Mathematics

Content Standards, Benchmarks, and Performance Standards

Approved by the New Mexico State Board of Education, June 2002 Reformatted January 2008

This document includes:

New Mexico Curriculum Framework Mathematics Guiding Principles Processes of Problem Solving K-12 Content Standards, Benchmarks, and Performance Standards Glossary

The 9-12 Standards included here are a final draft of a revision to align them with American Diploma Project(ADP)Benchmarks for College and Workplace Readiness.

Framework

New Mexico Mathematics Content Standards, Benchmarks, and Performance Standards

State Board of Education Goal: Establish clear and high standards in all academic and vocational subjects and ensure that assessments are aligned with content, benchmarks, and performance standards; require alignment of school curricula with performance standards and revise on a regular basis.

This document groups the Mathematics Content Standards, Benchmarks, and Performance Standards into five strands:

- Numbers and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

The Mathematics Content Standards, Benchmarks, and Performance Standards has been designed to:

- Establish an articulated, coordinated, and comprehensive description of the content and skills students should learn at specific grade levels in the study of mathematics;
- Help teachers create classroom instruction and authentic assessments that address a substantive mathematics curriculum that can be applied to learning across all disciplines;
- Serve as the basis for a statewide assessment of student learning; and
- Stimulate thoughtful conversations and policy development regarding the acquisition and application of essential math skills and concepts.

New Mexico Mathematics Content Standards, Benchmarks, and Performance Standards identify what students should know and be able to do across all grade levels. They form a spiraling framework in the sense that many skills, once introduced, develop over time. While the Performance Standards are set forth at grade-specific levels, they do not exist as isolated skills; each exists in relation to the others.

Each Content Standard is elaborated into three grade-span Benchmarks (K-4, 5-8, and 9-12) that are further defined by specific grade level Performance Standards. They illustrate how learners at every level apply mathematical concepts with increasing sophistication, refinement, and independence. In the 9-12 grade band, the five thematic strands have been focused into three fields of mathematical study to reflect more accurately how students engage in mathematics study during those grades. While the 9-12 Performance Standards do describe essential learning for high school students, they do not indicate grade-specific requirements because students are not required to enroll in specific courses in any particular grade.

High school students interested in various career directions may need to pursue mathematics topics beyond what is specified in the Standards. While all students will learn the mathematics content defined in the Standards, additional course offerings may provide a comprehensive continuum of learning experiences to prepare students to achieve their academic and career goals. Guidance for further mathematics study, in the form of Topics for Further Study, accompanies the Standards for grades 9-12. These topics are not required of all students and are not part of the State assessment system. Their purpose is to extend the depth and sophistication of students' knowledge and skills.

GUIDING PRINCIPLES

The New Mexico Mathematics Content Standards, Benchmarks, and Performance Standards provide a guide for focused, aligned, and sustained efforts to ensure that all students have access to high-quality mathematics education. The New Mexico Standards are based upon the framework developed by the National Council of Teachers of Mathematics (NCTM) and presented in Principles and Standards for School Mathematics. The NCTM recommends the following Guiding Principles that influence the development and delivery of successful mathematics programs. These Guiding Principles, although not unique to mathematics, establish the foundation for developing students' capabilities to mathematically reason, and solve problems.

Equity Excellence in mathematics requires equity, including high expectations and strong support for all students.

Mathematics can and must be learned by all students. Teachers and schools should encourage high expectations in their interactions with students. They determine students' opportunities to learn and succeed in mathematics.

Curriculum

A curriculum is more than a set of activities – it must be coherent, focused on important mathematics content, and clearly articulated across grades.

In a coherent curriculum, mathematical ideas are linked and build upon each another so that students' understanding and knowledge deepens, and their ability to apply mathematics expands. An effective curriculum focuses on important mathematics that will prepare students for continued study and for solving problems in a variety of school, home, college and work settings. An articulated curriculum challenges students to learn increasingly more sophisticated mathematical ideas as they progress.

Teaching

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Students' understanding of mathematics, their ability to use it to solve problems, and their confidence and disposition toward mathematics, are all shaped by the learning opportunities they encounter in school. To be effective, teachers must know and understand the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks. They need to be committed to their students as learners and as human beings, and be skillful in choosing and using a variety of pedagogical and assessment strategies. Effective teaching requires a challenging and supportive classroom learning environment. Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. Teachers' actions encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions.

GUIDING PRINCIPLES

Learning

Students must learn mathematics with understanding, actively acquiring new knowledge from experience and prior knowledge.

Mathematical proficiency requires conceptual understanding. In addition to factual knowledge and procedural facility, conceptual understanding allows students to use their knowledge flexibly, knowing when and how to use what they know, making sense of mathematics, remembering what they have learned, and connecting new knowledge to existing knowledge in meaningful ways. Learning with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future.

Assessment

Multiple and varied assessments should support the learning and furnish useful information to both teachers and students.

Assessment should enhance students' learning. Good assessment conveys messages to students about what kinds of mathematical knowledge and performances are important. These messages influence the decisions students make. Teachers need to move beyond a simple 'right or wrong' judgment and discern how students are thinking about the problems. When teachers use assessment techniques that include observation, conversations and interviews with students, and interactive journals, students are more likely to learn and remember by articulating their ideas and communicating their thinking. Assessment that is a routine part of ongoing classroom activity rather than an interruption, helps students in setting goals, assuming responsibility for their own learning, and becoming more independent learners. Assessment can also help teachers make decisions about the content or form of the instruction as well as the students' mastery of the content. Exemplary mathematics assessment should:

- Measure the essential mathematics that students should know and be able to do
- Be an open process
- Promote valid inference, and

• Enhance mathematics learning

• Be a coherent process.

• Promote equity

Technology

Technology is essential; it influences the mathematics that is taught and enhances students' learning.

Electronic technologies such as calculators and computers are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, facilitate organizing and analyzing data, and compute efficiently and accurately. They support investigation by students in every area of mathematics and allow students to focus on decision-making, reflection, reasoning, and problem solving. Technology also supports effective mathematics teaching and can dramatically increase the possibilities for engaging students with challenging content using visualization, simulation, graphing, and advanced computing. In this context, technology is not used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster greater understanding. Technology provides a means of viewing mathematical ideas from multiple perspectives by enriching the range and quality of investigations, by assisting with feedback, and by providing an opportunity for students to discuss with one another the mathematical representations they view on the screen and the various dynamic transformations. When mention is made to "technological tools" in the Performance Standards it includes (but not limited to various kinds of calculators, spreadsheets, dynamic software, computer algebra systems and various web-based applications.

PROCESS STANDARDS Problem Solving

Problem solving is an integral part of all mathematics learning. To solve problems, students must draw upon their knowledge of the concepts and skills they have learned and apply them to a novel situation; through this process, students develop new mathematical understanding. Problem solving should not be an isolated part of the program. Rather, problem solving should involve all content areas, numbers and operations, algebra, geometry, measurement, and data analysis and probability.

Instructional programs from kindergarten through grade 12 should enable students to:

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and other contexts
- Apply and adapt a variety of appropriate strategies to solve problems, and
- Monitor and reflect on the process of problem solving.

To build mathematical knowledge through problem solving, teachers must first be able to choose good problems that give students the chance to solidify and extend what they know and that stimulate mathematics learning. In selecting worthwhile problems that help develop important mathematical ideas, teachers need to understand how to analyze and adapt problems, be able to focus on mathematical ideas that can be illuminated by working on the problem, and learn to anticipate students' questions to decide if particular problems will further develop the mathematical goals for the class.

Good problem solvers tend naturally to analyze situations carefully in mathematical terms and to properly pose problems based on particular situations. They consider simple cases before trying something more complicated, and they look for and examine patterns and relationships. They develop a disposition to analyze more deeply that leads to a more complete understanding of the situation and the correct solution. Throughout the grades, teachers build this disposition by asking questions that help students find the mathematics in their experiences, and by encouraging students to persist with interesting but challenging problems.

Students who can successfully solve problems are able to apply and adapt a variety of appropriate strategies. These strategies must receive instructional attention to assist students to learn them, and opportunities to use these strategies must be embedded naturally in the curriculum across the content areas. In the earliest grades, students first learn to express, categorize, and compare their strategies. In the middle grades, students should be skilled in recognizing when various strategies are appropriate to use and should be capable of deciding how to use them. By high school, students should have access to a wide range of strategies, be able to decide which one to use, and be able to adapt and invent strategies.

Effective problem solvers constantly monitor and adjust what they are doing. They make sure they understand the problem and they plan frequently, periodically taking stock of their progress to see whether they seem to be on the right track. If they are not making progress, they stop to consider alternatives and do not hesitate to take a completely different approach. Poor problem solving is often due not to lack of knowledge but to the ineffective use of what they do know. Good problem solvers become aware of what they are doing and frequently use reflective skills. Teachers who create classroom environments that support the development of reflective habits provide opportunities where students are more likely to monitor their understanding and more likely to make productive adjustments necessary when solving problems.

PROCESS STANDARDS Reasoning and Proof

Reasoning is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas at all grade levels, students will learn that mathematics makes sense. Reasoning and proof cannot simply be taught in a single unit on logic or by 'doing proofs' in geometry. Reasoning and proof should be a consistent part of students' mathematical experience in kindergarten through grade 12. Reasoning mathematically is a habit of mind, and is developed through consistent use in many contexts.

Systematic reasoning is a defining feature of mathematics. It is found in all content areas and, with different degrees of rigor, at all grade levels. Effective instructional programs should enable students to:

- Recognize reasoning and proof as fundamental aspects of mathematics,
- Make and investigate mathematical conjectures,
- Develop and evaluate mathematical arguments and proofs, and
- Select and use various types of reasoning and methods of proof.

Conjecture, which is informed guessing, is a major pathway to discovery. Students can learn to make, refine, and test conjectures in very early grades, and they can develop their abilities to investigate their conjectures using concrete materials, calculators and other tools, and increasingly through the grades, mathematical representations and symbols. Beginning in the elementary grades, students can learn to disprove conjectures by finding counterexamples. At all levels, students will reason inductively from specific cases to larger patterns. Increasingly, they should learn to make effective deductive arguments. Students also need to work with other students to formulate their conjectures and to listen to and understand conjectures and explanations offered by classmates.

Early elementary students tend to justify general claims using specific cases. By the upper elementary grades, justifications can become more generalized and can draw on other mathematical results. In high school, students should be expected to construct relatively complex chains of reasoning. As students move through the grades they compare their ideas with other's ideas, which may cause them to modify, consolidate, or strengthen their arguments or reasoning.

Students need to learn how to select and use various types of reasoning and methods of proof to solve problems in a wide array of contexts. Students begin with informal reasoning, compared to formal logical deduction, and gradually become adept at various types of reasoning – algebraic and geometric reasoning, proportional reasoning, probabilistic reasoning, statistical reasoning, and so forth. Students need to encounter and build proficiency in all these areas with increasing sophistication as they move through the curriculum.

PROCESS STANDARDS Communication

Though communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process helps build meaning and permanence for ideas and makes them public. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to others' explanations gives students opportunities to develop their own understanding. Conversations in which mathematical ideas are explored from multiple perspectives help students sharpen their thinking and make connections. Such activity also helps students to develop a language for expressing mathematical ideas and an appreciation for the need for precision in that language.

Instructional programs from kindergarten through grade 12 should enable students to:

- Organize and consolidate their thinking through communication,
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others,
- Analyze and evaluate the mathematical thinking and strategies of others,-
- Use the language of mathematics to express mathematical ideas precisely, and
- Describe mathematical concepts using developmentally appropriate definitions.

Reflection and communication are intertwined processes in mathematical learning. Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed. Examining and discussing both exemplary and problematic pieces of mathematical writing can be beneficial at all levels. Students also need to test their ideas on the basis of shared knowledge and see whether they can be understood and are convincing.

Learning what is acceptable as evidence in mathematics should be an instructional goal from kindergarten through grade 12. Students benefit from analyzing and evaluating the mathematical thinking and strategies of others. Since not all methods and ideas have equal merit, students must learn to examine the methods and ideas of others to determine their own strengths and limitations.

As students articulate their mathematical understanding, they begin by using everyday, familiar language. Building upon this base, teachers can help students see that some words are also used in mathematics with different or more-precise meanings. While it is important that students have experiences that help them appreciate the power and precision of mathematical language, it is important to avoid imposing formal mathematical language prematurely. Students must first be allowed to grapple with their ideas and develop their own informal means of expressing them. Technology affords other opportunities and challenges for the development and analysis of language. Spreadsheets, algebraic symbols, and geometric shapes all contribute to building the vocabulary and an understanding of the language of mathematics. Graphing calculators, spreadsheets, dynamic software, computer algebra systems and various web-based applications can contribute to mathematical understanding and communication.

PROCESS STANDARDS Connections

Understanding involves making connections. Students that can connect their mathematical ideas develop a deeper and more lasting understanding of the rich interplay among mathematical topics and about the utility of mathematics. Viewing mathematics as a whole highlights the need for studying and thinking about the connections within the discipline, as reflected within the grade-level curriculum and across grades. Teachers help students build a disposition to recognize and use connections among mathematical ideas by asking guiding questions and providing opportunities for students to integrate mathematics in many contexts. Students begin to see the connections between arithmetic operations, understanding, for example, how multiplication can be thought of as repeated addition. As they see how mathematical operations can be used in different contexts, they develop an appreciation for the abstraction of mathematics.

