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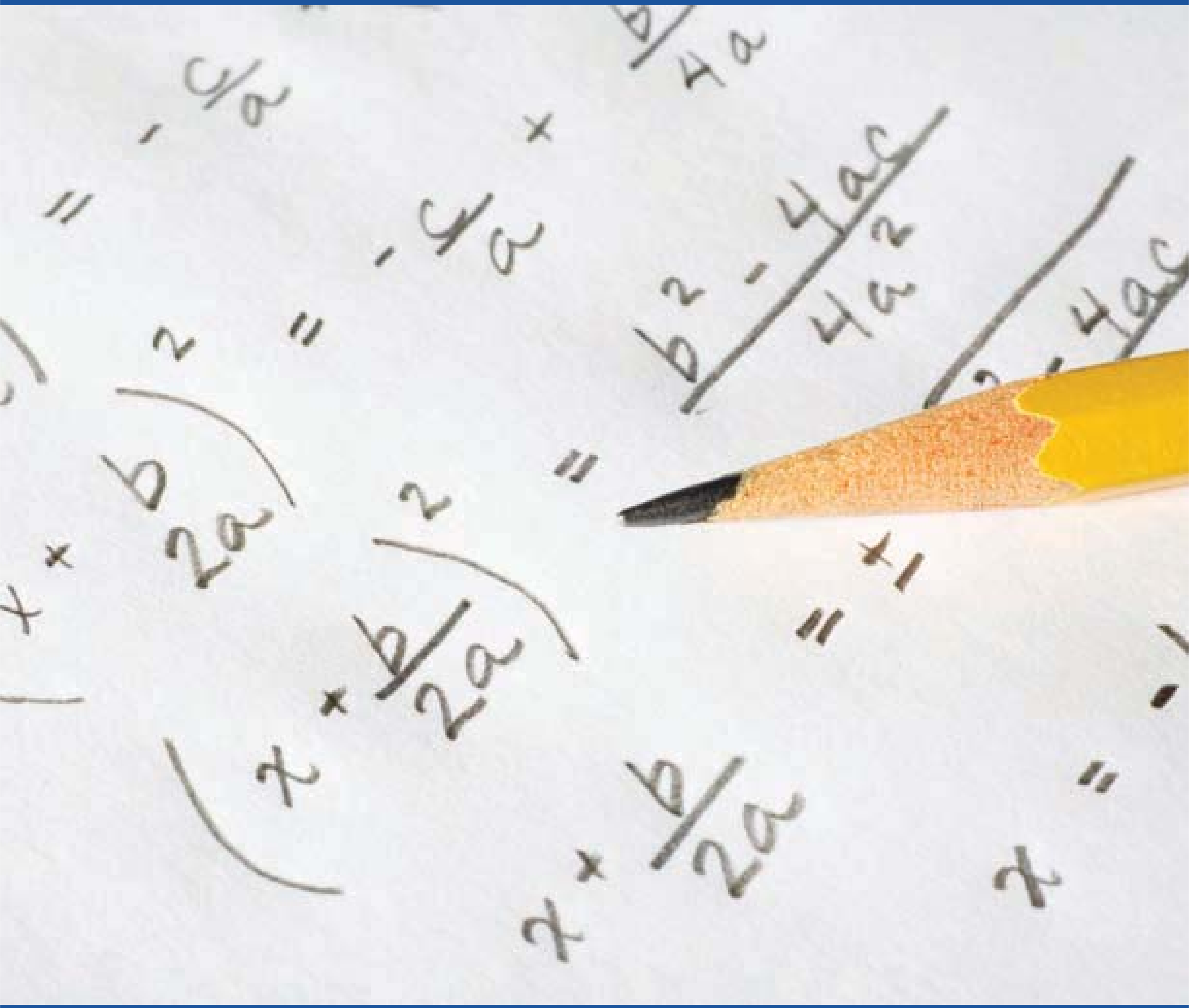
Evidence from the Education Longitudinal Study of 2002 (ELS:2002)

Statistical Analysis Report



NCES 2008-319

U.S. DEPARTMENT OF EDUCATION

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Evidence from the Education Longitudinal Study of 2002 (ELS:2002)

Statistical Analysis Report

January 2008

U.S. Department of Education
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Executive Summary

Recent research shows that U.S. 15-year-olds are behind their international counterparts in problem solving and mathematics literacy, ranking 24th of 29 nations (Lemke et al. 2004). Therefore, a key concern among policy makers and educators is improving the quantitative and analytical skills of American youth, who face job prospects in an economy that increasingly values a strong foundation in mathematics and science. One policy response has been to raise mathematics coursetaking requirements for graduation. For example, between 1987 and 2004, the number of states requiring at least 2.5 credits in mathematics for graduation increased from 12 to 26 (Council of Chief State School Officers 2004). Despite the focus on overall credit requirements, less is known about particular types of courses and their relationship with learning different types of mathematics skills and concepts—a critical piece of information for those interested in preparing American students for postsecondary training and the labor market.

Using data from the Education Longitudinal Study of 2002 (ELS:2002) this report is one of the first to examine both the course sequences that students follow during the last 2 years of high school and the level of mathematics proficiency they acquire during that period. ELS:2002 is a nationally representative longitudinal study of American students who were in the 10th grade in 2002. Students, their parents, teachers, and school administrators were interviewed and mathematics assessments were administered to students in the spring of 2002. Students were reinterviewed and retested in mathematics in the spring of 2004. Their transcripts were collected in the 2004–05 school year.

In this analysis, high school transcript information and mathematics assessment scores are used to examine coursetaking patterns and learning gains across sociodemographic characteristics of students and the types of schools they attend. These coursetaking patterns are then linked with learning gains to identify the concepts and skills learned by students who follow a particular course sequence. Differences are only reported if the comparisons were statistically significant (using *t* statistics with an *alpha* criterion of .05) and met the effect size criteria (using effect sizes [standardized mean differences] that are greater than 0.20 standard deviations for continuous variables and 5 percentage points for categorical variables). Findings from regression analyses are only reported if the coefficients have a *p* value of .05 or less. The main findings are summarized below.

Over the last 2 years of high school, students improved their mathematics skills. At the end of their senior year, students gave an average of 51.2 correct answers (out of 81 possible correct answers) on the mathematics assessment, compared to an average of 46.7 correct answers during their sophomore year—a gain of about 5 correct answers (about a third of a standard deviation). Because most students (94 percent) entered the second half of high school with a mastery of basic mathematics skills such as simple arithmetic and operations, most of their learning during this time was in intermediate-level mathematics skills and concepts. Specifically, the percentage of students with an understanding of simple problem solving skills grew from 53 to 65 percentage points over the second half of high school. Students learned very little of the most advanced skills such as solving multistep word problems and applying analytic logic: 96 percent of the students in the sample left high school without proficiency at this advanced level. As with many educational outcomes, learning levels and learning gains were associated with the sociodemographic characteristics of students and the types of schools they attended. High socioeconomic status (SES) students, students who attended Catholic or other private schools, and students who expected to earn a bachelor's degree exhibited gains in the most advanced areas and showed levels of proficiency at the most advanced levels at the end of high school.

Next, student transcripts were examined to understand both the types of courses that students were taking and how they relate to learning mathematics. Course sequences were identified in terms of the types of courses taken during the 2002–03 and 2003–04 school years—the 2 academic years between the mathematics assessments. The most common mathematics sequences taken during this time period were algebra II–no mathematics, followed by 13 percent of students; geometry–geometry/no mathematics, followed by 8 percent of students; and algebra II–precalculus, followed by 7 percent of students. In accord with previous research on coursetaking patterns, the most advanced course sequences—precalculus–calculus and precalculus–Advanced Placement/International Baccalaureate calculus—were more likely to be followed by Asian and White students, high SES students, students who live with both parents in the family, students who attended Catholic schools, and students who expected to earn a bachelor's degree.

While past research has shown that more advanced courses or curricular tracks are associated with aggregate gains in learning, it has not identified the specific courses related to this growth. Toward this end, this analysis links course sequences with gains in mathematics proficiencies at different levels. The findings show that the largest overall gains are made by students who take precalculus paired with another course during the last 2 years of high school. In terms of learning in specific content areas, the largest gains in intermediate skills such as simple operations and problem solving were made by those who followed the geometry–algebra II sequence. The largest gains in advanced skills such as derivations and making inferences from algebraic expressions were made by students who took precalculus paired with another course. The smallest gains were made by students who took one mathematics course or no mathematics courses during their last 2 years.

While the findings reported here corroborate other research on the topic, readers should keep in mind that without an experimental design, establishing a causal link between coursetaking and learning is not possible. Also, ELS:2002 provides only observational data: students were not randomly assigned to schools, classrooms, course sequences, or teachers. As a consequence, establishing a causal link between coursetaking and achievement is not possible. Additionally, the analysis requires test scores at two different periods of time (sophomore and senior years), thereby excluding students who had dropped out, transferred schools, or started homeschooling.

The resulting analytic sample includes a higher proportion of students who are White, a higher proportion of students who expect to receive a bachelor's degree or higher, and a higher proportion of students living with both their father and their mother than the full sophomore panel. Thus, the findings may not generalize to all students, particularly those who are non-White, those who have educational expectations that do not include college completion, and those who are not living with their mother and father. Readers should keep these caveats in mind when interpreting the results.

Foreword

The Education Longitudinal Study of 2002 (ELS:2002) provides a wealth of information from multiple sources about the factors and circumstances related to the performance and social development of the American high school student over time. This statistical analysis report uses information from student surveys, mathematics assessments, and high school transcripts to examine how mathematics achievement across the last 2 years of high school is patterned across American high school students. This analysis also provides new information on the mathematics courses that students take during their junior and senior years, and how these courses relate to their acquisition of different mathematics skills and concepts.

We hope that the information provided in this report will be useful to a wide range of readers, including policymakers and educators interested in improving the mathematics proficiency of American youth. Additionally, we hope that the results reported here will encourage other researchers to use the ELS:2002 data.

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Chapter 1
Introduction

A major focus of education policy in the United States is improving both the quality and rigor of core courses taught in schools and ensuring that all students have access to these courses. Mathematics in particular has received extensive attention, both because of its importance in an increasingly technical and global economy and because of the performance of American youth when compared with their international peers. Recent research shows that U.S. 15-year-olds continue to lag behind their international peers in mathematics—ranking 24th of 29 nations in problem solving and mathematics literacy on the 2003 Program for International Student Assessment (Lemke et al. 2004). As a means to improve proficiency in this area, many states have increased their course requirements for graduation. For example, between 1987 and 2004, the number of states requiring at least 2.5 credits in mathematics for graduation increased from 12 to 26 (Council of Chief State School Officers 2004). Accordingly, contemporary students are receiving more mathematics training than their predecessors. In 2004, high school seniors left high school with an average of 3.6 credits in mathematics, up from 2.7 in 1982 (Dalton et al. 2007). Further, contemporary students are more likely to take advanced mathematics courses. For example, 6 percent of high school seniors were taking calculus in 1982. By 2004, 14 percent of high school seniors were doing so (Dalton et al. 2007).

If students are enrolling in more mathematics courses and more high-level courses, are they necessarily developing an advanced comprehension of mathematics? The existing research indicates that mathematics achievement is associated with advanced mathematics coursetaking (Leow et al. 2004; Rock, Owings, and Lee 1994; Rock and Pollack 1995a; Scott et al. 1995; Wang and Goldschmidt 2003). However, the bulk of existing research on the topic is limited in two respects: first, possibly dissimilar courses are placed into broad categories for analytic convenience; and second, the scores used to assess achievement growth preclude the identification of specific concepts and skills students are developing and/or lacking. The implications these methodological setbacks have for understanding the relationship between curricular structures and learning are discussed in turn.

First, most studies bundle courses into broad categories to make comparisons.[1] Two common methods used to assess achievement growth for those in different curricular tracks, such as an honors track, a general track, or a vocational track (Carbonaro 2005; Hallinan 1994) or to assess achievement growth for those reaching different levels of mathematics, such as calculus, algebra II, or geometry (Lee et al. 1998; Rock, Owings, and Lee 1994; Rock and Pollack 1995a; Scott et al. 1995). For example, Rock and Pollack's (1995b) analysis of the National Education Longitudinal Study of 1988 (NELS:88) found that those whose highest mathematics course was calculus gained 5.61 points between the 10th and 12th grade on the mathematics assessment. What is obscured here are the other mathematics courses the student had taken prior to calculus. While mathematics is largely hierarchical and sequential, some students may have taken precalculus prior to calculus, others may have jumped directly into calculus from algebra II, and others may have taken another advanced course (e.g., statistics, trigonometry). These different

[1] This classification approach is useful in documenting aggregate trends in coursetaking, as is in done in Dalton et al. (2007). However, it is less useful in assessing the relationship between coursetaking and achievement gains, the focus of this study.

pathways may provide students with different foundational skills for learning more advanced concepts. As a consequence, the gains attributed to the highest course—in this instance, calculus—may be under- or overstated.

Second, most use an aggregate measure of mathematics achievement and consequently, overlook the content of the learning involved. For example, Rock and Pollack's finding that students who reach calculus gain 5.61 points on the NELS:88 mathematics assessment reveals little about the *content* of that learning. That is, are students who take calculus developing fluency in operations with real numbers, vectors, and matrices, or are they augmenting their base understanding of algebra and geometry? As most research relies on aggregated outcomes—for example, standardized composite scores or the number of correct answers on an assessment—the depth and breadth of learning and its relationship to curricular pathways is unclear. There may be differential advantages and disadvantages associated with taking a certain set of courses, and/or differential gains in learning certain mathematics skills and concepts. In most of the research, these contingencies are obscured.

This study, which uses data from the Education Longitudinal Study of 2002 (ELS:2002), improves upon past research by using information from high school transcripts to identify the exact course sequences students take and links them with achievement test scores that have been scaled to indicate different levels of mathematics proficiency. This linkage provides a more detailed understanding of the curricular pathways students travel and the types of proficiencies they acquire along the way. This chapter provides a brief background description of the pattern of mathematics coursetaking in the United States and lists the research questions. Chapter 2 describes the ELS:2002 data and the measures used in the analysis. Chapter 3 provides the results of the analysis. Chapter 4 concludes with a discussion of the findings and their limitations.

1.1 The Correlates of Mathematics Coursetaking

Courses are the building blocks of schooling and the larger system of education. Linked together across school years, courses teach students knowledge and skills unique to a particular subject area and how concepts relate to other areas of the curriculum. While courses, particularly in mathematics, are the primary means through which students learn advanced subject material, not all students take the same courses and/or course sequences.

As with many indicators of educational success, coursetaking in the United States has differed with socioeconomic status. In broad terms, students from more affluent backgrounds—families with high incomes and more highly educated parents—tend to take more advanced courses than their peers (Lee et al. 1998; Stevenson, Schiller, and Schneider 1994). For example, using transcript data in NELS:88, Hoffer, Rasinski, and Moore (1995) found that students from the highest socioeconomic quartile earned an average of 3.5 Carnegie units in mathematics during high school.[2] In contrast, those in the lowest socioeconomic quartile earned an average of 2.1 Carnegie units in mathematics during high school. Other analyses of NELS:88 have revealed that students from wealthy families and students with college-educated parents are most likely to enroll in an advanced mathematics course like calculus or trigonometry (Schneider, Swanson, and Riegle-Crumb 1998). Though these studies do not identify the specific mechanism linking

[2] A Carnegie unit is a standard of measurement used for secondary education that represents the completion of a course that meets one period per day for 1 year.

socioeconomic status with coursework, they provide firm evidence that, on average, affluence and higher parental education translate into a curricular advantage in high school mathematics.

