective domain, followed by well structured cognitive experiences, which are often introduced and learnt through student-engagement in psychomotor activities. In this regard the teachers preferred to engage students in situations which developed their ability to cope with the demands of the problems, the learning environment, and with being self-dependent as a means of becoming more self-confident. This sort of adaptive behaviour was seen as being of paramount importance in getting students to first rely on their own ability (self-help), to seek help from their peers only when they could not solve a problem on their own and to solicit the teacher's help only if the first two avenues were unsuccessful (communication). This was achieved through a teaching style which comprised of an amalgamation of very well prepared hands-on mathematical experiences (psychomotor domain), through which students were led to extracting concepts, symbolising and extending them (cognitive domain), all the while interacting within an environment which emphasised the development of appropriate non-confrontational and partnership-reliant social skills (affective domain). The most striking aspect of this system was the amount of emphasis placed upon working through the affective domain and encouraging students to freely desire to move from the concrete to the abstract as soon as possible. Hence, instead of attempting to cater for individual learning styles, the teachers relied on readiness behaviour cues from the students coupled with what could be termed as 'gentle pushing' by the teacher. The purpose for the latter was that teachers did not believe in simply waiting for the student to be ready to move to the next stage, although the students' learning pace seemed to dominate the decision to work on more advanced or new topics.

A third consequence of using such a teaching style was being able to make effective and appropriate use of both traditional and contemporary teaching methods, teaching and learning theories, and mathematics activities. Hence, no new methodology or pedagogy was accepted without careful scrutiny and comparison with those already in use or tried. There again this called for the teacher to be very knowledgeable and up-to-date with both traditional and contemporary theories, practices, mathematics content, processes and context.

5.2.2 The need for a relevant learning style inventory

It was also believed that the learning style inventory used would show clearly that learning style was related to success in solving mathematics problems. However, except for the Processing Information dimension, there was no statistically significant relationship pertaining to any of the other three learning style dimensions. Yet the Think Aloud Stimulation Interview (TASRI) revealed that students with different number sense performance levels tended to exhibit certain categorical styles while solving the problems given. For example, the TASRI indicated that the LnsLps students were also aware that they needed to check the reasonableness of their solutions, but did not do so in the case of NSIP because it seemed that such students believed that having to make sense of the numbers in a problem made it even more difficult to check whether their result was reasonable or not. Since these same students expressed an appreciation for having improved in solving NSIP, which was validated through analysis of their improvement on the same item from the problem solving pre- and post-tests, it could be that the learning style inventory used was not detecting certain specific beliefs pertaining to solving both NSIP and DNSP. Hence, it could be that a learning style inventory aimed specifically at discovering a student's number sense problem solving learning style could help in identifying possible issues of preference which could be influencing their belief. For instance, one possible item that would be included in such a learning style inventory, would be "I find it: (a) difficult (b) easy to know what to do with the numbers in a problem? This could be counter checked through another item of the form: I find it: (a) easy (b) difficult to check for the reasonableness of the answer if the problem involved making sense of numbers. Moreover, students' preference for solving NSIP, DNSP or both was also found to be related to their motivation to solve particular types of problems, although this warrants further research.

5.2.3 Students' success in solving number sense problems

It was expected that there was a very strong relationship between number sense and problem solving performance, and the analysis indicated that these two variables were quite highly correlated. In an attempt to be more precise about the nature of the relationship between the students' number sense and problem solving performance, some factors were identified which could have been instrumental in helping students to improve in their performance.

The first factor may be the eclectic nature of the teachers' teaching style combined with their insistence on teaching through a student-centred approach which incorporated a lot of affective considerations and interactions. As pointed out by Nuckles (2000), being student-centred engages teachers in a humanistic approach to education in which they function as facilitators of learning. It is interesting to note that the teaching style inventory analysis revealed that these teachers belonged to a minority group of teachers who saw themselves as facilitators, delegators and experts (Grasha & Yangarber-Hicks, 2000). The evidence from this study is in tandem with Grasha's (1994) statement that this cluster of teaching style presents the teacher as a supportive, cooperative and resourceful consultant. As observed in this study these teachers created a teaching-learning environment in which the teacher and students exchanged information in a warm emotional climate without any rigid formal boundaries to hinder free communication.

A second possible factor could be that a large majority of in-class learning experience time was devoted to engaging students to solving number sense inherent problems where a lot of emphasis was placed upon getting students to value the importance of working through the concrete and the abstract in a nearly simultaneous manner. Contrary to popular belief that teachers should not move too fast to get students to work in the abstract, these teachers encouraged students to move to the abstract and work with symbols as soon as they felt confident enough to do so. The teachers had very high expectations of their students although these were in line with the teacher's perceived performance level and ability of each student. Hence, high number sense problem solvers were expected to work more in the abstract, while the less able were expected to work more with concrete objects. Nevertheless, this was done in a collaborative manner between teacher and students so that the latter were the ones expressing their need to go to the abstraction phase. Another important point to note in that respect is that concrete manipulatives were always accessible to all students, although the teachers used them mainly when the situation was such that it was deemed to be the most appropriate method to use. Hence, it was often possible to observe students preferring to struggle with the problem until they felt that maybe using concrete aids might help.

A third possibility may be that the teachers laid much emphasis on getting students to use both physical and mental visual aids. Students were widely observed closing their eyes or staring into space trying to picture the problem through visual representation. This was so apparent that it seemed to be second nature to the students to describe equations, formulae or other mathematical relationships in terms of visual entities. This was usually accompanied by a lot of reflection which could be a result of students always having to justify their results whether these were correct or not. It could be that this was the reason why the reflective modality was the only one which was significantly and positively related to number sense performance, while the visual modality was the most popular singular modality. This could also be one reason why most of the students, regardless of whether their number sense problem solving performance was high or low, preferred to work mentally before setting pen to paper. It should be noted that the teachers placed a lot of emphasis on getting students to solve problems mentally and then compare with solving them on paper or vice versa.

A fourth possible reason may be that teachers always tried to encourage students to try and work in another mode, or use an alternative or invented method of computation or problem solving strategy. In this way students were made to reflect both inductively and deductively, which could be the main reason why the most balanced of all the four learning style dimensions was the understanding information one, where the majority of students indicated a bimodal preference which incorporated both the sequential and global learning modalities.

A fifth possible reason could be the way that learning of the basic facts and mental computation were incorporated into the main fabric of each learning experience. Hence, mental computation and knowledge of basic facts were not seen as being separate to the other necessary number sense skills to be developed in the learners, but rather as intertwining components which merged together to form one body of inter-related facts, concepts and processes. The teaching-learning experience which was observed incorporated an element which was reminiscent of Skemp's (1971, 1976) notion of a 'procept' in that the teachers tended to combine experience of the process, concept and symbol in that order, while encouraging students to always think of the process in terms of a concept and attempt to symbolise this concept as immediately as possible. Gray and Tall (1993) define a 'procept' as:

> a combined mental object consisting of a process, a concept produced by that process, and a symbol which may be used to denote either or both. (p. 2)

5.2.4 Limitations to generalisability

One of the limitations of this study is the size of the sample which could have influenced the extent to which generalisations could be made. Another limitation could be the type of learning style inventory used. Had the learning style inventory been designed specifically for looking at number sense problem solving style, a more accurate picture of students' learning preferences might have been discovered. Then there is the issue of controversy pertaining to theories of learning style (Coffield, Moseley, Hall, & Ecclestone, 2004b). The only core assumption of learning style theory which was supported by the results of this study is that there are individual differences in learning. All other core assumptions such as: (i) an individual's style of learning is fairly stable across time; (ii) an individual's style of learning is fairly stable across tasks/problems/situations; and (iii) we can effectively measure an individual's learning style, were refuted by both the teachers and the data. Finally one possible limitation is that learning style was never controlled or manipulated. Perhaps if this variable were manipulated, clearer evidence and support for the hypothesis that learning style is related to number sense problem solving performance could have been found.

5.2.5 Issues of reliability

Although according to a document by the Multimedia Educational Resource for Learning and Online Teaching (MERLOT, 2002) the Index of Learning Style Inventory (ILS) used in this research was peer reviewed by the MERLOT in August 30, 2001 as an "excellent all around" material in three rating dimensions (Quality of Content, Potential Effectiveness, and Ease of User), its reliability for this research was very low. Hence, it could be that the results of this research in terms of the relationship between learning style and the other major variables, would be enhanced if a more appropriate learning style instrument is used. A possible solution would be to use the Grasha-Riechmann Student Learning Style Scales, since the data obtained could be more easily comparable to the data obtained from the teaching style inventory used in this research, although it will not necessarily ensure that the reliability of the instrument would be higher.

It should also be noted that since the students' interview-questionnaire was created *in situ* without any prior piloting, and due to its purpose being solely to validate information gathered previously, no attempt was made to assess its reliability. Hence results stemming from this questionnaire should be treated with caution.

5.3 Implications

5.3.1 Implications for teaching and learning

The results of this present research pertaining to learning styles could be interpreted in various ways depending on which side of the match or mismatch proponents one belongs. Those who believe that teachers must match learning style and teaching style might conclude that the Index of Learning Style Inventory used in this study was not designed for Year 7 students, hence it did not yield a totally satisfactory result. On the other hand those who do not believe that there is such a thing as one's learning style or that it is best when there is a mismatch between a teacher's teaching style and the learning style of the students, might interpret these results as evidence for their claims. Nevertheless, the data revealed that there are some forms of preferred methods of doing and learning, but these could be so specific for each individual that for any learning style measuring instrument to be authentically accurate it must be designed *in situ*. The researcher believes that this could be the main reason why to date there are more than 70 learning style models which have been proposed; it could be that people are still searching for the right instrument. Moreover, according to a major report of Coffield, Moseley and Ecclestone (2004) most of the main tests used to identify an individual's learning style are doubtful in terms of the results they produce. Hence, it would be wiser for the mathematics teacher to take into consideration the findings of this present study which suggest that:

- Learning style, if it does exist, is not a fixed state; it varies according to students' mood of the moment, the nature of subject area and topics, the influence of the teacher's philosophy and methods, and other influential issues;
- Number sense problem solving requires certain relative tendencies, attributes, skills and know-how which cannot be gleaned from an inventory not designed for the specific purpose of discovering a student's number sense problem solving style;
- A student's preference seems to be partially dependent upon success on particular tasks;
- Weaker students tend to be more dependent learners while the more able ones are more independent, and the student's state of self-confidence could be masquerading as a learning style, when in reality the right teaching approach and strategies might go a long way towards enhancing a student's learning performance; and

- The teacher's persistent use of certain methods and aids — such as the considerable use of visual representations and getting students to continuously reflect upon all aspects of their learning — could have a great bearing on how students view what they prefer.

Hence, it seems that much more collaborative research in learning preferences needs to be carried out so that the nature of learning strategies, preferences and styles could be better understood before a unified theory pertaining to these three components is proposed. The results of this study indicate that since the time available is most often not sufficient for the teacher to cater for the individual differences of each particular student, the teacher may use very well thought-out cooperative group work to get students to help each other out while he or she works on an individual basis with other students. Therefore, instead of trying to discover a student's learning style through an existing inventory the teacher should first of all focus on satisfying individual differences through discussion with students as to what they think works for them. The teacher should then use a combination of common sense, observation data, academic performance results, and well designed interviews to inform his or her future teaching-learning interactions with each student. From there the teacher could work in collaboration with other teachers and researchers to come up with specific information which could be used in informing the construction of an instrument for gauging a student's learning-related preferences. Given that there is still much debate about learning style itself, whether this instrument should be called a learning style inventory or not could then be considered.

Encouraging students to work mentally should become a priority since most students found it easier to work mentally than using pen and paper. What could be lacking at times is the knowledge and appropriate mental computation skills. Hence, the teacher needs to be very conversant with various effective ways of enhancing students' mental computation awareness and proficiency, and he or she should create opportunities for students to learn how to use such strategies. According to Swan (2002) students in his study tended to favour mental computation as the first computation choice for most items, although he expressed his concern at "the poor performance recorded by students adopting mental methods" (p. 198). Just like it was revealed through the TASRI data in this present study, in regard to some of the students with a low number sense performance, Swan (2002) also claimed that "some students used a mental form of the written algorithm, which is an inefficient mental method, and therefore they became cognitively overburdened" (p. 198). Nevertheless, this present study also revealed that students who used efficient mental computation strategies were mainly those who preferred to solve number sense inherent problems (NSIP). Such students were less prone to committing errors, and if they did so it was easier for them to discover and rectify these than it was for those students with a combined preference for solving devoid of number sense problems (DNSP) and low number sense performance scores. Hence, it is deemed necessary for teachers to engage students in activities which promote the development of efficient mental computation strategies, and one possible way is to help them become more appreciative of solving NSIP and increasing their number sense ability through enhancing their performance. This study has further revealed that mental computation exercises cannot and should not be separated from the normal teaching-learning experience, where it would be treated as a special component of number sense development. It should permeate and merge into the fabric of each individual teaching-learning experience so that students would develop a culture of using efficient mental computation strategies through appreciation of its central role to solving both number sense inherent and devoid of number sense problems.