As students progress through their school mathematics experience, their ability to see the same mathematical structure in different settings should increase. Students in Kindergarten through grade 2 recognize instances of counting, number, and shape. Upper elementary school students look for instances of arithmetic operations, and middle-grade students look for examples of rational numbers, proportionality, and linear relationships. High school students look for connections among the many mathematical ideas they are encountering.

As students develop a view of mathematics as a connected and integrated whole, they will have less of a tendency to view mathematical skills and concepts separately. When conceptual understanding is linked to procedures, students will not perceive mathematics as an arbitrary set of rules. This integration of procedures and concepts should be central in school mathematics.

Instructional programs from kindergarten through grade 12 should enable students to:

- Recognize and use connections among mathematical ideas,
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole, and
- Recognize and apply mathematics in contexts outside of mathematics.

The opportunity for students to experience mathematics in a context is important. Mathematics is used in science, the social sciences, medicine, engineering, construction, business, government, arts and architecture, finance and commerce, and many other fields. These links are not only through content, but also, as in the case of science, through process. The processes and content of science can inspire an approach to solving problems that applies to mathematics, or even result in the creation of new mathematical fields. Equally, students who see the connection of mathematics to the world and to other disciplines are better able to apply knowledge from several different areas and are more likely to be successful problem solvers.

PROCESS STANDARDS Representation

The term representation refers both to process and product, meaning the act of capturing a mathematical concept or relationship in some form or the form itself. Representing applies to externally observable processes and products as well as to those that occur 'internally' in the minds of students as they are doing mathematics. When students have access to mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically.

Although some forms of representation, such as diagrams, graphical displays, and symbolic expressions, have long been a part of school mathematics, they have often been taught and learned in isolation. Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships, in communicating mathematical approaches, arguments, in understanding one's self and others, in recognizing connections among related mathematical concepts, and in applying mathematics to realistic problem situations through modeling. It is important to encourage students to represent their ideas in ways that makes sense to them, first in ways that are not conventional and later, in conventional form.

Instructional programs from kindergarten through grade 12 should enable all students to:

- Create and use representations to organize, record, and communicate mathematical ideas,
- Select, apply, and translate among mathematical representations to solve problems, and
- Use representations to model and interpret physical, social, and mathematical phenomena.

Teachers gain valuable insight into students' ways of interpreting and thinking about mathematics by looking at their representations. Teachers can then build bridges from students' personal representations to more conventional ones when appropriate. It is important that students have opportunities to not only learn conventional forms of representation, but also to construct, refine, and use their own representations as tools to support learning and doing mathematics.

Computers and calculators change what students can do with conventional representations and expand the set of representations with which they can work. A variety of technological tools allow students to manipulate, visualize, and simulate more complex data and therefore represent and investigate mathematical ideas and situations not otherwise possible. As students' representational repertoire expands, it is important for students to reflect on their use of representations to develop an understanding of the relative strengths and weaknesses of various representations for various purposes.

Different representations can illuminate different aspects of a complex concept or relationship. To become deeply knowledgeable about many aspects of mathematics, students need a variety of representations to support their understanding. As they move through grades, students' repertoires of representations should expand to include more complex pictures, tables, graphs, and words to model problems and situations. As students become more mathematically sophisticated, they develop an increasingly large array of mathematical representations as well as the knowledge of how to use them productively.

The term model has many meanings. It can refer to physical materials, that is, manipulative models. It also refers to providing example behavior, such as when a teacher demonstrates a problem-solving process, or model can be used synonymously with representation. The term mathematical

model means a mathematical representation of elements and relationships in an idealized version of a complex phenomenon. Mathematical models can be used to clarify and interpret the phenomenon and to solve problems.

In the early grades, students model situations using physical objects and simple pictures. As middle grade students model and solve problems that arise in the real and the mathematical worlds, they learn to use variables to represent unknowns and also learn how to employ equations, graphs, and tables to represent and analyze situations. High school students create and interpret models of phenomena drawn from a wider range of contexts by identifying the essential elements of the context and by devising representations that capture mathematical relationships among those elements. With technology tools, students can explore and understand complex concepts. These tools now allow students to explore iterative models for situations that were once studied in much more advanced courses.

Adapted from the National Council of Teachers of Mathematics. *Principals and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2000.

Strand: NUMBER AND OPERATIONS

Standard: Students will understand numerical concepts and mathematical operations.

K-4 Benchmark N.1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Performance Standards		
N.1.1 Demonstrate an understanding of the place-value structure of the base-ten number system:		
a. count with understanding and recognize "how many" in sets of objects up to 20		
b. read and write whole numbers up to 20		
c. compare and order whole numbers up to 20		
d. connect numerals to the quantities they represent using various physical models		
e. use an organized counting method to keep track of quantities while counting (one-to-one correspondence) (e.g., touch		
object once and only once as counting a set)		
f. order sets of objects and numbers from least to most or most to least		
1.1.1 Demonstrate an understanding of the place-value structure of the base-ten number system:		
a. read, write, model, and sequence whole numbers up to 100 (including filling in missing numbers in a sequence)		
b. count with understanding and recognize "how many" in sets of objects up to 50		
c. count orally by 2s to 20 and by 5s and 10s to 100		
d. count orally backward from 100		
e. compare and order numbers up to 100		
f. decompose and recombine numbers using manipulatives (e.g., by breaking numbers apart and recombining) to create		
and construct equivalent representations for the same number (e.g., $10 = 3 + 7$ or $1 + 2 + 7$ or $3 + 2 + 5$)		
g. group objects by 10s and 1s to explore place value (e.g., 24 equals two tens and four ones)		
h. use ordinal numbers (e.g., what position?) and cardinal numbers (e.g., how many?) appropriately		
i. connect number words and numbers to the quantities they represent		
1.1.1 Understand the relationship between numbers, quantities, and place value in whole numbers up to 1,000 and develop		
flexible ways of thinking about numbers:		
a. use multiple models to explore place value and the base-ten number system		
b. represent whole numbers and use them in flexible ways including decomposing and recombining numbers and see their		
relationships (e.g., 3 is one less than 4, one more than 2, two less than 5)		
c. identify whether a set of objects has an odd or even number of elements		
d. compare and order numbers using a variety of terms (e.g., tens, less than, odd numbers)		
e. apply strategies for computation utilizing an understanding of place value (e.g., 48 + 25 would be 40 + 20 is 60, 8 + 5 is 13, 60 + 13 is 73)		

c. connect number words and numerals to the quantities they represent using physical models and other representations

d. model how many parts make a whole using equal fractional parts (e.g., 1/2, 1/3, 1/4, and 1/6 as equal parts of a whole)

3.N.1.1	Exhibit an understanding of the place-value structure of the base-ten number system by:
	a. reading, modeling, writing, and interpreting whole numbers up to 10,000
	b. comparing and ordering numbers up to 1,000
	c. recognizing the position of a given number in the base-ten number system and its relationship to benchmark numbers
	such as 10, 50, 100, 500
3.N.1.2	Use whole numbers by using a variety of contexts and models (e.g., exploring the size of 1,000 by skip-counting to 1,000
	using hundred charts or strips 10 or 100 centimeters long).
3.N.1.3	Identify some representations for some numbers and generate them by decomposing and recombining numbers (e.g., 853 =
	$8 \ge 100 + 5 \ge 10 + 3$; $85 \ge 10 + 3 = 853$; $853 = 900 - 50 + 3$).
3.N.1.4	Identify the relationship among commonly encountered factors and multiples (e.g., factor pairs of 12 are 1 x 12, 2 x 6, 3 x
	4; multiples of 12 are 12, 24, 36).
3.N.1.5	Use visual models and other strategies to recognize and generate equivalents of commonly used fractions and mixed
	numbers (e.g., halves, thirds, fourths, sixths, eighths, and tenths).
3.N.1.6	Demonstrate an understanding of fractions as parts of unit wholes, parts of a collection or set, and as locations on a number
	line.
3.N.1.7	Use common fractions for measuring and money (e.g., using fractions and decimals as representations of the same concept,
	such as half of a dollar = 50 cents).
4.N.1.1	Exhibit an understanding of the place-value structure of the base-ten number system by reading, modeling, writing, and
	interpreting whole numbers up to 100,000; compare and order the numbers:
	a. recognize equivalent representations for the same number and generate them by decomposing and combining numbers
	$(e.g., 853 = 8 \times 100 + 5 \times 10 + 3; 853 = 85 \times 10 + 3; 853 = 900 - 50 + 3)$
	b. identify the numbers less than 0 by extending the number line and using negative numbers through familiar
	applications (e.g., temperature, money)

(e.g., 23 can be twenty-three 1s, one 10 and thirteen 1s, or two 10s and three 1s)

4.N.1.2 Identify fractions as parts of unit wholes, as parts of groups, and as locations on number lines:

- a. use visual models and other strategies to compare and order commonly used fractions
- b. use models to show how whole numbers and decimals (to the hundredths place) relate to simple fractions (e.g., $\frac{1}{2}$, $\frac{5}{10}$, 0.5)
- c. identify different interpretations of fractions:

ratio

division of whole numbers by whole numbers

2.N.1.2 Apply counting skills and number sense through meaningful activities:a. count and recognize "how many" in sets of objects up to 1,000b. count forward and backward from given numbers to 1,000

- equivalence
- parts of a whole or parts of a set

- ordering of fractions

4.N.1.3 Add and subtract fractions with common and uncommon denominators using a variety of strategies (e.g., manipulatives, numbers, pictures):

- a. recognize and generate equivalent decimal forms of commonly used fractions (e.g., halves, quarters, tenths, fifths)
- b. identify the numbers less than 0 by extending the number line and using negative numbers through familiar applications (e.g., temperature, money)
- **4.N.1.4** Recognize classes of numbers (e.g., odd, even, factors, multiples, square numbers) and apply these concepts in problem-solving situations.

Strand: NUMBER AND OPERATIONS

Standard: Students will understand numerical concepts and mathematical operations.

K-4 Benchmark N.2: Understand the meaning of operations and how they relate to one another.

Performance Standards

K.N.2.1 Represent numbers using pictures, objects, or numerals.

K.N.2.2 Use concrete objects to solve simple addition and subtraction story problems (e.g., oral not written).

1.N.2.1 Use a variety of models to demonstrate an understanding of addition and subtraction of whole numbers.

- **1.N.2.2** Solve addition and subtraction problems with one- and two-digit numbers (e.g., $5 + 58 = \Box$).
- 1.N.2.3 Find the sum of three one-digit numbers to the sum of 15.
- **1.N.2.4** Understand and use the inverse relationship between addition and subtraction to solve problems and check solutions (e.g., 8 + 6 = 14 is related to 14 6 = 8).
- **1.N.2.5** Use concrete materials to investigate situations that relate to multiplication and division (e.g., equal groupings of objects, sharing equally).

1.N.2.6 Given simple story problems, explain verbally how to select and use appropriate operations.

2.N.2.1 Find the sum of two whole numbers up to three digits long (e.g., $235 + 476 = \Box$; $564 - 273 = \Box$).

2.N.2.2 Find the difference of two whole numbers up to three digits long.

2.N.2.3 Understand and use the inverse relationships between addition and subtraction to solve problems and check solutions (28 + 31 = 59; therefore, 59 - 31 = 28).

2.N.2.4 Identify and describe situations that require multiplication and division and develop strategies to solve problems for repeated joining of groups and partitioning into equal subgroups or shares (e.g., repeated addition and subtraction, counting by multiples, equal sharing).

- **3.N.2.1** Use a variety of models to show an understanding of multiplication and division of whole numbers (e.g., charts, arrays, diagrams, and physical models [i.e., modeling multiplication with a variety of pictures, diagrams, and concrete tools to help students learn what the factors and products represent in various contexts]).
- **3.N.2.2** Find the sum or difference of two whole numbers between 0 and 10,000.
- **3.N.2.3** Solve simple multiplication and division problems (e.g., $135 \div 5 = \Box$).
- **3.N.2.4** Identify how the number of groups and the number of items in each group equals a product.
- **3.N.2.5** Demonstrate the effects of multiplying and dividing on whole numbers (e.g., to find the total number of legs on 12 cats, 4 represents the number of each [cat] unit, so $12 \times 4 = 48$ [leg] units).
- **3.N.2.6** Identify and use relationship between multiplication and division (e.g., division is the inverse of multiplication) to solve problems.
- 3.N.2.7 Select and use operations (e.g., addition, multiplication, subtraction, division) to solve problems.
- **4.N.2.1** Demonstrate an understanding of and the ability to use:
 - a. standard algorithms for the addition and subtraction of multi-digit numbers
 - b. standard algorithms for multiplying a multi-digit number by a two-digit number and for dividing a multi-digit number by a one-digit number
- 4.N.2.2 Select and use appropriate operations (addition, subtraction, multiplication, and division) to solve problems.
- **4.N.2.3** Extend the uses of whole numbers to the addition and subtraction of simple decimals (positive numbers to two places).
- **4.N.2.4** Demonstrate commutative, associative, identity, and zero properties of operations on whole numbers (e.g., $37 \times 46 = 46 \times 37$ and $(6 \times 2) \times 5 = 6 \times (2 \times 5)$).
- **4.N.2.5** Demonstrate the concept of distributivity of multiplication over addition and subtraction (e.g., 7 x 28 is equivalent to (7 x 20) + (7 x 8) or (7 x 30) (7 x 2)).