In addition to differences according to socioeconomic status, racial/ethnic and sex differences in education are well documented. With respect to race/ethnicity, Hispanic and Black students on average tend to lag behind their White and Asian peers in school (Kao and Thompson 2003). These broad patterns are evident in the mathematics coursetaking patterns of high school students. For example, a recent analysis of ELS:2002, the data used in this paper, showed that 87 percent of Asian and 79 percent of White high school seniors reached algebra II in high school, compared to 75 percent of Black and 67 percent of Hispanic high school seniors (Dalton et al. 2007). These disparities in coursetaking have implications for learning—a substantial portion of racial/ethnic differences in student achievement has been linked to differences in coursetaking patterns. For example, differences between Black and White students on standardized tests are minimized when comparing students who have taken advanced courses at comparable rates (Berends, Lucas, and Briggs 2002; Gamoran 1987).

While the largest differences in coursetaking are along socioeconomic and racial/ethnic lines, sex differences in mathematics are also apparent. Historically, girls have trailed behind boys in mathematics coursetaking (U.S. Department of Education 1997), and some researchers maintain that this difference explains sex differences in academic achievement among elementary and high school-aged boys and girls (Oakes 1990; Pallas and Alexander 1983). However, there is evidence that the sex gap has been closing in recent years: recent research shows that rates of mathematics coursetaking and mathematics achievement performance among high school aged boys and girls is reaching parity (Dalton et al. 2007; Perkins et al. 2004). As young women make strides in this traditionally male dominated subject area, assessing how the distribution of learning opportunities varies between the sexes is currently of interest.

Though sociodemographic characteristics have been shown to be associated with coursetaking patterns and curricular experiences, different school structures affect this relationship. Most schools in the United States follow the comprehensive high school model, wherein the curriculum remains flexible and diversified to accommodate the needs of a wide range of students with different interests, skills, and aptitudes (Oakes 1985). Not all schools follow this model. Catholic high schools, for example, tend to adhere to a constrained curriculum, offering higher level academic courses, such as intermediate and advanced mathematics, to all their students (Lee et al. 1998). Additionally, when compared with their public school peers, students who enroll in Catholic schools tend to have families who are better educated and more involved in their children’s education (Coleman and Hoffer 1987; Morgan and Sorenson 1999).

Taken together, these research findings show that both background characteristics of students and the types of schools they attend are associated with their curricular experiences. Accordingly, the present analysis will explore how mathematics course sequences are distributed along these dimensions in contemporary American high schools.

1.2 Research Questions

This study addresses the following questions:

- How much does mathematics achievement change during the last 2 years of high school and are these changes related to student background and school characteristics?
- What are the most common mathematics course sequences taken by students in the 11th and 12th grades and are these sequences related to student background and school characteristics?
- What mathematics course sequences are associated most closely with mathematics achievement?

The first question will examine learning gains in mathematics over the latter half of high school. Changes in both the level of mathematics achievement (as measured by IRT-estimated number-right scores) and the type of mathematics knowledge (as measured by proficiency probability scores) will be examined. Exploring this research question reveals whether students have improved their overall mastery of mathematics skills and identifies the content areas in which students are making (or not making) gains. Additionally, the variation of learning gains among students from different backgrounds enrolled in different kinds of schools will be assessed. This will update previous research on sociodemographic and school sector differences in achievement using a recent cohort of high school students. The second question will identify the number and types of mathematics courses most frequently taken by high school juniors and seniors and will link these course patterns with sociodemographic characteristics of students and their schools. The final question will address which course sequences are associated with the largest gains in mathematics achievement and which are associated with the smallest gains in mathematics achievement between the 10th and 12th grade, apart from student background and school characteristics.

Chapter 2
Data and Methods

This analysis uses data from the Education Longitudinal Study of 2002 (ELS:2002), which was designed to provide trend data about critical transitions experienced by students as they proceed through high school and into young adulthood.[3] This nationally representative study of 17,590 students who were 10th-graders in 2002 was conducted by the National Center for Education Statistics (NCES).[4] ELS:2002 used a two-stage sampling procedure. In the first stage, a sample of 750 high schools, both public and private, were selected with probabilities proportional to their size. In the second stage, approximately 26 students were randomly sampled from each school on the condition that they were in the 10th grade in the spring term. Of the 17,590 eligible students, 15,360 completed a survey about their school and home experiences (87 percent weighted response rate), of which 14,540 completed cognitive assessments in mathematics and reading (95 percent, weighted response rate). Their parents, teachers, principals, and librarians were surveyed as well. In the spring of 2004, about 14,710 of the originally selected sample members were reinterviewed and comprise the target population for this study: sophomores in the spring of 2002 who were respondents in both the base-year (BY) and first follow-up (F1) interviews. Their high school transcripts were collected in 2004–05 and they were reinterviewed in the spring of 2006 (2 years post on-time high school graduation).[5]

To be included in this analysis, sample members had to have been an in-school sophomore in 2001–02, participated in both the BY and F1 interviews, completed the mathematics assessment in the BY and F1 interviews, and had complete transcript information for the 2002–03 and 2003–04 academic years. Of the 14,710 base-year sophomores who participated in both the BY and F1 interviews, 13,330 participated in the BY mathematics assessment, of whom 9,920 participated in the F1 mathematics assessment.[6] Only students who remained in their base-year schools were administered the F1 mathematics assessment. Scores were imputed for students who transferred to a new school or were still enrolled in their base-year school but were unable to participate during the in-school test administration. However, because mathematics achievement is the key variable in this analysis, these cases with imputed test scores were excluded to prevent any error in estimating learning and learning gains. Lastly, 330 cases were excluded because they had no transcript information and 130 cases were excluded because they lacked evidence of both a mathematics course and complete transcript

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[3] The study design and data collection was undertaken by RTI International.

[4] The sample sizes are approximate because restricted-use data are used. In accordance with NCES Statistical Standards (Seastrom 2003), exact sample sizes from restricted-use data files cannot be published unless the data are perturbed in some way. The perturbation approach taken here was to round the exact sample sizes of cells to 10s or 100s.

[5] More information on the design and collection procedures of ELS:2002 can be found in appendix A and in the *Education Longitudinal Study of 2002: Base-Year to First Follow-up Data File Documentation* (Ingels et al. 2005).

[6] Scores were missing for sample members in the F1 interview because they had dropped out, transferred schools, or started homeschooling. RTI only tested students who were enrolled in their BY school in the spring of 2004. For this analysis, examining students who were continuously exposed to only one curriculum and school environment, however, provides a clearer portrait of the relationship between coursework and learning.

information for both the 2002–03 and 2003–04 years.[7] The final analytic sample includes 9,460 respondents, or about 64 percent of the approximately 14,710 members of the sophomore cohort who participated in both the base-year and first follow-up interviews. A bias analysis comparing the sociodemographic composition of the analytic sample (n = 9,460) with the full sophomore panel sample (n = 14,710) is presented in section A.6 of appendix A. Compared with the full sophomore panel sample, there are higher proportions of White students, students who expect a bachelor's degree or higher, and students living with both their father and their mother in the analytic sample. All estimates are weighted with the panel weight (F1PNLWT) and generalize (with the above qualifications about inclusion conditions) to the population of students who were sophomores in the spring of 2002. In all analyses, standard errors were adjusted for the clustered and stratified sampling design using Taylor-series linearization methods (StataCorp 2004).

2.1 Mathematics Achievement Assessments

Assessments in mathematics were administered to students in their schools during the BY and F1 survey administrations. There were multiple forms of the test. In the BY, assignment of form was based on a routing test, and in the F1, on the BY ability estimate. These tests, designed and scored using Item Response Theory (IRT), serve as "bookends" to learning that took place during the 2002–03 and the 2003–04 academic years—that is, approximately the end of sophomore year to approximately the end of senior year for on-time students.[8] The BY assessment can be thought of as a pretest, or baseline, to academic experiences that take place during the second half of high school, while the F1 assessment can be thought of as a posttest. IRT uses patterns of correct, incorrect, and omitted answers to obtain achievement estimates that are comparable across different test forms within a domain.[9] In estimating a student's achievement, IRT also accounts for each test question's difficulty, discriminating ability, and a guessing factor. For this analysis, two measures of mathematics achievement based on their performance on this test are used: IRT-estimated number-right scores and proficiency probability scores.

The IRT-estimated number-right score is an overall measure of mathematical knowledge and skill. The IRT-estimated number-right score used in this analysis is an IRT-based estimate of the number of items an examinee would have answered correctly if he or she had taken all of the items in the item pool on the multiform assessment administered to 10th-graders in ELS:2002's predecessor study, the National Education Longitudinal Study of 1988 (NELS:88). Using common item calibration techniques for linking scales, results between NELS:88 and ELS:2002 are comparable.[10] There were 81 items in the vertically scaled 10th- to 12th-grade ELS:2002 item pool. For the analytic sample used in this study, students answered an average of 47 questions correctly on the 10th-grade assessment and 51 questions correctly on the 12th-grade assessment.

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[7] Complete transcript information is defined in this analysis as having a transcript showing enrollment in any four courses in both the 2002–03 and 2003–04 school years. However, if the transcript did not have four courses in one school year but had information on mathematics courses, they were included in the analysis.
[8] Less than 1 percent of students included in the analysis were not in the 12th grade at the time of the F1 survey administration (n = 60), likely due to grade retention. As this time span captures the academic experiences in the junior and senior years for almost the entire sample (99 percent), the phrases "junior and senior year of high school," "latter half of high school" and "2002–03 and 2003–04 academic years" will be used interchangeably in this report.
[9] For an account of IRT, see Embretson and Reise (2000) or Hambleton, Swaminathan, and Rogers (1991).
[10] Development of the 1992 NELS:88 mathematics scale is documented in Rock and Pollack (1995b). The linkage of the NELS:88 scale to ELS:2002 through IRT methods is documented in Ingels et al. (2005, p. 39).

A proficiency probability score is a criterion-referenced score measuring how well an examinee performs relative to some set criterion representing mastery of knowledge and skills assessed. There are five distinct scores corresponding to five hierarchical levels (level 1 through level 5). Mastery of a higher level typically implies proficiency at lower levels. In contrast to the IRT-estimated number-right scores, which indicate overall achievement, the proficiency probability scores indicate what knowledge and skills the student does or does not possess. The five ordinal levels of mathematics proficiency include:

1. simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers;
2. simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents;
3. simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram;
4. understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and
5. complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

The proficiency probability score at each level ranges from 0 to 1 and indicates the likelihood that a student has mastered the skills and knowledge described above (0 = no mastery, 1 = compete mastery). The mean of a proficiency probability score aggregated over a subgroup of students is analogous to an estimate of the percentage of students in the subgroup who have displayed mastery of the particular skill.[11] For example, in this study, the analytic sample has a mean score of .73 for level 2 in the 10th grade. This can be interpreted as “73 percent of 10th-graders have mastered the skills and concepts of level 2.” The proficiency probabilities were computed using IRT-estimated item parameters originally calibrated in NELS:88. Appendix A provides more detailed information about the assessment framework, the distribution of the item pool across its elements, and the scaling techniques for the different scores. For the purposes of presentation and discussion, throughout this report, level 1 is considered basic skills, levels 2 and 3 are considered intermediate skills, and levels 4 and 5 are considered advanced skills.

2.2 Student and School Characteristics

This analysis uses the following student background characteristics: sex, race/ethnicity, socioeconomic status, family composition, school sector, and student’s educational expectations. Descriptions of these measures are provided in appendix A.

2.3 Statistical Testing

Bivariate comparisons drawn in the text of this report have been tested for statistical significance at the .05 level using *t* statistics to ensure that the differences are larger than those

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[11] Although probabilities of proficiency have been placed on a 0–1 scale, when aggregated they can be interpreted as a proportion. On the interpretation of a probability as a proportion, see Fleiss, Levin, and Paik (2003, p. 1).

that might be expected due to sampling variation. In analyses using a large sample, such as the one used in this report, standard errors accompanying estimates are often small and thus small differences between groups are often found to be statistically significant. Since tests of statistical significance reveal whether a relationship between variables is statistically reliable—but tell us little about the strength of the relationship—strength-of-effect measures were obtained to accompany all statistical tests and used as a second criterion to determine whether a result could be reported.

One measure of strength of effect is the effect size. Effect size is the estimated difference between the mean of population A and the mean of population B divided by the pooled standard deviation. The effect size indicates the magnitude of the estimated difference in terms of the number of standard deviations separating the means of the two groups. A standard deviation is the statistical measure of the extent to which values are spread around the mean. The reporting criterion applied to differences in means was an effect size (Cohen's d) of 0.2, or one-fifth of a standard deviation (Cohen 1988). When evaluating effect sizes, the proficiency probability scores, like the IRT-estimated number-right scores, have been treated as means, and are subject to the 0.2 standard deviation criterion. Tables in this report, however, supply estimated proportions as well as means. Therefore for comparisons involving percentage differences between subgroups a strength-of-effect criterion was also set: the criterion for percentages was set at a minimum of 5 percentage points difference.

2.4 Limitations of the Analysis

There are a number of limitations to this analysis that readers should keep in mind when evaluating the findings reported in the next chapter. First, since ELS:2002 provides observational data, where students were not randomly assigned to schools, classrooms, or teachers, establishing a causal link between coursetaking and learning is not possible. Additionally, the analysis requires test scores at two different periods of time (sophomore and senior years), thereby excluding students who had dropped out, transferred schools, or started homeschooling. The resulting analytic sample includes a higher proportion of students who are White, a higher proportion of students who expect to receive a bachelor's degree or higher, and a higher proportion of students living with both their father and their mother than the full ELS:2002 sophomore panel. Thus, the findings may not generalize to all students, particularly those who are non-White, those who have educational expectations that do not include college completion, and those who are not living with their mother and father.

Chapter 3
Findings

As noted earlier, this report addresses three principal research questions. The first of these was:

How much does mathematics achievement change during the last 2 years of high school and are these changes related to student background and school characteristics?

Table 1 shows the IRT-estimated number-right scores in 10th and 12th grade and changes in those scores by student background and school characteristics, and the effect sizes associated with the changes. Table 2 shows the proficiency probability in the 10th and 12th grade, changes in those scores by student background and school characteristics, and the effect sizes associated with the changes. Standard deviations corresponding to the effect sizes are shown in table B-1b and in table B-2b.

On average, students improved their performance on the mathematics assessment by about five correctly answered questions—about a third of a standard deviation (e.g., an effect size of 0.33). Additionally, all subgroups yield gains (table 1). Students in the Catholic sector made the largest gain: the effect size associated with their learning gain is a little more than half a standard deviation (0.54). The proficiency scores augment this information by detailing the levels of skills that were learned. The averages at the top of table 2 indicate that the smallest gains were made at the lowest and highest levels while the largest gains took place at levels 3 and 4. In 10th grade, 53 percent of students were proficient at level 3 and 25 percent were proficient at level 4. By the end of 12th grade, 65 percent and 38 percent were proficient at level 3 and level 4, respectively. At the highest level, gains across the second half of high school were smaller than those made in levels 2, 3, and 4. By the end of senior year, only about 4 percent of seniors had mastered the skills to be considered proficient at level 5.[12]

Before entering the last 2 years of high school, there are no detectable differences between boys and girls in either their number-right scores or in their level of mastery at all five proficiency levels. As measured by both gains in the number-right scores and increases in proficiency levels, on average, there are no detectable differences between girls and boys in their rate of learning across the final 2 years of high school. Additionally, upon leaving high school, there are no detectable differences between boys and girls in their number-right score nor in the level of mastery at all five levels of proficiency. These findings corroborate other contemporary research that finds increasing parity in mathematics achievement (for information on trends, see Bae et al. 2000 and Cahalan et al. 2006; for a review of the scientific evidence of gender differences in mathematics learning, see Halpern et al. 2007.