Teaching through the affective domain might require considerable commitment and personal training from the teacher, but this study has shown that it could be the kingpin when it comes to helping students improve in their number sense problem solving performance. Many students come to school with emotional scars which include a fear for anything mathematical. Hence, pumping them full of mathematical content through academic exercises devoid of genuine empathy and respect of the individual as a valued human being might hamper the learner's mathematical growth. To facilitate the creation of an affectively proficient mathematics classroom ethos the teacher must be ready to prepare students to become autonomous learners through providing communication pathways based on trust, valuing of each student's contributions and encouraging them to set high achievable standards. Furthermore, such a system cannot exist within the confines of a forceful mechanism which is controlled totally by the teacher. Students have to be empowered to question their own learning of number facts, processes, concepts and symbols, and to know the function and importance of these in their own learning.

Nevertheless, the teacher is not being called upon to act as a regulator of a student's emotional states and the one eliciting emotional reactions in the student, but rather to help the student achieve a balance in dealing with his or her emotions towards mathematics as a subject. This should be done with the hope of heightening the student's willingness and capacity to voluntarily get involved in the acquisition, organisation and use of mathematics information, notably in regard to number sense problem solving. For this to happen, the teacher must find the most appropriate ways to activate each student's level of motivation. This in turn calls for the teacher to be very observant in monitoring how the student make his or her choice of courses of action, and the intensity and persistence of the student's effort. As it was shown through the analysis and results the students with high number sense problem solving ability preferred to persevere regardless of the difficulty level of the task at hand. What the teacher is being called upon to do relates more to how he or she can affect each student's belief about their capabilities for changing their mathematical performance situation to a higher level. The aim of the teacher should always be to relinquish control to the student so that the latter can exercise personal influence over his or her own motivation, thought processes, emotional states and patterns of behaviour in relation to developing his or her number sense and problem solving performance. A starting point would be for teachers to be very conversant with the notions of self-efficacy, as proposed by Bandura (1994).

Teaching through a problem-based approach should be a priority for every teacher of mathematics who endeavours to enhance his or her students' number sense problem solving proficiency. As pointed out by the NCTM *Standards* (2000), both number sense and problem solving are crucial to the learning of mathematics. This calls for a change towards creative teaching which requires careful planning mixed with quality preparation to deal with the unexpected. The latter situation is bound to become more abundant, as shown in this study, as the students become more autonomous and confident in interacting with each other, their teacher and the mathematics they are learning and teaching each other. Hence, the teacher must be very well prepared to cope with such high demands; the teacher must engage in teacher-knowledge-growth enterprises so that he or she would gain a profound understanding of fundamental mathematics as proposed by Ma (1999). In teaching through a problem-based approach the teacher would necessarily make use of enrichment activities and games. Paterson (2004) reiterates the importance of games as a "wonderfully motivating avenue to re-enforce number facts and mental strategies" (p. 174). According to Oldfield (1991) mathematical games can be a valuable resource for the stimulation and support of student collaboration and cooperative classroom mathematical discussion. When it comes to improving mental skills, games are able to provide students with the required repetitive practice without causing them to become easily bored, which would have been the case if they were made to go through repetition exercises similar to the traditional ones (Hatch, 1998).

Another very important consideration for enhancing teaching through a problem-based approach is the use of effective strategies and activities which students enjoy and which in turn help them in producing quality performance. In this present study the teachers were able to successfully teach through a carefully planned amalgamation of traditional and contemporary teaching approaches, which implies that teachers need to be wise in selecting the approaches they want to use, and that there is nothing wrong with using traditional approaches as long as it adds something important to proven contemporary methods. Moreover, the teachers spent more time mentally planning the teaching-learning experience, which made it easier for them to be flexible in changing the course of the lesson if students' motivation were not to the level required. Although written lesson plans are important, it could be that preparing mentally has more impact on the teacher being able to interact not only with the academic mathematics contributions of the students, but also the latter's enthusiasm level. Moreover, the teachers pointed out that extremely detailed lesson plans are not emotionally easy to discard or adapted since it seems that the teacher might feel guilty for having wasted valuable time preparing so much and then not being able to achieve what was planned. Furthermore, the results of this study have shown that the possible argument that without extremely detailed written lesson plans it would be difficult to achieve the objectives set is an invalid argument, since the gain or learning scores of students' performance from the pre-tests to the post-tests indicates that there was significant improvement.

5.3.2 Implications for curriculum

Flexibility of the Curriculum

The results of this study suggest that there is a need for teachers to view the curriculum as being flexible. It is often a complaint of teachers that the curriculum is not flexible enough. The teachers who participated in this research felt that the present outcomes based mathematics curriculum is accessible to all students as the statements are open-ended. Nevertheless, they also expressed a dissatisfaction with the amount of paper work that they had to do which they felt robbed them of valuable time, which could have been spent in quality planning and evaluation of their teaching and the students' learning. Still they planned in ways that were beneficial to the students. The sort of planning implication being considered here relies heavily upon the teacher having a very good mathematics, pedagogical and methodological background. This calls for the teacher to be very experienced as well, hence the teachers thought that teacher training was too theory-centred; trainee teachers lack both micro and macro teaching experience. Hence, although these teachers' lesson plans were done mainly in the head, with the written format being more in note form, it is not advisable that beginning and inexperienced teachers do the same. Nevertheless, this research has indicated that mental preparation could be very important in ensuring that the teacher can cope with sudden unexpected 'knee-jerk' reactions, which sometimes come in the form of very challenging questions for the teacher. Another advantage of preparing the lesson mentally is that it can be reviewed instantly at any time and virtually anywhere as opposed to a teacher relying mainly on a written plan. Furthermore, flexibility is an inherent aspect of mental preparation, which makes it easy to make necessary changes whenever the need arises. This could be one reason why it seemed easier than normal for these teachers to teach according to the moods of the students without losing sight of what students needed to learn. It also helped them in becoming more cooperative partners with their students in the teaching-learning process.

Planning and Implementing the Curriculum around Students' Needs

All students interviewed expressed considerable appreciation for how important the teacher was in terms of making students feel at ease and appreciate mathematics. The fun element that the teacher brought to the mathematics lessons was regarded as a key aspect that students felt good about. Contrary to popular belief none of the students agreed that their teachers talked too much. In fact they thought it was very important that the teacher spent time clarifying issues through pertinent and very clear explanations. It could be that the teacher's preference of allowing students to do most of the talking motivated the students to in turn want to listen to the teacher. Another possible related motivating factor could be the quality of the explanation, most of which saw the teacher engaging students in discussions about what they were doing, why they were doing it in a certain way and not in another way, and where they might use such mathematical information and skills in everyday life. Another possible reason for the students' expressed appreciation of the teacher's explanation could be due to the applied curriculum being programmed around students' interest. For instance, the teachers spent a lot of time talking to individual students, and this was also very much appreciated by all students. Students also pointed out that getting them to discuss the mathematics they were doing increased their learning capacity. They expressed much appreciation for being allowed to work on their own, but preferred when the teacher encouraged them to work in pairs or in groups. The students' appreciation also extended to the teacher's willingness to allow them to participate in suggesting ideas for some of the lessons that they did, and a major implication is that they felt they still needed more say in the content and organisation of the lessons. The main reason that students advanced was that they were more motivated to learn when they knew how the mathematics they were learning could be used in real life contexts and how these also took into consideration their feelings, likes and dislikes. They enjoyed both the hands-on and more abstract type of work, especially since both were most often linked together, although they did complain about not having enough time learning mathematics through the computer. Hence, the official curriculum, the teacher's hidden curriculum and the practised curriculum need to take into consideration the students' input, their likes and dislikes.

All three teachers were keen participants in professional development programmes, which was encouraged by the school principal, deputy principal and curriculum coordinator. As pointed out by the National Council of Teachers of Mathematics (NCTM, 2000) teachers must be encouraged to be more active in taking the initiative in their own professional development. In that respect NCTM (2000) stipulate that:

> Mathematics teachers must develop and maintain the mathematical and pedagogical knowledge they need to teach their students well. One way to do this is to collaborate with their colleagues and to create their own learning opportunities where none exist. They should also seek out high-quality professional development opportunities that fit their learning needs. By pursuing sources of information, building communities of colleagues, and participating in professional development, teachers can continue to grow as professionals. (p. 373)

One factor which could be responsible for some of the successes of these teachers, in helping their students improve in their number sense performance, could be the school ethos itself. Handal, Handal and Herrington (2003) reported that the type of school management system can also play a great role in the way that the curriculum is implemented. According to Handal, Handal and Herrington (2003):

> There is also evidence that teachers participating in courses where externally-based 'withdrawal' (Schiller, 1985) or 'top-down authority-based' methods are used (Dynan, 1983, p. 42), have little success in implementing change when they return to their school. (p. 3)

The response from management personnel and teachers from all three schools participating in this present study showed that the school as a whole was very receptive to positive change. In each case the school was run on the basis that teachers were responsible to implement, monitor and evaluate the curriculum according to informed decisions. Hence, there were forums for discussion about possible ongoing amendments to the curriculum, and the teacher was viewed and treated as a respected agent of change. That is the type of atmosphere that is required in order for a good number sense problem solving curriculum programme to be implemented successfully.

The Reciprocal Relationship among Teaching, Learning and Assessment

Assessment of student work cannot be in only one form or rely on the use of a single instrument. The teacher must design various strategies and instruments for assessing the students. One key assessment method used by these teachers was observation. The teacher has to set aside ample time during the teaching-learning experience for him or her to be able to observe the students at work and take note of pertinent issues. Another valuable assessment strategy which was used quite extensively by these teachers was the one-on-one interview. In answer to McIntosh and Dole's (2000) question "where, if at all, does the assessment of mental computation occur within assessment at school, system or national level?" (p. 402), the data collected in this present study reveal that the teachers employed an ongoing system of assessment. Mental and written notes were being taken throughout the lesson. These notes pertained to issues of particular students' strengths and weaknesses. Clarke and Stephens (1998) brought up the issue that, "what is assessed defines what is taught" (p. 77), which prompted Patterson (2004) to claim "therefore pencil and paper tests test only for a student's ability with pencil and paper methods" (p. 181). As early as 1988 Sowder had claimed that "teachers must examine more than answers and must demand from students more than answers" (p. 227). Sowder (1988) took such a stance because in her opinion correct answers cannot be taken as being safe indicators of good thinking by anyone. Yang (1995) proved that Sowder's (1988) comments were true since he could not find any good correlation between pen and paper scores, and high number sense or level of understanding scores. Moreover, in her research thesis Patterson (2004) reported that:

> In the absence of mental strategies being taught, mental computation strategies occur naturally and informally, either self-devised or borrowed. In the current study, oral explanations for these sorts of strategies were readily given. When students had obviously borrowed written methods and applied them mentally, oral explanations were harder to give. (p. 181)

This is in tandem with the TASRI data obtained through the present study, which showed that students who scored low on the written tests managed to solve certain respective and semi-parallel problems mentally, because now they did not feel under pressure to write. In fact many students were writing down what they were doing in their heads for the researcher to have a record of their mental procedures. Therefore, it could be that student improvement in this study pertaining to number sense and problem solving was partially due to how they were assessed, since all three teachers were constantly interviewing the students. They never relied solely on the results from a written test. Another format was to assess students through drawings as a means of allowing those who were more adept at expressing themselves through drawings to have a chance to show what they had learnt. That is one more reason why the process scoring system has a great advantage over the basic scoring system. Number sense is akin to problem solving especially when it comes to the nature of the assessment items. Hence, it cannot be ascertained through a basic scoring system. In addition to what has been discussed so far Anghileri (2000) referred to Askew and Wiliam's (1995) report that even rote learning can be turned into something good if as research results "show … 'knowing by heart' and 'figuring out' support each other in children's learning about numbers" (p. 129). Therefore, it is an imperative duty of any curriculum designer to incorporate suggestions for various modes of assessment and effective ways in which these may be used, while making allowance for the teacher's and students' input. It is also imperative that teachers become creative and wise in the way they assess students, which would be possible if the curriculum is flexible in terms of adaptation. If assessment is done only at the end of a lesson, a series of lessons or each term, then it defeats the purpose of providing an authentically accurate picture of a learner's ability. Furthermore, assessment can even be used to defeat the impulse to engage in rote learning of facts if the assessment items also require students to show understanding through analysis, application and synthesis.

5.3.3 Implications for research

Research pertaining to number sense is gaining momentum while problem solving has been for a long time under the scrutiny of many researchers. On the other hand, with the controversy surrounding research on learning style and teaching style, it has become increasingly difficult to establish credibility of research results pertaining to the two subjects in the wider research community. Coffield, Moseley, Hall and Ecclestone (2004) made extensive reviews of thirteen most popular learning style inventories, after which they made some very disturbing statements. They discovered that “the learning styles field is not unified, but instead is divided into three linked areas of activity: theoretical, pedagogical and commercial” (p. 1).

