Teaching High School Age Students with Special Needs the Four Situations for Subtraction

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Abstract

All learners, including those who qualify for special education services, should have access to learning mathematical concepts. This study examined the efficacy of using hands-on sets of materials to teach two high school students with mental retardation the four situations for subtraction. This is a mixed methods study using a simple pretest-posttest design to determine correct interpretation of subtraction story problems for different situations (take-away, comparison, completion, and whole-part-part) and correct regrouping during dynamic subtraction. Eight story problems were presented to the two participants eight weeks prior to instruction; students answered an identical posttest after twelve half-hour lessons on subtraction story problems and regrouping, which occurred during a two-and-a-half week period. Qualitative teacher observations of the two students were used to triangulate the data. On the pretest, students scored poorly, with one student completing none of the eight items correctly and the other completing three. The non-take-away subtraction situations confused the students and they made many regrouping errors during dynamic subtraction. On the posttest, both students solved all the problems correctly. Teacher observations indicated that students found the hands-on materials engaging and motivating, using the manipulatives to help them during regrouping. The materials and method of teaching were effective for the following reasons: the sets of objects suited the concrete developmental stage of the students; a variety of colorful and exciting materials were used to pique and sustain student interest; the teacher modeled and verbally explained how to solve the problems using the hands-on materials; the teacher encouraged interaction and elicited students explanations; and students were moved from the concrete manipulatives to the more abstract nature of word problems. The authors recommend that these materials not only be used with primary grade students to teach subtraction situations, but with other students with special needs (19 references; 2 tables, one containing photographs of materials and the other showing the pretest/posttest questions).

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Introduction

All Learners Need Access to Mathematics

The National Council of Teachers of Mathematics (2000) supports providing all youth equal access to mathematical concepts. Similarly, the ultimate goal for a person with cognitive disabilities is the same as the goal for any other person: the ability to solve practical problems on that person's ability level (Gable, Evans & Evans, 1993 as cited in Thomas, 1996). To achieve this goal, problem solving must be emphasized at every level. Our study addresses the mathematical needs of two high school students with mental retardation enrolled in a life skills class who required practice in discerning everyday situations in which subtraction may be used to solve problems.

The Importance of Manipulatives to Learning Mathematics

The National Council of Teachers of Mathematics' theme for the 2006-07 school year was "Show me the math: Learning through representation" (2006). The process of representation includes using models to organize, record, and communicate mathematical ideas. These models may be manipulative materials that can be used to "show math." The students in our study used sets of manipulatives to understand the different situations that call for subtraction: take-away, comparison, completion, and whole-part-part.

Students with cognitive disabilities often have difficulties with mathematics (Algozzine, O'Shea, Crews, & Stoddard, 1987; Cawley, Baker-Kroczynski, & Urban, 1992). Devlin (2000) stated that for students to learn abstract concepts more readily, it is important for them to learn concepts in a familiar and concrete manner first. One way to do this is through concrete, hands-on activities. Numbers can be experienced and the relationships between them can be made concrete by using manipulatives (Bell & Tuley, 2003).

Manipulatives are defined as materials that are physically handled by students to help them see actual examples of mathematical principles at work. Manipulatives, though most commonly employed in elementary grades, can be very useful in middle and secondary education, if they are wisely planned and executed to build a firm, concrete model for abstract mathematical concepts (Jones, 2003). Individual students learn in different ways. When manipulatives are used, the senses are brought into learning: students can touch and move objects to make visual

representations of mathematical concepts. They can be used to represent both numbers and operations on those numbers (Teacher2Teacher - T2T, 2007).

The manipulatives used in this study were attractive, colorful sets of small items and accompanying story problems that illustrated subtraction situations sometimes neglected by teachers who emphasize subtraction as take-away. However, subtraction is used in situations which do not call for the removal of objects or quantities. Because these situations occur in everyday life and because students need to know how to use subtraction to solve comparison, completion, and whole-part-part problems, we focused our work in this area.

Motivating Students Intrinsically to Learn Mathematics

We modeled our hands-on subtraction situation activities after interesting sets of subtraction materials featured in an article in a *Mathematics Teaching* (Rule, 2005). We wanted our two students to be intrinsically motivated in their mathematics work; therefore, we tried to design our activities to satisfy the four criteria identified by Malone and Lepper (1987). These four components can be present in any individual learning situation: challenge, control, curiosity, and fantasy. These aspects of our materials and activities are discussed in the following paragraphs.