[12] At the time of the F1 interview, 720 sample members who had participated in the BY interview were dropouts. These sample members are excluded from the analysis because they lack a F1 test score and thus cannot contribute to an analysis of learning gains. In accord with research in the dropout literature, these dropouts fared worse on the base-year mathematics assessment than their peers who were enrolled at the time of both the BY and F1 interviews. In the spring of 2002, these dropouts had a number-right score of 32.6. Their proficiency probability scores for the five levels were 80.7 (level 1), 37.9 (level 2), 17.1 (level 3), 4.4 (level 4), and 0.1 (level 5).

With respect to race/ethnicity, differences noted in other national studies of achievement—for example, Hoffer, Rasinski, and Moore's (1995) analysis of the National Education Longitudinal Study of 1988 (NELS:88)—were detected here as well. Compared with their Black and Hispanic counterparts, Asian and White students had higher number-right scores at both time points. However, there were no differences detected among racial/ethnic groups in their number-right score gains across the second half of high school. In addition to overall aggregate differences, there are disparities in the content of their learning. Asian and White students entered the second of half of high school having mastered basic mathematics skills and concepts—for example, 83 percent of Asians and 82 percent of Whites were proficient in level 2 at the end of sophomore year (table 2). Black and Hispanic students, on the other hand, entered the second half of high school with a less solid foundation in mathematics: 46 percent of Black students and 54 percent of Hispanic students were proficient at level 2 at the end of their sophomore year. Given this disadvantage, Black and Hispanic students may be less likely than their Asian and White peers to acquire the most advanced mathematics skills before graduation. Indeed, the evidence from ELS:2002 suggests this is the case: when leaving high school, 52 percent of Asian students and 45 percent of White students are proficient at level 4, compared to 20 percent of Hispanic students and 12 percent of Black students.

Table 1. Average mathematics IRT-estimated number-right scores, by selected student characteristics: 2002 and 2004

Student characteristic	10th grade	12th grade	Change (ES)
Total	46.7	51.2	4.5 (0.33)
Sex			
Female	45.8	50.2	4.4 (0.33)
Male	47.6	52.2	4.6 (0.34)
Race/ethnicity			
White	49.8	54.3	4.5 (0.35)
Black	36.8	41.2	4.4 (0.37)
Hispanic	39.7	44.2	4.5 (0.34)
Asian	51.6	56.4	4.8 (0.34)
American Indian	39.6	42.8	3.2 (0.26)
More than one race	45.2	50.2	5.0 (0.37)
Socioeconomic status			
Quartile 1 (low)	39.4	43.5	4.1 (0.32)
Quartile 2	43.7	47.6	3.9 (0.30)
Quartile 3	47.9	52.5	4.6 (0.36)
Quartile 4 (high)	53.6	58.8	5.2 (0.43)
School sector			
Public	46.2	50.6	4.4 (0.32)
Catholic	51.7	57.7	6.0 (0.54)
Other private	53.3	58.8	5.5 (0.45)
Family composition			
Mother and father	48.5	53.2	4.7 (0.36)
Mother or father and guardian	45.2	49.3	4.1 (0.32)
Single parent	43.2	47.6	4.4 (0.31)
Other	40.0	42.9	2.9 (0.22)
Student's educational expectations[1]			
High school or less	33.6	37.3	3.7 (0.33)
Some college	38.6	42.6	3.9 (0.33)
Bachelor's degree or more	49.0	53.6	4.6 (0.35)
Don’t know	42.8	46.9	4.1 (0.29)

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. IRT = Item Response Theory. ES = Effect Size (i.e., standardized mean difference).

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Patterns across socioeconomic status (SES) follow suit. For the most part, students in the highest SES quartile make greater gains than students in the lowest SES quartile. Moreover, compared with students in the lowest SES quartile, high SES students are making gains in more advanced subject matter. For example, those in the highest SES quartile improved their proficiency at level 4 by 17 percentage points while those in the lowest SES quartile improved their proficiency at level 4 by 7 percentage points (table 2). High SES students are ahead in their learning of mathematics, with higher levels of achievement and growth in their understanding of advanced topics, while less affluent students are less prepared. The gains low SES students do make are concentrated in intermediate-level skills.

Table 2. Average mathematics proficiency probability scores, by selected student characteristics: 2002 and 2004

	10th grade					12th grade					Change				
Student characteristic	Level 1	Level 2	Level 3	Level 4	Level 5	Level 1	Level 2	Level 3	Level 4	Level 5	Level 1	Level 2	Level 3	Level 4	Level 5
Total	0.94	0.73	0.53	0.25	0.01	0.96	0.80	0.65	0.38	0.04	0.03	0.06	0.11	0.13	0.03
Sex															
Female	0.94	0.72	0.51	0.23	0.01	0.96	0.79	0.63	0.35	0.03	0.03	0.07	0.12	0.12	0.02
Male	0.94	0.75	0.56	0.28	0.02	0.97	0.81	0.66	0.41	0.06	0.03	0.06	0.11	0.13	0.04
Race/ethnicity															
White	0.96	0.82	0.63	0.31	0.01	0.98	0.87	0.74	0.45	0.05	0.01	0.05	0.11	0.14	0.04
Black	0.85	0.46	0.23	0.06	#	0.92	0.58	0.35	0.12	#	0.07	0.12	0.12	0.06	#
Hispanic	0.88	0.54	0.31	0.12	#	0.93	0.65	0.44	0.20	0.01	0.05	0.11	0.13	0.08	0.01
Asian	0.96	0.83	0.67	0.36	0.05	0.97	0.87	0.76	0.52	0.12	0.01	0.04	0.09	0.16	0.07
American Indian	0.91	0.55	0.35	0.06	#	0.93	0.62	0.40	0.16	0.02	0.02	0.07	0.05	0.09	0.02
More than one race	0.93	0.71	0.48	0.21	0.01	0.96	0.79	0.62	0.34	0.03	0.04	0.07	0.15	0.13	0.02
Socioeconomic status															
Quartile 1 (low)	0.88	0.54	0.32	0.10	#	0.93	0.63	0.42	0.18	0.01	0.06	0.10	0.11	0.07	0.01
Quartile 2	0.92	0.67	0.44	0.17	#	0.95	0.74	0.55	0.27	0.02	0.03	0.07	0.12	0.10	0.01
Quartile 3	0.95	0.78	0.57	0.26	0.01	0.98	0.84	0.70	0.39	0.04	0.02	0.06	0.13	0.13	0.03
Quartile 4 (high)	0.98	0.89	0.74	0.42	0.03	0.99	0.93	0.84	0.60	0.10	0.01	0.04	0.10	0.17	0.07
School sector															
Public	0.93	0.72	0.52	0.24	0.01	0.96	0.79	0.63	0.36	0.04	0.03	0.07	0.11	0.12	0.03
Catholic	0.98	0.87	0.70	0.34	0.01	0.99	0.93	0.83	0.55	0.06	0.01	0.06	0.13	0.21	0.05
Other private	0.98	0.88	0.73	0.40	0.03	0.99	0.93	0.84	0.59	0.11	0.01	0.05	0.11	0.18	0.07
Family composition															
Mother and father	0.95	0.78	0.59	0.29	0.01	0.97	0.84	0.70	0.43	0.05	0.02	0.06	0.11	0.14	0.04
Mother or father and guardian	0.93	0.71	0.48	0.20	0.01	0.96	0.77	0.59	0.31	0.03	0.03	0.06	0.12	0.11	0.02
Single parent	0.91	0.64	0.43	0.18	0.01	0.95	0.72	0.54	0.28	0.03	0.04	0.08	0.11	0.10	0.02
Other	0.87	0.56	0.34	0.11	#	0.91	0.63	0.43	0.16	0.01	0.04	0.07	0.08	0.05	0.01
Student's educational expectations[1]															
High school or less	0.78	0.35	0.15	0.04	#	0.87	0.44	0.24	0.08	#	0.09	0.09	0.09	0.04	#
Some college	0.87	0.53	0.28	0.08	#	0.93	0.64	0.40	0.14	#	0.06	0.11	0.12	0.06	#
Bachelor's degree or more	0.96	0.79	0.60	0.29	0.01	0.98	0.85	0.72	0.43	0.05	0.02	0.06	0.11	0.14	0.04
Don't know	0.91	0.62	0.40	0.17	0.01	0.95	0.70	0.51	0.27	0.03	0.04	0.08	0.10	0.10	0.02

Rounds to zero.

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Much like the volume of literature comparing private and public schools (Coleman and Hoffer 1987; Morgan and Sorenson 1999), evidence from ELS:2002 shows that students in Catholic schools fare better in mathematics than their public school counterparts. On average, Catholic school students outgained public school students on the mathematics assessment (6 versus 4) and left high school with higher overall scores (58 versus 51) (table 1). There were no detected differences in the gains of students attending other private schools and public school students (5 versus 4). However, other private school students left high school with higher overall scores than their public school peers (59 versus 51). With the exception of level 4, there were no differences detected in the gains across different levels of proficiency between public school students and Catholic school students during the last 2 years of high school. At level 4, Catholic school students were about 10 percentage points ahead of their public school counterparts (table 2). There were no differences detected in the gains at the three lowest levels of proficiency between public school students and other private school students. However, other private school students outpaced their public school peers at level 4 and level 5.

Compared with public school students, other private school students left high school with greater proficiency at all five levels and Catholic school students left high school with greater proficiency at levels 1–4. There were no differences at the end of high school detected between Catholic school students and other private school students at levels 1–4. Other private school students, however, were more proficient than were Catholic school students at level 5 (11 percent versus 6 percent) (table 2).

Like school sector, family composition is also associated with learning in mathematics. While there were no differences detected in the gains of students who live with both parents in the family, single parent families, and stepparent families, students who live with both parents in the family left high school with higher number-right scores and greater proficiency at levels 2–4. For example, 43 percent of students who live with both parents in the family were proficient at level 4 compared with 31 percent of students living in stepfamilies (i.e., mother or father and guardian) and 28 percent of students living with single parents (table 2). Compared with students who live with both parents in the family, students in other family forms left high school with lower number-right scores and lower proficiencies at all five levels.

Lastly, the educational expectations of students were linked with mathematics learning. Those who expected a college degree performed better on the mathematics assessments than their peers who expected to complete high school or less—for example, those expecting a college degree answered an average of 53.6 questions correctly on the 12th-grade assessment compared with an average of 37.3 correct answers for those who expected high school or less (table 1). During the final 2 years of high school, those who expected to attend college outpaced their peers at the highest levels (level 4 and level 5), and finished high school with a greater understanding of mathematics concepts at all levels than their peers who expected a high school degree or less.

In the aggregate, these descriptive findings are neither new nor novel—educational researchers have long documented differences in mathematics achievement along key dimensions of student background. What this analysis does highlight, however, is that by mastering basic mathematics skills by their sophomore year, Asians, Whites, High-SES students, students who live with both parents in the family, students attending private schools, and students expecting a college degree are in a better position to acquire more intermediate and advanced skills than their peers. As the learning of mathematics skills and concepts—both basic and advanced—rests on the content and instruction received in different courses, this analysis now turns its focus to the curricular pathways that students follow.

What are the most common mathematics course sequences taken by students in the 11th and 12th grades and are these sequences related to student background and school characteristics?

As mentioned in the introduction, this study improves upon previous research on learning and coursetaking by identifying the actual *course sequences* students take between test administrations, rather then simply counting the number of credits earned or using broad curricular classification such as a track placement or a level (e.g., "highest mathematics"). The analysis in this report was accomplished using data from the ELS:2002 High School Transcript Study. First, all mathematics courses for which the student earned credit during 2002–03 and 2003–04,[13] the 2 academic years between the 2 mathematics assessments, were classified in one of the following 16 hierarchical categories:

1. No Mathematics;
2. Basic Mathematics;
3. General Mathematics;
4. Applied Mathematics;
5. Prealgebra;
6. Algebra I;
7. Geometry;
8. Algebra II;
9. Trigonometry;
10. Other Advanced Mathematics;
11. Precalculus;
12. Statistics;
13. Advanced Placement/International Baccalaureate (AP/IB) Mathematics;
14. Calculus;
15. Advanced Placement/International Baccalaureate (AP/IB) Calculus; or
16. Other Mathematics.[14]

The course titles that comprise these categories are listed in appendix A. As the typical school curriculum permits one mathematics course each academic year, this study operationalizes course sequences in terms of a two-course sequence: mathematics course (if any) for which credit was earned in 2002–03 and mathematics course (if any) for which credit was earned in 2003–04.[15] Of the 256 possible combinations of two course sequence courses based on

[13] Earning course credit is defined by receiving a letter grade higher than an F or a "pass."

[14] "Other mathematics" is a residual category and is not considered more or less rigorous than the other 15 course categories.

[15] Given the structure of most school calendars and curricula, the average student enrolls in two courses during his or her junior and senior years. However, due to summer school, dual enrollment (e.g., business mathematics and geometry in the same term), or semester-long courses students could possibly take more than two courses in this time period. In the analytic sample, 12.7 percent (n = 1,200) had taken three or more mathematics courses during the 2002–03 and 2003–04 academic years. To classify these students in a way that was consistent with the majority of students who had a two-course sequence, these students were classified based sequentially on their two highest courses. For example, if the student had taken geometry, applied mathematics, and algebra II, his or her course sequence was classified as "geometry–algebra II." The effect of this classification rule on the estimates of learning should be negligible for two reasons. First, 62 percent of these students (n = 750) have sequences that fall under "all other patterns"—the residual group excluded from key comparisons; and second, all multiple regression models control for students who took more than two courses.

the classifications above, 180 sequences were followed by ELS:2004 sample members. Only six sequences were followed by more than 5 percent of students.

This process of constructing mathematics sequences reveals that the secondary mathematics curriculum in the United States is more diverse than once thought. For instance, analyses of NELS:88 using broader categories to classify courses found that 75 percent of students fit into one of five predefined coursetaking patterns (Burkam and Lee 2003). Aggregating course titles into broad patterns obscures some of the heterogeneity of mathematics coursetaking. Despite the relatively sequential and hierarchical nature of the subject matter, students enroll in a range of mathematics courses in their final years of high school.

The large number of course sequences precludes a succinct analysis of the learning gains for the entire analytic sample. To facilitate interpretation and to produce efficient estimates of learning, course sequences followed by more than 200 students (approximately 3 percent of the unweighted sample) form the basis of this study.[16] Nine course sequences meet this criterion and are listed in table 3. In preliminary analyses (not shown), students who had followed a geometry–no mathematics sequence had similar mathematics gains as those in geometry–geometry.[17] Given that these students were only exposed to geometry during the interval and that geometry is sometimes taught over the course of 2 years, they were combined into one group. This course sequence is herein referred to as geometry–geometry/no mathematics. Similarly, as trigonometry is often embedded in the content of algebra II and preliminary analyses (not shown) find no differences in their learning gains, students who had followed an algebra II–trigonometry sequence were combined with students who followed an algebra II–algebra II sequence.[18] This course sequence is herein referred to as algebra II–algebra II/trigonometry. Since these two course sequences were constructed by combining courses, they should be interpreted with caution.

[16] Selecting the most common course sequences yields the most pertinent information as these courses were experienced by a majority of students. These course sequences account for 55 percent of the sequences taken by students in ELS:2002. Including course sequences followed by fewer students would potentially threaten the efficiency of the estimates, particularly in a multiple regression analysis.
[17] Differences in number-right score gains and differences in the proficiency probability score gains between students who followed a geometry–no mathematics sequence and students who followed a geometry–geometry sequence did not meet the .05 level for statistical significance required for this study.
[18] Differences in number-right score gains and differences in the proficiency probability score gains between students who followed an algebra II–trigonometry sequence and students who followed an algebra II–algebra II sequence did not meet the .05 level for statistical significance required for this study.