Strand: NUMBER AND OPERATIONS

Standard: Students will understand numerical concepts and mathematical operations.

K-4 Benchmark N.3: Compute fluently and make reasonable estimates.

Performance Standards

K.N.3.1 Estimate quantities of objects up to 20.

- **1.N.3.1** Use strategies for whole-number computation, with a focus on addition and subtraction (e.g., counting on or counting back, doubles, sums that make 10, direct modeling with pictures or objects, numerical reasoning based on number combinations and relationships).
- 1.N.3.2 Demonstrate a variety of methods to compute (e.g., objects, mental computation, paper and pencil, and estimation).

1.N.3.3 Perform addition and subtraction with whole number combinations.

1.N.3.4 Use and explain estimation strategies to determine the reasonableness of answers involving addition and subtraction.

- **2.N.3.1** Use and explain strategies for addition and subtraction of multi-digit whole numbers.
- 2.N.3.2 Model and solve problems representing adding and subtracting amounts of money using dollars and coins.
- **2.N.3.3** Use addition combinations (addends through 10) and related subtraction combinations, and develop strategies for computing based on number sense (e.g., 25 + 37: Take 3 from the 25 and use it to turn 37 into 40; then add 40 and 22 to get 62).
- **2.N.3.4** Select and use a variety of appropriate strategies methods to compute (e.g., objects, mental computation, estimation, paper and pencil).
- **2.N.3.5** Skip-count by 2, 5, and 10 to develop multiplicative reasoning and notational representations (e.g., 5, 10, 15, 20; $4 \times 5 = 20$; four groups of 5 equals 20).
- **3.N.3.1** Choose computational methods based on understanding the base-ten number system, properties of multiplication and division, and number relationships.
- **3.N.3.2** Use strategies (e.g., 6×8 is double 3×8) to become fluent with the multiplication pairs up to 10×10 .

3.N.3.3 Compute with basic number combinations (e.g., multiplication pairs up to 10 x 10 and their division counterparts).

3.N.3.4 Demonstrate reasonable estimation strategies for measurement, computation, and problem solving.

- **4.N.3.1** Demonstrate multiplication combinations through 12 x 12 and related division facts, and use them to solve problems mentally and compute related problems (e.g., 4 x 5 is related to 40 x 50, 400 x 5, and 40 x 500).
- **4.N.3.2** Add, subtract, and multiply up to two double-digits accurately and efficiently.
- **4.N.3.3** Use a variety of strategies (e.g., rounding and regrouping) to estimate the results of whole number computations and judge the reasonableness of the answers.
- **4.N.3.4** Use strategies to estimate computations involving fractions and decimals.

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

K-4 Benchmark A.1: Understand patterns, relations, and functions.

Perforn	Performance Standards	
K.A.1.1	Identify the attributes of objects (e.g., the ability to identify attributes is a foundational skill for sorting and classifying).	
K.A.1.2	Sort, classify, and order objects by size, number, and other properties.	
K.A.1.3	Recognize, reproduce, describe, extend, and create repeating patterns (e.g., color, shape, size, sound, movement, simple	
	numbers).	
1.A.1.1	Recognize, reproduce, describe, extend, and create repeating patterns (e.g., color, shape, size, sound, movement, simple	
	numbers) and translate from one representation to another (e.g., red, red, blue, blue to step, step, clap, clap).	
1.A.1.2	Skip-count on a hundreds chart (e.g., by 2s up to 20 and 5s and 10s up to 100) to identify, describe, and predict	
	number patterns.	
	Identify number patterns on the hundreds chart.	
2.A.1.1	Recognize, reproduce, describe, extend, and create repeating and growing patterns, and translate from one representation to	
	another.	
2.A.1.2	Skip-count using calculators or a hundreds chart to identify, describe, predict, and make generalizations about number	
	patterns to differentiate rote counting versus the meaning of the numbers.	
	Construct and solve open sentences that have variables (e.g., $10 = +7$).	
2.A.1.4	Relate everyday problem situations to number sentences involving addition and subtraction (e.g., 25 students are going to	
	the store. Five students can ride in a car. How many cars will be needed?).	
	Represent relationships of quantities in the form of mathematical expressions, equations, or inequalities.	
	Solve problems involving numeric equations.	
3.A.1.3	Select appropriate operational and relational symbols to make an expression true (e.g., "If $4 = 3 = 12$, what operational symbol goes in the box?").	
3 4 1 4	Use models of feet and inches to express simple unit conversions in symbolic form (e.g., 36 inches = feet x 12) that	
0	develop conceptual understanding versus procedural skills.	
3.A.1.5	Recognize and use the commutative property of multiplication (e.g., if $5 \ge 7 = 35$, then what is $7 \ge 5$?).	
	Create, describe, and extend numeric and geometric patterns including multiplication patterns.	
	Represent simple functional relationships:	
	a. solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple	
	items given the cost per unit)	
	b. extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be	
	calculated by counting by 4s, by multiplying the number of horses by 4, or through the use of tables)	

4.A.1.1 Represent and analyze patterns and simple functions using words, tables, and graphs.

4.A.1.2 Create and describe numeric and geometric patterns including multiplication and division patterns.

4.A.1.3 Express mathematical relationships using equations.

4.A.1.4 Use and interpret variables, mathematical symbols, and properties to write and simplify expressions and sentences:

- a. use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding of the concept of a variable)
- b. interpret and evaluate mathematical expressions using parentheses
- c. use and interpret formulas (e.g., Area = Length x Width or A = L x W) to answer questions about quantities and their relationships

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

K-4 Benchmark A.2: Represent and analyze mathematical situations and structures using algebraic symbols.

Perform	Performance Standards	
K.A.2.1	Use concrete, pictorial, and verbal representation to develop an understanding of invented and conventional symbols.	
1.A.2.1	Write number sentences that use concrete objects, pictorial, and verbal representations to express mathematical situations	
	using invented and conventional symbols (e.g., +, -, =).	
1.A.2.2	Demonstrate and describe the concept of equal (e.g., using objects, balance scales).	
	Solve open number sentences that have variables representing numbers up to 10 (e.g., $10 = +2$).	
2.A.2.1	Use mathematical language to describe a variety of representations and mathematical ideas and situations.	
	Explain the concept of equal (e.g., quantities on both sides of equation are the same) by using objects or giving examples.	
2.A.2.3	Construct and solve open number sentences that have variables representing numbers up to 20 (e.g., $20 = +6$).	
	Use objects, words, and symbols to explain the concept of addition.	
	Determine the value of variables in missing part problems (e.g., $139 + = 189$).	
3.A.2.2	Recognize and use the commutative and associative properties of addition and multiplication (e.g., "If $5 \times 7 = 35$, then what	
	is 7 x 5? And if 5 x 7 x $3 = 105$, then what is 7 x 3×5 ?").	
3.A.2.3	Explore the ways that commutative, distributive, identity, and zero properties are useful in computing with numbers.	
4.A.2.1	Identify symbols and letters that represent the concept of a variable as an unknown quantity.	
4.A.2.2	Explore the uses of properties (commutative, distributive, associative) in the computation of whole numbers.	
4.A.2.3	Express mathematical relationships using equations.	
4.A.2.4	Determine the value of variables in simple equations (e.g., $80 \times 15 = 40 \times \Box$).	
4.A.2.5	Develop simple formulas in exploring quantities and their relationships (e.g., $A = L \times W$).	

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

K-4 Benchmark A.3: Use mathematical models to represent and understand quantitative relationships.

Performance Standards

K.A.3.1 Model situations that involve whole numbers using objects or pictures.

- **1.A.3.1** Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions to 20 (e.g., 3 + 5 = 8, 2 + 6 = 8).
- **1.A.3.2** Describe situations that involve addition and subtraction of whole numbers including objects, pictures, and symbols (e.g., Robert has four apples, Maria has five more).
- 2.A.3.1 Model situations of addition and subtraction of whole numbers using objects, pictures, and symbols.
- 2.A.3.2 Solve problems related to trading (e.g., coin trading, measurement trading).
- 2.A.3.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.
- **3.A.3.1** Model problem situations with objects and use representations such as pictures, graphs, tables, and equations to draw conclusions.
- **3.A.3.2** Solve problems involving proportional relationships including unit pricing (e.g., four apples cost 80 cents; therefore, one apple costs 20 cents).
- 3.A.3.3 Describe relationships of quantities in the form of mathematical expressions, equations, or inequalities.
- **3.A.3.4** Select appropriate operational and relational symbols to make an expression true (e.g.," If 4 = 3 = 12, what operational symbol goes in the box?").
- **4.A.3.1** Solve problems involving proportional relationships (including unit pricing and map interpretations; e.g., one inch = five miles; therefore, five inches = miles).
- **4.A.3.2** Model problem situations and use graphs, tables, pictures, and equations to draw conclusions (e.g., different patterns of change).
- **4.A.3.3** Use and interpret formulas (e.g., Area = Length x Width or A = L x W) to answer questions about quantities and their relationships.

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

K-4 Benchmark A.4: Analyze changes in various contexts.

Performance Standards

K.A.4.1 Verbally describe changes in various contexts (e.g., plants or animals growing over time).

1.A.4.1 Describe qualitative change (e.g., a student growing taller, trees getting bigger, ice melting).

2.A.4.1 Describe quantitative change (e.g., a student growing two inches in one year, water heating up to boil).

3.A.4.1 Demonstrate how change in one variable can relate to a change in a second variable (e.g., input-output machines, data tables).

4.A.4.1 Identify and describe situations with constant or varying rates of change and compare them.

4.A.4.2 Determine how a change in one variable relates to a change in a second variable (e.g., data tables, input-output machines).

4.A.4.3 Find and analyze patterns using data tables (e.g., T tables).

4.A.4. Demonstrate and describe varying rates of change in relation to real-world situations (e.g., plant growth, students' heights).

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

K-4 Benchmark G.1: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Perforn	nance Standards	
K.G.1.1	I Identify common objects in their environments and describe their geometric features:	
	a. describe, identify, model, and draw common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere,	
	cone)	
	b. compare familiar plane and solid objects by common attributes (e.g., shape, size, number of corners)	
1.G.1.1		
	a. recognize, name, build, and draw both polygonal (up to six sides) and curved shapes	
	b. sort two- and three-dimensional shapes into categories based on common attributes	
	c. use the attributes of shapes to analyze and identify examples and non-examples of geometric shapes	
	d. participate in discussions comparing, identifying, and analyzing attributes to develop the vocabulary needed to describe	ibe
	two- and three-dimensional geometric shapes and their attributes (e.g., sides, corners, edges, faces)	
2.G.1.1	Identify and describe the attributes of common figures in a plane and common objects in space:	
	a. sort, describe, and analyze plane and solid geometric shapes (e.g., circle, triangle, square, rectangle, sphere, pyramid,	,
	cube, rectangular prism) based on various attributes (e.g., faces, edges, and corners)	
	b. put shapes together and take them apart to form other shapes (e.g., two congruent right triangles can be arranged to	
	form a rectangle)	
	c. explore lines of symmetry in two-dimensional shapes	
3.G.1.1	Describe and compare the attributes of plane and solid geometric figures to show relationships and solve problems:	
	a. identify, describe, and classify polygons (e.g., pentagons, hexagons, and octagons)	
	b. identify lines of symmetry in two-dimensional shapes	
	c. explore attributes of quadrilaterals (e.g., parallel and perpendicular sides for the parallelogram, right angles for the	
	rectangle, equal sides and right angles for the square)	
	d. identify right angles	
	e. identify, describe, and classify common three-dimensional geometric objects (e.g., cube, rectangular solid, sphere,	
	prism, pyramid, cone, cylinder)	

- **4.G.1.1** Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes:
 - a. build, draw, create, and describe geometric objects
 - b. identify lines that are parallel or perpendicular
 - c. identify and compare congruent and similar figures
- **4.G.1.2** Classify two- and three-dimensional shapes according to their properties and develop definitions of classes like triangles and pyramids:
 - a. visualize, describe, and make models of geometric solids in terms of the number of faces, edges, and vertices
 - b. interpret two-dimensional representations of three-dimensional objects

4.G.1.3 Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

K-4 Benchmark G.2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Performance Standards
K.G.2.1 Follow simple directions to find a specific location in space.
K.G.2.2 Use spatial vocabulary (e.g., left, right, above, below) to describe relative position.
1.G.2.1 Participate in group and individual activities based on the concepts of space and location:
a. describe direction, location, space, and shape (e.g., left, right, over, under, near, far, between)
b. visualize, describe, and record directions for navigating from one location to another to develop the vocabulary needed
to describe direction, distance, location, and representation
c. use materials to create representations of the surrounding environment (e.g., three-dimensional models, maps of the
classroom)
d. develop estimates and measure distances using nonstandard measurements
2.G.2.1 Find and name locations with simple relationships like "near to" and apply ideas about relative position.
2.G.2.2 Describe, name, and interpret direction in navigating space and apply ideas about direction and distance.
2.G.2.3 Use maps to locate points and navigate through mazes or maps.
2.G.2.4 Visualize, justify, and create paths using landmarks, space, shapes, and descriptive language.
2.G.2.5 Make and draw rectangular arrays of squares.