Hence, it is high time that all those involved in one way or another with research in learning style come together to decide on a course of action which will result in a more scientifically acceptable theory. As it is right now everyone seems to be doing his or her own research and whether the intention is for good or bad, there is a need for a unified theory which is credible and which can hold the water of criticism being poured into it. The plethora of inventories has brought some sharp criticism from reviewers such as Coffield, Moseley, Hall and Ecclestone (2004). It is within this same contextual dilemma that the results of the current study have revealed a need to create a specific number sense problem solving style inventory. Hence, instead of focusing on learning style it would be more appropriate to look at a less illusive variable such as the preferred style that students use to solve number sense inherent and devoid of number sense inherent problems.

Compared to how they reacted towards the notion of a learning style, the teachers did not mention anything negative about the teaching style inventory. Furthermore, the observation and interview data were very much in accord with the teaching style inventory data. When one considers the fact that teaching style has received much less criticism, if any, than learning style due to the various modes of data collection and triangulation, it would make sense for the teacher to engage in activities to discern his or her learning style through a reliable inventory. For instance, comparing data from a teacher’s written lesson plan, observing him or her while teaching, getting the teacher to fill in a teaching style inventory and finally interviewing the teacher to validate the data produces a more credible picture than gauging such information through a paper and pencil inventory only. Hence it is being proposed that a researcher could get teachers to keep a diary of all their activities, maybe even of how they feel, and recording their actions and verbal interactions. The researcher could then analyse the recordings and even observe the teacher while teaching, and compare this data with the other data from the recordings so as to get a more accurate picture of a teacher’s teaching style.

The ease with which the large majority of students managed to go through solving the problems, while simultaneously trying to say what they were doing to the problem, could be attributed to factors such as: the interviewer making them feel comfortable; the environment being non-threatening; and the students being used to verbally relating their thinking and problem solving processes to their teacher while on a problem solving experience. Nevertheless, the latter factor could be the most important, since the students were used to being interviewed in this way by their teachers. Hence, this provides another reason why teachers should incorporate the interview as an important method of assessing their students' number sense and other mathematics sense making performances.

5.4 Recommendations for Further Study

The discussion presented previously as to implications for research has shown that there is a need for more research in the areas of number sense, problem solving, teaching style, and especially learning style. What follows are suggestions for further study in these four main areas.

- Is it possible that students perform better on a particular type of problem based on how much emphasis is laid on solving such problems in class? For instance, if the teacher focuses a lot more on getting students to solve devoid of number sense problems, as opposed to less emphasis being placed on number sense inherent problems, would this motivate students to prefer DNSP to NSIP?
- Could it be that effective teachers of mathematics generally use a behaviourist-oriented extrinsic reward and reinforcement approach on their first encounter with new students, which they then gradually start reducing its use, and replacing it by a system which encouraged students to be intrinsically motivated, as both teacher and the students become more acquainted with each other?
- A number sense and problem solving style questionnaire needs to be developed as a means of having an instrument which specifically gauges students' learning preference vis-à-vis solving number sense inherent and devoid of number sense problems.
- More research is needed about what kind of teaching helps develop sound number sense and problem solving skills.
- What are good/effective/useful ways of assessing number sense and problem solving?
- There is more scope for collaborative large scale scientific research as to the real nature of learning style, and how to measure it effectively.
- There is a need for research to compare students' progress on number sense and problem solving using: (i) control class(es) who are taught by 'ordinary' teachers who are also not trained in any new programme being tested in the

research; (ii) experimental class(es) where the teachers would have gone through a programme designed to enhance their effectiveness as number sense problem solving teachers; and (iii) class(es) taught by teachers identified as 'effective' teachers of mathematics.

- To what extent are students involved in the planning and delivery of teaching-learning experiences? Is this a subliminal practice of effective teachers of mathematics only or is it universal among teachers?

5.5 Concluding Comments

Finally it is important to reiterate certain elements resulting from this study which could prove to be very important for the average mathematics teacher who is aspiring to improve upon his or her teaching of number sense and problem solving. These elements have to do with how confident the teachers were, how much freedom they were prepared to give to the students in terms of the latter's input, and how to ascertain that the whole teaching-learning experience was effective.

First of all it is important to note that all three teachers participating in this research were regarded as effective teachers of mathematics. This is supported through the data resulting from the Teaching Style Inventory, the classroom observations, and the interviews with the teachers and the students, which situated these teachers as belonging to a small minority of teachers who could be classified as being simultaneously experts in the subject area and its pedagogical implications; acting as facilitators instead of giver of knowledge and skills; and competent delegators who gradually empower students to take control of their own learning. Hence, these were not ordinary teachers who relied on a strict lesson-plan oriented teaching, but rather flexible human beings who were prepared to change the course of the lesson according to how students responded. Thus, they managed to help students in raising their personal performance and achieving high standard goals. It is hoped that this would serve not only as a reminder but also as a guide for other teachers who want to provide students with quality learning experience opportunities.

A second point which warrants special mention is how these teachers catered for individual differences through the affective domain, getting students to help each other out in group work, attending to individual students as per their immediate needs and allowing students to voice their opinion and make suggestions through both open and guided discussions. There again this calls for the teacher to be ready to deal with novel challenging ideas from the students. Hence, a teacher who lacks confidence as to his or her subject matter knowledge and skills; limitations as a professional; and how to interact with the students and their queries, could find it very difficult to make allowance for students to suggest how they want to learn, what they want to learn and how best these could be made to happen. It is also important to note that in this study the teachers were always in control of the whole situation as a delegators; while as facilitators they allowed students to feel free to interact with the mathematics they were learning, their peers; and as experts they were ready to provide sound advice and authentic information, or help students work towards finding the answers to questions that the teachers could not immediately answer with certainty.

Thirdly it is interesting to note that as a result of the high standard of expectation that they had for their students, none of these teachers attempted to prepare special lessons for individual students according to their perceived level of performance or ability. Nevertheless, the close monitoring of the students' progress resulted in the teachers adjusting the learning experience *in situ* to accommodate any difficulty that individual students could be encountering.

A fourth point is the issue of monitoring students' progress through various modes of assessment so that a more accurate picture could be obtained of a student's performance. Since these effective teachers expressed their discontentment with having too much paper work to do, it seemed that using a variety of assessment methods and instruments helped in easing the pressure which they had to endure. Of these methods, the interview was used quite extensively in a setting where the teacher would be one-on-one with a particular student while others work at their tasks in groups.

Finally, although there is a need for more research into how effective teachers of mathematics operate, especially when it comes to teaching number sense and problem solving, it should be possible to use the results of this study of such effective teachers of mathematics to help all teachers improve their performance.

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Appendix I: Letters and Consent Forms

Principal's Letter and Consent Form

Dear __________________________,

I am a PhD student currently studying at ECU under the supervision of Dr Jack Bana, and I intend to research the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students.

The purpose of this study is to examine the type of teaching and learning involved in number sense development and problem solving, and the perceptions of primary school mathematics teachers, students, principals, deputy principals and curriculum coordinators to discover what they feel are the skills and traits required by effective teachers of mathematics who endeavour to teach for the development of number sense and problem solving through the context of working mathematically. It is also hoped that observing teachers and students as they engage in such a teaching discourse will provide the mathematics community, parents, teachers and students with useful information about the relationship between number sense and problem solving in the context of working mathematically.

The study will involve pre-tests before any instruction takes place, followed by observations and selected interviews during terms one to four of 2004. As the program wraps up, there will be post-tests and selected interviews. During this collection of data, I will be reviewing all of the information that I am presented with in order to form the basis of my research. Teacher and student participation in this study, related to involvement in interviews, answering test items (for students only), and filling in inventories, will involve a time commitment of approximately three to six hours spanning four school terms.

I am hereby requesting your consent for me to:

- carry out this project at your school;
- observe, a lesson involving the chosen teacher-participant and his/her students, once per week;
- interview both you and your deputy principal once during the study;
- interview the chosen teacher-participant once every term, and hold short five to ten-minute informal interviews with him/her as per his/her availability;
- formally interview selected students from his/her class, once during the fourth term;
- involve selected/available students, from his/her class, in brief 5 to 10-minute informal interviews or 'chats' as per their availability;
- have the teacher-participant fill in a teaching style and a learning style inventory;
- have the students in the teacher-participant's class do a number sense and a problem solving pre- and post-tests
- have students in the his/her class do a number sense and a problem solving pre- and post-tests; and

- analyse relevant school policy and mathematics program documents kept by the school which could shed light upon the information being sought through this project.

The participation of the teacher is sought because he/she is currently identified as a very effective teacher of mathematics, which implies that I will be examining how he/she teaches for the development of number sense and problem solving ability.

Hopefully this project could be beneficial to all concerned, and it may lead to some improvement in mathematics education.

Thank you for participating in this research project. Could you please sign and return the following consent form.

Jemmy Louange
Date

Questions concerning the project can be directed to:
Jemmy Louange
Edith Cowan University
2 Bradford St
Mt Lawley WA 6050
phone 9370 65 62/9370 6337
email – jlouange@student.ecu.edu.au
If you have any concerns about the project or would like to talk to an independent person, you may contact Dr Jack Bana on 9370 6468 or email j.bana@ecu.edu.au
………………………………………………………………………………………

Consent Form (Principal)

Project Title: An examination of the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students

I ________________________________ have read the information above (or, "have been informed about all aspects of the above research project") and any questions I have asked have been answered to my satisfaction.

I agree to participate in these activities, realising I may withdraw at any time.

I also give my approval for the following teacher ____________________________ and her class, and the deputy principal ____________________________ to participate in this study

I agree that the research data gathered for this study may be published provided that neither the school nor any individuals are identifiable.

Participant _______________________ Date: ________________

Investigator ______________________ Date: ________________

Deputy Principal/Curriculum Coordinator's Letter and Consent Form

Dear __________________________,

I am a PhD student currently studying at ECU under the supervision of Dr Jack Bana, and I intend to research the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students.

The purpose of this study is to examine the type of teaching and learning involved in number sense development and problem solving, and the perceptions of primary school mathematics teachers, students, principals, deputy principals and curriculum coordinators to discover what they feel are the skills and traits required by effective teachers of mathematics who endeavour to teach for the development of number sense and problem solving through the context of working mathematically. It is also hoped that observing teachers and students as they engage in such a teaching discourse will provide the mathematics community, parents, teachers and students with useful information about the relationship between number sense and problem solving in the context of working mathematically.

The study will involve pre-tests before any instruction takes place, followed by observations and selected interviews during terms one to four of 2004. As the program wraps up, there will be post-tests and selected interviews. During this collection of data, I will be reviewing all of the information that I am presented with in order to form the basis of my research. Teacher and student participation in this study, related to involvement in interviews, answering test items (for students only), and filling in inventories, will involve a time commitment of approximately three to six hours spanning four school terms.

I have requested the consent of the school principal for me to:

- carry out this project at your school;
- observe, a lesson involving the chosen teacher-participant and his/her students, once per week;
- interview you once during the study;
- interview the chosen teacher-participant once every term, and hold short five to ten-minute informal interviews with him/her as per his/her availability;
- formally interview selected students from his/her class, once during the fourth term;
- involve selected/available students, from his/her class, in brief 5 to 10-minute informal interviews or 'chats' as per their availability;
- have the teacher-participant fill in a teaching style and a learning style inventory;
- have the students in the teacher-participant's class do a number sense and a problem solving pre- and post-tests
- have students in his/her class do a number sense and a problem solving pre- and post-tests; and
- analyse relevant school policy and mathematics program documents kept by the school which could shed light upon the information being sought through this project.

The participation of the teacher is sought because he/she is currently identified as an effective teacher of mathematics, which implies that I will be examining how he/she teaches for the development of number sense and problem solving ability.

Hopefully this project could be beneficial to all concerned, and it may lead to some improvement in mathematics education.

I am hereby requesting your consent for you to participate in this study.

Thank you for participating in this research project. Could you please sign and return the following consent form.

Jemmy Louange
Date

Questions concerning the project can be directed to:
Jemmy Louange
Edith Cowan University
2 Bradford St
Mt Lawley WA 6050
Phone – 9370 65 62/9370 6337
Email – jlouange@student.ecu.edu.au
If you have any concerns about the project or would like to talk to an independent person, you may contact Dr Jack Bana, on 9370 6468 or email j.bana@ecu.edu.au
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Consent Form (Deputy principal/curriculum coordinator)

Project Title: An examination of the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students

I ________________________________ have read the information above (or, "have been informed about all aspects of the above research project") and any questions I have asked have been answered to my satisfaction.

I agree to participate in these activities, realising I may withdraw at any time.

I agree that the research data gathered for this study may be published provided I am not identifiable.

Participant _______________________ Date: ________________

Investigator ______________________ Date: _________________

Teacher's Letter and Consent Form

Dear ____________________,

I am a PhD student currently studying at ECU under the supervision of Dr Jack Bana, and I intend to research the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students.