Table 3. Weighted percentage and unweighted frequency of students taking mathematics course sequences: 2002 and 2004

Course sequence	Weighted percentage	Unweighted frequency
Total	100.0	9,460
Algebra II–no mathematics	12.7	1,160
Geometry–geometry/no mathematics	7.8	650
Algebra II–precalculus	6.8	730
Algebra II–algebra II/trigonometry	6.1	680
Precalculus–AP/IB calculus	6.1	610
No mathematics–no mathematics	5.6	480
Geometry–algebra II	4.6	420
Precalculus–no mathematics	3.6	330
Precalculus–calculus	2.0	240
All other patterns	44.6	4,160

NOTE: The nine course sequences listed refer to those followed by more than 200 students in the sample and are ordered hierarchically. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. Details may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

These nine course sequences account for a little more than half of the sample (55 percent).[19] The most common course sequence undertaken in the second half of high school is algebra II during junior year followed by no mathematics course during the senior year. This course pattern was undertaken by 13 percent of students. The most common course sequence involving two separate courses was algebra II during junior year followed by precalculus taken during senior year. This course pattern was undertaken by 7 percent of the students. Despite its centrality to preparation for postsecondary studies, 6 percent of students do not take any mathematics courses during the final 2 years of high school. Readers should note that with the exception of no mathematics, none of these sequences contain courses lower than geometry, and thus this study does not address the learning gains made by students who take general or basic mathematics courses. About 45 percent of students took course sequences taken by fewer than 200 students. The most common courses taken by students following these other course sequences were algebra II, no mathematics, and other advanced mathematics. The most common course sequences taken by students following these other course sequences were algebra I–no mathematics, other advanced mathematics–no mathematics, and algebra II–other advanced mathematics. Given the heterogeneity of courses in the "all other patterns" sequence, the learning gains of these students are not discussed in this report.[20]

As discussed earlier, numerous studies have found that the types of courses students take are related to both their background characteristics and the types of schools they attend. To assess whether these patterns are present when using a new classification of curricular

[19] There are no detectable differences in the sociodemographic composition of students who follow these nine course sequences (n = 5,300) and the sociodemographic composition of the analytic sample (n = 9,460). This is shown in section A.6 (Bias Analysis) of appendix A.

[20] As discussed earlier, dropouts are not included in the analysis due to a lack of an F1 mathematics test score and incomplete coursetaking records. Of the 720 dropouts with a BY test score, 130 have coursetaking information for the 2002–03 and 2003–04 school year. The most common coursetaking sequences for these dropouts include: no mathematics–no mathematics, no mathematics–basic mathematics, and algebra II–no mathematics.

experiences and this cohort of high school students, table 4 shows the percentage of students taking mathematics course sequences by student and school characteristics.

Table 4. Percentage of students taking mathematics course sequences, by selected student characteristics: 2002 and 2004

Student characteristic	Course sequence									
	No mathematics–no mathematics	Geometry–geometry/no mathematics	Geometry–algebra II	Algebra II–no mathematics	Algebra II–algebra II/trigonometry	Algebra II–precalculus	Precalculus–no mathematics	Pre-calculus–calculus	Pre-calculus–AP/IB calculus	All other patterns
Total	5.6	7.8	4.6	12.7	6.1	6.8	3.6	2.0	6.1	44.6
Sex										
Female	4.8	7.2	4.6	12.7	6.7	7.2	3.9	1.9	5.8	45.2
Male	6.5	8.5	4.6	12.8	5.4	6.4	3.3	2.1	6.4	44.0
Race/ethnicity										
White	5.7	7.4	3.8	12.1	6.1	7.0	3.8	2.6	6.9	44.5
Black	4.8	7.1	8.7	14.3	9.4	6.7	2.1	0.5	2.3	44.0
Hispanic	6.1	10.8	5.0	15.2	3.8	6.4	4.5	0.5	3.8	44.0
Asian	2.8	5.7	3.4	10.4	3.8	7.8	1.8	2.2	13.4	48.7
American Indian	14.7	8.3	6.7	9.5	1.3	6.6	#	#	5.1	48.0
More than one race	6.7	9.1	4.7	12.7	6.1	4.9	4.4	1.3	3.8	46.4
Socioeconomic status										
Quartile 1 (low)	8.7	12.4	5.5	12.4	5.2	5.6	2.1	0.3	2.5	45.3
Quartile 2	6.6	10.3	5.1	15.1	6.0	4.9	3.3	1.2	2.7	44.9
Quartile 3	5.4	7.0	4.2	13.7	6.6	7.6	3.6	2.3	5.2	44.3
Quartile 4 (high)	2.7	3.2	3.9	10.1	6.3	8.6	5.0	3.6	12.4	44.3
School sector										
Public	6.0	8.4	4.7	13.1	5.6	6.4	3.6	1.7	5.8	44.8
Catholic	1.4	1.9	5.0	9.0	11.2	10.8	2.9	5.5	9.0	43.2
Other private	1.7	2.4	2.2	7.6	11.9	12.9	3.6	4.9	9.1	43.7
Family composition										
Mother and father	5.2	7.2	4.0	12.4	5.8	6.9	4.3	2.5	7.5	44.2
Mother or father and guardian	5.8	8.8	4.4	13.3	6.4	7.8	2.7	0.9	3.3	46.7
Single parent	6.3	8.9	6.5	13.6	6.6	6.1	2.4	1.4	4.2	43.9
Other	8.3	10.0	5.1	11.5	6.6	4.1	2.1	1.0	2.4	48.7

See notes at end of table.

Table 4. Percentage of students taking mathematics course sequences, by selected student characteristics: 2002 and 2004—Continued

Student characteristic	Course sequence									
	No mathematics–no mathematics	Geometry–geometry/no mathematics	Geometry–algebra II	Algebra II–no mathematics	Algebra II–algebra II/trigonometry	Algebra II–precalculus	Precalculus–no mathematics	Pre-calculus–calculus	Pre-calculus–AP/IB calculus	All other patterns
Student's educational expectations[1]										
High school or less	13.5	14.6	4.5	12.5	3.2	0.8	1.0	0.4	0.1	49.4
Some college	12.7	16.2	5.9	13.3	2.6	2.5	1.1	0.1	0.9	44.8
Bachelor's degree or more	3.9	6.4	4.3	12.7	7.0	8.0	4.3	2.4	7.3	43.7
Don't know	9.4	8.4	5.6	12.9	2.7	4.2	1.9	1.2	4.1	49.6

Rounds to zero.

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. The nine course sequences listed refer to those followed by more than 3 percent of the sample. Details may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Much like the analysis of mathematics assessment scores, sequences of mathematics courses taken during junior and senior year of high school differed across student characteristics. With respect to sex, there were no detectable differences between boys and girls in taking course sequences containing precalculus during the last 2 years of high school: 18 percent of boys and 19 percent of girls did so. With respect to race/ethnicity, 25 and 20 percent of Asian and White students, respectively, followed course sequences that contained precalculus, compared to 15 percent of their Hispanic peers and 12 percent of their Black peers (table 4). The differences are greater when examining socioeconomic status. Almost 30 percent of students in the highest SES quartile followed pathways that included precalculus, while 11 percent of those in the lowest SES did so. This descriptive evidence shows that Blacks, Hispanics, and less affluent students were reaching advanced mathematics courses less often than their Asian, White, and more affluent peers.

Turning next to school type, 28 percent of Catholic school students have taken a sequence that included precalculus, compared to 18 percent of their public school peers. Similar patterns were found for other private school students, who were more likely than public school students to have taken a sequence containing precalculus (30 percent versus 18 percent). At the most advanced course sequence in the present analysis, precalculus-AP/IB, there were no detectable differences among the three school sectors: 6 percent of public school students and 9 percent of Catholic school students and other private school students followed this sequence during the second half of high school.

In terms of family structure, there were no detectable differences between students living with both parents and students living in a stepfamily or with a single parent in taking course sequences containing precalculus during the last 2 years of high school: 21 percent of students living with both parents, 15 percent of students living in a stepfamily, and 14 percent of students living with a single parent. However, more students living with both parents (21 percent) have taken course sequences containing precalculus during the last 2 years of high school than their peers living in other family forms (10 percent).

Lastly, students' expectations for their future educational attainment were linked with differential curricular pathways: those who hold higher educational plans tend to take more advanced courses while those who set lower educational goals tend to take fewer advanced courses. For example, 7 percent of those who expected a college degree when enrolled in 10th grade later followed a precalculus–AP/IB calculus sequence while less than 1 percent of those who expected to attain some college and those who expected a high school degree or less did so. Moreover, 4 percent of students who expected a bachelor's degree took no mathematics courses during the latter part of high school, compared with 14 percent of students who expected a high school diploma or less.

What mathematics course sequences are associated most closely with mathematics achievement?

To answer this research question, both bivariate and multivariate techniques are used. First, learning gains in mathematics are examined descriptively for students taking different course sequences. Next, regression techniques are used to measure the association of mathematics courses with mathematics achievement at the end of high school, apart from key factors related with these curricular pathways. Together, these analyses identify course sequences most often associated with a student's mastery of advanced mathematics skills and concepts.

Table 5 shows average scores and gains for the number-right scores. Effect sizes for these gains are shown in parentheses; corresponding standard deviations are shown in table B-5b. In terms of number-right scores, students improved the most during their junior and senior years when they reached at least algebra II and took another advanced course, while students who took less than two mathematics courses or were still taking the geometry series improved the least. The gains for students in the most advanced course sequences—precalculus–calculus and precalculus–AP/IB calculus—are particularly large, with effect sizes nearing almost one standard deviation.

To highlight the relationships between coursework and learning, consider algebra II, the modal course taken by high school juniors.[21] On average, students who took algebra II during their junior year followed by precalculus in their senior year improved by an average of 6.9 correct answers (table 5). On the other hand, those who took algebra II in their junior year but did not take a mathematics course during their senior year improved by an average of 4 correct answers. Their overall number-right scores at the end of high school were about 9 points apart (57.6 and 48.7, respectively).

Table 5. Average mathematics IRT-estimated number-right scores, by mathematics course sequences: 2002 and 2004

Course sequence	10th grade	12th grade	Change (ES)
Total	46.7	51.2	4.5 (0.33)
No mathematics–no mathematics	41.1	42.5	1.4 (0.10)
Geometry–geometry/no mathematics	39.1	42.1	3.0 (0.28)
Geometry–algebra II	40.6	45.8	5.2 (0.52)
Algebra II–no mathematics	44.7	48.7	4.0 (0.37)
Algebra II–algebra II/trigonometry	47.2	55.3	6.2 (0.85)
Algebra II–precalculus	50.7	57.6	6.9 (0.71)
Precalculus–no mathematics	55.2	60.0	4.8 (0.56)
Precalculus–calculus	59.4	66.2	6.8 (0.95)
Precalculus–AP/IB calculus	62.5	69.2	6.8 (0.95)
All other patterns	45.9	50.1	4.2 (0.29)

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample. IRT = Item Response Theory. ES = Effect Size (i.e., standardized mean difference). AP/IB calculus = Advanced Placement/International Baccalaureate calculus.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

By and large, these results complement past research that finds that more advanced coursetaking is associated with greater gains (Leow et al. 2004; Rock, Owings, and Lee 1994; Rock and Pollack 1995a; Scott et al. 1995; Wang and Goldschmidt 2003). However, are students who follow different mathematics course sequences learning different mathematics concepts and skills? The proficiency probability scores in ELS:2004 allow for such an assessment. Table 6 shows average scores, gains, and effect sizes for the gains for the proficiency probability scores for each of the nine major mathematics sequences. Standard deviations associated with the effect sizes are shown in table B-6a. The estimates in table 6 show that while advanced courses are

[21] Algebra II is the modal course taken in the 11th grade. In the analytic sample, 35 percent of students earned credit in algebra II in the 2003–04 school year.

associated with learning in the aggregate (as evidenced when using number-right scores), curricular pathways differentially predict the acquisition of mathematics skills and concepts.

At level 1, the basic level, improvements are highest for those students who took the geometry-geometry/no mathematics sequence. This is not surprising as this level measures the most basic mathematical concepts such as arithmetic and whole numbers. As most of the other major pathways shown are beyond basic mathematics and algebra I, which require a solid foundation in arithmetic, student mastery is near the ceiling for students in the other major course sequences—leaving little room for growth.

At the intermediate levels—level 2 and level 3—improvements are the highest for those who take algebra II paired with geometry or trigonometry. For example, the percentage of students who took geometry paired with algebra II who were proficient at level 2 improved by 14 percentage points. The percentage of these students who were proficient at level 3 improved by 23 percentage points (table 6). Learning gains at level 2 and level 3 are smaller for those in the most advanced course sequences as these students have already mastered these skills—for example, before starting their junior year, 99 percent of students taking the precalculus–calculus sequence were already proficient at level 2 and 93 percent were already proficient at level 3.

For the most part, the largest gains at the advanced levels are made by students who take precalculus paired with another course. For example, improvements at level 4 are highest for those who follow the algebra II-precalculus sequence. These students improved their proficiency at this level by 28 percentage points. At level 5, improvements are highest for those who follow the precalculus–AP/IB calculus sequence. Students following this sequence improved their proficiency by 22 percentage points. As these courses expose students to the most challenging skills and concepts in the high school mathematics curriculum, it is not surprising that the students taking them learn the most. Despite their rigor, however, there is still substantial room for learning: the majority of students (71 percent) taking precalculus–AP/IB calculus, arguably the most advanced sequence, are not proficient at level 5.[22]

Table 7 summarizes the key relationships in table 6 by showing the two course sequences associated with the largest gain in learning at each proficiency level. Taken together, these descriptive findings suggest that gains in mathematics achievement and an understanding of more advanced topics are best achieved by juniors and seniors who take precalculus along with another course. However, it is possible that these observed improvements in learning are a reflection of the types of students who follow these course sequences. In other words, students who enroll in a precalculus–calculus sequence may be faring well in mathematics because they are affluent, ambitious, and attend private schools, not because of the courses they are taking. To separate out the influence of course sequences from background characteristics, a series of regression analyses are performed. The advantage of regression analysis is that it can show the relationship between a dependent variable and any individual independent variable, while holding the other independent variables constant. Despite this advantage, the regression analysis reported here cannot establish a causal relationship between coursetaking and mathematics achievement. Readers should keep this in mind when interpreting the results reported herein.

[22] Though the scaling of the test was designed to prevent a "ceiling effect" at this level, that no course sequence yields proficiency over 50 percent suggests that even the most highly prepared students leaving high school may need more training in mathematics to master advanced skills such as multistep problem solving and derivations.

Table 6. Average mathematics proficiency probability scores, by mathematics course sequences: 2002 and 2004

	10th grade					12th grade					Change				
Course sequence	Level 1	Level 2	Level 3	Level 4	Level 5	Level 1	Level 2	Level 3	Level 4	Level 5	Level 1	Level 2	Level 3	Level 4	Level 5
Total	0.94	0.73	0.53	0.25	0.01	0.96	0.80	0.65	0.38	0.04	0.03	0.06	0.11	0.13	0.03
No mathematics–no mathematics	0.88	0.59	0.37	0.14	#	0.92	0.61	0.40	0.16	0.01	0.03	0.02	0.03	0.02	#
Geometry–geometry/no mathematics	0.90	0.55	0.27	0.07	#	0.95	0.64	0.38	0.09	#	0.05	0.09	0.11	0.03	#
Geometry–algebra II	0.94	0.62	0.29	0.06	#	0.98	0.76	0.52	0.15	#	0.04	0.14	0.23	0.09	#
Algebra II–no mathematics	0.96	0.75	0.46	0.14	#	0.98	0.83	0.63	0.23	#	0.02	0.08	0.17	0.10	#
Algebra II–algebra II/trigonometry	0.98	0.82	0.56	0.17	#	1.00	0.92	0.78	0.36	0.01	0.02	0.10	0.22	0.20	#
Algebra II–precalculus	0.98	0.88	0.72	0.28	#	1.00	0.95	0.86	0.56	0.03	0.02	0.07	0.15	0.28	0.02
Precalculus–no mathematics	0.99	0.95	0.84	0.45	0.02	1.00	0.98	0.92	0.65	0.03	0.01	0.04	0.08	0.20	0.02
Precalculus–calculus	1.00	0.99	0.93	0.62	0.02	1.00	1.00	0.98	0.86	0.15	#	0.01	0.05	0.24	0.13
Precalculus–AP/IB calculus	1.00	0.99	0.95	0.75	0.07	1.00	1.00	1.00	0.93	0.29	#	0.01	0.05	0.18	0.22
All other patterns	0.92	0.69	0.52	0.26	0.01	0.95	0.75	0.61	0.37	0.04	0.03	0.06	0.09	0.12	0.03

Rounds to zero.