3.G.2.1 Describe location and movement using common language and geometric vocabulary (e.g., directions from classroom to gym).

3.G.2.2 Use ordered pairs to graph, locate specific points, create paths, and measure distances within a coordinate grid system.

3.G.2.3 Use a two-dimensional grid system (e.g., a map) to locate positions representing actual places.

4.G.2.1 Describe location and movement using common language and geometric vocabulary.

4.G.2.2 Use ordered pairs to graph, locate, identify points, and describe paths in the first quadrant of the coordinate plane.

4.G.2.3 Use a variety of methods for measuring distances between locations on a grid.

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

K-4 Benchmark G.3: Apply transformations and use symmetry to analyze mathematical situations.

Performance Standards

K.G.3.1 Use manipulatives (e.g., puzzles, tangrams, blocks) to demonstrate rotation (i.e., turns), translations (i.e., slides), and reflection (i.e., flips).

K.G.3.2 Investigate the symmetry of two-dimensional shapes (e.g., by folding or cutting paper, using mirrors).

1.G.3.1 Predict the results of changing a shape's position or orientation by using rotation (i.e., turns), reflection (i.e., flips), and translations (i.e., slides).

1.G.3.2 Create simple symmetrical shapes and pictures.

1.G.3.3 Recognize and describe the symmetric characteristics of designs (e.g., geometric designs made with pattern blocks).

2.G.3.1 Use systematic thinking to solve geometric puzzles (e.g., pentominoes).

2.G.3.2 Use materials to investigate rotational and line symmetry and create shapes that have symmetry.

3.G.3.1 Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.

3.G.3.2 Identify and describe the line of symmetry in two- and three-dimensional shapes.

4.G.3.1 Create and describe rotational designs using language of transformational symmetry.

4.G.3.2 Describe a motion or set of motions that will show that two shapes are congruent.

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

K-4 Benchmark G.4: Use visualization, spatial reasoning, and geometric modeling to solve problems.

Perforn	ance Standards
K.G.4.1	Describe how to get from one location to another (e.g., how to get to the library).
K.G.4.2	Find and describe geometric shapes in nature or architecture.
1.G.4.1	Use combinations of shapes to make a new shape to demonstrate relationships between shapes (e.g., a hexagon can be made from six triangles).
1.G.4.2	Create three-dimensional shapes based on two-dimensional representations.
	Participate in activities to develop mental visualization and spatial memory (e.g., "quick image" activities that require students to recall or reproduce a configuration of dots on a card or to determine the number of dots without counting).
1.G.4.4	Describe how to get from one location to another by visualizing the landmarks along the route.
	Identify structures from different views or match views of the same structure portrayed from different perspectives.
2.G.4.1	Demonstrate relationships of different attributes with concrete materials (e.g., change one characteristic of a shape while preserving others such as increasing number of sides while perimeter stays the same).
2.G.4.2	Select and use visualization skills to create mental images of geometric shapes.
	Describe geometric shapes and structures from different perspectives.
	Relate geometric ideas to numbers (e.g., seeing rows in array as a model of repeated addition).
2.G.4.5	Recognize geometric shapes and structures in the environment and specify their location.
3.G.4.1	Visualize, build, and draw geometric objects.
	Create and describe mental images of objects, patterns, and paths.
	Recognize geometric shapes and structures (e.g., in the environment).
	Use geometric models to solve problems in other areas of mathematics (e.g., using arrays as models of multiplication or area).
3.G.4.5	Identify and build three-dimensional objects from two-dimensional representations of that object.
	Investigate two-dimensional representations of three-dimensional shapes.
	Explore geometric ideas and relationships as they apply to other disciplines and to problems that arise in the classroom or in everyday life.

- **4.G.4.1** Develop and use mental images of geometric shapes to solve problems (e.g., represent three-dimensional shapes in two dimensions).
- **4.G.4.2** Use geometric models such as number lines, arrays, and computer simulations to investigate number relationships (e.g., patterns).
- **4.G.4.3** Explore relationships involving perimeter and area:
 - a. measure area of rectangular shapes and use appropriate units
 - b. recognize that area can have the same perimeter but different areas and vice versa
 - c. use models and formulas to solve problems involving perimeter and area of rectangles and squares (e.g., arrays)

Strand: MEASUREMENT

Standard: Students will understand measurement systems and applications.

K-4 Benchmark M.1: Understand measurable attributes of objects and the units, systems, and process of measurement.

erformance Standards
.M.1.1 Describe and compare, using appropriate concepts and vocabulary, the measurable properties of length (e.g., shorter,
longer, taller), volume (e.g., full, empty), weight (e.g., heavy, light), and time (e.g., before, after, morning, afternoon, days
of week).

it to check the prediction).
.M.1.3 Measure using non-standard units of measurement (e.g., use pencils to measure desk top, use different lengths of rope to
measure distance in classroom).
.M.1.4 Use digital and analog (face) clocks to tell time to the hour.
M.1.1 Develop an understanding of measurable properties (e.g., length, volume, weight, area, and time) using appropriate
concepts and vocabulary:
a. length by measuring and estimating (e.g., longer, shorter, meter, centimeter, inch, yard)
b. weight by measuring, estimating, and weighing (e.g., heavy [-ier], light [-er])
c. volume by measuring, estimating, and weighing (e.g., full, empty)
d. area by measuring and estimating (e.g., perimeter, rectangles, squares)
e. time by estimating (e.g., minutes, hours, days, weeks)
M.1.2 Use digital and analog (face) clocks to tell time to the half hour.
M.1.1 Identify a unit of measure (e.g., nearest inch) and repeat that unit comparing it to the item being measured.
M.1.2 Use direct comparison to compare and order objects according to length, mass, and area.
M.1.3 Measure and compare common objects using standard and non-standard units of length.
M.1.4 Find and represent the value of a collection of coins and dollars up to \$5.00, using appropriate notation.
M.1.5 Identify and use time intervals (e.g., hours, days, weeks, months).
M.1.6 Select and use appropriate measurement tools (e.g., ruler, yardstick, meter stick)
M.1.7 Tell time to the nearest quarter hour.
M.1.1 Demonstrate understanding of the need for measuring with standard units and become familiar with standard units in the
U.S. customary system.
M.1.2 Choose and use the appropriate units and measurement tools to quantify the properties of objects (e.g., length [ruler], width
[ruler], or mass [balance scale]).
M.1.3 Identify time to the nearest minute (elapsed time) and relate time to everyday events.
M.1.4 Identify and use time intervals (e.g., hours, days, weeks, months, years).

3.M.1.5 Identify properties (e.g., length, area, weight, volume) and select the appropriate type of unit for measuring each property.
3.M.1.6 Demonstrate understanding that measurements are approximations, investigate differences in units and their effect on precision, and consider the degree of accuracy for different situations.

- **4.M.1.1** Select the appropriate type of unit for measuring perimeter and size of an angle.
- **4.M.1.2** Understand the need for measuring with standard units and become familiar with the standard units in customary and metric system.
- 4.M.1.3 Identify the inverse relationship between the size of the units and the number of units.
- 4.M.1.4 Develop formulas to determine the surface areas of rectangular solids.
- 4.M.1.5 Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms.

4.M.1.6 Carry out simple conversions within a system of measurement (e.g., hours to minutes, meters to centimeters).

Strand: MEASUREMENT

Standard: Students will understand measurement systems and applications.

K-4 Benchmark M.2: Apply appropriate techniques, tools, and formulas to determine measurements.

Perforn	nance Standards
K.M.2.1	Explore measuring objects using a repeating non-standard unit of measurement (e.g., paper clips, cubes, etc.).
1.M.2.1	Measure with multiple copies of units the same size (e.g., paper clips).
1.M.2.2	Use repetition of a single unit to measure something larger than the unit (e.g., a yardstick/meterstick to measure a room).
	Develop common referents to make comparisons and estimates of length, volume, weight, area, and time.
2.M.2.2	Develop an understanding that different measuring tools will yield different numerical measurements of the same object
	(e.g., ruler, yardstick, meterstick, paper clip).
2.M.2.3	Estimate measurements and develop precision in measuring objects.
3.M.2.1	Find the area of rectangles using appropriate tools (e.g., grid paper, tiles).
3.M.2.2	Estimate measurements.
3.M.2.3	Use appropriate standard units and tools to estimate, measure, and solve problems (e.g., length, area, weight).
3.M.2.4	Recognize a 90-degree angle and use it as a strategy to estimate the size of other angles.
4.M.2.1	Estimate perimeters, areas of rectangles, triangles, and irregular shapes.
4.M.2.2	Find the area of rectangles, related triangles, and parallelograms.
4.M.2.3	Estimate, measure, and solve problems involving length, area, mass, time, and temperature using appropriate standard units
	and tools.
4.M.2.4	Identify common measurements of turns (e.g., 360 degrees in one turn, 90 degrees in a quarter-turn).
4.M.2.5	Compute elapsed time and make and interpret schedules.
4.M.2.6	Use tools to measure angles (e.g., protractor, compass).

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

K-4 Benchmark D.1: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Perform	nance Standards
K.D.1.1	Collect data about objects and events in the environment to answer simple questions (e.g., brainstorm questions about self
	and surroundings, collect data, and record the results using objects, pictures, and pictographs).
1.D.1.1	Collect, organize, represent, and compare data by category on graphs and charts to answer simple questions:
	a. answer questions about "how" data can be gathered
	b. gather data by interviewing, surveying, and making observations
	c. organize data into appropriate categories by sorting based on shared properties
	d. participate in discussions about selecting an appropriate way to display the data
	e. represent data using objects, pictures, tables, and simple bar graphs
2.D.1.1	Collect numerical data systematically.
2.D.1.2	Represent data by using concrete objects, pictures, tables, numbers, tallies, and graphs (e.g., pictographs).
	Pose questions about students' selves and their surroundings and gather data by interviewing, surveying, and making
	observations to answer the questions posed.
2.D.1.4	Identify patterns and explain the relationships of the units in the pattern (e.g., the number of ears on one dog, two dogs, etc.,
	or linear numerical patterns).
3.D.1.1	Collect and organize data using observations, measurements, surveys, or experiments.
3.D.1.2	Represent data using tables and graphs (e.g., line plots, bar graphs, and line graphs).
3.D.1.3	Conduct simple experiments by determining the number of possible outcomes and make simple predictions:
	a. identify whether events are certain, likely, unlikely, or impossible
	b. record the outcomes for a simple event and keep track of repetitions
	c. summarize and record the results in a clear and organized way
	d. use the results to predict future events
4.D.1.1	Organize, represent, and interpret numerical and categorical data and clearly communicate findings:
	a. choose and construct representations that are appropriate for the data set
	b. recognize the differences in representing categorical and numerical data
4.D.1.2	Design investigations and represent data using tables and graphs (e.g., line plots, bar graphs, line graphs).

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

K-4 Benchmark D.2: Select and use appropriate statistical methods to analyze data.

Performance Standards	
K.D.2.1 Describe simple data and pose questions about the data.	
1.D.2.1 Analyze simple data:	
a. interpret what the graph or other representation shows	
b. determine whether or not the data gathered helps answer the specific question that was posed	
c. compare parts of the data (e.g., "How many students have lost none, one, two, or three teeth?") to make statements	
about the data as a whole (e.g., "Most students in the class have lost only two teeth")	
2.D.2.1 Describe and interpret data by drawing conclusions and making conjectures based on the data collected.	
2.D.2.2 Display data in a variety of formats.	
3.D.2.1 Apply and explain the uses of sampling techniques (e.g., observations, polls, tally marks) for gathering data.	
4.D.2.1 Compare and describe related data sets.	
4.D.2.2 Use the concepts of median, mode, maximum, minimum, and range and draw conclusions about a data set.	
4.D.2.3 Use data analysis to make reasonable inferences/predictions and to develop convincing arguments from data described in a	
variety of formats (e.g. bar graphs, Venn diagrams, charts, tables, line graphs, and pictographs).	

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

K-4 Benchmark D.3: Develop and evaluate inferences and predictions that are based on data.

Performance Standards

K.D.3.1 Make simple predictions.

1.D.3.1 Make conclusions based on data (e.g., whether or not other groups would reach similar conclusions based on the same data).

2.D.3.1 Discuss events related to students' experiences as "likely" or "unlikely" and "possible" or "certain".

2.D.3.2 Recognize appropriate conclusions generated from the data collected.

2.D.3.3 Recognize inappropriate descriptions of the data set.

3.D.3.1 Analyze data displayed in a variety of formats to make reasonable inferences and predictions, answer questions, and make decisions.

4.D.3.1 Propose and justify conclusions and predictions based on data.