The purpose of this study is to examine the type of teaching and learning involved in number sense development and problem solving, and the perceptions of primary school mathematics teachers, students, principals, deputy principals and curriculum coordinators to discover what they feel are the skills and traits required by effective teachers of mathematics who endeavour to teach for the development of number sense and problem solving through the context of working mathematically. It is also hoped that observing teachers and students as they engage in such a teaching discourse will provide the mathematics community, parents, teachers and students with useful information about the relationship between number sense and problem solving in the context of working mathematically.

The study will involve pre-tests before any instruction takes place, followed by observations and selected interviews during terms one to four of 2004. As the program wraps up, there will be post-tests and selected interviews. During this collection of data, I will be reviewing all of the information that I am presented with in order to form the basis of my research. Your participation in this study, related to involvement in interviews, and inventories, will involve a time commitment of approximately three to six hours spanning four school terms. The students are expected to spend the same amount of time participating in these same activities including doing a number sense test and a problem solving test, both at the start and at the end of the research.

I am hereby requesting your consent for me to:

- carry out this project in your class;
- observe, a lesson involving you and your students, at least once per week;
- formally interview you once every term, and hold short five to ten-minute informal interviews with you as per your availability;
- formally interview selected students from your class, once during the fourth term;
- involve selected/available students in brief 5 to 10-minute informal interviews or 'chats' as per their availability;
- have the students fill in a learning style inventory;
- have you fill in a teaching style and a learning style inventory;
- have the students in your class do a number sense and a problem solving pre- and post-tests; and
- analyse relevant school policy and mathematics program documents kept by the school, which could contain your contributions, as a means of shedding light upon the information being sought through this project.

Your participation is sought because you are currently identified as a very effective teacher of mathematics, which implies that I will be examining how you teach for the development of number sense and problem solving.

Hopefully this project could be beneficial to all concerned, and it may lead to some improvement in mathematics education.

Thank you for participating in this research project. Could you please sign and return the following consent form.

Jemmy Louange
Date

Questions concerning the project can be directed to:
Jemmy Louange
Edith Cowan University
2 Bradford St
Mt Lawley WA 6050
Phone – 9370 65 62/9370 6337
Email – jlouange@student.ecu.edu.au
If you have any concerns about the project or would like to talk to an independent person, you may contact Dr Jack Bana, on 9370 6468 or email j.bana@ecu.edu.au
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Consent Form (Teacher)

Project Title: An examination of the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students

I ________________________________ have read the information above (or, "have been informed about all aspects of the above research project") and any questions I have asked have been answered to my satisfaction.

I agree to participate in these activities, realising I may withdraw at any time.

I also give my approval for the students in my class to participate in this study

I agree that the research data gathered for this study may be published provided that neither the school nor any individuals are identifiable.

Participant _______________________ Date: _______________

Investigator _____________________ Date: ________________

Parents/Guardians' Letter and Consent Form

Dear Parent/Guardian,

I am a PhD student currently studying at ECU under the supervision of Dr Jack Bana, and I intend to research the relationships between teaching and learning styles, and the number sense and problem solving ability of Year 7 students.

The purpose of this study is to examine the type of teaching and learning involved in number sense development and problem solving, and the perceptions of primary school mathematics teachers, students, principals and deputy principals to discover what they feel are the skills and traits required by effective teachers of mathematics who endeavour to teach for the development of number sense and problem solving through the context of working mathematically. It is also hoped that observing such teachers and their students, as they engage in such a teaching discourse, will provide the mathematics community, parents, teachers and students with useful information about the relationship between number sense and problem solving in the context of working mathematically.

The study will involve pre-tests before any instruction takes place, followed by observations and selected interviews during terms one to four of 2004. As the course wraps up, there will be post-tests and selected interviews. During this collection of data, I will be reviewing all of the information that I am presented with in order to form the basis of my research. Student participation in this study, related to involvement in interviews, answering test items and inventories, will involve a time commitment of approximately three to six hours spanning all four terms.

Through this project I wish to:

- observe a lesson in your child's mathematics class, once per week;
- formally interview selected students from your child's class, once during the fourth term;
- involve selected/available students from your child's class in brief 5 to 10-minute 'chats' about what was observed in class, as per availability of students and time;
- ask students in your child's class to fill in a learning style inventory; and
- have students in your child's class do a number sense and a problem solving pre- and post-tests.

Your child's participation is sought because he/she is currently a student in the class of a teacher who has been identified as very effective teacher of mathematics, which implies that I will be examining how that teacher teaches for the development of number sense and problem solving. By participating in this project it is hoped that your child could gain a greater understanding of his/her learning style and maybe improve his/her performance in developing his/her number sense and problem solving abilities. It is also expected that he/she could be more willing to share and ask for help in this important educational process. He/she would also be more likely to use his/her acquired knowledge in the learning process and he/she could be more willing to personally monitor and make positive changes in his/her learning of mathematics. Moreover, your child will know about his/her number sense and problem solving ability in terms of results obtained from a well-structured test. As he/she will be directly involved in the project, his/her opinions and perceptions will directly influence the findings of this study.

Hence, my supervisor and I are seeking your approval for your child to participate in this project.

Thank you for your contribution towards this research project. Could you please sign and return the following consent form.

Jemmy Louange
Date:

Questions concerning the project
Can be directed to:
Jemmy Louange
Edith Cowan University
2 Bradford St
Mt Lawley WA 6050
phone 9370 65 62/9370 6337
email – jlouange@student.ecu.edu.au

If you have any concerns about the project or would like to talk to an independent person, you may contact Dr Jack Bana, on 9370 6468 or email j.bana@ecu.edu.au

. ………………………………………………………………………………………

Consent Form (Parent/Guardian)

Project Title: An examination of the relationships between teaching and learning styles, and the number sense and problem solving abilities of Year 7 students.

I ______________________________________(the parent/guardian of the participant) have read the information provided with this consent form and any questions I have asked have been answered to my satisfaction.

I agree to allow my child ______________________________ (name) to participate in the activities associated with this research and understand that I can withdraw consent at any time.

I agree that the research data gathered in this study may be published providing my child is not identified in any way.

Signed _____________________________

Date: ______________________________

Student Copyright Clearance Authorisation

Authorisation for Copying Student Work

Dear Teacher, Student, Principal, Deputy Principal,

Thank you for your response and co-corporation with this research project. There will be situations where your comments are made, such as during interviews or in completing questionnaires, and/or materials you have produced be utilised and/or these comments might be quoted anonymously to demonstrate a point.

This form seeks your copyright permission to use your work for the following purposes:

- Research and PhD study regarding the relationship between number sense and problem solving; and

- Development of greater understanding of the factors which affect the practice of effective teachers of number sense and problem solving through the context of working mathematically;.

If you are happy for your work to be used for the above purpose, please return this consent form via return e-mail with the following details completed.

As a/the student/teacher/principal/deputy principal of -------------------------- Primary School, I declare that information/material provided in my name is my own work.

I authorise this work or part of this work to be:

- Communicated;
- Copied;
- Annotated both hardcopy and electronic;
- Published in research and PhD study; and
- Where appropriate to be broken-up to highlight aspects of the assignment /exam requirements.

Note:

I understand I will retain copyright of this work.

The published works will not show your name unless you specifically indicate below:

Do not attach / attach (please indicate by deleting the inappropriate response) my name to all pieces of my work.

The following details completed and sent back via return e-mail, will represent your permission to use the above-mentioned work in the above-mentioned ways.

Full name:

Student Number:

And Contact detail: Address: Email:

Phone Number: Mobile:

Thank you for your cooperation. If you would like more information or to discus this consent form please e-mail me at jlouange@student.ecu.edu.au

Jemmy Louange

Faculty: CSESS

School: Education

email: jlouange@student.ecu.edu.au

phone: 9370 6562/9370 6337

Date:

Appendix II: Year 7 Number Sense Tests

Administration Protocol

1. Tell students: Today I want to check how good your number sense is.

2. Have students clear their desk and get a pen/pencil ready.

3. Give out test papers and ask students write in their names on it, and to not turn over until told.

4. Preparation for test.

Tell students: Today I want you to do the maths number problems mentally. That is, do all the calculations (working) in your head. Only write the answer or circle the right answer. Don't do any other writing. In many questions you will be asked to estimate rather than calculate the answer exactly. Be sure to follow those directions. I will read each question while you follow me. Then I'll give you half a minute - 30 seconds - to do it, before asking you to go on to the next question.

4. Practice questions.

- Tell students: The left side of the page has the questions and the right side is where you show your answers. I now want you to try the first practice question. I'll read it for you while you follow on your sheet, and when I finish reading you'll have half a minute - 30 seconds - to do it.
- Read the question out loud. Allow half a minute. Indicate and justify the correct answer and the need to circle the matching letter rather than the whole answer.
- Say: Now we'll try the second practice question. Read the question out loud, allow half a minute, then indicate and explain the correct response and how this was to be recorded.
- Say: There are 45 questions in this test and they are all set out like these two. I'll read each question and then allow half a minute - 30 seconds. This should be plenty of time for each question. If you make a mistake cross it out and try again. Don't forget to only write the answer - no other writing is allowed. Are there any questions about the test? Answer as appropriate.

5. The test.

Say: Now we are ready to start. Turn to the next page. Question 1 says . . . Read the question out loud, making sure to emphasise any underlined words, and allow 30 seconds. Then say: Question 2 . . . and so on until the test is complete.

Number Sense Test

[The cover page included student details]

Practice Questions

1. Without counting exactly, about how many children are there in your school? (Circle the nearest answer.)	A 3 B 30 C 300 D 3000
2. What is the missing number to make this sentence true? **30 + ? = 50**	______________

DO NOT turn over the page until you are told.

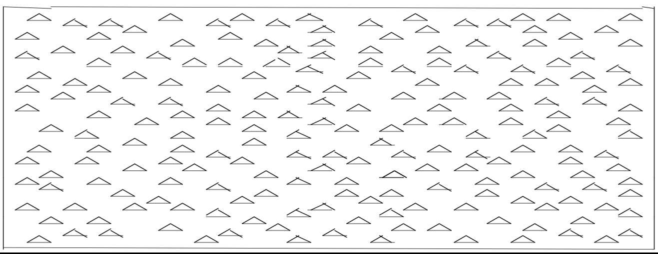
DO NOT write anything except your answer.

There are 45 questions. You will have 30 seconds for each question.

Item 1

About how many days have you lived? (Circle the nearest answer.)	A 300 B 3000 C 30 000 D 300 000

Item 2

About how many triangles are there here? (Circle the nearest answer.) 	A 20 B 50 C 100 D 200 E 500

Item 3

The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Put one digit in each box so that the answer will be as big as possible.	4□ - 231 = ?

Item 4

Put one digit in each box so that the answer will be as big as possible	431 - 2□ = ?

Item 5

Here are five digits: **2, 6, 3, 5, 1**. Arrange <u>all</u> these digits to make the smallest number possible.	__________________

Item 6

Here are five digits, **2, 6, 3, 5, 1**. Arrange them to make the number nearest to 20 000.	__________________________________

Item 7

The farmer has stored all his apples in 80 boxes with 40 apples in each box. He now needs to repack them all into 40 new boxes. How many apples will there be in each new box?	A. 2 B. 40 C. 80 D. 120

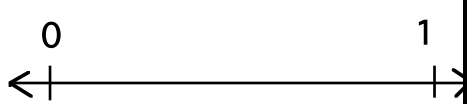
Item 8

For a long time Jane has been putting only 10 cent coins in her piggy bank. Last night she opened it and counted her money. She had \$46.70. How many 10 cent coins were in the bank?	_______________________

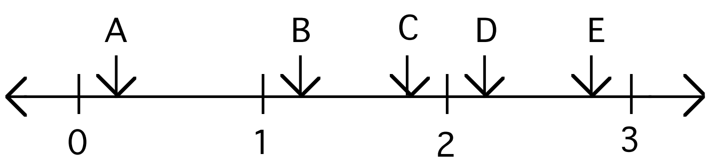
Item 9

Place the numbers 0.1 and 0.8 in their correct positions on this number line:	

Item 10

Place the numbers $\frac{1}{10}$ and $\frac{4}{5}$ in their correct positions on this number line:	

Item 11

 Which letter on the number line above best represents 2.19?	_________________

Item 12

Circle the fraction which represents the largest amount.