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Table 7. Mathematics course sequences associated with the two largest gains in mathematics learning, by proficiency level: 2002 and 2004

Proficiency level[1]	Course sequence associated with the largest gain	Course sequence associated with the second largest gain
Level 1	Geometry–geometry/no mathematics	Geometry–algebra II
Level 2	Geometry–algebra II	Algebra II–algebra II/trigonometry
Level 3	Geometry–algebra II	Algebra II–algebra II/trigonometry
Level 4	Algebra II–precalculus	Precalculus–calculus
Level 5	Precalculus–AP/IB calculus	Precalculus–calculus

[1] Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

NOTE: AP/IB calculus = Advanced Placement/International Baccalaureate calculus.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

The availability of two mathematics assessments, one before students enter their junior year and one at the end of senior year, permits a more stringent assessment of learning gains. This study estimates a set of regression models where the test score from the 12th grade is used as the dependent variable and the test score from the 10th grade is used as a control variable. Using the 10th-grade test score as a control variable conditions the effects of the rest of the predictor variables on students’ initial level of mathematics proficiency (i.e., the estimated relationship for a particular covariate is conditional upon the other variables in the model). This is often referred to as the covariate adjustment or regressor variable approach to analyzing change.[23]

Two sets of models are estimated. First, the relationships between the course sequences and the 12th-grade number-right score are estimated. Second, the relationships between the course sequences and the five proficiency levels from the 12th grade are estimated. Both outcomes are continuous and thus, ordinary least squares (OLS) estimation is used. When a lagged version of the dependent variable (i.e., the 10th-grade score) is included in the model, as is the case in this study, the coefficients associated with the other predictor variables represent the amount of change in the dependent variable associated with a unit change in each

[23] There are two general approaches to modeling change in a regression framework when the dependent variable is measured at two points in time: the change score approach and the covariate adjustment approach. In the change score approach, the dependent variable would be the difference between the F1 test score and the BY test score. This difference score would be regressed on the measures of course sequences and background characteristics. In the covariate adjustment approach, the F1 test score is used as the dependent variable and the BY test score is used as a predictor variable alongside the measures of course sequences and background variables. When the “treatment” occurs between the pretest and the posttest, and assignment to the treatment group is affected by the pretest, the covariate adjustment approach is preferable to the change score approach (Allison 1990; Maris 1998). In this study, course sequences are considered to be the “treatment” and they occur between the BY and F1 test administration. As evidenced in the descriptive statistics in tables 5 and 6, course sequences that students take at the end of high school are associated with their initial scores on the pretest. Additionally, the change score approach constrains the coefficient associated with the baseline score to 1, which is too restrictive. For these reasons, this study uses the covariate adjustment approach rather than the change score approach. In the absence of an experimental design, the covariate adjustment method greatly reduces the threat of endogeneity (i.e., that the omission of other unmeasured factors influencing both course sequences and 12th-grade test scores will bias the estimates) (Allison 1990; Maris 1998).

independent variable in the model (Finkel 1995). With the level of achievement in 10th grade already controlled, any differences detected by the rest of the coefficients reflect achievement *beyond* what is already measured at the end of 10th grade—hence, the change interpretation. The first set of models is shown in table 8.[24]

[24] None of the variables in any of the models presented in tables 8 and 9 yielded a variance inflation factor greater than 5.

Table 8. Coefficients from an ordinary least squares (OLS) regression of 12th-grade IRT mathematics scores, by characteristic: 2002 and 2004

Characteristic	Model 1[1]		Model 2	
	Coefficient	Standard error	Coefficient	Standard error
10th-grade IRT mathematics score	0.9*	0.01	0.9*	0.01
Sex				
Female (reference)	†	†	†	†
Male	–0.4*	0.17	–0.6*	0.16
Race/ethnicity				
White (reference)	†	†	†	†
Black	–0.8*	0.25	–1.3*	0.25
Hispanic	–0.3	0.27	–0.6*	0.27
Asian	0.5	0.38	0.2	0.38
American Indian	–1.6	1.02	–1.6	0.94
More than one race	0.4	0.42	0.4	0.40
Socioeconomic status				
Quartile 1 (low) (reference)	†	†	†	†
Quartile 2	#	†	#	†
Quartile 3	0.9*	0.23	0.7*	0.22
Quartile 4 (high)	1.7*	0.26	1.3*	0.25
School sector				
Public (reference)	†	†	†	†
Catholic	1.4*	0.22	1.0*	0.21
Other private	0.9*	0.36	0.5	0.39
Family composition				
Mother and father (reference)	†	†	†	†
Mother or father and guardian	–0.5*	0.26	–0.5	0.25
Single parent	–0.2	0.19	–0.2	0.20
Other	–1.7*	0.51	–1.7*	0.48
Student's educational expectations[2]				
High school or less (reference)	†	†	†	†
Some college	0.7	0.52	0.8	0.50
Bachelor's degree	1.9*	0.43	1.5*	0.42
Don't know	1.1*	0.51	1.0*	0.49
Course sequence				
No mathematics–no mathematics	†	†	–2.8*	0.41
Geometry–geometry/no mathematics	†	†	–1.5*	0.38
Geometry–algebra II	†	†	0.6	0.42
Algebra II–no mathematics (reference)	†	†	†	†
Algebra II–algebra II/trigonometry	†	†	3.2*	0.33
Algebra II–precalculus	†	†	2.1*	0.37
Precalculus–no mathematics	†	†	1.8*	0.41
Precalculus–calculus	†	†	3.7*	0.45
Precalculus–AP/IB calculus	†	†	4.2*	0.37
All other patterns	†	†	#	†

See notes at end of table.

Table 8. Coefficients from an ordinary least squares (OLS) regression of 12th-grade IRT mathematics scores, by characteristic: 2002 and 2004—Continued

Characteristic	Model 1[1]		Model 2	
	Coefficient	Standard error	Coefficient	Standard error
Three mathematics courses or less 2002–2004 (reference)	†	†	†	†
More than 3 mathematics courses 2002–2004	†	†	1.3*	0.24
Constant	7.8*	0.620	9.9*	0.68
N	9,456	†	9,456	†
Adjusted R-squared	0.81	†	0.83	†

† Not applicable.
# Rounds to zero.
* $p < .05$.
[1] The course sequence variable was not included in Model 1.
[2] Educational expectations of 10th-grade students.
NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. IRT = Item Response Theory. AP/IB calculus = Advanced Placement/International Baccalaureate calculus.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Model 1 includes only the 10th-grade number-right score and the student background characteristics, which are used as control variables in addition to the 10th-grade mathematics scores. All of the characteristics are categorical and are entered as a series of indicator (0, 1) variables. Reference categories include: White students (race/ethnicity), female students (sex), SES quartile 1 (low) (socioeconomic status), public school students (school sector), students who live with both parents in the family (family composition), and expecting a high school diploma or less (educational expectations).

As indicated by the adjusted R-squared, 81 percent of the variation in 12th-grade number-right scores is explained by sociodemographic characteristics and 10th-grade number-right scores—revealing that a substantial amount of the variation in learning at the end of high school is explained by factors that precede enrollment in mathematics courses during the last 2 years of high school. Most relationships shown in Model 1 are in accord with other multivariate analyses of learning gains using NCES data sets (e.g., Carbonaro 2005; Morgan and Sorenson 1999). When controlling for other background factors, most racial/ethnic differences disappear, with the exception of differences between Black and White students’ test scores: all else equal, Black students’ correct-answer gains were 0.8 less than their White counterparts (Model 1, table 8). Socioeconomic disparities persist: students in SES quartile 3 and quartile 4 had significantly higher gains in mathematics learning than their peers in the lowest quartile. Though there were no bivariate differences detected in the learning rates of boys and girls, once student and school background characteristics were controlled for, the correct-answer gains made by boys were significantly less than the gains made by girls. School sector effects are still evident, even with other student background characteristics controlled: Catholic school students gained 1.4 correct answers and other private school students gained 0.9 correct answers more than their public school peers. Lastly, students from stepfamilies and other family forms had smaller gains than did students who live with both parents in the family, and students who expected to earn a bachelor’s degree had significantly higher gains than those who expected a high school degree or less.

Mathematics course sequences are added to student background characteristics in Model 2 on table 8. These are added as a set of indicator (0,1) variables indicating the different course sequences. The reference category is "algebra II–no mathematics" so that all comparisons could be made with respect to the modal pattern taken by contemporary high school students. Also included is a control for students who had taken more than three courses during the 2002–03 and 2003–04 school years.

Before looking at the effects of the individual coursetaking sequences, note that at a very general level, curricular pathways are important predictors of mathematics learning at the end of high school. F-tests are used to compare the fit of different models. An F-test comparing the fit of Model 1 (10th-grade scores + background characteristics) with the fit of Model 2 (10th-grade scores + background characteristics + course sequences) yields a significant test statistic ($F = 64.61$, $df = 10$, $p < .01$), indicating that course sequences significantly improve the fit of the model. Substantively, however, the explanatory power of course sequences is limited, explaining only 2 percent of the variation beyond what is accounted for by previous learning and background factors. Course sequences do matter, as evidenced by the significant F statistic, but achievement at the end of high school is largely predicted by factors that precede these years.

What course sequences are associated with the largest learning gains in mathematics at the end of high school? In accord with the descriptive analysis shown in table 5, with previous learning and background characteristics controlled, students who take more advanced course sequences gain more than do students taking lower-level course sequences. The relationship is mostly linear as the magnitude and direction of coefficients change from – to + when moving from sequences containing less advanced math courses to sequences containing more advanced math courses.[25]

To provide a direct example, consider algebra II—the modal course taken by high school juniors. Some students will continue on to precalculus during their senior year while others will refrain from any mathematics course during their senior year. Though these different pathways are associated with students' achievement and sociodemographic characteristics, the relationship between an additional mathematics course and learning gains is better assessed using multiple regression as these possibly confounding factors are controlled. The estimates on table 8 show that all else equal, students who take precalculus following algebra II gain 2.1 correct answers on the mathematics assessment more than their peers who follow an algebra II–no mathematics sequence (Model 2, table 8). Students who are further along in the mathematics sequence have even higher gains. For example, those who take precalculus in their junior year and take either calculus or AP/IB calculus in their senior year gain about 4.2 correct answers more than their peers who follow an algebra II–no mathematics sequence.

Though the use of number-right scores provides a portrait of mathematics gains in the aggregate, the relationship between different course sequences and particular skills and concepts are unknown. The use of proficiency probability scores as dependent variables permits such an analysis. Table 9 shows five multiple regression models—one predicting each of the proficiency

[25] Establishing a hierarchy of less advanced to more advanced course sequences is not straightforward given that two separate courses comprise a sequence. Thus, some course sequences are not necessarily more or less advanced than one another (e.g., algebra II–precalculus and precalculus–no mathematics). However, it is largely the case that course sequences containing more (less) advanced courses correspond to larger (smaller) gains in contrast to algebra II–no mathematics.

levels. As was the case for Models 1 and 2 on table 8, all models control for proficiency measured in the 10th grade as well as background characteristics.[26]

Similar to the descriptive analysis, mathematics course sequences are generally unrelated to gains in proficiency at level 1. For example, with the exception of the coefficient for no mathematics–no mathematics, none of the coefficients associated with the major course sequences are significantly different from algebra II–no mathematics, the modal course sequence. This is likely because most students enter their junior year with a mastery of basic arithmetic and whole numbers; thus, there is little if any room for improvement. At the intermediate levels—level 2 and level 3—there is evidence that when compared to those who followed an algebra II–no mathematics sequence, the largest gains were made by those who followed an algebra II–precalculus sequence. They outgained their algebra II–no mathematics peers by 4 and 7 percentage points at levels 2 and 3, respectively (table 9).

In levels 1, 2, and 3, the most advanced course sequences—precalculus–calculus and precalculus–AP/IB calculus—have negative coefficients. While at first this may seem counterintuitive, recall that students in these sequences enter the second half of high school with almost complete mastery of these topics (see table 6). Therefore, they make fewer gains than their peers who are in the algebra II–no mathematics sequence.

At level 4, the largest gains relative to algebra II–no mathematics were made by those who followed the algebra II-algebra II/trigonometry sequence and those who followed one of the precalculus sequences. At the most advanced level—level 5—the largest gains were made by students who followed one of the precalculus sequences. For example, at level 5 students who followed the precalculus–calculus sequence outgained those who were in the algebra II–no mathematics sequence by 11 percentage points and students who followed the precalculus–AP/IB calculus sequence outgained those who were in the algebra II–no mathematics sequence by 20 percentage points. This complements the bivariate findings presented earlier: mastery of the most advanced skills is associated with credits earned in courses beyond algebra II, such as trigonometry, precalculus, and calculus.

Finally, in 4 of the 6 models in which course sequences were used to predict mathematics achievement outcomes in 12th grade (Model 2 in table 8 and models for levels 2, 3, and 4 in table 9), students who followed an algebra II–precalculus sequence had higher gains than their peers who followed an algebra II–no mathematics sequence. Moreover, none of the models showed any measurable differences between students who followed a geometry–algebra II sequence and those who followed an algebra II–no mathematics sequence. In essence, students who took algebra II in their junior year and did not take any mathematics courses in their senior year learned no more or less than their peers who did not reach algebra II until their senior year. On the other hand, students who reached algebra II in their junior year and then continued on to precalculus in their senior year made the most sizeable gains.

[26] Since background characteristics are not the main focus of this report, the models predicting the proficiency levels including only background characteristics are not shown. Similar to the model comparisons shown in table 8, the addition of the coursetaking terms explain little of the variation in levels 1–4 beyond what is accounted for by previous learning and background factors. When the coursetaking terms are added, the adjusted R-squared remains the same (.49) for level 1, remains the same (.69) for level 2, improves from .62 to .64 for level 3, and improves from .69 to .72 for level 4. The largest growth in adjusted R-squared occurs for level 5, where the inclusion of the coursetaking terms improves 9 percent—from .38 to .47.