4.D.3.2 Develop convincing arguments from data displayed in a variety of formats.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

K-4 Benchmark D.4: Understand and apply basic concepts of probability.

Performance Standards

K.D.4.1 Answer questions that relate to the possibility of familiar events happening or not.

- **1.D.4.1** Discuss the likelihood of events (based on student experiences or from books) using terminology such as "more likely", "less likely", "possible", or "certain".
- **1.D.4.2** Observe, explore, and discuss whether some events occur more often than others (e.g., tossing two die and recording the sum after each toss to explore whether or not certain sums occur more frequently than others).

2.D.4.1 Investigate concepts of chance (e.g., outcomes of a simple experiment).

2.D.4.2 Investigate whether outcomes of a simple event are equally likely to occur.

3.D.4.1 Discuss the degree of likelihood of events and use terminology such as "certain," "likely," "unlikely".

3.D.4.2 Predict the outcomes of simple experiments (e.g., coin tossing) and test the predictions using concrete objects (e.g., coins, counters, number cubes, spinners).

3.D.4.3 Record the probability of a specific outcome for a simple probability situation (e.g., probability is three out of seven for choosing a black ball; 3/7).

4.D.4.1 Describe events as "likely," "unlikely," or "impossible" and quantify simple probability situations:

a. represent all possible outcomes for a simple probability situation in an organized way (e.g., tables, grids, tree diagrams)

- b. express outcomes of experimental probability situations verbally and numerically (e.g., three out of four, ³/₄)
- **4.D.4.2** List all the possible combinations of objects from three sets (e.g., spinners, number of outfits from three different shirts, two skirts, and two hats).

Strand: NUMBER AND OPERATIONS

Standard: Students will understand numerical concepts and mathematical operations.

5-8 Benchmark N.1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Performance Standards

5.N.1.1 Compare and order using concrete or illustrated models:

- a. whole numbers (to millions)
- b. common fractions (halves, thirds, fourths, eighths)
- c. decimals (thousandths)
- **5.N.1.2** Demonstrate understanding of the magnitude of the value of numbers from thousandths to millions, including common fractions.
- 5.N.1.3 Represent place value using concrete or illustrated models up to one billion (1,000,000,000).
- **5.N.1.4** Interpret percents as part of a hundred (i.e., find decimal and percent equivalents for common fractions, explain how they represent the same value, and compute a given percent of a whole number).
- 5.N.1.5 Identify and represent on a number line decimals, fractions, and mixed numbers.

5.N.1.6 Identify prime and composite numbers to 50.

6.N.1.1 Compare and order rational numbers.

- **6.N.1.2** Use equivalent representations for rational numbers (e.g., integers, decimals, fractions, percents, ratios, numbers with whole-number exponents).
- **6.N.1.3** Use appropriate representations of positive rational numbers in the context of real-life applications.
- 6.N.1.4 Identify greatest common factor and least common multiples for a set of whole numbers.

6.N.1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.

7.N.1.1 Determine the absolute value of rational numbers.

7.N.1.2 Illustrate the relationships among natural (i.e., counting) numbers, whole numbers, integers, rational and irrational numbers.

- 7.N.1.3 Use properties of the real-number system to explain reasoning and to formulate and solve real-world problems.
- **7.N.1.4** Read, write, and compare rational numbers in scientific notation (e.g., positive and negative powers of 10) with approximate numbers using scientific notation.

7.N.1.5 Simplify numerical expressions using order of operations.

8.N.1.1 Sort numbers by their properties (e.g., prime, composite, square, square root).

8.N.1.2 Demonstrate the magnitude of rational numbers (e.g., trillions to millions).

Strand: NUMBER AND OPERATIONS

Standard: Students will understand numerical concepts and mathematical operations.

5-8 Benchmark N.2: Understand the meaning of operations and how they relate to one another.

Performance Standards

- **5.N.2.1** Explain and perform whole number division and express remainders as a whole number or a fractional part as appropriate to the context of real-life problems.
- **5.N.2.2** Add and subtract decimals.
- 5.N.2.3 Add and subtract fractions and mixed numbers without regrouping and express answers in simplest form.
- **5.N.2.4** Find the factors and multiples of whole numbers.
- **5.N.2.5** Use arithmetic operations and inverse relationships to represent and solve real-world problems.
- **5.N.2.6** Identify and represent on a number line decimals, fractions, and mixed numbers.
- 5.N.2.7 Demonstrate proficiency with division, including one- and two-digit divisors.
- 5.N.2.8 Solve simple problems involving the addition and subtraction of fractions and mixed numbers.
- **5.N.2.9** Represent and use fractions and decimals in equivalent forms.
- 6.N.2.1 Calculate multiplication and division problems using contextual situations.
- 6.N.2.2 Factor a whole number into a product of its primes.
- **6.N.2.3** Demonstrate the relationship and equivalency among ratios and percents.
- **6.N.2.4** Use proportions to solve problems.
- **6.N.2.5** Explain and perform:
 - a. whole number division and express remainders as decimals or appropriately in the context of the problem
 - b. addition, subtraction, multiplication, and division with decimals
 - c. addition and subtraction with integers
 - d. addition, subtraction, and multiplication with fractions and mixed numerals
- **6.N.2.6** Determine the least common multiple and the greatest common divisor of whole numbers and use them to solve problems with fractions.
- **7.N.2.1** Add, subtract, multiply, and divide rational numbers (e.g., integers, fractions, terminating decimals) and take positive rational numbers to whole-number powers.
- 7.N.2.2 Convert terminating decimals into reduced fractions.
- **7.N.2.3** Calculate given percentages of quantities and use them to solve problems (e.g., discounts of sales, interest earned, tips, markups, commission, profit, simple interest).
- **7.N.2.4** Add and subtract fractions with unlike denominators.
- 7.N.2.5 Multiply, divide, and simplify rational numbers by using exponent rules.
- **7.N.2.6** Understand the meaning of the absolute value of a number:

- a. interpret the absolute value as the distance of the number from zero on a number line
- b. determine the absolute value of real numbers
- **7.N.2.7** Find square roots of perfect whole-number squares.
- 7.N.2.8 Simplify and evaluate positive rational numbers raised to positive whole number powers.
- **7.N.2.9** Solve addition, subtraction, multiplication, and division problems that use positive and negative integers and combinations of these operations.
- 8.N.2.1 Use real number properties (e.g., commutative, associative, distributive) to perform various computational procedures.
- **8.N.2.2** Perform arithmetic operations and their inverses (e.g., addition/subtraction, multiplication/division, square roots of perfect squares, cube roots of perfect cubes) on real numbers.
- **8.N.2.3** Find roots of real numbers using calculators.

Strand: NUMBER AND OPERATIONS

Standard: Students will understand numerical concepts and mathematical operations.

5-8 Benchmark N.3: Compute fluently and make reasonable estimates.

Performance Standards

5.N.3.1 Add, subtract, multiply, and divide whole numbers.

- **5.N.3.2** Add and subtract decimals.
- 5.N.3.3 Use estimation strategies to verify the reasonableness of calculated results.
- 5.N.3.4 Explain how the estimation strategy impacts the result.
- 5.N.3.5 Relate the basic arithmetic operations to one another (e.g., multiplication and division are inverse operations).
- **5.N.3.6** Simplify numerical expressions using order of operations.
- 5.N.3.7 Recognize and explain the differences between exact and approximate values.

6.N.3.1 Estimate quantities involving rational numbers using various estimations.

6.N.3.2 Use estimates to check reasonableness of results and make predictions in situations involving rational numbers.

- 6.N.3.3 Determine if a problem situation calls for an exact or approximate answer and perform the appropriate computation.
- 6.N.3.4 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.
- 6.N.3.5 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- **6.N.3.6** Interpret and use ratios in different contexts.

6.N.3.7 Compute and perform multiplication and division of fractions and decimals and apply these procedures to solving problems.

7.N.3.1 Use estimation to check reasonableness of results, and use this information to make predictions in situations involving rational numbers, pi, and simple algebraic equations.

- 7.N.3.2 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- **7.N.3.3** Read, write, and compare rational numbers in scientific notation (e.g., positive and negative powers of 10) with approximate numbers using scientific notation.
- **7.N.3.4** Calculate the percentage of increases and decreases of a quantity.
- 7.N.3.5 Add and subtract fractions with unlike denominators.
- 7.N.3.6 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer.
- **8.N.3.1** Formulate algebraic expressions that include real numbers to describe and solve real-world problems.
- 8.N.3.2 Use a variety of computational methods to estimate quantities involving real numbers.
- **8.N.3.3** Differentiate between rational and irrational numbers.
- 8.N.3.4 Use real number properties to perform various computational procedures and explain how they were used.
- **8.N.3.5** Perform and explain computations with rational numbers, pi, and first-degree algebraic expressions in one variable in a variety of situations.
- **8.N.3.6** Select and use appropriate forms of rational numbers to solve real-world problems including those involving proportional relationships.
- **8.N.3.7** Approximate, mentally and with calculators, the value of irrational numbers as they arise from problem situations.
- 8.N.3.8 Express numbers in scientific notation (including negative exponents) in appropriate problem situations using a calculator.
- **8.N.3.9** Estimate answers and use formulas to solve application problems involving surface area and volume.

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

5-8 Benchmark A.1: Understand patterns, relations, and functions.

Performance Standards

5.A.1.1 Identify and graph ordered pairs in the first quadrant of the coordinate plane.

- 5.A.1.2 Describe, represent, and analyze patterns and relationships.
- **5.A.1.3** Identify, describe, and continue patterns presented in a variety of formats (e.g., numeric, visual, oral, written, kinesthetic, pictorial).
- **5.A.1.4** Generate a pattern using a written description.
- 6.A.1.1 Solve problems involving proportional relationships.
- **6.A.1.2** Graph ordered pairs in the coordinate plane.
- 6.A.1.3 Explain and use symbols to represent unknown quantities and variable relationships.
- 6.A.1.4 Explain and use the relationships among ratios, proportions, and percents.
- **6.A.1.5** Make generalizations based on observed patterns and relationships.
- 7.A.1.1 Identify and continue patterns presented in a variety of formats.
- **7.A.1.2** Represent a variety of relationships using tables, graphs, verbal rules, and possible symbolic notation, and recognize the same general pattern presented in different representations.
- 7.A.1.3 Simplify numerical expressions by applying properties of rational numbers, and justify the process used.
- 7.A.1.4 Interpret and evaluate expressions involving integer powers and simple roots.
- 7.A.1.5 Graph and interpret linear functions.
- 7.A.1.6 Solve problems involving rate, average speed, distance, and time.

8.A.1.1 Move between numerical, tabular, and graphical representations of linear relationships.

- **8.A.1.2** Use variables to generalize patterns and information presented in tables, charts, and graphs:
 - a. graph linear functions noting that the vertical change per unit of horizontal change (the slope of the graph) is always the same
 - b. plot the values of quantities whose ratios are always the same, fit a line to the plot, and understand that the slope of the line equals the quantities

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

5-8 Benchmark A.2: Represent and analyze mathematical situations and structures using algebraic symbols.

erformance Standards
.A.2.1 Compute the value of the expression for specific numerical values of the variable.
.A.2.2 Use a letter to represent an unknown number.
A.2.3 Understand the differences between the symbols for "less than", "less than or equal to", "greater than", and "greater than or
equal to".
A.2.1 Solve problems involving proportional relationships.
A.2.2 Use letters to represent an unknown in an equation.
A.2.3 Solve one-step linear equations and inequalities in one variable with positive whole-number solutions.
A.2.4 Demonstrate that a variable can represent a single quantity that changes.
A.2.5 Demonstrate how changes in one variable affect other variables.
A.2.1 Write verbal expressions and sentences as algebraic expressions and equations:
a. evaluate algebraic expressions
b. solve simple linear equations
c. graph and interpret results
.A.2.2 Use variables and appropriate operations to write an expression, an equation, or an inequality that represents a verbal
description.
A.2.3 Use the order of operations to evaluate algebraic expressions.
A.2.4 Simplify numerical expressions by applying properties of rational numbers.
A.2.5 Graph linear functions and identify slope as positive or negative.
A.2.6 Use letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes.
A.2.1 Demonstrate the difference between an equation and an expression.
A.2.2 Solve two-step linear equations and inequalities in one variable with rational solutions.
.A.2.3 Evaluate formulas using substitution.
.A.2.4 Demonstrate understanding of the relationships between ratios, proportions, and percents and solve for a missing term in a
proportion.
A.2.5 Graph solution sets of linear equations in two variables on the coordinate plane.
.A.2.6 Formulate and solve problems involving simple linear relationships, find percents of a given number, variable situations,
and unknown quantities.
.A.2.7 Use symbols, variables, expressions, inequalities, equations, and simple systems of equations to represent problem
situations that involve variables or unknown quantities.

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

5-8 Benchmark A.3: Use mathematical models to represent and understand quantitative relationships.