Students prefer to work at an optimal level of challenge of intermediate difficulty, one with a goal that is attainable but uncertain. This concept of challenge at the right level corresponds to Strong, Silver, Perini, and Tuculescu's (2003) intrinsic drives toward mastery and understanding. Activities that are too easy do not generate interest, just as those that are too difficult cause students to give up and shut down. Activities that are developmentally appropriate – that build on underlying skills, but increase the difficulty by adding something new, provide an appropriate level of challenge.

Control was achieved by allowing students choice in the set of materials with which they worked, by providing several sets of materials within each box, and by providing modeling and support from the teacher during the exercises. Curiosity was generated when the teacher introduced the six boxes of colorful and unusual hands-on materials (glossy ribbons, shiny plastic pieces, glittery pompoms in colorful eggs, glassy flat marbles, bright metallic coins, striped straws of varying lengths). Fantasy was addressed as students read the story problems and used the manipulatives to enact the situations or as students pretended or imagined extensions of the activities.

Finally, learners are motivated when they see the value of their learning (Pintrich & Schrunk, 1996). After working with the materials, students thought of everyday situations in which these same subtraction situations occur, applying the concepts to their own lives.

Method

This study is a mixed methods case study of two high school age students with special needs learning the different situations for subtraction through hands-on materials. Students took identical pretest-posttests on subtraction story problems and the teacher made observations of student progress. Both sources were used to triangulate the data and determine the efficacy of this approach to teaching subtraction story problems.

Students took the pretest about eight weeks prior to instruction, participated in twelve half-hour lessons using the materials over a period of two and a half weeks, and then took the posttest after the final lessons.

Two male students with mental retardation who attended social skills classes at a rural high school in New York State participated in this study. The students were ages 16 and 17 and were a high school sophomore and junior respectively. Permission was obtained from the parents, school principal, and the Committee on Research Regarding Human Subjects of the overseeing university. Pseudonyms are used for the two students in this article.

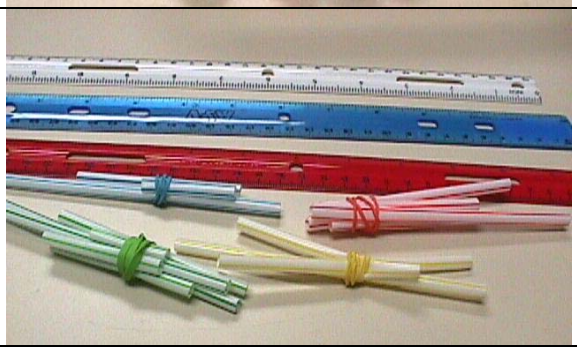
Curriculum Materials

The teacher developed six sets of curriculum materials to illustrate the less-commonly taught subtraction situations of comparison, completion, and whole-part-part. She followed recommended guidelines for developing and using manipulatives for students with special needs (Friend & Bursuck, 2006; Marzola, 1987; Ross & Kurtz, 1993). She incorporated dynamic subtraction into the problems so that students would have ample practice with regrouping. These teacher-made materials are explained in Table 1.

Table 1. Curriculum materials developed to teach subtraction situations

Animal Print Ribbons Subtraction Situation: Comparison Directions: Choose one ribbon of each color: one black and one white. Compare the number of paw prints and bones on the two ribbons. How many more paw prints there are than bones?	
Treasure Bags Subtraction Situation: Completion Choose a treasure bag. Write on your paper the number that is on the tag that goes with the bag. Open the bag and count the number of coins. How many more coins do you need to have the number written on the bag's tag?	

Table 1. Continued: Curriculum materials developed to teach subtraction situations

Marbles Subtraction Situation: Whole-Part-Part Select a small container of marbles. Count the total number of marbles in the container. This is the number of marbles in the whole set. Then count just the clear marbles (a part of the set). Now put the marble back in the container. Determine how many blue marbles are in the set by subtraction.	
Circles Subtraction Situation: Whole-Part-Part Select a small bag of circles. Count the total number of circles in the bag. This is the number of circles in the whole set. Then count just the green circles (a part of the set). Now put the circles back in the bag. Determine how many purple circles are in the set by subtraction.	
Eggs Subtraction Situation: Completion Choose an egg. Write the number that is on the egg on your paper. Open the egg and count the number of pompoms. How many more pompoms do you need to have the number written on the egg?	
Straws Subtraction Situation: Comparison Directions: Choose one bundle of straws. Arrange the straws in order from shortest to longest. How much longer is the longest straw than the shortest straw?	