Table 9. Coefficients from an ordinary least squares (OLS) regression of 12th-grade proficiency probability mathematics scores, by characteristic: 2002 and 2004

Characteristic	Level 1 Co-efficient	Level 1 Standard error	Level 2 Co-efficient	Level 2 Standard error	Level 3 Co-efficient	Level 3 Standard error	Level 4 Co-efficient	Level 4 Standard error	Level 5 Co-efficient	Level 5 Standard error
10th-grade level proficiency	0.43*	0.018	0.72*	0.011	0.66*	0.010	0.82*	0.011	1.01*	0.056
Sex										
Female (reference)	†	†	†	†	†	†	†	†	†	†
Male	#	†	#	†	–0.01	0.007	–0.03*	0.006	–0.02*	0.003
Race/ethnicity										
White (reference)	†	†	†	†	†	†	†	†	†	†
Black	–0.01	0.004	–0.02*	0.010	–0.10*	0.013	–0.10*	0.008	–0.02*	0.003
Hispanic	–0.01*	0.004	–0.01	0.011	–0.05*	0.012	–0.05*	0.009	–0.01*	0.003
Asian	#	†	–0.01	0.011	–0.01	0.012	0.01	0.014	0.02	0.010
American Indian	–0.02	0.024	–0.03	0.030	–0.10*	0.037	–0.05	0.037	–0.01	0.013
More than one race	#	0.005	0.01	0.013	#	†	–0.01	0.016	–0.01*	0.007
Socioeconomic status										
Quartile 1 (low) (reference)	†	†	†	†	†	†	†	†	†	†
Quartile 2	#	†	#	†	0.02	0.011	0.02*	0.008	#	†
Quartile 3	0.01	0.003	0.02	0.009	0.06*	0.011	0.04*	0.007	0.01*	0.003
Quartile 4 (high)	#	†	0.02	0.009	0.06*	0.011	0.07*	0.009	0.03*	0.004
School sector										
Public (reference)	†	†	†	†	†	†	†	†	†	†
Catholic	#	†	0.01*	0.005	0.03*	0.009	*0.05	0.009	#	†
Other private	#	†	0.01	0.008	0.01	0.014	0.01	0.015	0.02*	0.010
Family composition										
Mother and father (reference)	†	†	†	†	†	†	†	†	†	†
Mother or father and guardian	#	†	–0.01	0.008	–0.01	0.011	–0.02	0.008	#	†
Single parent	#	†	–0.01	0.007	–0.02*	0.009	–0.01	0.008	#	†
Other	–0.02	0.010	–0.03	0.019	–0.05*	0.020	–0.05*	0.013	#	†
Student's educational expectations[1]										
High school or less (reference)	†	†	†	†	†	†	†	†	†	†
Some college	0.02*	0.010	0.06*	0.020	0.06*	0.023	0.02	0.013	#	†
Bachelor's degree	0.03*	0.008	0.07*	0.017	0.12*	0.020	0.08*	0.011	0.01*	0.002
Don't know	0.02*	0.009	0.05*	0.018	0.08*	0.023	0.05*	0.013	0.01*	0.004

See notes at end of table.

Table 9. Coefficients from an ordinary least squares (OLS) regression of 12th-grade proficiency probability mathematics scores, by characteristic: 2002 and 2004—Continued

Characteristic	Level 1		Level 2		Level 3		Level 4		Level 5	
	Co-efficient	Standard error	Co-efficient	Standard error	Co-efficient	Standard error	Co-efficient	Standard error	Co-efficient	Standard error
Course sequence										
No mathematics–no mathematics	–0.03*	0.008	–0.09*	0.014	–0.15*	0.018	–0.06*	0.013	0.01*	0.002
Geometry–geometry/no mathematics	#	†	–0.04*	0.014	–0.10*	0.018	–0.07*	0.011	#	†
Geometry–algebra II	0.01	0.004	0.02	0.018	0.02	0.023	–0.01	0.014	#	†
Algebra II–no mathematics (reference)	†	†	†	†	†	†	†	†	†	†
Algebra II–algebra II/trigonometry	0.01	0.003	0.02	0.010	0.04*	0.015	0.18*	0.015	0.01*	0.004
Algebra II–precalculus	0.01	0.003	0.04*	0.012	0.07*	0.018	0.09*	0.016	#	†
Precalculus–no mathematics	0.01	0.004	#	†	0.02	0.016	0.13*	0.020	0.01	0.009
Precalculus–calculus	#	†	–0.02*	0.008	–0.01	0.016	0.17*	0.020	0.11*	0.020
Precalculus–AP/IB calculus	#	†	–0.02*	0.007	–0.01	0.013	0.14*	0.017	0.20*	0.016
All other patterns	–0.01*	0.003	–0.04*	0.008	–0.07*	0.012	0.03*	0.010	0.02*	0.003
Three mathematics courses or less 2002–2004 (reference)	†	†	†	†	†	†	†	†	†	†
More than 3 mathematics courses 2002–2004	0.01*	0.003	0.02*	0.007	0.03*	0.010	0.04*	0.010	0.01*	0.006
Constant	0.54*	0.020	0.23*	0.022	0.23*	0.024	0.09*	0.016	0.01*	0.005
N	9,456	†	9,456	†	9,456	†	9,456	†	9,456	†
Adjusted R-squared	0.49	†	0.70	†	0.64	†	0.72	†	0.47	†

† Not applicable.
# Rounds to zero.
* $p < .05$.
[1] Educational expectations of 10th-grade students.
NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Chapter 4
Conclusion

This study finds that at the end of their high school years, American students increase their knowledge in mathematics—mostly in intermediate-level skills and concepts. The majority of students enter their junior year with a solid proficiency in lower-level skills such as whole numbers, fractions, and decimals, and improve their understanding of intermediate skills and concepts such as algebraic relationships and logic. Despite these overall gains, most do not graduate with a solid foundation in the most advanced skills, such as derivations and multistep problems. At this level, only 4 percent are proficient, up 3 percentage points from the end of sophomore year.

This study, using data from the Education Longitudinal Study of 2002 (ELS:2002), yields mixed information on the relationships between coursetaking and learning. On the one hand, students who reach the most advanced courses such as precalculus and calculus before leaving high school are more likely to learn the most advanced skills and concepts. On the other hand, the lion's share of learning that takes place at the end of high school is predicted by background characteristics and curricular experiences that precede the start of junior year.[27] Each of these is discussed in turn.

Much like the bulk of previous research, this study finds that students who take advanced courses such as precalculus and algebra II learn more than their peers who take intermediate courses such as geometry and those who do not take mathematics courses. However, this study extends previous research by using scaled scores that permit an assessment of the specific skills and concepts being learned. While there is evidence of learning gains in mathematics during the last 2 years of high school across the board, students who follow a geometry–algebra II sequence or an algebra II–trigonometry sequence show the greatest improvement in intermediate skills, such as operations with whole numbers and basic algebraic expressions, while students who follow an algebra II–precalculus sequence or a precalculus–calculus sequence show the greatest improvements in advanced skills such as multistep analytical problems. These effects were robust when included in regression models that controlled for prior learning and key background characteristics.

While mathematics course sequences are associated with learning, most of 12th-grade achievement is explained by background factors and previous learning; mathematics courses at the end of high school explain little of the variation in achievement once background factors and previous achievement are included as controls. This is likely because these background factors and previous learning experiences are linked with their coursetaking patterns. Indeed, differences in coursetaking follow a pattern that is well documented by social scientists and replicated here in the ELS:2004 data. Students with more socioeconomic and educational resources—students from affluent families, students in two-parent homes, students who attend private schools, and students who have ambitious educational expectations—are more likely to reach the most advanced mathematics courses. Their experiences and instruction at earlier stages in schooling

[27] Eighty-one percent of the variation in 12th-grade number-right scores is explained by sociodemographic characteristics and 10th-grade number-right scores.

likely gives them the foundational skills to move through the hierarchy at a pace that ensures their enrollment in courses like precalculus and calculus.

Despite the strengths of this study—for example, the longitudinal design, course information from administrative records, and assessments scaled to different proficiency levels—ELS:2002, like many National Center for Education Statistics (NCES) data sets, is observational. As such, students were not randomly assigned to schools, classrooms, or course sequences—limiting the ability to establish a causal link between coursetaking and learning. Regression procedures were used to estimate the relationship between coursetaking and learning controlling for prior achievement and other observed characteristics known to shape students' placement in different tracks/courses. Though the effects of course sequences were robust when all control variables were included, there may have been other unmeasured characteristics (e.g., student motivation, teacher engagement, etc.) that may have caused both course sequence placement and learning, thus making the relationship between coursetaking and learning spurious.

Another limitation to the study is that the course sequences for 45 percent of the analytic sample could not be examined because there were too few cases to make valid generalizations. It could well be the case that some of these course sequences are more beneficial at improving mathematics proficiency than those identified in this report. The small sample sizes, however, preclude a thorough examination of every curricular pathway undertaken by American high school students. Additionally, by creating course sequences from two separate courses, it is not possible to assess the strength of the relationship of each individual course with mathematics achievement. Lastly, compared with the target population, the analytic sample used in this study has higher proportions of White students, students who expect to attain a bachelor's degree or higher, and students living with both their father and their mother. Readers should keep these caveats in mind when interpreting the results and conclusions of this report.

In closing, this study shows that in the last 2 years of high school, American students are improving in mathematics, but that there is room for further improvement. Proficiency scores show that by the end of high school some 38 percent had mastered intermediate mathematics skills, as compared to 25 percent of the sample 2 years before. However, only 4 percent had mastered advanced skills and concepts, compared to 1 percent 2 years earlier. This study shows that the mathematics achievement gains registered between sophomore and senior year are associated with specific coursetaking sequences. In turn, different sequences are related to the proficiency level at which the gains are taking place. The largest gains in intermediate skills were made by students who followed an algebra II-geometry sequence. The largest gains in advanced skills were made by students who followed a sequence that included precalculus paired with algebra II, calculus, or Advanced Placement/International Baccalaureate calculus. At the same time, this study shows that despite the significant relationship between mathematics coursetaking sequences and achievement gain, a greater amount of the variation in learning at the end of high school is explained by factors that precede enrollment in these coursetaking sequences.

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Appendix A
Technical Notes and Glossary

A.1 Design and Implementation of the Education Longitudinal Study of 2002 (ELS:2002)

The National Center for Education Statistics (NCES) of the U.S. Department of Education has collected longitudinal data for more than 30 years. Starting in 1972 with the National Longitudinal Study of the High School Class of 1972 (NLS:72) and continuing to the most recent study, the Education Longitudinal Study of 2002 (ELS:2002), NCES has collected longitudinal and trend data on the achievement and experiences of high school students, as well as postsecondary outcomes such as entry into the labor market and college enrollment and persistence.

The base year of ELS:2002 was the first stage of a new effort designed to provide data about critical transitions experienced by a cohort of sophomores in 2002 as they proceed through high school and into postsecondary education and/or their careers. The 2002 sophomore cohort was surveyed again in 2004. Starting in 2006, future follow-ups will collect data about postsecondary access and choices, postsecondary attainment, entry into the work force, family formation, voting, volunteerism, and life goals and values.

This appendix provides an overview of the base-year (BY) and first follow-up study (F1) design and methodology. This appendix also provides information on the statistical procedures employed and supplies a glossary that documents the analysis variables used in this report. Appendix B includes tables of standard errors. In addition, it reports weighted standard deviations and raw sample sizes for all means reported in tables of estimates.

A.2 Overview of ELS:2002

A.2.1 Study Objectives

ELS:2002 is a longitudinal study, in which the same individuals are surveyed repeatedly over time. Individual students are expected to be followed until about age 30; the base-year schools have been surveyed twice (they were surveyed in 2002 and again in 2004). In the high school years, ELS:2002 is an integrated multilevel study, involving multiple respondent populations, including students, their parents, their teachers, and their schools (from which data are collected at three levels: from the principal, the librarian, and a facilities checklist). This multilevel focus supplies researchers with a comprehensive picture of the students' home, community, and school environments. This multiple-respondent perspective is unified by the fact that, for most purposes, the student is the basic unit of analysis.

The analyses presented in this report use data from the base-year and first follow-up components. Key elements from these components are summarized below.

Base Year (2002)

- Baseline survey of high school sophomores.
- Cognitive tests in reading and mathematics.
- Surveys of parents, school administrators, English teachers, and mathematics teachers.

- Additional components for this study included a school facilities checklist and a media center/library questionnaire.
- Sample sizes of approximately 750 participating schools and over 15,000 participating sophomores.
- Oversampling of Asians and private schools.
- Design linkages (test score equating in reading and mathematics, some questionnaire items in common) with the Program for International Student Assessment (PISA) and score reporting linkages to the prior longitudinal studies (the High School and Beyond longitudinal study [HS&B] and the National Education Longitudinal Study of 1988 [NELS:88]).

First Follow-up (2004)

- Follow-up in spring-term 2004, when most sample members were seniors but some were dropouts, early graduates, or in other grades.
- Student questionnaire, dropout questionnaire, assessment in mathematics, and school administrator questionnaire administered; specially tailored instruments for early graduates and homeschooled students.
- Interviewed students who remained in their base-year school as well as those who transferred, dropped out, graduated early, or transitioned into a homeschool setting.
- High school transcript component with data collection late 2004 through early 2005 (coursetaking records for grades 9–12). Course offerings information was also collected, for the base-year school sample.

A.2.2 Base-Year and First Follow-up Study Design and Content

Base-year Study Design. Seven study components comprised the base-year design: assessments of students (achievement tests in mathematics and reading); a survey of students; surveys of parents, teachers, school administrators, and librarians; and a facilities checklist (completed by survey administrators, based on their observations at the school). The student assessments measured achievement in mathematics and reading; the baseline scores can serve as a covariate or control variable for later outcomes. Mathematics achievement was reassessed in the first follow-up, so that achievement gain over the last 2 years of high school can be measured and related to school processes and mathematics coursetaking. The student questionnaire gathered information about the student's background, school experiences and activities, plans and goals for the future, employment and out-of-school experiences, language background, and motivation toward learning.

One parent of each participating sophomore was asked to respond to a parent survey. The parent questionnaire was designed to gauge parental aspirations for the child, home background and the home education support system, the child's educational history prior to 10th grade, and parental interactions with and opinions about the student's school. For each student enrolled in English or mathematics, a teacher was also selected to participate in a teacher survey. Teachers typically (but not always) reported on multiple ELS:2002 sophomores. The teacher questionnaire collected the teacher's evaluation of the student and provided information about the teacher's background and activities. The head librarian or media center director at each school was asked to complete a library media center questionnaire, which inquired into the school's library media center facility, its staffing, its technological resources, collection and expenditures, and scheduling and transactions. Finally, the facilities checklist was a brief observational form completed for each school. The form collected information about the condition of school buildings and facilities.

First Follow-up Study Design. In the first follow-up, the base-year schools were surveyed by means of an administrator questionnaire. Base-year students were surveyed whether in the base-year school, in a new school, or out of school. A mathematics assessment was administered to first follow-up students who were still attending their original (base-year) schools. When possible, students who had dropped out, transferred schools, or entered a homeschool setting were surveyed, but were not administered mathematics assessments. Further details on the instrumentation, sample design, data collection results, data processing, weighting and imputation, and data files available for analysis may be found in the *Education Longitudinal Study of 2002: Base-Year to First Follow-up Data File Documentation* (Ingels et al. 2005).[1]

Transcript Study Design. Transcripts were collected from sample members in late 2004 and early 2005, about 6 months to 1 year after most students had graduated from high school. Collecting the transcripts in the 2004–05 academic year allowed for more complete high school records. Transcripts were collected from the school that the students were originally sampled from in the base year (which was the only school for most sample members) and from their last school of attendance if it was learned during the first follow-up student data collection that they had transferred.

The ELS:2002 high school transcript data collection sought key pieces of information about coursetaking from the student's official high school record—including courses taken while attending secondary school, information on credits earned, year and term a specific course was taken, and final grades. When available, other information was collected, including dates enrolled, reason for leaving school, and standardized test scores. Once collected, information (e.g., course name, credits earned, course grades) was transcribed and linked back with the student's questionnaire and assessment data. Due to the size and complexity of the file, and because of reporting variation by school, additional variables were constructed from the raw transcript file. Further details on the instrumentation, sample design, data collection results, data processing, weighting and imputation, and data files available for analysis may be found in the *Education Longitudinal Study of 2002: First Follow-up Transcript Component Data File Documentation* (Bozick et al. 2006).

A.3 Study Design

A.3.1 Sampling

The ELS:2002 base-year sample design began with a nationally representative, two-stage stratified probability sample. The first stage of selection was schools; schools were selected with probability proportional to size (PPS). The public school sample was stratified by the nine U.S. Census divisions and by urbanicity (metropolitan status of urban, suburban, or rural). Private

[1] See appendix reference list for full citation. The base-year first follow-up Data File Documentation can be downloaded from the NCES website at http://nces.ed.gov/pubsearch. For more comprehensive information about the base year, see Ingels et al. (2004).

schools (Catholic and other private) were stratified by four levels of geography (Census region) and urbanicity; private schools were oversampled. The target sample size was 800 schools. Cooperation was sought from 1,220 eligible selections. The realized sample comprised 750 participating 10th-grade schools (67 percent participation rate). The second stage of selection was students. Of 17,590 sampled students in the schools, 15,360 students participated. Some groups (e.g., Asians, students in nonpublic schools) were oversampled. The weighted student response rate was 87 percent.