Performance Standards

- **5.A.3.1** Use mathematical models to represent and explain mathematical concepts and procedures.
- **5.A.3.2** Understand and use mathematical models such as:
 - a. the number line to model the relationship between rational numbers and rational number operations
 - b. pictorial representation of addition and subtraction of rational numbers with regrouping
 - c. manipulatives or pictures to model computational procedures
 - d. graphs, tables, and charts to describe data
 - e. diagrams or pictures to model problem situations
- **5.A.3.3** Demonstrate how a situation can be represented in more than one way.
- **6.A.3.1** Develop and use mathematical models to represent and justify mathematical relationships found in a variety of situations.
- 6.A.3.2 Create, explain, and use mathematical models such as:
 - a. Venn diagrams to show the relationships between the characteristics of two or more sets
 - b. equations and inequalities to model numerical relationships
 - c. three-dimensional geometric models
 - d. graphs, tables, and charts to interpret and analyze data
- 7.A.3.1 Create scale models and use them for dimensional drawings.
- 7.A.3.2 Understand and use the coordinate plane to graph ordered pairs and linear equations.
- **7.A.3.3** Select and use an appropriate model for a particular situation.
- **8.A.3.1** Generate different representations to model a specific numerical relationship given one representation of data (e.g., a table, a graph, an equation, a verbal description).

Strand: ALGEBRA

Standard: Students will understand algebraic concepts and applications.

5-8 Benchmark A.4: Analyze changes in various contexts.

Performance Standards

5.A.4.1 Recognize and create patterns of change from everyday life using numerical or pictorial representations.

5.A.4.2 Generalize patterns of change and recognize the same general patterns presented in different representations.

- 6.A.4.1 Represent and explain changes using one-step equations with one variable.
- 6.A.4.2 Solve problems that involve change using proportional relationships.
- **6.A.4.3** Use ratios to predict changes in proportional situations.
- **6.A.4.4** Use tables and symbols to represent and describe proportional and other relationships involving conversions, sequences, and perimeter.
- 6.A.4.5 Generate formulas to represent relationships involving changes in perimeter.
- **7.A.4.1** Use variables and appropriate operations to write an expression, an equation, and/or an inequality that represents a verbal description involving change.
- 7.A.4.2 Interpret and evaluate expressions involving integer powers and simple roots as they relate to change.
- **7.A.4.3** Graph and interpret linear functions as they are used to solve problems.
- **7.A.4.4** Solve two-step equations and inequalities with one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
- **8.A.4.1** Use graphs, tables, and algebraic representations to make predictions and solve problems that involve change.
- **8.A.4.2** Estimate, find, and justify solutions to problems that involve change using tables, graphs, and algebraic expressions.
- **8.A.4.3** Use appropriate problem-solving strategies (e.g., drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table or graph, working a simpler problem, writing an algebraic expression or working backward) to solve problems that involve change.
- **8.A.4.4** Solve multi-step problems that involve changes in rate, average speed, distance, and time.
- **8.A.4.5** Analyze problems that involve change by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing, and observing patterns.

8.A.4.6 Generalize a pattern of change using algebra and show the relationship among the equation, graph, and table of values.

8.A.4.7 Recognize the same general pattern of change presented in different representations.

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

5-8 Benchmark G.1: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematics arguments about geometric relationships.

Performance Standards

5.G.1.1 Identify, describe, and classify two-dimensional shapes and three-dimensional figures by their properties.

5.G.1.2 Recognize and describe properties of regular polygons having up to ten sides.

5.G.1.3 Identify faces, edges, and bases on three-dimensional objects.

6.G.1.1 Identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:

- a. measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software)
- b. understand that the sum of angles of any triangle is 180 degrees and the sum of the angles of any quadrilateral is 360 degrees and use this information to solve problems
- c. visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids
- **6.G.1.2** Classify angles as right, obtuse, or straight.
- **6.G.1.3** Describe the properties of geometric figures that include regular polygons, circles, ellipses, cylinders, cones, spheres, and cubes.
- **6.G.1.4** Classify polygons as regular or irregular.
- 6.G.1.5 Classify triangles as scalene, isosceles, or equilateral and by angles (i.e., right, acute, and obtuse).
- 6.G.1.6 Identify angle, line, segment, and ray and use the symbols for each.
- 6.G.1.7 Describe the relationship between radius, diameter, and circumference of a circle.
- 7.G.1.1 Classify geometric figures as similar or congruent.

7.G.1.2 Understand the concept of a constant (e.g., pi) and use the formulas for the circumference and area of a circle.

7.G.1.3 Explain and use the Pythagorean theorem.

- 7.G.1.4 Determine the radius, diameter, and circumference of a circle and explain their relationship.
- 7.G.1.5 Use properties to classify solids including pyramids, cones, prisms, and cylinders.
- **8.G.1.1** Recognize, classify, and discuss properties of all geometric figures including point, line, and plane.
- 8.G.1.2 Identify arc, chord, and semicircle and explain their attributes.
- **8.G.1.3** Use the Pythagorean theorem and its converse to find the missing side of a right triangle and the lengths of the other line segments.

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

5-8 Benchmark G.2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Performance Standards
5.G.2.1 Recognize perpendicular and parallel lines.
6.G.2.1 Use coordinate geometry to describe location on a plane.
6.G.2.2 Recognize skewed lines in space.
7.G.2.1 Construct and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine the
image under translations and reflections.
8.G.2.1 Represent, formulate, and solve distance and geometry problems using the language and symbols of algebra and the
coordinate plane and space (e.g., ordered triplets).

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

5-8 Benchmark G.3: Apply transformations and use symmetry to analyze mathematical situations.

Performance	Standards
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5.G.3.1 Identify line of symmetry in simple geometric figures.

6.G.3.1 Identify line of symmetry with rotation and scaling.

7.G.3.1 Determine how perimeter and area are affected by changes of scale.

8.G.3.1 Describe the symmetry of three-dimensional figures.

8.G.3.2 Describe and perform single and multiple transformations that include rotation, reflection, translation, and dilation (i.e., shrink or magnify) to two-dimensional figures.

Strand: GEOMETRY

Standard: Students will understand geometric concepts and applications.

5-8 Benchmark G.4: Use visualization, spatial reasoning, and geometric modeling to solve problems.

	nance Standards
5.G.4.1	Understand and compute the perimeter of regular polygons.
5.G.4.2	Identify and explain circumference, radius, and diameter.
6.G.4.1	Use appropriate technology, manipulatives, constructions, or drawings to recognize or compare geometric figures.
7.G.4.1	Compute the perimeter and area of common geometric shapes and use the results to find measures of less common objects.
7.G.4.2	Identify and describe the properties of two-dimensional figures:
	a. identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms
	b. use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle
	c. draw quadrilaterals and triangles from given information
8.G.4.1	Understand angle relationships formed by parallel lines cut by a transversal.
8.G.4.2	Recognize and apply properties of corresponding parts of similar and congruent triangles and quadrilaterals.
8.G.4.3	Represent and solve problems relating to size, shape, area, and volume using geometric models.
	Develop and use formulas for area, perimeter, circumference, and volume.
8.G.4.5	Construct two-dimensional patterns for three-dimensional models (e.g., cylinders, prisms, cones).

Strand: MEASUREMENT

Standard: Students will understand measurement systems and applications.

5-8 Benchmark M.1: Understand measurable attributes of objects and the units, systems, and processes of measurement.

Performance Standards

5.M.1.1	Understand properties (e.g., length, area, weight, volume) and select the appropriate type of unit for measuring each using
	both U.S. customary and metric systems.

- **5.M.1.2** Select and use appropriate units and tools to measure according to the degree of accuracy required in a particular problem-solving situation.
- **5.M.1.3** Solve problems involving linear measurement, weight, and capacity (e.g., measuring to the nearest sixteenth of an inch or nearest millimeter; using ounces, milliliters, or pounds and kilograms) to the appropriate degree of accuracy.
- 5.M.1.4 Perform one-step conversions within a system of measurement (e.g., inches to feet, centimeters to meters).
- **6.M.1.1** Perform multi-step conversions of measurement units to equivalent units within a given system (e.g., 36 inches equals 3 feet or 1 yard).
- 6.M.1.2 Estimate measurement in both U.S. customary and metric units.
- **6.M.1.3** Select and use units of appropriate size and type to measure angles (e.g., degrees, radians), perimeter, area, and capacity in both U.S. customary and metric systems.
- **6.M.1.4** Use standard units of linear measurement to the nearest sixteenth of an inch; metric measurements to the nearest millimeter.
- **7.M.1.1** Choose appropriate units of measure and ratios to recognize new equivalences (e.g., 1 square yard equals 9 square feet) to solve problems.
- **7.M.1.2** Select and use the appropriate size and type of unit for a given measurement situation.
- 7.M.1.3 Compare masses, weights, capacities, geometric measures, times, and temperatures within measurement systems.
- **7.M.1.4** Approximate the relationship between standard and metric measurement systems (e.g., inches and centimeters, pounds and kilograms, quarts and liters).
- **7.M.1.5** Use measures expressed as rates and measures expressed as products to solve problems, check the units of the solutions, and analyze the reasonableness of the answer.
- **8.M.1.1** Understand the concept of volume and use the appropriate units in common measuring systems (e.g., cubic centimeter, cubic inch, cubic yard) to compute the volume of rectangular solids.
- **8.M.1.2** Use changes in measurement units (e.g., square inches, cubic feet) to perform conversions from one-, two-, and three-dimensional shapes.

Strand: MEASUREMENT

Standard: Students will understand measurement systems and applications.

5-8 Benchmark M.2: Apply appropriate techniques, tools, and formulas to determine measurements.

Performance Standards

5.M.2.1 Solve measurement problems using appropriate tools involving length, perimeter, weight, capacity, time, and temperature.

5.M.2.2 Select and use strategies to estimate measurements including length, distance, capacity, and time.

5.M.2.3 Apply strategies and use tools for estimating and measuring the perimeter of regular and irregular shapes.

6.M.2.1 Apply various measurement techniques and tools, units of measure, and degrees of accuracy to find accurate rational number representations for length, liquid, weight, perimeter, temperature, and time.

6.M.2.2 Select and use formulas for perimeters of squares and rectangles.

6.M.2.3 Select and use strategies to estimate measurements including angle measure and capacity.

6.M.2.4 Select and justify the selection of measurement tools, units of measure, and degrees of accuracy appropriate to the given situation.

7.M.2.1 Apply strategies and formulas to find missing angle measurements in triangles and quadrilaterals.

7.M.2.2 Select and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles.

7.M.2.3 Solve problems involving scale factors, ratios, and proportions.

- **8.M.2.1** Use ratios and proportions to measure hard-to-measure objects.
- **8.M.2.2** Use estimation to solve problems.

8.M.2.3 Use proportional relationships in similar shapes to find missing measurements.

8.M.2.4 Apply strategies to determine the surface area and volume of prisms, pyramids, and cylinders.

8.M.2.5 Perform conversions with multiple terms between metric and U.S. standard measurement systems.

8.M.2.6 Estimate volume in cubic units.

8.M.2.7 Solve simple problems involving rates and derived measurements for such properties as velocity and density.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

5-8 Benchmark D.1: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Performance Standards

- 5.D.1.1 Construct, read, analyze, and interpret tables, charts, graphs, and data plots.
- **5.D.1.2** Construct, interpret, and analyze data from graphical representations and draw simple conclusions using bar graphs, line graphs, circle graphs, frequency tables, and Venn diagrams.
- 5.D.1.3 Display, analyze, compare, and interpret different data sets, including data sets of different sizes.
- 5.D.1.4 Organize and display single-variable data in appropriate graphs and representations.
- **5.D.1.5** Organize, read, and display numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including correct titles, labels, and intervals or categories including:
 - a. frequency tables
 - b. stem and leaf plots
 - c. bar, line, and circle graphs
 - d. Venn diagrams
 - e. pictorial displays
 - f. charts and tables

5.D.1.6 Formulate questions and identify data to be collected to correctly answer a question.

- **6.D.1.1** Use statistical representations to analyze data.
- **6.D.1.2** Draw and compare different graphical representations of the same data.
- 6.D.1.3 Use mean, median, mode, and range to describe data.
- **6.D.1.4** Sketch circle graphs to display data.
- **6.D.1.5** Solve problems by collecting, organizing, displaying and interpreting data.
- 6.D.1.6 Compare different samples of a population with the entire population and determine the appropriateness of using a sample.
- 6.D.1.7 Conduct and explain sampling techniques such as observations, surveys, and random sampling for gathering data.
- 6.D.1.8 Determine the median for a rational number data set containing an odd number of data points.
- 6.D.1.9 Calculate and explain the median for a whole number data set containing an even number of data points.
- 6.D.1.10 Explain advantages and disadvantages of using various display formats for a specific data set.
- 6.D.1.11 Formulate and solve problems by collecting, organizing, displaying, and interpreting data.
- 7.D.1.1 Describe how data representations influences interpretation.
- 7.D.1.2 Select and use appropriate representation for presenting collected data and justify the selection.
- 7.D.1.3 Use measures of central tendency and spread to describe a set of data.