A $\frac{5}{6}$ B $\frac{5}{7}$

C $\frac{5}{8}$ D $\frac{5}{9}$

Item 13

<u>Without calculating the exact answer</u>, circle the best estimate for:

29 x 0.98

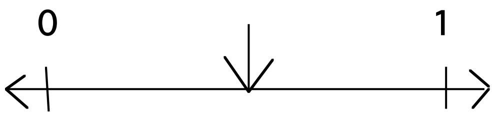
A more than 29

B less than 29

C impossible to tell without working it out

Item 14

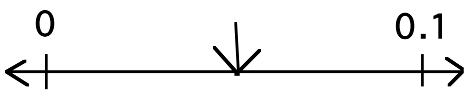
<u>Estimate</u> the <u>decimal</u> shown by the arrow on the number line:



____

Item 15

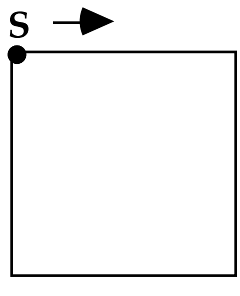
<u>Estimate</u> the <u>decimal</u> shown by the arrow on the number line:



____

Item 16

You are going to walk once around a square-shaped field. You start at the corner marked **S** and move in the direction shown by the arrow. Mark with an **X** where you will be after $^{1}/_{3}$ of your walk.



Item 17

<u>Without calculating the exact answer</u>, circle the largest product.

A. 18 x 17

B. 16 x 18

C. 17 x 19

Item 18

When a 3-digit number is added to a 3-digit number the result is:	A always a 3-digit number B always a 4-digit number C always a 5-digit number D either a 3, 4 or 5-digit number E either a 3 or 4 digit number

Item 19

Without calculating, circle the expression which represents the larger amount.	A. 145 x 4 B. 144 + 146 + 148 + 150

Item 20

Without calculating the exact answers, circle the best estimate for: 18 x 19	A. 290 B. 390 C. 490

Item 21

Which two numbers multiplied together give an answer closest to the target number? **4** **18** **50** **37** Target Number $\boxed{75}$	________ and _________

Item 22

Which two numbers multiplied together give an answer closest to the target number? **4** **18** **50** **37** Target Number $\boxed{1000}$	________ and _________

Item 23

Scott ran 100 metres in 14.52 seconds. Kelly took 2 tenths of a second longer. How long did it take Kelly to run 100 metres? Circle your answer.	A 34.52 seconds B 16.52 seconds C 14.72 seconds D 14.54 seconds E 14.50 seconds

Item 24

How many different decimals are there between 1.52 and 1.53?

Circle your answer and then fill in the blank.

A None. Why? ____________________

B One. What is it? ___________________

C A few. Give two: ________ and ________

D Lots. Give two: _______ and _______

Item 25

How many different <u>fractions</u> are there between $\frac{2}{5}$ and $\frac{3}{5}$?

Circle your answer and then fill in the blanks.

A None. Why? ____________________

B One. What is it? ___________________

C A few. Give two: ________ and ________

D Lots. Give two: _______ and _______

Item 26

Circle all the statements that are <u>true</u> about the number $\frac{2}{5}$.

A	It is greater than	$\frac{1}{2}$
B	It is the same as	2.5
C	It is equivalent to	0.4
D	It is greater than	$\frac{1}{3}$

Item 27

Circle the decimal which best represents the amount of the box shaded.

A 0.018

B 0.15

C 0.4

D 0.801

E 0.52

Item 28

Write a number in the box to make a fraction which represents a number between 2 and 3.

$$\frac{\boxed{}}{\boxed{8}}$$

Item 29

0.5 x 840 is the same as:

A 840 ÷ 2

B 5 x 840

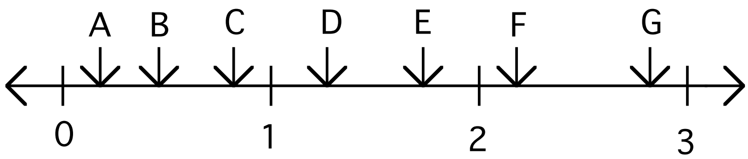
C 5 x 8400

D 840 ÷ 5

E 0.50 x 84

Item 30

In the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator.



Which letter in the number line above names a fraction where the numerator is slightly more than the denominator?

____

Item 31

<u>Without calculating the exact answer</u>, circle the best estimate for:

87 x 0.09

A a lot less than 87

B a little less than 87

C a little more than 87

D a lot more than 87

Item 32

<u>Without calculating</u>, which total is more than 1?

(Circle the correct answer.)

A $\frac{2}{5} + \frac{3}{7}$

B $\frac{1}{2} + \frac{4}{9}$

C $\frac{3}{8} + \frac{2}{11}$

D $\frac{4}{7} + \frac{1}{2}$

Item 33

Write 'is greater than', 'is equal to' or 'is less than' to make this a true statement:

$5 \times 7\frac{1}{2}$ __________ $35 + \frac{1}{2}$

Item 34

<u>Without calculating</u>, decide which one of these answers is reasonable, and circle it:	A	45 x 1.05 = 39.65
	B	4.5 x 6.5 = 292.5
	C	87 x 1.076 = 93.61
	D	589 x 0.95 = 595.45

Item 35

Circle the number which can be put in <u>both</u> boxes to make this sentence true:	A	0
	B	0.1
$243 \times \square = \square \times 24$	C	1
	D	10

Item 36

93 x 134 is equal to 12462.	
Use this to write the answer to the following:	
93 x 135	______________

Item 37

93 x 134 is equal to 12462.	
Use this to find the answer to the following:	
12462 ÷ 930	______________

Item 38

Circle the number you can put in the box to make this sentence true:	A	$\frac{2}{4}$
	B	$\frac{2}{3}$
$\frac{1}{2} \times \square = \frac{3}{6}$	C	1
	D	3

Item 39

A tank holds 1000 fish. If I increase the number by 50%, how many fish will there be now in the tank?	A	500
(Circle the correct answer.)	B	1050
	C	1500
	D	2000

Item 40

Dale had \$150. She spent 100% of it. How much money did she have left? (Circle the correct answer.)	A \$0 B \$50 C \$100 D \$150 E \$250 F \$300

Item 41

<u>Without calculating the exact answer</u>, circle the best estimate for: $29 \div 0.8$	A less than 29 B equal to 29 C greater than 29 D impossible to tell without calculating

Item 42

$\frac{3}{4}$ is a fraction between $\frac{1}{2}$ and 1. Name another fraction between $\frac{1}{2}$ and 1.	________________

Item 43

Put two of the numbers **4, 9, 12** in the boxes to make a fraction as close as possible to $\frac{1}{2}$.	$\frac{\square}{\square}$

Item 44

If I have \$378 in my savings account and withdraw all my money, how many 10-dollar notes would the bank be willing to give me?	________________

Item 45

Mary had \$426 and spent 0.9 of it on clothes. <u>Without calculating the exact answer</u>, circle the best estimate for how much she spent.	A. slightly less than \$426 B. much less than \$426 C. slightly more than \$426 D. impossible to tell without calculating

Appendix III: Year 7 Problem Solving Test

Administration Protocol

1. Tell students: Today I want to check how good your problem solving is.

2. Have students clear their desks and get the following ready:
   - a pen/pencil; and
   - a simple calculator

3. Give out test papers and ask students write in their names on it, and to not turn over until told.

4. Preparation for test.
   Tell students: Today I want you to do the maths problems mentally, in writing and or using a calculator. That is, you can do all the calculations (working) in your head, or by working it out on paper, or by using a calculator, or by combining two or all three methods.

- You may use any strategy you like to answer a question.
- If you have a specific name for the strategy you have used write it in the space on the right of the question.
- All essential working and solution must be shown in the space underneath the respective question.
- You are not obliged to follow any format to explain how you arrived at the solution. You might decide to write only a few lines to show the exact calculations, or you might decide to write short sentences, or some of you might just draw a diagram to explain how you solved the problem.
- If you worked mentally, you must use the What I did mentally to explain how you solved the problem in your head.
- Once you have the correct answer write it in the Answer space.
- If you have anything else that you feel is important about how you solved the problem or how you felt about it you may write In the Any other comment space.
- Your explanations or working could be in writing, drawings such as diagrams or a combination of all of these.
- Calculator: You are allowed to use a calculator, but you must show the essential calculation steps you used to solve the problem.
- After solving a problem answer the three questions at the bottom of the page. If you do not completely solve all the problems you MUST ensure that you answer these three questions before handing in your paper.

Say: There are eight (8) problem-questions for you to solve. You might take different amounts of time to solve each of them. You will be given 56 minutes to solve all of the eight problems. I will read each question while you follow me. Then I'll give you seven minutes to do it, before asking you to go on to the next question.

If you finish solving a problem and writing in all the appropriate spaces for that question you may try to answer another problem even before I have read it to the class.

If you are finding it difficult to solve a problem and wish to go to another problem you may do so at any time.

If there is still time (i.e. the 56-minute period is not over) you may spend the rest of the time answering those problems that you might not have been able to solve yet.

5. Problem example.
   - Tell students: The left side of the page has the questions and below it is the space in which you write your working and show your answers. The right hand side column is for you to write in the strategy/strategies you have used and information about how you solved the problem mentally (if you did solve it mentally). I now want you to look at the **example problem** and how a student solved it. I'll read it for you while you follow on your sheet, and when I am reading and explaining it you may ask questions.
   - Read the question out loud. Allow them to ask questions. Indicate and justify the solution steps, mental work and correct answer and the need to write in the appropriate spaces.
   - Say: I'll read each question and then allow about seven minutes for you to answer each one. This should be plenty of time for each question. If you make a mistake cross it out and try again. Don't forget to only write your problem solving steps. Are there any questions about the test?

Answer as appropriate.

6. The test.

Say: Now we are ready to start. Turn to the next page. Question 1 says . . . Read the question out loud and allow seven minutes. Then say: Question 2 . . . and so on until the test is complete.

Problem Solving Test

Example problem

Did you use a calculator?

YES		Briefly **Explain why** Because I could solve the problem in my head and also by calculating with a pen on paper.
NO	√	

In this column write any strategy that you have used and any comments you want to make about how you solved the problem.

A shop repaired 40 vehicles (cars and motorcycles) in a month. The total number of wheels on the vehicles was 100.
How many cars and motorcycles were repaired?

Working & Solution:

Number of vehicles		**TOT**	**Number of wheels**		**Tot**	**Difference**
Cars	**Motor cycles**		**Cars**	**Motor Cycles**		
0	**40**	**0+40=40**	**0x4=0**	**40x2=80**	**0+80=80**	**100−80=20**
1	**39**	**1+39=40**	**1x4=4**	**39x2=78**	**4+78=82**	**100−82=18**
2	**38**	**2+38=40**	**2x4=8**	**38x2=76**	**8+76=84**	**100−84=16**

If all vehicles were motorcycles → 80 wheels
100 − 80 = 20 wheels difference

Adding one car:
wheels added = 4; wheels deducted =2
4 -2 = 2 wheels more than before

For the difference of 20 (100 – 80) to equal 0: 20 ÷ 2 = 10
So, I will need to replace 10 motorcycles by 10 cars

40 vehicles – 10 cars = 30 motorcycles

10 cars = 10x4 = 40 wheels
30 motorcycles = 30x2=60 wheels

40 + 60 = 100 wheels

Answer:
Therefore, there were 10 cars and 30 motorcycles.

Strategy or strategies used :
Draw a table

What I did Mentally (To be filled ONLY if you worked mentally):
I thought like this in my head:
If all the vehicles were motorcycles, what would be the total number of wheels? I worked this out mentally and got 80.
How many wheels should there be in reality? 100.
So, there will be 20 wheels less than in reality.
I estimated that replacing a single motorcycle by a car increases the total number of wheels by about 2. This will decrease the difference between the actual number of wheels (100) and 80 wheels by 2. Since the difference is 20 (100 – 80), 20 subtract 2 is 18.
How many such replacements are needed to make the difference zero? 10.
I estimated that there would be about 10 cars, which means that there would be 30 motorcycles.

Any other comment
At first I was finding it hard to solve the problem because I did not know where to start. So I could not 'see' what was being asked. When I started working in my head I was not sure that I would get the answer. The problem would be difficult to solve if you don't try to solve it bit by bit, step by step.

(i)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
(e) ***Other reason:***			

DO NOT turn over the page until you are told. There are **8 questions**. You will have six minutes for each question.