Results

Pretest and Posttest Performance

Table 2 shows the pretest and posttest responses and scores of the two students.

Table 2. Pretest/posttest questions and responses.

Question	Situation	Correct Response	Student 1 = Hal			Student 2 = Tommy		
			Pretest Response	Pretest Correct?	Posttest Correct?	Pretest Response	Pretest Correct?	Posttest Correct?
1. What is 10 take away 3?	Take-away	10-3=7	10-3=13	No	Yes	2	No	Yes
2. What is 24 take away 16?		24-16=8	24-16=12	No	Yes	Than 25	No	Yes
3. How much larger is 27 than 8?	Comparison	27-8=9	27+8 = 21	No	Yes	30	No	Yes
4. How much larger is 32 than 24?		32-24=8	32+24=56	No	Yes	8	Yes	Yes
5. Joe has 23 kids in his class. If he has made 14 cupcakes, how many more does he need to give one to each person?	Completion	23-14=9	23+14=37	No	Yes	9	Yes	Yes
6. Steve wants 21 books. If Sara gave him 12, how many more does he need?		21-12=9	21+12=33	No	Yes	12	No	Yes
7. Megan has 42 red or green holiday candies. If 23 are red, how many are green?	Whole-Part-Part	42-23=19	23	No	Yes	20	No	Yes
8. Robin has a charm bracelet with 13 charms. If 4 are broken, how many are not broken?		13-4=9	13	No	Yes	13-4=9	Yes	Yes
Total Correct out of 8 questions				0/8	8/8		3/8	8/8

On the pretest, both students' performances were poor because of their lack of understanding of the subtraction situations and their inability to regroup during subtraction. The first student, Hal, was not able to solve any of the problems because he interpreted four of the problems as addition (questions 3-6), or subtracted incorrectly by taking the smallest digit in each place value column and subtracting it from the larger digit (question 2), or added the minuend and subtrahend (question 1), or guessed one of the given parts (questions 7-8). He was confused by wording in two of the problems that asked "which is larger" or "how much more?" and concluded that addition was the correct operation. Even when this student wrote an equation as a subtraction problem for question 1, he added the minuend and subtrahend instead of subtracting them, perhaps because he was more confident in adding two numbers. Another possibility is that all of the problems required regrouping and the student may have been unsure of how to regroup.

The second student, Tommy, was able to solve three of the problems correctly

on the pretest. This student's incorrect responses did not always show a discernable pattern partly because he did not usually record the equations used, but many errors were the result of incorrect counting on the fingers. On the third question, he used his fingers to determine the difference and made an error in counting, writing "20" instead of "19". Questions 4 and 5 were correctly answered. Again on question 7, Tommy used his fingers to calculate the answer and was off by one. He answered the final question correctly, writing the equation correctly also.

Therefore, in summary, the problems these two students exhibited were not understanding that subtraction was called for by the problems or making errors in regrouping.

On the posttest, both students scored 8 out of 8, demonstrating their mastery of identifying situations calling for subtraction and regrouping during subtraction. The concrete manipulatives provided assisted students in understanding the necessary regrouping and the teacher gave mini-lessons on regrouping as students used the materials.

Teacher's Observations during the Math Activities

Student 1, Hal, (who scored zero on the pretest) was not motivated to participate in the lessons at first. Hal exhibited this same attitude to most subjects; the only subject he liked was physical education. This student is older (a junior) and aware of the differences between himself and more typical students his age. He would not participate in the activities unless he was alone with the teacher in the room because he didn't want others to know of his low performance level. The first time the teacher asked him to work with the subtraction situation materials, it took a long time to convince him to even get up off the floor and look at the materials. The teacher began by asking him to just work one problem from the box and several times he continued to another box, working additional problems without being asked to do so. This showed his interest in the attractive materials and his subsequent motivation. At first he had a lot of difficulty writing the equations and solving the problems. The teacher wrote on paper, with Hal dictating what to write. Finally, Hal began to write the problems himself. Although Hal wanted to use the calculator on his cell phone, the teacher insisted that he work them out himself. Hal was more willing to participate after he had tried each of the activities. He still made many careless errors, but continued to improve.