- 7.D.1.4 Choose between median and mode to describe a set of data and justify the choice for a particular situation.
- 7.D.1.5 Determine the quartiles of a data set.
- 7.D.1.6 Identify ordered pairs of data from a graph and interpret the data in terms of the situation depicted by the graph.
- 7.D.1.7 Use various scales and formats to display the same data set.
- 7.D.1.8 Identify and explain the misleading representations of data.
- **7.D.1.9** Collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set.
- 7.D.1.10 Compute the minimum, lower quartile, median, upper quartile, and maximum of a data set.
- 7.D.1.11 Identify and explain the effects of scale and/or interval changes on graphs of whole number data sets.

7.D.1.12 Use and explain sampling techniques (e.g., observations, surveys, and random sampling) for gathering data.

- **7.D.1.13** Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, and selecting, collecting, and displaying appropriate data to address the problem.
- **8.D.1.1** Represent two numerical variables on a plot, describe how the data points are distributed, and identify relationships that exist between the two variables.
- 8.D.1.2 Generate, organize, and interpret real numbers in a variety of situations.
- **8.D.1.3** Organize, analyze, and display appropriate quantitative and qualitative data to address specific questions including:
 - a. frequency distributions b. plots
 - c. histograms
- d. bar, line, and pie graphs
- e. diagram and pictorial displays
- f. charts and tables
- **8.D.1.4** Select the appropriate measure of central tendency to describe a set of data for a particular problem situation.
- **8.D.1.5** Simulate an event selecting and using different models.
- **8.D.1.6** Develop an appropriate strategy using a variety of data from surveys, samplings, estimations, and inferences to address a specific problem.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

5-8 Benchmark D.2: Select and use appropriate statistical methods to analyze data.

Performance Standards

5.D.2.1 Organize and display single-variable data in appropriate graphs and representations and determine which types of graphs are appropriate for various data sets.

5.D.2.2 Use fractions and percentages to compare data sets of different sizes.

5.D.2.3 Correctly rank the values of a numerical data set containing simple fractions and decimals, identify maximum and minimum data values, and calculate the range for a data set.

- 6.D.2.1 Choose an appropriate graphical format to organize and represent data.
- **6.D.2.2** Describe the effects of missing or incorrect data.
- 6.D.2.3 Compute and analyze statistical measurements for data sets:
 - a. understand how additional data added to data sets may affect the computations of central tendency
 - b. understand how the inclusion or exclusion of outliers affects measures of central tendency
 - c. know why a specific measure of central tendency provides the most useful information in a given context
- 6.D.2.4 Use data samples of a population and describe the characteristics and limitations of the sample.
- **6.D.2.5** Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.
- 6.D.2.6 Explain how the way a question is asked in a survey might influence the results obtained.
- 6.D.2.7 Identify data that represent sampling errors and explain why the sample and the display might be biased.

6.D.2.8 Identify claims based on statistical data and, in sample cases, evaluate the validity and usefulness of the claims.

- 7.D.2.1 Choose and justify appropriate measures of central tendencies (e.g., mean, median, mode, range) to describe given or derived data.
- **7.D.2.2** Know various ways to display data sets (e.g., stem and leaf plot, box and whisker plot, scatter plots) and use these forms to display a single set of data or to compare two sets of data.
- 7.D.2.3 Use the analysis of data to make convincing arguments.
- **7.D.2.4** Use appropriate technology to gather and display data sets and identify the relationships that exist among variables within the data set.
- 7.D.2.5 Use data samples of a population and describe the characteristics and limitations of the sample.
- 7.D.2.6 Identify data that represent sampling errors and explain why the sample and the display might be biased.
- 7.D.2.7 Identify claims based on statistical data and evaluate the validity of the claims.
- 8.D.2.1 Use changes in scales, intervals, or categories to help support a particular interpretation of data.
- 8.D.2.2 Generate, organize, and interpret real number and other data in a variety of situations.

8.D.2.3 Analyze data to make decisions and to develop convincing arguments from data displayed in a variety of formats including:

a. plots

b. distributions

c. graphs

d. scatter plots

e. diagrams

f. pictorial displays

g. charts and tables

- h. Venn diagrams
- 8.D.2.4 Interpret and analyze data from graphical representations and draw simple conclusions (e.g., line of best fit).
- **8.D.2.5** Evaluate and defend the reasonableness of conclusions drawn from data analysis.
- **8.D.2.6** Use appropriate central tendency and spread as a means for effective decision-making in analyzing data and outliers.
- **8.D.2.7** Identify simple graphic misrepresentations and distortions of sets of data (e.g., unequal interval sizes, omission of parts of axis range, scaling).
- **8.D.2.8** Use appropriate technology to display data as lists, tables, matrices, graphs, and plots and to analyze the relationships of variables in the data displayed.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

5-8 Benchmark D.3: Develop and evaluate inferences and predictions that are based on data.

	nance Standards
5.D.3.1	Make and justify valid inferences, predictions, and arguments based on statistical analysis.
5.D.3.2	Compare a given prediction with the results of an investigation.
5.D.3.3	Use counting strategies to determine all the possible outcomes of a particular familiar event.
5.D.3.4	Find all possible outcome sets involving four or more sets of objects.
5.D.3.5	Evaluate the reasonableness of inferences that are based on data in the context of the original solution.
5.D.3.6	Identify the method used to make an inference and/or a prediction on a given data set and solve similar problems.
5.D.3.7	Determine the accuracy of a prediction or an inference based on the accuracy of the data in a given data set.
5.D.3.8	List all possible outcomes of simple events.
5.D.3.1	Identify claims based on statistical data and evaluate the validity of the claim
D.3.2	Conduct observations, surveys, experiments and/or simulations, record the results in charts, tables, or graphs, and use the
	results to draw conclusions and make predictions.
	Find all possible combinations in a given set (e.g., the number of ways a set of books can be arranged on a shelf).
5.D.3.4	Compare expected results with actual results in a simple experiment.
'.D.3.1	Formulate and justify mathematical conjectures based on data and a general description of the mathematical question or problem posed.
.D.3.2	Analyze data to make accurate inferences, predictions, and to develop convincing arguments from data displayed in a variety of forms.
.D.3.3	Approximate a line of best fit for a data set in a scatter plot form and make predictions using the simple equation of that line.
.D.3.1	Describe how changes in scale, intervals, or categories influence arguments for a particular interpretation of the data.
3.D.3.2	Describe how reader bias, measurement errors, and display distortion can affect the interpretation of data, predictions, and inferences based on data.
.D.3.3	Conduct simple experiments and/or simulations, record results in charts, tables, or graphs, and use the results to draw
	conclusions and make predictions.
.D.3.4	Compare expected results with experimental results and information used in predictions and inferences.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

5-8 Benchmark D.4: Understand and apply basic concepts of probability.

Performance Standards

5.D.4.1 Determine probabilities through experiments and/or simulations and compare the results with mathematical expressions.

- 5.D.4.2 Make predictions from the results of student-generated experiments of single events.
- **5.D.4.3** Identify simple experiments where the probabilities of all outcomes are equal.
- 5.D.4.4 Describe and predict the results of a probability experiment.
- 5.D.4.5 Use fractions to describe the results of an experiment.
- 5.D.4.6 Use probability to generalize from a simple pattern or set of examples and justify why the generalization is reasonable.
- **6.D.4.1** List all possible outcomes for a compound event composed of two independent events and recognize whether an outcome is certain, impossible, likely, or unlikely.
- **6.D.4.2** Determine and compare experimental (empirical) and mathematical (theoretical) probabilities (e.g., flipping two color counters).
- **6.D.4.3** Determine theoretical and experimental probabilities and use them to make predictions about events.
- **6.D.4.4** Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.
- 6.D.4.5 Use data to estimate the probability of future events (e.g., batting averages).
- **6.D.4.6** Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1- P is the probability of the event not occurring.
- **6.D.4.7** Describe the difference between independent and dependent events and identify situations involving independent or dependent events.
- 7.D.4.1 Determine the probability of a compound event composed of two independent events.
- 7.D.4.2 Identify examples of events having the probability of one or zero.
- 7.D.4.3 Describe the probability of events using fractions, decimals, and percents.
- **7.D.4.4** Express probability as a fraction, zero, or one.
- 7.D.4.5 Use probability to generate convincing arguments, draw conclusions, and make decisions in a variety of situations.
- 7.D.4.6 Make predictions based on theoretical probabilities of compound events.

7.D.4.7 Determine the probability of a simple event or a compound event composed of a simple, independent events.

8.D.4.1 Calculate the odds of a desired outcome in a simple experiment.

8.D.4.2 Design and use an appropriate simulation to estimate the probability of a real-world event (e.g., disk toss, cube toss).

8.D.4.3 Explain the relationship between probability and odds and calculate the odds of a desired outcome in a simple experiment.

8.D.4.4 Use theoretical or experimental probability to make predictions about real-world events.

8.D.4.5 Use probability to generate convincing arguments, draw conclusions, and make decisions in a variety of situations.

8.D.4.6 Understand that the probability of two unrelated events occurring is the sum of the two individual possibilities and that the

probability of one event following another, in independent trials, is the product of the two probabilities.

The 9-12 Standards are the final draft of a sanctioned revision that is awaiting formal approval.

Strand: ALGEBRA, FUNCTIONS, AND GRAPHS

Standard: Students will understand algebraic concepts and applications.

9-12 Benchmark.A.1: Represent and analyze mathematical situations and structures using algebraic symbols.

Performanc	Performance Standards	
9-12.A.1.1	Use the special symbols of mathematics correctly and precisely.	
9-12.A.1.2	Classify and use equivalent representations of natural, whole, integer, rational, irrational numbers and complex	
	numbers, and choose which type of number is appropriate in a given context.	
9-12.A.1.3	Determine the relative position on the number line and the relative magnitude of integers, decimals, rationals,	
	irrationals, and numbers in scientific notation.	
9-12.A.1.4	Explain that the distance between two numbers on the number line is the absolute value of their difference.	
9-12.A.1.5	Use a variety of computational methods, recognize when an estimate or approximation is more appropriate than an	
	exact answer, and understand the limits on precision of approximations.	
9-12.A.1.6	Simplify numerical expressions using the order of operations, including integer exponents.	
9-12.A.1.7	Translate verbal statements into algebraic expressions or equations.	
9-12.A.1.8	Solve formulas for specified variables.	
9-12.A.1.9	Solve quadratic equations in one variable.	
9-12A.1.10	Solve radical equations involving one radical.	
9-12.A.1.11	Describe the properties of rational exponents and apply these properties to simplify algebraic expressions.	
9-12.A.1.12	Explain and use equivalent representations for algebraic expressions (e.g., simplify using the distributive property).	
9-12.A.1.13	Simplify rational expressions by factoring and reducing to lowest terms.	
	Evaluate polynomial, rational, radical, and absolute value expressions for one or more variables.	
	Compare and order polynomial expressions by degree.	
9-12.A.1.16	Factor polynomials of various types (e.g., difference of squares, perfect square trinomials, sum and difference of	
	cubes).	
9-12.A.1.17	Solve linear equations and inequalities in one variable including those involving the absolute value of a linear function.	
	Use the four basic operations $(+, -, \times, \div)$ with linear, polynomial, and rational expressions in contextual situations.	
9-12.A.1.19	Use the four basic operations $(+, -, \times, \div)$ in contextual situations with numbers in scientific notation, and express the	
	results with the appropriate number of significant figures.	

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Strand: ALGEBRA, FUNCTIONS, AND GRAPHS

Standard: Students will understand algebraic concepts and applications.

9-12 Benchmark A.2: Understand patterns, relations, functions, and graphs.

Performance Standards

- **9-12.A.2.1** Distinguish between the concept of a relation and a function.
- **9-12.A.2.2** Determine whether a relation defined by a graph, a set of ordered pairs, a table of values, an equation, or a rule is a function.
- 9-12.A.2.3 Translate among tabular, symbolic, and graphical representations of functions and relations.
- 9-12.A.2.4 Construct a linear function that represents a given graph.
- **9-12.A.2.5** Explain and use function notation in both abstract and contextual situations and evaluate a function at a specific point in its domain.
- 9-12.A.2.6 Graph a linear equation and demonstrate that it has a constant rate of change.
- **9-12.A.2.7** Graph a linear inequality in two variables.
- 9-12.A.2.8 Graph a quadratic function and understand the relationship between its real zeros and the x-intercepts of its graph.
- 9-12.A.2.9 Graph exponential functions and identify their key characteristics as related to contextual situations.
- 9-12.A.2.10 Identify and describe symmetries of graphs.
- **9-12.A.2.11** Use the quadratic formula and factoring techniques to determine whether the graph of a quadratic function will intersect the *x*-axis in zero, one, or two points (include quadratic functions that represent real phenomena).
- 9-12.A.2.12 Explain the meaning of the real and complex roots of quadratic functions in contextual situations.
- **9-12.A.2.13** Read information and draw conclusions from graphs, and identify properties of a graph that provide useful information about the original problem.
- **9-12.A.2.14** Understand the relationship between the coefficients of a linear equation and the slope and x- and y- intercepts of its graphs.
- 9-12.A.2.15 Evaluate estimated rate of change in a contextual situations.

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Strand: ALGEBRA, FUNCTIONS, AND GRAPHS

a slope.

Standard: Students will understand algebraic concepts and applications.

9-12 Benchmark A.3: Use mathematical models to represent and understand quantitative relationships.