Did you use a calculator?	In this column write any strategy that you have used and any

YES		Briefly **Explain why**	comments you want to make about how you solved the problem.
NO			

1. Peter, Paul and Pat divide $120 so that Peter gets three times as much as Paul, who gets half as much as Pat. How much does Peter get? **Working & Solution:** **Answer:**	**Strategy or strategies used:** **What I did Mentally (To be filled ONLY if you worked mentally):** **Any other comment**

(i)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)			
This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) ***Other reason:***		

[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator? **YES** / **NO** — Briefly **Explain why**	In this column write any strategy that you have used and any comments you want to make about how you solved the problem.
2. Four holes are drilled in a straight line in a rectangular steel plate. The distance between hole 1 and hole 4 is 35 mm. The distance between hole 2 and hole 3 is twice the distance between hole 1 and hole 2. The distance between hole 3 and hole 4 is the same as the distance between hole 2 and hole 3. What is the distance, in millimetres, between hole 1 and hole 3? **Working & Solution:** **Answer:**	**Strategy or strategies used:** **What I did Mentally (To be filled ONLY if you worked mentally):** **Any other comment**

(i)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) ***Other reason:***		

[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator?	In this column write any strategy that you have used and any comments you want to make about how you solved the problem.
YES / **NO** — Briefly **Explain why**	
3. A laboratory has a total of three rats in cages. One cage has one rat, a second cage has two rats and a third cage has three rats. How can this be? **Working & Solution:** **Answer:**	**Strategy or strategies used:** **What I did Mentally (To be filled ONLY if you worked mentally):** **Any other comment**

(i)Circle the most appropriate answer

I found this problem	(a) very easy	(b) easy	(c) neither easy nor difficult	(d) difficult	(e) very difficult

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) ***Other reason:***		

[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator?			In this column write any strategy that you have used and any comments you want to make about how you solved the problem.
YES		Briefly **Explain why**	
NO			

4. Alan, Brett, Carol and Dianne went to basketball, cricket, hockey and athletics. Carol didn't go to basketball; Brett couldn't go to cricket; the girl who went to hockey would like to have gone to cricket; and the person who went to basketball was upset she couldn't go to athletics. Who went where?

Working & Solution:

Answer:

Strategy or strategies used:

What I did Mentally (To be filled ONLY if you worked mentally):

Any other comment

(i)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) *Other reason:*		

[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator? **YES** **NO** Briefly **Explain why**	In this column write any strategy that you have used and any comments you want to make about how you solved the problem.
5. A jellybean jar contains 12 black, 18 green, and 30 red ones. If you shake the jar and pick out 10 jellybeans without looking, how many of each colour are you most likely to get? **Working & Solution:** **Answer:**	**Strategy or strategies used:** **What I did Mentally (To be filled ONLY if you worked mentally):** **Any other comment**

(i)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

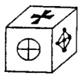
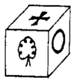
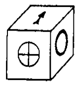
(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)			
This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) *Other reason:*		

[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator?			In this column write any <u>strategy</u> that you have used and any <u>comments</u> you want to make about how you solved the problem.
YES		Briefly **Explain why**	
NO			

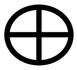
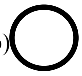
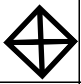
6. I have a die with pictures on it.

   I rolled it three times and this is how it landed.

first time	second time	third time
		

The drawings in the table below show which pictures were on the top of the die when it landed, on four other occasions.

In the empty box below each picture draw the respective picture that was on the bottom.

Picture on top	(a) 	(b) 	(c) 	(d) 
Picture on the bottom	(a)	(b)	(c)	(d)

Working & Solution:

Strategy or strategies used:

What I did Mentally (To be filled ONLY if you worked mentally):

Any other comment

(i)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) *Other reason:*		

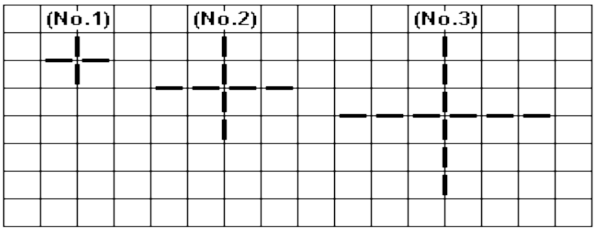
[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator?

YES		**Briefly Explain why**
NO		

In this column write any strategy that you have used and any comments you want to make about how you solved the problem.

7. The diagram below shows the first three shapes of a pattern made from matches. There are 4 matches in shape 1. There are 12 matches in shape 3.



(a) How many matches will there be in shape 10 (No.10)?
(c) Which shape will have 100 matches in it?

Working & Solution:

Answer:

(a)____

(b)____

Strategy or strategies used:

What I did Mentally (To be filled ONLY if you worked mentally):

Any other comment

)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) *Other reason:*		

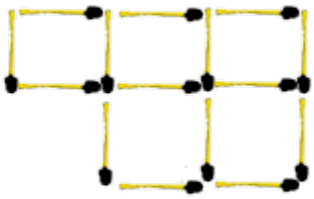
[redacted] There are [redacted]. You will have [redacted] for each question.

Did you use a calculator?

YES		Briefly **Explain why**
NO		

In this column write any strategy that you have used and any comments you want to make about how you solved the problem.

8. There are six squares in the pattern. Remove three of the matches, without moving any others, and turn the six squares into three squares. (You should end up with complete squares only. No matchstick should be on its own, not forming part of a square).



Working & Solution:

Answer: **[Draw your three squares in the space below]**

Strategy or strategies used:

What I did Mentally (To be filled ONLY if you worked mentally):

Any other comment

)Circle the most appropriate answer

I found this problem	**(a) very easy**	**(b) easy**	**(c) neither easy nor difficult**	**(d) difficult**	**(e) very difficult**

(ii). Circle either (a) or (b) then circle either (c), (d) or (e) N.B. Provide some details if you circle (e)

This problem was made	**(a) easier**	**(b) more difficult**	to solve…
…because it	**(c) involves number**	**(d) does not involve number**	
	(e) ***Other reason:***		

[redacted] There are [redacted]. You will have [redacted] for each question.

Appendix IV: Student and Teacher Learning Style Inventories

Administration Protocol

Instruction for filling in the on-line inventory.

Provide instructions to turn on the computer and open the internet browser.

Say to the students:

1. Open the Google home page.
2. Type “Index of Learning Styles Questionnaire” and then click search.
3. Click on the site title “Index of Learning Styles Questionnaire”. This will open the electronic questionnaire for you.
4. This is not a test. It might seem like one, but it is not. Therefore, there is no right or wrong answers. There are only answers that are true for you.
5. What you see in front of you is a questionnaire designed to give you some important ideas to help you become a better, more successful learner. The result will also help the school and your teachers adjust their teaching methods and learning environment to help YOU learn better.
6. Please type in your full name in the rectangle labelled ‘name'. Your name will be printed on the information that is returned to you.
7. There are 44 questions in all. Each question has two possible answers labeled “a” or “b”. [Ensure that they can all identify these]
8. When I ask you to start you are to answer each of the 44 questions;
9. To do that, for each of the 44 questions below, select either "a" or "b" to indicate your answer. You do this by clicking in one of the little ‘radio buttons’ before “a” or “b”.
10. Please choose only one answer for each question. If both "a" and "b" seem to apply to you, choose the one that applies more frequently. When you are finished selecting answers to each question please select the submit button at the end of the form.
11. You have to answer each question as honestly as possible. Otherwise, we won’t be able to use your results to successfully help you improve in your learning.
12. Do you have any question?
13. So, we will do the first one together.
14. Do you have any question?
15. If you have any questions during this activity just raise your hand and I’ll come and assist you.
16. You may now begin answering each question honestly.

When they have finished they are to be instructed to print a copy of their filled in questionnaire. They must then wait for the signal for them to click “SUBMIT”.
This will give them an electronic output, which they will also print.

Both the hard copies of the filled-in questionnaire and results will be picked up for analysis by the teacher or researcher.

Learning Style Inventory

Directions

Please provide your name, surname, age and school.

Name: Age: School:

This learning style inventory will help me in identifying your preferred learning style for validation against your teaching style and the students' learning styles.

For each of the 44 questions below select (By clicking in the little circle before the letter) either "a" or "b" to indicate your answer. Please choose only one answer for each question. If both "a" and "b" seem to apply to you, choose the one that applies more frequently.

1. I understand something better after I
   (a) try it out.
   b) think it through.
2. I would rather be considered
   (a) realistic.
   (b) innovative.
3. When I think about what I did yesterday, I am most likely to get
   (a) a picture.
   (b) words.
4. I tend to
   (a) understand details of a subject but may be fuzzy about its overall structure.
   (b) understand the overall structure but may be fuzzy about details.

5 When I am learning something new, it helps me to
   (a) talk about it.
   (b) think about it.

6. If I were a teacher, I would rather teach a course
   (a) that deals with facts and real life situations.
   (b) that deals with ideas and theories.
7. I prefer to get new information in
   (a) pictures, diagrams, graphs, or maps.
   (b) written directions or verbal information.
8. Once I understand
   (a) all the parts, I understand the whole thing.
   (b) the whole thing, I see how the parts fit.
9. In a study group working on difficult material, I am more likely to
   (a) jump in and contribute ideas.
   (b) sit back and listen.
10. I find it easier
    (a) to learn facts.
    (b) to learn concepts.
11. In a book with lots of pictures and charts, I am likely to
    (a) look over the pictures and charts carefully.
    (b) focus on the written text.
12. When I solve math problems
    (a) I usually work my way to the solutions one step at a time.
    (b) I often just see the solutions but then have to struggle to figure out the steps to get to them.
13. In classes I have taken
    (a) I have usually gotten to know many of the students.
    (b) I have rarely gotten to know many of the students.
14. In reading nonfiction, I prefer
    (a) something that teaches me new facts or tells me how to do something.
    (b) something that gives me new ideas to think about.

15. I like teachers
    (a) who put a lot of diagrams on the board.
    (b) who spend a lot of time explaining.

16. When I'm analyzing a story or a novel
    (a) I think of the incidents and try to put them together to figure out the themes.
    (b) I just know what the themes are when I finish reading and then I have to go back and find the incidents that demonstrate them.

17. When I start a homework problem, I am more likely to
    (a) start working on the solution immediately.
    (b) try to fully understand the problem first.

18. I prefer the idea of
    (a) certainty.
    (b) theory.

19. I remember best
    (a) what I see.
    (b) what I hear.

20. It is more important to me that an instructor
    (a) lay out the material in clear sequential steps.
    (b) give me an overall picture and relate the material to other subjects.

21. I prefer to study
    (a) in a study group.
    (b) alone.

22. I am more likely to be considered
    (a) careful about the details of my work.
    (b) creative about how to do my work.

23. When I get directions to a new place, I prefer
    (a) a map.
    (b) written instructions.

24. I learn
    (a) at a fairly regular pace. If I study hard, I'll "get it."
    (b) in fits and starts. I'll be totally confused and then suddenly it all "clicks."

25. I would rather first
    (a) try things out.
    (b) think about how I'm going to do it.

26. When I am reading for enjoyment, I like writers to
    (a) clearly say what they mean.
    (b) say things in creative, interesting ways.

27. When I see a diagram or sketch in class, I am most likely to remember
    (a) the picture.
    (b) what the instructor said about it.

28. When considering a body of information, I am more likely to
    (a) focus on details and miss the big picture.
    (b) try to understand the big picture before getting into the details.

29. I more easily remember
    (a) something I have done.
    (b) something I have thought a lot about.

30. When I have to perform a task, I prefer to
    (a) master one way of doing it.
    (b) come up with new ways of doing it.

31. When someone is showing me data, I prefer
    (a) charts or graphs.
    (b) text summarizing the results.

32. When writing a paper, I am more likely to
    (a) work on (think about or write) the beginning of the paper and progress forward.
    (b) work on (think about or write) different parts of the paper and then order them.

33. When I have to work on a group project, I first want to
    (a) have "group brainstorming" where everyone contributes ideas.
    (b) brainstorm individually and then come together as a group to compare ideas.

34. I consider it higher praise to call someone
    (a) sensible.
    (b) imaginative.

35. When I meet people at a party, I am more likely to remember
    (a) what they looked like.
    (b) what they said about themselves.

36. When I am learning a new subject, I prefer to
    (a) stay focused on that subject, learning as much about it as I can.
    (b) try to make connections between that subject and related subjects.

37. I am more likely to be considered
    (a) outgoing.
    (b) reserved.

38. I prefer courses that emphasize
    (a) concrete material (facts, data).
    (b) abstract material (concepts, theories).

39. For entertainment, I would rather
    (a) watch television.
    (b) read a book.

40. Some teachers start their lectures with an outline of what they will cover. Such outlines are
    (a) somewhat helpful to me.
    (b) very helpful to me.

41. The idea of doing homework in groups, with one grade for the entire group,
    (a) appeals to me.
    (b) does not appeal to me.

42. When I am doing long calculations,
    (a) I tend to repeat all my steps and check my work carefully.
    (b) I find checking my work tiresome and have to force myself to do it.

43. I tend to picture places I have been
    (a) easily and fairly accurately.
    (b) with difficulty and without much detail.

44. When solving problems in a group, I would be more likely to
    (a) think of the steps in the solution process.
    (b) think of possible consequences or applications of the solution in a wide range of areas.

Grid of Teachers' Learning Style Preferences

	Dimensions of learning styles							
Teacher	Active	Reflective	Sensing	Intuitive	Visual	Verbal	Sequential	Global
Bob (T_2)	1			1		3	3	
	Fairly well balanced		Fairly well balanced		Fairly well balanced		Fairly well balanced	
Amanda (T_1)		1		7		7		5
	Fairly well balanced		moderate preference for INTUITIVE		moderate preference for VERBAL		moderate preference for GLOBAL	
Chantal (T_3)		1	5			1	7	
	Fairly well balanced		moderate preference for SENSING		Fairly well balanced		moderate preference for SEQUENTIAL	

- Score on a scale is 1-3: fairly well balanced on the two dimensions of that scale.
- Score on a scale is 5-7: a moderate preference for one dimension of the scale and will learn more easily in a teaching environment which favours that dimension.
- Score on a scale is 9-11: a very strong preference for one dimension of the scale. You may have real difficulty learning in an environment which does not support that preference.