The first follow-up returned to the same schools to seek their cooperation, and to base-year sophomore respondents and a sample of base-year nonrespondents, regardless of whether they had remained in the base-year school. Although 5 of the 750 base-year schools were ineligible because they no longer enrolled ELS:2002 sample members or seniors, of the eligible schools, 700 (93 percent) participated. Overall, there were 16,520 sample members (students, dropouts, homeschooled, or early graduates), of whom 14,990 participated. This analysis uses the sample of 14,710 students who were sophomores in 2001–02, participated in both the BY and F1 interviews, completed the mathematics assessment in the BY and F1 interviews, and had complete transcript information for the 2002–03 and 2003–04 academic years. Students were in the same school in the base year and follow-up survey.

A.3.2 Weighting and Imputation

Weighting. A number of weights are included on the ELS:2002 data file to compensate for unequal probabilities of selection of schools and students into the base-year sample and to adjust for the fact that not all schools and students selected into the sample actually participated. The analyses in this report are weighted with the panel weight (F1PNLWT), which accommodates analyses using sample members who participated in both the base year and first follow-up.

Imputation. For key classification variables used in this analysis, missing data were replaced with imputed values. These include: sex, race/ethnicity, family composition, educational expectations, and socioeconomic status. Single imputation (by means of a weighted sequential hot deck procedure) was implemented for missing key questionnaire variables. Multiple imputation of the mathematics ability estimate *theta* (theta is the point on the test scale that marks the ability of the test taker) was used to treat missing assessment data. Although (for several classes of respondents) missing test scores were imputed in ELS:2002, imputed test data have not been used in this report. Only students with two unimputed test scores are included in the analysis sample.

A.3.3 Base-Year and First Follow-up Response Rates

Base-year Response Rates. Of 1,220 eligible contacted schools, 750 participated in the study, for an overall weighted school participation rate of approximately 68 percent (62 percent unweighted). Of 17,590 selected eligible students, 15,360 participated, for a weighted student response rate of approximately 87 percent.[2] (School and student weighted response rates reflect use of the base weight [design weight] and do not include nonresponse adjustments.) School and student unit nonresponse bias analyses were performed, as well as an item nonresponse bias

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[2] Stage 1 (school) response rates can be multiplied by stage 2 (student) response rates for a combined two-stage response rate: 68 percent * 87 percent = 59 percent.

analysis for the questionnaires. The school-level bias due to nonresponse prior to and after computing weights was estimated based on the data collected from both respondents and nonrespondents, as well as sampling frame data. At the unit level (but not the item level), weighting techniques were employed to reduce detected bias; after final nonresponse adjustments, the remaining relative bias ranged from 0 percent to 0.2 percent for schools and from 0 percent to 0.07 percent for students. For details of the bias analyses, see the *Education Longitudinal Study of 2002: Base Year Data File User's Manual* (Ingels et al. 2004). Unweighted and weighted school-level response by stratum is summarized in table A-1. Table A-2 summarizes base-year response rates by instrument.

First Follow-up Response Rates. First follow-up weighted response rates are reported at the student level only (the school sample was not strictly representative of the nation's high schools with 12th grades in 2003–04). Overall, 14,990 of 16,520 sample members participated, for a weighted response rate of 89 percent. Further details of first follow-up coverage and completion rates are provided in table A-3.

High School Transcript Response and Coverage Rates. A total of 1,550 out of 1,950 schools (base-year schools and transfer schools) participated in the request for transcripts for an unweighted participation rate of 79 percent. The base-year school weighted response rate is 95 percent. The course offerings response rate for base-year schools is 88 percent. Ninety-one percent of the entire student sample have some transcript information (14,920 out of 16,370). Note that for transcripts, a coverage rate—indicating the number of students who participated in one of the two rounds who have transcript data—is given rather than a response rate. Table A-4 provides coverage rates for base-year students in the high school transcript study.

Table A-1. Unweighted base-year school sampling and eligibility, and unweighted and weighted participation, by sampling stratum: 2004

School sampling stratum	Sampled schools		Eligible schools		Participating schools		
	Number	Unweighted percentage[1]	Number	Unweighted percentage[2]	Number	Unweighted percentage[3]	Weighted percentage
Total	1,270	100.0	1,220	96.3	750	61.6	67.8
School sector							
Public	950	75.2	930	97.2	580	62.6	69.1
Catholic	140	11.0	140	100.0	100	67.9	74.0
Other private	180	13.8	160	88.6	80	49.7	62.9
School urbanicity							
Urban	430	34.2	410	95.4	250	60.4	67.3
Suburban	630	49.7	610	96.7	360	59.3	59.8
Rural	200	16.1	200	97.1	140	71.2	79.3

[1] Percentage is based on overall total within column. Details may not sum to totals due to rounding.
[2] Percentage is based on number sampled within row.
[3] Percentage is based on number eligible within row.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "First Follow-up, 2004."

Table A-2. Summary of ELS:2002 base-year response rates, by instrument: 2002

Instrument	Selected	Participated	Weighted percentage	Unweighted percentage
Student (sophomore) questionnaire	17,590	15,360	87.3	87.3
Student (sophomore) assessment	15,360	14,540	95.1	94.7
Parent questionnaire	15,360	13,490	87.5	87.8

NOTE: Response rates for the student assessment are based on the percentage of cases for which a student questionnaire was obtained and for which a mathematics test was also obtained. Note that test scores have been imputed where missing so that test scores are available for all 15,360 questionnaire completers. Coverage rates are calculated for the parent questionnaire. These indicate the percentage of participating students with a parent report. These completion rates reflect the number of records in the public-use data file, where parent (and teacher) data were excluded for students who did not complete a base-year student questionnaire.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002."

Table A-3. Summary of ELS:2002 first follow-up response rates, by instrument: 2004

Instrument	Selected	Participated	Weighted percentage	Unweighted percentage
Total sample for public-use file	16,520	14,990	88.7	90.8
Student questionnaire	13,090	12,430	93.4	94.9
Student mathematics assessment[1]	12,430	11,000	87.4	88.5

[1] Indicates a coverage rate: percentage of cases for which a student questionnaire was obtained and for which a mathematics test was also obtained. When a test was not obtained, test results were imputed.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "First Follow-up, 2004."

Table A-4. Transcript component coverage rates: Percentage of base-year students with a complete transcript, by selected characteristics (weighted): 2002 and 2004

Student characteristic	10th-grade (G10) cohort[1]
Total	90.6
Unweighted (N)	16,171
Sex	
Male	89.8
Female	91.4
Race/ethnicity	
White	92.1
Black	88.2
Hispanic	86.9
Asian	90.5
American Indian	92.3
More than one race	91.4
School control	
Public	90.6
Catholic	94.9
Other private	85.6

[1] G10 cohort indicates the nationally representative, cross-sectional population of the 2002 spring-term sophomore class.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. All race categories exclude individuals of Hispanic or Latino origin.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002," "First Follow-up, 2004," and "Base-Year High School Transcript Study, 2002."

A.3.4 Quality of Estimates: Reliability and Validity of ELS:2002 Questionnaire and Transcript Data

Most of the items used in the ELS:2002 questionnaires were taken from prior studies, particularly HS&B and NELS:88. Given their past use with large, nationally representative samples, their measurement characteristics are well established. A number of data quality studies have been conducted using these items. Interested readers should see, in particular, Fetters, Stowe, and Owings (1984), Kaufman and Rasinski (1991), and McLaughlin and Cohen (1997). Data quality analyses for the subset of new questionnaire items used in ELS:2002 (as well as the reading and mathematics assessments) can be found in the base-year field test report (Burns et al. 2003). The base-year and base-year to first follow-up data documentation manuals (Ingels et al. 2004, 2005) also address issues of questionnaire and assessment data quality for both the ELS:2002 baseline and its first follow-up, while Bozick et al. (2006) address similar issues for the high school transcript component of the study. Data quality for the mathematics assessments is discussed in section A.5 of this appendix. While transcript data are assumed to be superior to student self-report data (the degree of difference between records sources and questionnaire responses is set out in Fetters, Stowe and Owings [1984]), archival records are not infallible data sources. Apart from problems of nonresponse (although response rates for the ELS:2002 transcript collection were high), a major records-gathering problem with a mobile longitudinal cohort is incompleteness of data. Some 14 percent of transcript respondents do not have 4 "complete" years of high school records information (Bozick et al. 2006). However, the problem of incompleteness in part reflects the fact that some records are necessarily incomplete (dropouts and those who failed to graduate with their cohort members by definition have incomplete high school records). Apart from the necessarily incomplete records of students who dropped out or did not advance in modal sequence of the cohort, full records were substantially more difficult to obtain for transfer students than for students who did not move to a new school. Since dropouts, held back, and transfer students are not represented in the analyses in this report, these nonresponse factors are substantially mitigated.

A.3.5 Survey Standard Errors

Because the ELS:2002 sample design involved stratification, the disproportionate sampling of certain strata, and clustered (i.e., multistage) probability sampling, the resulting statistics are more variable than they would have been if they had been based on data from a simple random sample of the same size. In all analyses in this report, standard errors were adjusted for the clustered and stratified sampling design using Taylor-series linearization methods.

A.4 Statistical Procedures

A.4.1 Statistical Significance: Student *t* Statistics

Comparisons that have been discussed in the text of this report have been tested for statistical significance (set at a probability of .05) to ensure that the differences are larger than those that might be expected due to sampling variation. The statistical comparisons in this report were based largely on the t statistic. Whether the statistical test is considered significant is determined by calculating a t value for the difference between a pair of means or proportions and comparing this value to published tables of values, called critical values. The alpha level is an *a priori* statement of the probability that a difference exists in fact rather than by chance.

The *t* statistic between estimates from various subgroups presented in the tables can be computed by using the following formula:

$$t = \frac{x_1 - x_2}{\sqrt{\left(SE_1^2 + SE_2^2\right)}},$$

where x_1 and x_2 are the estimates to be compared (e.g., the means of sample members in two groups), and SE_1 and SE_2 are their corresponding standard errors. This formula is valid only for independent estimates.

An F-test was used to compare the fit of two regression models. This test is computed using the following formula:

$$F = \frac{SSE(Reduced) - SSE(Full) \Big/ df(Reduced) - df(Full)}{SSE(Full) \Big/ df(Full)}$$

where Reduced is the model with fewer independent variables, Full is the model with all the independent variables, SSE is the sum square of errors, and df is the degrees of freedom. For this test the degrees of freedom are n – k , where n = the sample size and k = the number of parameters in the model. A significant F statistic indicates that the Full model is a better fit to the data than the Reduced model.

A.4.2 Effect Sizes

For means (which in this report are scores from the ELS:2002 mathematics assessment), an effect size (or standardized mean difference) has been calculated. The effect size stands as a measure of the magnitude of a difference. For purposes of comparisons drawn in this report, effect sizes (Cohen's *d*) were calculated as the change in mean test scores divided by their pooled standard deviation using the following formula

$$d = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1 - 1)\sigma_1^2 + (n_2 - 1)\sigma_2^2}{n_1 + n_2 - 2}}}$$

A criterion of one-fifth ($d = 0.2$) of a standard deviation was set as the minimum effect size. Differences were only reported in the text if the comparison met two criteria: (1) statistical significance at the .05 level; and (2) the difference was greater than one-fifth of a standard deviation. For purposes of evaluating effect sizes, the proficiency probability scores, like the Item Response Theory (IRT) number-right scores, have been treated as means, and are subject to the 0.2 required effect size. For proportions, this report has adopted a simple convention of reporting differences only if they are 5 percentage points or more. The effect size criterion was used because with large samples, such as the one in ELS:2002, a level of statistical significance can be reached based on differences that may be small in magnitude.[3]

A.4.3 Multivariate Analysis: Ordinary Least Squares Regression

Ordinary least squares (OLS) regression analyses were performed to describe the relationship between math coursetaking and achievement after controlling for student and school characteristics (tables 8 and 9). The regression coefficients generated by the OLS procedure are interpreted as a slope. The regression coefficients, or slope, indicate how many units of change in the dependent variable occur for each unit change in the independent variable controlling for all other factors included in the model. A significant positive coefficient means that for every unit change in the independent variable there is a b units increase in the dependent variable. Conversely, a significant negative coefficient means that for every unit change in the independent variable there is a b units decrease in the dependent variable. T-test comparisons were conducted using the regression coefficients produced in the analyses. The same statistical significance criteria used in the bivariate analyses (*p*-value of .05 or less) was used. For example, in Model 2 in table 8, the regression coefficient for precalculus–AP/IB calculus is 4.2 and is significant at the .05 level—meaning that students who follow a precalculus–AP/IB calculus course sequence improve on the math achievement exam by 4.2 more correct answers than their peers who followed an algebra II–no mathematics course sequence.

A.5 Base-Year to First Follow-up Mathematics Tests

A.5.1 The 2002 and 2004 Mathematics Assessments

The purpose of the ELS:2002 assessment battery is to provide measures of student achievement in mathematics (and reading, tested in the base year only) that can be related to student background variables and educational processes, for individuals and for population subgroups. The reading and mathematics tests must provide accurate measurement of the status of individuals at a given point in time. In addition, the mathematics test must provide accurate measurement of the acquisition of mathematics skills over time.

Test Design and Format

Test specifications for the ELS:2002 base year and first follow-up were adapted from frameworks used for NELS:88. There were two levels to the framework: content areas and cognitive processes. Mathematics tests contained items in arithmetic,[4] algebra, geometry, data/probability, and advanced topics. The tests also reflected cognitive process categories of

[3] For more information about effect sizes, see Cohen (1988), Murphy and Myers (2004), and Seastrom (2003, Guideline 5-1-4F).

[4] For those familiar with National Assessment of Educational Progress (NAEP) terminology, it should be noted that the "arithmetic" content area in NELS:88 and ELS:2002 has essentially the same meaning as "number sense" or "number properties and operations" in NAEP. The NAEP 2005 "content strands" at grade 12 are similar to but subtly different from those in NELS:88 and ELS:2002—number, geometry and measurement, algebra, and data analysis and probability (National Assessment Governing Board [NAGB] 2004). The second dimension of the NAEP framework—akin to the cognitive process or skill categories in NELS:88 and ELS:2002—was historically known as "mathematical abilities" but in the 2005 framework appears as "mathematical complexity of items." This framework dimension encompasses, at three levels of complexity, procedural knowledge and conceptual understanding as well as problem solving, reasoning, and communication.

skill/knowledge, understanding/comprehension, and problem solving. The test questions were selected from previous assessments: NELS:88, NAEP, and PISA. Most, but not all base-year items, were multiple choice (about 10 percent of the base-year mathematics items were open-ended). In the first follow-up, all items were multiple choice.

Both 10th-grade and 12th-grade items were field tested in 2001, and 12th-grade items were field tested again in 2003.[5] Items were selected or modified based on field test results. Final forms were assembled based on psychometric characteristics and coverage of framework categories. On the NELS:88 mathematics framework, see Rock and Pollack 1991 (chapter 2); on its adaptation to ELS:2002, see Ingels et al. 2004 (section 2.2.2.1).