Student 2, Tommy, was willing and motivated to try the activities even as the teacher was assembling them at her desk several days in advance of the lessons. He asked what they were for and he asked to help put them together. When he had free time in his schedule, he offered to work on the subtraction situation projects rather than work on the computer or go for a walk through the building, his usual preferred activities. This was unusual and demonstrated his high level of interest in the hands-on materials. Tommy enjoyed the materials and remarked that the treasure coins were so shiny that they were almost too bright for his eyes. Tommy liked being able to choose the box with which he worked. His favorite was the treasure bag activity because this was one that he had helped assemble. The day the teacher had brought the bag of coins into the classroom in preparation for making the materials, he noticed them and became almost obsessed with them. No other work could be accomplished until Tommy had helped the teacher put the coins in the treasure bags. He asked if he could take one home and the

teacher let him keep one. This shows the intense appeal that interesting materials have for students and the motivating effect they can have.

Malone and Lepper (1987) identified four sources of intrinsic motivation: challenge at the right level, control through choice and a sense of having needed supports, curiosity, often for interesting materials, and fantasy or pretending. These components were present in the activities. These exercises represented challenge at the right level for both students because both were proficient at addition and ready to master subtraction. Both students had a sense of control in the activities because they were able to choose the box with which they worked and the specific ribbons or set of marbles, etc. that were most appealing. The teacher supported the students by modeling and explaining what to do the first time and helping them as needed. Certainly, both students were curious about the materials, although Student 1, Hal, was concerned about how others might view him. The fact that Student 2, Tommy, enjoyed the materials so much was another reason that Hal tried to avoid them; he did not want to like the things that other students who qualify for special education like. Tommy really enjoyed the colorful and exciting items, particularly the coins. He exhibited fantasy when he asked about the paw prints on the ribbons and began to suggest the types of animals that might make them.

Conclusion

Throughout this project, it was observed that hands-on materials were motivating to the students. The teacher developed six sets of curriculum materials to illustrate the less-commonly taught subtraction situations of comparison, completion, and whole-part-part. She addressed all the following guidelines for using manipulatives when working with special populations (Friend & Bursuck, 2006; Marzola, 1987; Ross & Kurtz, 1993).

- Select materials that suit the concept and the developmental stage of the students. These exercises represented challenge at the right level for both students because both had mastered addition and were able to solve static subtraction problems, but need to improve their skills with subtraction word problems related to situations other than take-away and dynamic subtraction.
- Use a variety of materials. The teacher selected visually appealing and sensorial materials in a variety of textures and colors to appeal to the students. She included two different example sets of materials for each subtraction situation.
- Use verbal explanations to accompany manipulatives. The teacher gave mini-lessons on regrouping as students used the materials.
- Encourage active interaction. The teacher assisted Hal when he was reluctant to write the problems by scribing for him until he had the confidence to do so independently. She discussed and explained the context of each set of materials. Both students eventually told their insights into the problems as they worked.
- Elicit student explanations. The teacher encouraged both Tommy and Hal to verbalize what they were doing with the manipulatives.
- Present clear guidelines for handling manipulatives. The teacher

asked students to treat the materials she had made with care so that they would be available to future students.

- Move students beyond concrete level when they are ready. The teacher encouraged students to think of other examples of comparison, completion, and whole-part-part in their daily lives. Both students were able to solve new subtraction story problems without manipulatives and correctly answer the abstract questions on the posttest that required regrouping.

These hands-on story-problems for different subtraction situations helped our high school students with special needs recognize that subtraction is required to solve problems in many different contexts. We think these materials would help students at a variety of ages and abilities, especially when the previously elaborated guidelines for manipulatives are employed. These materials have potential benefits for students with special needs who may lack knowledge and reasoning skills to understand abstractions (Smith, 2004). Therefore, we recommend that students be given the opportunity to work with hands on materials whenever possible to improve their math skills.

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