Appendix V: Teaching Style Inventory

Instructions

This survey will help me in evaluating your attitude towards instructional behaviour and also to validate what I have observed about your teaching style. Forty questions will probe assumptions about method in mathematics teaching at Year 7 level.

- First you will be asked to supply some basic information about yourself and your teaching (especially mathematics).
- Then you can take the questionnaire by responding to a seven point scale for each item. The scale is given at the top of the Inventory, and each item also contains a pop-up menu of this scale.
- N.B. if the pop-up menu does not work just type your preference of 1, 2, 3, 4, 5, 6 or 7 underneath the respective item.

Your Name:		
Your Gender:		

(i.) How long have you been teaching at Year 7?
(ii.) Which subjects do you teach to the class?
Do you teach at any other Year level?
If yes, what do you teach at this/these other level(s)?

On a rating scale of [1 2 3 4 5 6 7] where a 1 indicates *I do not enjoy teaching mathematics* and a 7 indicates *I really enjoy teaching mathematics*, rate the extent to which you like teaching this subject area:						
I do not enjoy teaching mathematics		I somewhat do not enjoy teaching mathematics		I somewhat enjoy teaching mathematics		I really enjoy teaching mathematics
1	2	3	4	5	6	7

Below is a Teaching Styles Inventory. You are required to answer each item as honestly and objectively as you can. Resist the temptation to respond as you believe you "should or ought to think and behave" or in relation to what you believe is the "expected or proper thing to do." Use the following rating scale:

1	2	3	4	5	6	7
Strongly Disagree		Somewhat Disagree	Neither Disagree or Agree	Somewhat Agree		Strongly Agree
Very Unimportant Aspect of My Approach to Teaching this Course					Very Important Aspect of My Approach to Teaching this Course	

Teaching Style Inventory

1. Facts, concepts, and principles are the most important things that students should acquire.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

2. I set high standards for students in my mathematics classes.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

3. What I say and do models appropriate ways for students to think about issues in the content.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

4. My teaching goals and methods address a variety of student learning styles.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

5. Students typically work on mathematics projects alone with little supervision from me.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

6. Sharing my knowledge and expertise with students is very important to me.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

7. I give students negative feedback when their performance is unsatisfactory.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

8. Students are encouraged to emulate the example I provide.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

9. I spend time consulting with students on how to improve their work on individual and/or group projects.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

10. Activities in this Year 7 maths class encourage students to develop their own ideas about content issues.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

11. What I have to say about a mathematics topic is important for students to acquire a broader perspective on the issues in that area.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

12. Students would describe my standards and expectations as somewhat strict and rigid.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

13. I typically show students how and what to do in order to master the mathematics lessons content.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

14. Small group discussions are employed to help students develop their ability to think critically.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

15. Students design one or more self-directed learning experiences.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

16. I want students to leave this course well prepared for further work in this subject area.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

17. It is my responsibility to define what students must learn and how they should learn it.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

18. Examples from my personal experiences often are used to illustrate points about the material.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

19. I guide students' work on course projects by asking questions, exploring options, and suggesting alternative ways to do things.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

20. Developing the ability of students to think and work independently is an important goal.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

21. Lecturing (direct exposition where the teacher does about 80% to 90% of the talking) is a significant part of how I teach each of the class sessions.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

22. I provide very clear guidelines for how I want tasks completed in relation to this subject area.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

23. I often show students how they can use various principles and concepts.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

24. The mathematics activities encourage students to take initiative and responsibility for their learning.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

25. Students take responsibility for teaching part of the class sessions.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

26. My expertise is typically used to resolve disagreements about content issues.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

27. This course has very specific goals and objectives that I want to accomplish.							
1	*2*	*3*	*4*	*5*	*6*	*7*	

28. Students receive frequent verbal and/or written comments on their performance.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

29. I solicit student advice about how and what to teach in this subject area.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

30. Students set their own pace for completing independent and/or group projects.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

31. **Students might** describe me as a "storehouse of knowledge" who dispenses the fact, principles, and concepts they need.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

32. My expectations for what I want students to do in this maths class are clearly defined in the WA curriculum and SOS documents.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

33. Eventually, many students begin to think like me about mathematics program content.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

34. Students can make choices among activities in order to complete mathematics program requirements.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

35. My approach to teaching is similar to a manager of a work group who delegates tasks and responsibilities to subordinates.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

36. There is more material in this mathematics program than I have time available to cover it.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

37. My standards and expectations help students develop the discipline they need to learn.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

38. Students might describe me as a "coach" who works closely with someone to correct problems in how they think and behave.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

39. I give students a lot of personal support and encouragement to do well in this mathematics program.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

40. I assume the role of a resource person who is available to students whenever they need help.							
1	**2**	**3**	**4**	**5**	**6**	**7**	

Appendix VI: Formal Teacher Interviews

Each term the researcher engaged each teacher in a one-hour interview. The first teacher interview was mainly about the teacher's perceptions and philosophies about teaching style, learning style, problem solving and number sense in their teaching and the students' learning, and how these were related. The next three interview questions were aimed at validating what was observed in class and the results of the first teacher interview. Many changes had taken place in the students' learning from one interview period to the next; hence it was important to ascertain the factors which could have contributed towards such changes. Each subsequent interview contained elements of what had been discovered up to that particular interview, and acted as a validation exercise of previously collected data.

First Teacher Interview Schedule

1. Which age level do you feel most comfortable teaching?
2. What is your perception of your teaching style?
3. Please describe your preferred teaching style/method/pattern/procedure?
4. What is the place of problem solving in your teaching and the students' learning?
5. What is the place of number sense in your teaching and the students' learning?
6. How do you cater for the differences in your students?
7. What is the learning style of:
   (a) high problem solvers;
   (b) low problem solvers?
8. **ARE THERE ANY STUDENTS IN YOUR CLASS WHO YOU FEEL SHOULD NOT BE IN A HETEROGENEOUS (REGULAR) CLASSROOM ENVIRONMENT? WHY?**
9. What qualities should a good problem solver have?
10. What main factors affect a student's problem solving performance/ability?
11. What main factors affect a student's development of number sense?
12. What link, if any, is there between number sense and problem solving ability? Describe it.
13. What is the learning style of students who have:
    (a) high number sense?
    (b) low number sense?

Second Teacher Interview Schedule

1. Where do you draw your content from?
2. Can you describe the children with above average mathematical ability in your class? What skills, attitudes or behaviours do you see them exhibit?
3. Can you describe the children who have lower mathematical ability in your class? What skills, attitudes do they exhibit?
4. What are the characteristics of a good mathematics teacher?
5. What would be the ideal way you would like to teach mathematics?
6. What changes would you like to see in your current mathematics programme?
7. Can you identify areas in which improvements could lift your current programme?
8. Are you aware of any underlying philosophy with which you strive to be consistent when teaching mathematics, or other subjects?
9. Who and what influence your professional practice?

10. What forms of professional development have you taken part in during the last 5 years?
11. Is there any area of your personal teaching on which you are currently focusing?
12. What do you believe about how children (a) develop number sense (b) learn to solve problems?
13. What do you believe about how children learn?
14. What impact do parents have on your teaching in general, and on your teaching of mathematics in particular?
15. What do you believe about how people learn mathematics?
16. What impact does school policy have on your teaching in general, and on your teaching of mathematics in particular?
17. What impact do the other staffs at your school have on your teaching in general, and on your teaching of mathematics in particular?
18 Are there any aspects of mathematics you feel form a necessary base to help support children who are about to enter year 8?
19 In what areas of the mathematics course that you currently teach do you feel most confident?
20 In what areas of the mathematics course that you currently teach do you feel less confident?
21 Are there any areas of primary/high school mathematics that you would feel hesitant to teach at this time?

Third Teacher Interview Schedule

1 To what extent is the instruction in this class planned to highlight connections between number sense and problem solving?
2 To what extent will this class involve the application of technologies (e-mail, cd's, computers, calculators, etc.)?
3 To what extent will you make significant attempts to access your students' prior knowledge of a topic before instruction? What techniques will you use?
4 To what extent do the tests and exams of this lesson/course/set of lessons stress reasoning, logic and understanding over memorisation of facts and procedures? Would you provide copies of these materials?
5 In what ways do you think your teaching in this lesson/course/set of lessons models the type of teaching that you believe should be done to improve students' number sense and problem solving ability?
6. To what extent will you explicitly encourage your students to reflect on changes in their ideas about topics in your lessons/course? Can you give an example? What techniques do you anticipate using?
7. What should be the role of the teacher in developing:
   (a) number sense?
   (b) problem solving sense?
8. What does it take for a student to be successful in mathematical problem solving?
9. What do you expect of a good teacher of math problem solving?
10. What does it take for a student to be successful in number sense?
11. What do you expect of a good teacher of number sense?
12. (a) Can a student do well in both general mathematics problem solving, and number sense?
    (b) What are the qualities of such students?

13. What does it take for a student to be successful in mathematics?
14. (a) Have you, as the teacher in this mathematics class helped your students make connections between number sense and problem solving?
    (b) How did you do that?
15. To what extent have your lessons involved the application of technologies (e-mail, cd's, computers, calculators, etc.)?
    (a) Do you think that you made significant attempts to understand your students' understanding of a topic before instruction?

    (b) How would you usually do that?

    (c) Did the tests/assignments/activities reflect this emphasis?

16. To what extent have your lessons stressed reasoning, logic, and understanding over memorisation of facts and procedures?

Fourth Teacher Interview Schedule

1. (a) Do you think the teaching you experienced in your lessons models the type of teaching that you believe should be done to improve students' number sense?

(b) How?
(c) Why?

2. (d) Do you think the teaching you experienced in your lessons models the type of teaching that you believe should be done to improve students' problem solving ability?
(e) How?
(f) Why?

3. Did you explicitly encourage your students to reflect on what they learned in this class?

How do you think you did this?

4. (a) After they have participated in your maths classes, what are your expectations regarding your students number sense and problem solving ability?
(b) What should be in the curriculum?

5. How should each be taught?
(a) Number Sense
(b) Problem solving

6. What sort of training did you go through to become a maths teacher?
7. Are you still training yourself as a maths teacher?
How?
8. What do you focus upon when highlighting number sense in your teaching and the students' learning?
9. How much do you encourage your students to use manipulatives (concrete materials/aids)?
10. (a) Should a maths teacher read or use as reference any number sense literature and teaching documents in preparing his/her maths lessons?
(b) Why?
(c) What number sense literature and teaching documents do you read or use as reference in preparing your lessons?

11. (a) How would you generally prepare a typical maths lesson?
(b) How much of this preparation is devoted to catering for development of number sense?
(c) How much of this preparation is devoted to catering for development of number sense?

12. What is the school's policy towards number sense and problem solving in mathematics?
13. How do you cater for differences in learning style?
14. What do you think is the relationship between your teaching style and the students learning style, and the number sense and problem solving ability of the students?
15. What mathematics competitions, clubs or extra curricular activities do your students participate in?

Appendix VII: Student Interviews

Interview Protocol

The interview with the students will employ a think aloud protocol. Its main purpose is to help in answering how number sense contributes towards problem solving success.

Establishing rapport to help the student feel comfortable

a) Researcher introduces himself and asks the student to introduce himself/herself.
b) Explain what the activity is about:
Point out in a natural way that his/her contribution by participating in this interview will help the researcher understand more about how Year 7 students solve different types of problems and enable the latter to provide information which will help students become better problem solvers and develop their number sense.

Practice protocol

Before solving the problems selected for the interview the student will be given some practice in solving a 'practice' problem as he/she thinks aloud. The researcher will guide the student into how he/she should respond during the interview.

1. As the student attempts to understand the problem question and conditions, observe him/her and if necessary, ask questions such as the following, as you find appropriate:
   a) What are you doing first when given the problem question and conditions?
   b) Describe what you are doing after that/next?
   c) What questions is asked in the problem? What are the important facts, conditions in the problem?
   d) Is there anything you don't understand about the problem?

2. As the student works on a solution to the problem, remind him/her again to talk about it, and ask him/her questions such as the following, if appropriate:
   a) What strategy are you using? Do you think it will lead to a solution? Have you thought about using other strategies? Which ones?
   b) Where are you having difficulty? What are your ideas about where to go from here?

3. As the student finds an answer to the problem, observe the ways, if any, in which he/she checks the answer and its reasonableness as a solution. If the student is not talking as he/she is solving the problem, ask prompting questions such as:
   a) What are you doing now? Why?
   b) What is going on in your head?
   c) What do you intend to do now?