9-12.A.3.1 Model real-world phenomena using linear equations and linear inequalities interpret resulting solutions, and use estimation to detect errors. 9-12.A.3.2 Model real-world phenomena using quadratic equations, interpret resulting solutions, and use estimation to detect errors. 9-12.A.3.3 Model real-world phenomena using exponential equations, interpret resulting solutions, and use estimation to detect errors. 9-12.A.3.4 Solve systems of linear equations in two variables algebraically and graphically 9-12.A.3.5 Solve applications involving systems of two equations in two variables. 9-12.A.3.6 Write an equation of the line that passes through two given points. 9-12.A.3.7 Verify that a point lies on a line, given an equation of the line, and be able to derive linear equations given a point and

9-12.A.3.8 Determine whether the graphs of two given linear equations are parallel, perpendicular, coincide or none of these.

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Strand: GEOMETRY AND TRIGONOMETRY

Standard: Students will understand geometric concepts and applications.

9-12 Benchmark G.1: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Performance Standards	
9-12.G.1.1	Understand that numerical values associated with measurements of physical quantities must be assigned units of
	measurement or dimensions; apply such units correctly in expressions, equations and problem solutions that involve
	measurements; and convert a measurement using one unit of measurement to another unit of measurement.
9-12.G.1.2	Find the area and perimeter of a geometric figure composed of a combination of two or more rectangles, triangles,
	and/or semicircles with just edges in common.
9-12.G.1.3	Draw three-dimensional objects and calculate the surface areas and volumes of these figures (e.g. prisms, cylinders,
	pyramids, cones, spheres) as well as figures constructed from unions of prisms with faces in common, given the
	formulas for these figures.
9-12.G.1.4	Identify the hypothesis and conclusion in examples of conditional statements.
9-12.G.1.5	Use definitions in making logical arguments.
9-12.G.1.6	Use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to
	refute a universal statement.
9-12.G.1.7	Explain the difference between inductive and deductive reasoning and provide examples of each.
9-12.G.1.8	Explain why, for inductive reasoning, showing a statement is true for a finite number of examples does not show it is
	true for all cases unless the cases verified are all possible cases.
9-12.G.1.9	
	related to: theorems involving the properties of parallel and perpendicular lines, circles, and polygons; theorems
	involving complementary, supplementary, and congruent angles; theorems involving congruence and similarity; and
	the Pythagorean theorem.
9-12.G.1.10	Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true.

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Strand: GEOMETRY AND TRIGONOMETRY

Standard: Students will understand geometric concepts and applications.

9-12 Benchmark G.2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Performance Standards	
9-12.G.2.1	Identify the origin, coordinate axes, and four quadrants on the Cartesian coordinate plane, and draw and label them
	correctly.
9-12.G.2.2	Determine the midpoint and distance between two points within a coordinate system and relate these ideas to geometric
	figures in the plane (e.g., find the center of a circle given the two points of a diameter of the circle).
9-12.G.2.3	Use basic geometric ideas (e.g., the Pythagorean theorem, area and perimeter) in the context of the Cartesian
	coordinate plane (e.g., calculate the perimeter of a rectangle with integer coordinates and with sides parallel to the
	coordinate axes, and of a rectangle with sides not parallel).

Strand: GEOMETRY AND TRIGONOMETRY

Standard: Students will understand geometric concepts and applications.

9-12 Benchmark G.3: Apply transformations and use symmetry to analyze mathematical situations.

Performance Standards

- **9-12.G.3.1** Use rigid motions (compositions of reflections, translations and rotations) to determine whether two geometric figures are congruent in a coordinate plane.
- 9-12.G.3.2 Sketch a planar figure that is the result of given transformations (i.e., translation, reflection, rotation, and/or dilation).
- 9-12.G.3.3 Identify similarity in terms of transformations.
- 9-12.G.3.4 Determine the effects of transformations on linear and area measurements of the original planar figure.

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Strand: GEOMETRY AND TRIGOMETRY

Standard: Students will understand geometric concepts and applications.

9-12 Benchmark G.4: Use visualization, spatial reasoning, and geometric modeling to solve problems.

Performance Standards	
9-12.G.4.1	Solve contextual problems using congruence and similarity relationships of triangles (e.g., find the height of a pole
	given the length of its shadow).
9-12.G.4.2	Solve problems involving complementary, supplementary, and congruent angles.
9-12.G.4.3	Know that the effect of a scale factor k on length, area and volume is to multiply each by k, k^2 and k^3 , respectively.
9-12.G.4. 4	Solve problems using the Pythagorean theorem.
9-12.G.4. 5	Understand how similarity of right triangles allows the trigonometric functions sine, cosine and tangent to be defined
	as ratios of sides and be able to use these functions to solve problems.
9-12.G.4.6	Apply basic trigonometric functions to solve right-triangle problems.
9-12.G.4.7	Use angle and side relationships in problems with special right triangles (e.g., 30-, 60-, 90-, and 45-, 45-, 90- degree
	triangles).
9-12.G.4.8	Describe the intersections of a line and a plane, intersections of lines in the plane and in space, or of two planes in
	space.

The 9-12 Standards are the final draft of a sanctioned revision that is awaiting formal approval.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

9-12 Benchmark D.1: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Performance Standards	
9-12.D.1.1	Explain the differences between various methods of data collection.
9-12.D.1.2	Describe the characteristics of a well-designed and well-conducted survey by differentiating between sampling and
	census, and a biased and unbiased sample.
9-12.D.1.3	Describe the characteristics of a well-designed and well-conducted experiment by differentiating between experiments
	and observational studies, and recognizing the sources of bias in poorly designed experiments.
9-12.D.1.4	Explain the role of randomization in well-designed surveys and experiments.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

9-12 Benchmark D.2: Select and use appropriate statistical methods to analyze data and make predictions.

Performance Standards

- **9-12.D.2.1** Distinguish measurement data from categorical data, and define the term *variable*.
- 9-12.D.2.2 Explain the meaning of *univariate* and *bivariate* data.
- **9-12.D.2.3** Display the distribution of univariate data, describe its shape using appropriate summary statistics, and understand the distinction between a statistic and a parameter.
- 9-12.D.2.4 Calculate and apply measures of variability (e.g., standard deviation).
- 9-12.D.2.5 Compare distributions of univariate data using back-to-back stem and leaf plots and parallel box and whisker plots.
- 9-12.D.2.6 Describe the characteristics of a normal distribution.
- **9-12.D.2.7** Compare and draw conclusions between two or more sets of univariate data using basic data analysis techniques and summary statistics.
- 9-12.D.2.8 Describe the shape of a scatterplot.
- 9-12.D.2.9 Use linear patterns in data to make predictions.
- 9-12.D.2.10 Use technological tools to find the line of best fit.

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9-12.D.2.11 Describe the relationship between two variables and determine its strength with and without technological tools.

- **9-12.D.2.12** Explain why correlation does not imply a cause-and-effect relationship.
- **9-12.D.2.13** Use the results of simulations to explore the variability of sample statistics from a known population and construct sampling distributions.
- **9-12.D.2.14** Describe how sample statistics, including the law of large numbers, reflect the values of population parameters and use sampling distributions as the basis for informal inference.
- **9-12.D.2.15** Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions.

Strand: DATA ANALYSIS AND PROBABILITY

Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

9-12 Benchmark D.3: Understand and apply basic concepts of probability.

Performance Standards	
9-12.D.3.1	Explain the concept of a random variable.
9-12.D.3.2	Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the
	outcome.
9-12.D.3.3	Use the results of simulations to compute the expected value and probabilities of random variables in simple cases.
9-12.D.3.4	Compute the probability of an event using the complement rule, addition rule for disjoint and joint events,
	multiplication rule for independent events, and rules for conditional probability.

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Strand: ALGEBRA, FUNCTIONS, AND GRAPHS Guidance for Further Study

9-12 Topics for Further Study

9-12.AX.1 Solving equations, inequalities and systems

(As students encounter ever more sophisticated mathematical situations, they will need to be able to generate and solve a variety of equations, inequalities, and systems. They begin by studying more complex linear and quadratic equations and systems. Students should understand that to solve certain problems and equations one must go outside of the real number system to the complex number system.) Students will be able to:

9-12.AX.1.1 Solve three-by-three linear systems.

9-12.AX.1.2 Solve two-by-two linear-quadratic and quadratic-quadratic systems.

9-12.AX.1.3 Solve and graph linear and quadratic equations and inequalities involving absolute value.

9-12.AX.1.4 Solve quadratic inequalities by factoring.

9-12.AX.2 Polynomials and conics

(Students will extend the concept of solving linear equations to higher degree polynomials. These polynomials can be used to more accurately describe real-world phenomena.) Students will be able to:

- **9-12.AX.2.1** Factor polynomials of degree higher than two using the fundamental theorem of algebra (i.e., an n^{th} degree polynomial has at most *n* distinct linear factors), integral and rational zero theorems, and factor and remainder theorems.
- **9-12.AX.2.2** Graph 2nd, 3rd and 4th degree polynomials using the intermediate value theorem.
- **9-12.AX.2.3** Graph, interpret, and find the equations for conic sections with axes parallel to the coordinate axes, and apply them to contextual situations.
- 9-12.AX.2.4 Describe Pascal's triangle and use it to expand binomial expressions that are raised to positive integer powers.
- **9-12.AX.2.5** Analyze the general shape of polynomial functions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials).
- **9-12.AX.2.6** Identify the relationship between the degree of a polynomial function and the maximum number of turning points in its graph, and the relationship between the leading term and the behavior of the function for large values of |x|.

9-12.AX.3 Functions

(The language and properties of functions are essential to understanding the components of higher mathematics. Functions are the fundamental objects on which students operate in some higher mathematics and are among the building blocks of higher mathematics.) Students will be able to:

- **9-12.AX.3.1** Determine the composition of functions (e.g., find f(g(x)) when f(x) = 2x 3 and g(x) = 3x 2), and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.
- 9-12.AX.3.2 Find and use inverse functions involving ordered pairs, graphs, and explicit statements of a function rule.
- **9-12.AX.3.3** Determine the domain and range of linear and quadratic functions expressed symbolically and graphically, and of functions defined by a finite set of ordered pairs in mathematical and contextual situations.
- 9-12.AX.3.4 Examine and graph piece-wise defined functions, including the use of the properties of continuity and discontinuity.

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- **9-12.AX.3.5** Analyze and describe middle and end (i.e., asymptotic) behavior of linear, quadratic, and exponential functions, and sketch their graphs.
- **9-12.AX.3.6** Graph rational functions with factorable numerators and denominators and locate zeros and horizontal and vertical asymptotes.
- **9-12.AX.3.7** Analyze the effects of parameter changes on these functions: quadratic (e.g., $f\{x-a\}$, changes in coefficients and constants), exponential (e.g., changes caused by changing the base or the exponent), and polynomial (e.g., changes caused by positive or negative values of $\pm a_n$, or in a constant a_0).

9-12.AX.4 Logarithms and exponential functions

(Logarithms and exponential functions provide tools for more sophisticated modeling and applications for understanding real-life phenomena. Higher mathematics requires regular and successful use of logs and exponents to move beyond polynomials.) Students will be able to:

9-12.AX.4.1 Prove basic properties of a logarithm using properties of its inverse.

9-12.AX.4.2 Apply exponential functions and common and natural logarithms to understand real-life situations (e.g., half-life, amortization, logistic growth).

9-12.AX.4.3 Apply logarithms and exponents to solve equations.

9-12.AX.5 Series and sequences

As students progress toward higher mathematics, they will need an understanding of sequences and functions whose domains are sets of whole numbers as opposed to sets of real numbers (e.g., discrete functions versus continuous functions). Infinite geometric series provide one way to begin a discussion about limits.) Students will be able to:

9-12.AX.5.1 Use algebraic techniques to generate the specific formulas for arithmetic and geometric sequences and series.

9-12.AX.5.2 Extend the concept of series to infinite geometric series.

9-12.AX.5.3 Use the language and notation of limits.

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Strand: GEOMETRY AND TRIGONOMETRY Guidance for Further Study

9-12 Topics for Further Study

9-12.GX.1 Trigonometry concepts

(Trigonometry allows a student to consider periodic functions.) Students will be able to:

- **9-12.GX.1.1** Graph all six trigonometric functions using radian measure, their domains and ranges, and the exact values of the six trigonometric functions at the five angles: $0, \pi/2, \pi, 3\pi/2, 2\pi$.
- **9-12.GX.1.2** Determine the exact values of the six trigonometric functions at 0°, 30°, 45°, 60° and 90° (and the corresponding radian measures).
- 9-12.GX.1.3 Explain trigonometric functions as circular functions using symmetry.
- **9-12.GX.1.4** Solve linear and quadratic trigonometric equations of a single function algebraically and other trigonometric equations with technological tools.
- 9-12.GX.1.5 Verify trigonometric identities, such as $\sin^2(x) + \cos^2(x) = 1$ and $\cos(\pi/2-x) = \sin(x)$, and formulas for sine and cosine, such as addition and double angle formulas.
- 9-12.GX.1.6 Apply trigonometric functions and the law of sines and cosines to solve contextual problems.

MATHEMATICS GLOSSARY

Being Revised