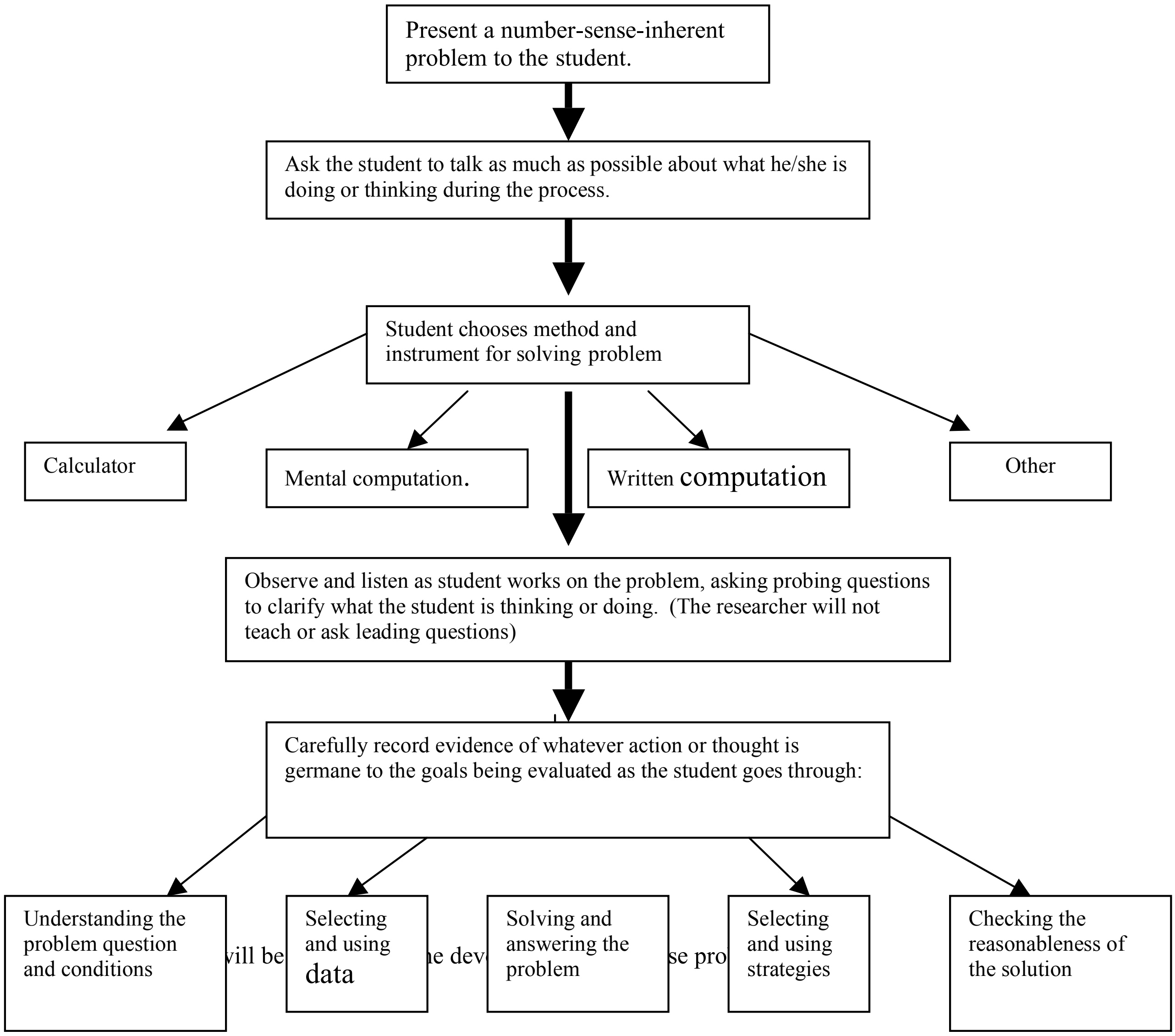
4. After the student has solved the problem, ask questions such as:
   a) Can you describe a solution to the problem and how you found it?
   b) Is this problem like any other problem you've solved before? How?
   c) Do you think this problem could be solved in another way? What are your ideas?
   d) How did you feel while you were solving this problem? How do you feel now that you have found a solution?

When all the problems have been solved, engage the student in a final discussion revolving around the questions:

a) Which problem did/didn't you enjoy doing? Why?
b) Which type of problem do you prefer?
e) Do you think it is important to check your answer? Why?

Flow diagram of the interview process

Then the main problem solving interview will take place. The audio tape will be turned on just before the start of the main interview process. The student will be asked to solve a number-sense-inherent problem and then a devoid-of-number-sense problem.



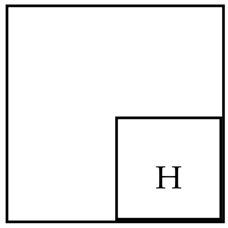
TASRI Problems

1. Henry McPenny had three daughters who were born at one-year intervals. Their combined ages are now just one-fifth of Henry's age. In six years their combined ages will be two thirds of Henry's age. How old is Henry now?

2. A farmer has 100 pigs that he feeds every day with 100 potatoes. The white pigs are very greedy and they each eat three potatoes. The black pigs have to share one potato between three of them. How many white pigs are there?

3. There were four runners in a sprint race. Bill finished as many places behind Chris as Kyle was before Theo. Chris wasn't first. Theo wasn't second. What was the finishing order?

4. Farmer Brown died and left a quarter of his square farm with the homestead (H) as in the diagram, to his wife, and left the remainder of his farm in equal-shaped and equal-sized portions to his four children. Sketch the four congruent portions in the diagram below.



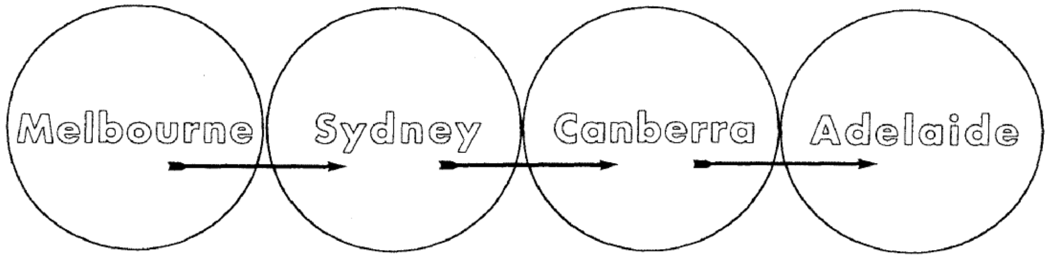
Appendix VIII: Sample Worksheets

Worksheet from Amanda's Lesson (Thursday 03/06/2004; 9:28 a.m.)

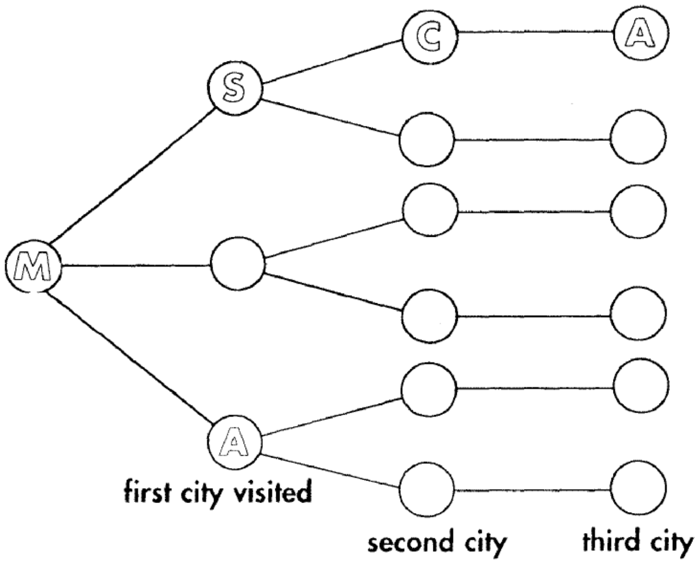
Chance variation 4

Tree diagrams showing possible outcomes.

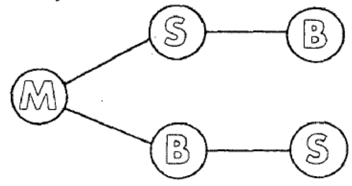
1 A businessman from Melbourne has to visit three cities: Sydney, Adelaide and Canberra.



How many different ways can he fly around the cities? (The diagram shows one way: fly to Sydney, then to Canberra and Adelaide.)
Make a tree diagram to show all possible flights.



2 If there were two cities, Sydney and Brisbane, to visit the tree diagram would look like this:



3 On a separate page draw a diagram showing all possible ways of visiting four cities (S, C, A and B) from the office in Melbourne (M).

Chance order in flights provides an opportunity to introduce and extend the use of tree diagrams when examining a range of outcomes.

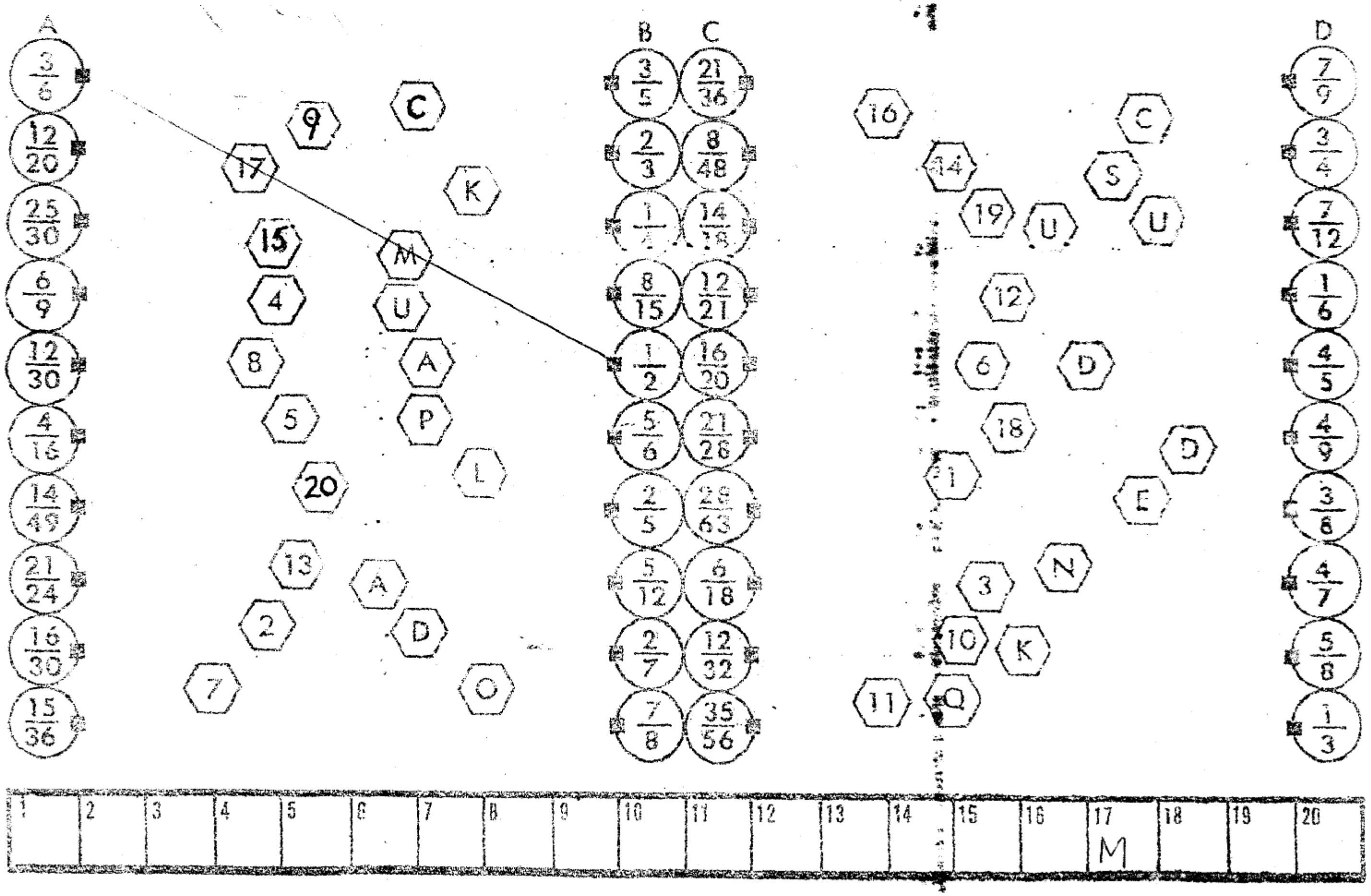
74 ©Michael J. Quinn, *Enrichment Mathematics Book 3*, Oxford University Press 2000

Worksheet from Chantal's class. (Wednesday 02/06/2004; 8:50 a.m.)

LINE UP

example

DIRECTIONS: *The fractions in column B are the reduced forms of the fractions in column A, and similarly for columns C and D. Draw a straight line connecting each pair of **equivalent fractions.** Each line will cross a letter and a number. The number tells you where to put the letter in the line of boxes at the bottom of the page.*



D14