The ELS:2002 assessments were designed to maximize the accuracy of measurement that could be achieved in a limited amount of testing time, while minimizing floor and ceiling effects, by matching sets of test questions to initial estimates of students' achievement. In the base year, this was accomplished by means of a two-stage test. In 10th grade, all students received a short multiple-choice routing test, scored immediately by survey administrators who then assigned each student to a low, middle, or high difficulty second-stage form, depending on the student's number of correct answers in the routing test. In the 12th-grade administration, students were assigned to an appropriate test form based on their performance in 10th grade. Cut points for the 12th-grade low, middle, and high forms were calculated by pooling information from the field tests for 10th and 12th grades in 2001, the 12th-grade field test in 2003, and the 10th-grade national sample. Item and ability parameters were estimated on a common scale. Growth trajectories for longitudinal participants in the 2001 and 2003 field tests were calculated, and the resulting regression parameters were applied to the 10th-grade national sample. Test forms were designed to match the projected achievement levels of the lowest and highest 25 percent, and the middle 50 percent, of the base-year sample 2 years later. Each of the test form contained 32 multiple-choice items.

In the four tables immediately below (A-5 through A-8), content and process information[6] is provided about the 73 unique items that comprise the base-year, and 59 items that comprise the first follow-up, mathematics assessments. Additional tables are presented later (A-9 and A-10) that break down assignments of items by content and process *by test form*, and thus show the extent of overlap (any given unique item may appear on one or more forms).[7] Tables A-5 and A-6 show the numbers and percentages of unique test items devoted to each content area for the base-year and first follow-up test batteries. Tables A-7 and A-8 show the number and percentages of unique test items devoted to each cognitive process area.

[5] For more details about the field tests, see Burns et al. (2003) (NCES 2003-03) and appendix J of the *Base-Year to First Follow-up Data File Documentation*, Ingels et al. (2005) (NCES 2006-344).

[6] Content by process (cognitive behavior) matrices can be useful for giving some sense of how tests have been constructed but must be interpreted with caution. Robitaille et al. (1993) make the point that such grids somewhat oversimplify the interrelatedness of elements in the scheme. Knowledge and abilities or behavior in one area of mathematics are not unconnected to knowledge and skills in other areas, and this caveat needs to be kept in mind. As the National Assessment Governing Board has remarked on its 2005 NAEP mathematics framework (NAGB 2004, p. 7), its divisions "are not intended to separate mathematics into discrete elements. Rather, they are intended to provide a helpful classification scheme that describes the full spectrum of mathematical content assessed by NAEP. Classifying items into one primary content area is not always clear cut, but doing so brings us closer to the goal of ensuring that important mathematical concepts and skills are assessed in a balanced way."

[7] There was also overlap across waves, in that some items were used both in the base year and first follow-up.

Table A-5. Number and percentage of unique mathematics items in ELS:2002 base year, by content area: 2002

Content area	Number of items	Percentage of items
Arithmetic	19	26.0
Algebra	17	23.3
Geometry/measurement	20	27.4
Data analysis, statistics/probability	9	12.3
Advanced topics[1]	8	11.0

[1] “Advanced topics” includes precalculus and analytic geometry.
NOTE: To provide overlap, some items appear on more than one test form.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002.”

Table A-6. Number and percentage of unique mathematics items in ELS:2002 first follow-up, by content area: 2004

Content area	Number of items	Percentage of items
Arithmetic	15	25.4
Algebra	17	28.8
Geometry/ measurement	17	28.8
Data analysis, statistics/probability	4	6.8
Advanced topics[1]	6	10.2

[1] “Advanced topics” includes precalculus and analytic geometry.
NOTE: To provide overlap, some items appear on more than one test form.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “First Follow-up, 2004.”

Table A-7. Number and percentage of unique mathematics items per skill/cognitive process area in ELS:2002 base year, by process/skill specifications: 2002

Process/skill specifications	Number of items	Percentage of items
Procedural skills/knowledge	23	31.5
Conceptual understanding	27	37.0
Problem solving	23	31.5

NOTE: To provide overlap, some items appear on more than one test form.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002.”

Table A-8. Number and percentage of unique mathematics items per skill/cognitive process area in ELS:2002 first follow-up, by process/skill specifications: 2004

Process/skill specifications	Number of items	Percentage of items
Procedural skills/knowledge	17	28.8
Conceptual understanding	26	44.1
Problem solving	16	27.1

NOTE: To provide overlap, some items appear on more than one test form.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “First Follow-up, 2004.”

Table A-9 shows the number of mathematics test items per form in the base year and first follow-up. Again, forms were assigned on the basis of performance on a routing test in the base year, but were assigned on the basis of the base-year ability estimate in the first follow-up. While all examinees received a 32-item form in 2004, the number of items ranged from 40 to 42 in the base year, except for a handful of students who received the single-stage 23-item version of the base year assessment (this abbreviated version of the test was used at two schools that had too limited testing time available to administer the full version).

Table A-9. Number of items in each ELS:2002 base-year and first follow-up tests for assessing achievement in mathematics, by form: 2002 and 2004

Form	Base year (2002)	First follow-up (2004)
Routing test	15	†
Second stage tests		
Form X (low difficulty)	25	32
Form Y (middle difficulty)	27	32
Form Z (high difficulty)	27	32
Form V (single stage in 2002; broad range in 2004)	23	32

† Not applicable.
NOTE: Some items overlap and appear on more than one test form.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

While the tables above show the content and process areas for the unique items that comprise the overall base-year and first follow-up mathematics tests, students took different forms of each test, and a given item could be used on more than one form. To see the number or proportion of items in a given content or skill area that students at various levels of form assignment in fact took, an additional set of tables is required. Table A-10 below shows content by cognitive process distributions of items across all test forms. Contents of the routing tests are shown separately, although for purposes of computation of the base-year ability estimate, *theta*, the two stages of the test (i.e., the routing test and the ability-tailored second stage test) were combined.

Table A-10. Number of mathematics items per content area, by cognitive skill/process, and form: ELS:2002 base year through first follow-up: 2002 and 2004

	Content area				
Cognitive process	Arithmetic	Algebra	Geometry/ measurement	Data analysis/ statistics probability	Advanced topics[1]
Skill/knowledge					
Routing test	3	†	1	†	†
10th-grade low (X)	7	3	1	3	†
10th-grade medium (Y)	1	1	2	3	1
10th-grade high (Z)	†	2	1	†	†
10th-grade 1-stage (V)	2	3	1	†	†
12th-grade low (X)	7	4	2	†	†
12th-grade medium (Y)	2	4	1	†	1
12th-grade high (Z)	†	2	2	†	1
12th-grade broad (V)	4	3	2	†	1
Understanding/comprehension					
Routing test	1	4	1	1	†
10th-grade low (X)	3	†	1	1	2
10th-grade medium (Y)	2	3	2	1	5
10th-grade high (Z)	3	2	1	5	5
10th-grade 1-stage (V)	2	3	1	1	3
12th-grade low (X)	5	4	2	2	†
12th-grade medium (Y)	2	7	4	1	2
12th-grade high (Z)	†	5	4	1	4
12th-grade broad (V)	3	3	3	1	2
Problem solving					
Routing test	†	2	2	†	†
10th-grade low (X)	2	†	1	1	†
10th-grade medium (Y)	1	†	3	1	1
10th-grade high (Z)	1	1	10	1	†
10th-grade 1-stage (V)	2	†	3	1	1
12th-grade low (X)	2	†	3	1	†
12th-grade medium (Y)	2	1	5	†	†
12th-grade high (Z)	1	2	9	1	†
12th-grade broad (V)	3	2	4	1	†

† Not applicable.

[1] "Advanced topics" includes precalculus and analytic geometry.

NOTE: Some of the 73 base year and 59 first follow-up items appear on more than one test form. Twelfth grade was the modal grade for sample members in 2004; all sample members were 10th graders in 2002.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year to First Follow-up, 2002" and "First Follow-up, 2004."

Table A-11 shows (by test form) numbers and percentage of items in each content area. The items in the base-year stage 1 test (routing test) have been combined with the items in the stage 2 test. Thus we see, for example, that in the first follow-up (2004) when most sample members were in their senior year, students assigned the low form had 44 percent arithmetic items and no advanced topics; while students assigned the high form had 3 percent arithmetic items and 16 percent advanced topics. Nonetheless, the different forms comprise a single test, and with IRT methods, proficiencies can be estimated for ELS:2002 items not assigned to the examinee. In other words, all ELS:2002 IRT scores (whether number-right or proficiency probability scores) measure student performance on the entire item pool regardless of which form they took.

Table A-11. Percentage distribution of ELS:2002 test items, by content area and mathematics test form: 2002

	Content area									
	Arithmetic		Algebra		Geometry/ measurement		Data analysis/ statistics/probability		Advanced topics	
Mathematics test form	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number	Percentage	Number
10th-grade low (X)	40.0	16	25.0	10	15.0	6	15.0	6	5.0	2
10th-grade medium (Y)	19.0	8	26.2	11	23.8	10	14.3	6	16.7	7
10th-grade high (Z)	11.9	5	31.0	13	38.1	16	7.1	3	11.9	5
10th-grade 1-stage (V)	26.1	6	26.1	6	21.7	5	8.7	2	17.4	4
12th-grade low (X)	43.8	14	21.9	7	25.0	8	9.4	3	0.0	0
12th-grade medium (Y)	18.8	6	37.5	12	31.3	10	3.1	1	9.4	3
12th-grade high (Z)	3.1	1	28.1	9	46.9	15	6.3	2	15.6	5
12th-grade broad (V)	31.3	10	25.0	8	28.1	9	6.3	2	9.4	3

NOTE: "Advanced topics" includes precalculus and analytic geometry. Detail may not sum due to rounding. Tenth-grade item summaries by forms X, Y, and Z combine the routing test and the second stage test. Twelfth grade was the model grade for sample members in 2004; all sample members were 10th-graders in 2002.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year to First Follow-up, 2002."

A.5.2 Scoring

A.5.2.1 IRT Scoring Procedures

The scores used to describe students' performance on the direct cognitive assessment are broad-based measures that report performance as a whole. The scores are based on Item Response Theory, which uses patterns of correct, incorrect, and omitted answers to obtain ability estimates that are comparable across different test forms.[8] In estimating a student's ability, IRT also accounts for each test question's difficulty, discriminating ability, and a guessing factor.

IRT has several advantages over raw number-right scoring. By using the overall pattern of right and wrong responses to estimate ability, IRT can compensate for the possibility of a low-ability student guessing several difficult items correctly. If answers on several easy items are wrong, a correct difficult item is assumed, in effect, to have been guessed. Omitted items are also less likely to cause distortion of scores, as long as enough items have been answered right and wrong to establish a consistent pattern. Unlike raw number-right scoring, which necessarily treats omitted items as if they had been answered incorrectly, IRT procedures use the pattern of responses to estimate the probability of correct responses for all test questions. Finally, IRT scoring makes it possible to compare scores obtained from test forms of different difficulty. The common items present in overlapping forms and in overlapping administrations (10th grade and 12th grade) allow test scores to be placed on the same scale.

In the ELS:2002 first follow-up survey, IRT procedures were used to estimate longitudinal gains in achievement over time by using common items present in both the 10th- and 12th-grade forms. Items were pooled from both the 10th- and 12th-grade administrations and anchored to the IRT scale of the NELS:88 survey of 1988–92. Item parameters were fixed at NELS:88 values for the items that had been taken from the NELS:88 test battery and to base-year values for non-NELS:88 items. In each case, the fit of the follow-up item response data to

[8] For an account of Item Response Theory, see Embretson and Reise (2000) or Hambleton, Swaminathan, and Rogers (1991).

the fixed parameters was evaluated, and parameters for common items whose current performance did not fit previous patterns were re-estimated, along with non-NELS:88 items new to the follow-up tests.

A.5.2.2 Score Descriptions and Summary Statistics

Two different types of IRT scores are used in this report to describe students' performance on the mathematics assessment. NELS:88-equated *IRT-estimated number-right scores* measure students' performance on the whole item pool. NELS:88-equated *proficiency probabilities* estimate the probability that a given student would have demonstrated proficiency for each of the five mathematics levels defined for the NELS:88 survey in 1992.[9]

ELS:2002-NELS:88 Equating. Equating the ELS:2002 scale scores to the NELS:88 scale scores was completed through common-item or *anchor equating*. The ELS:2002 and NELS:88 mathematics tests shared 44 mathematics items. These common items provided the link that made it possible to obtain ELS:2002 student ability estimates on the NELS:88 ability scale. (The ELS:2002 data for 12 additional mathematics items did not fit the NELS:88 IRT parameters, so these items were not treated as common items for the purpose of equating.) Parameters for the common items were fixed at their NELS:88 values, resulting in ability estimates consistent with the NELS:88 metric.

IRT-estimated Number-right. The NELS:88-equated IRT-estimated number-right scores for mathematics are estimates of the number of items students would have answered correctly had they taken the NELS:88 exam and responded to all items in the mathematics items pool. The NELS:88 item pool contained 81 mathematics items in all test forms administered in grades 8, 10, and 12. Table A-12 provides basic statistics for base-year (BYNELS2M) and first follow-up (F1NELS2M) IRT-estimated number-right scores for ELS:2002 students, on the NELS:88 score scale. These scores are not integers because they are sums of probabilities, not counts of right and wrong answers.

Proficiency Probability Scores. Table A-12 also provides basic statistics for the five base-year (BYTX1MPP–BYTX5MPP) and five first follow-up (F1TX1MPP–F1TX5MPP) continuous proficiency probability scores. The criterion-referenced NELS:88-equated proficiency probability scores are based on clusters of items that mark different levels on the mathematics scale. Clusters of four items were identified in the NELS:88 tests that marked five hierarchical levels in mathematics:

1. Simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers;
2. Simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents;
3. Simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram;

[9] For further information on the NELS:88 proficiency levels, see Rock and Pollack (1995b), *Psychometric Report for the NELS:88 Base Year Through Second Follow-up* (NCES 95-382).

4. Understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and
5. Complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

Note that while Level 5 is based on a measurement of advanced mathematical material, the ELS:2002 mathematics test contains no calculus items. To the extent that advanced mathematics content on the ELS:2002 is limited, the present study may understate the relationship between mathematics course sequences and the acquisition of the most advanced skills and concepts. A high school student enrolled in calculus may see improved ELS:2002 test performance indirectly: the course may help keep mathematics understanding fresh and hone problem-solving skills, but there will be no direct test benefit in learning calculus content, in that there are no calculus items on the mathematics assessment.

The proficiency levels are hierarchical in the sense that mastery of a higher level typically implies proficiency at lower levels. The NELS:88-equated proficiency probabilities in ELS:2002 were computed using IRT item parameters calibrated in NELS:88. Each proficiency probability represents the probability that a student would pass a given proficiency level defined as above in the NELS:88 sample.

Table A-12 shows variable names, descriptions, and summary statistics for the NELS:88-equated number-right and proficiency probability scores.

Table A-12. IRT-estimated number-right scores and proficiency probability scores: 2002 and 2004

Variable name	Description	Range	Weighted mean	Weighted standard deviation
BYNELS2M	Mathematics—NELS-equated estimated number right (1992 scale)	0–81	44.40	13.70
F1NELS2M	Mathematics—NELS-equated estimated number right (1992 scale)	0–81	50.10	14.20
BYTX1MPP	Mathematics—level 1	0–1	.92	.20
BYTX2MPP	Mathematics—level 2	0–1	.67	.42
BYTX3MPP	Mathematics—level 3	0–1	.46	.46
BYTX4MPP	Mathematics—level 4	0–1	.21	.33
BYTX5MPP	Mathematics—level 5	0–1	.01	.07
F1TX1MPP	Mathematics—level 1	0–1	.96	.12
F1TX2MPP	Mathematics—level 2	0–1	.78	.37
F1TX3MPP	Mathematics—level 3	0–1	.62	.45
F1TX4MPP	Mathematics—level 4	0–1	.35	.41
F1TX5MPP	Mathematics—level 5	0–1	.04	.14

NOTE: National Education Longitudinal Study of 1988 (NELS:88). IRT = Item Response Theory.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

The IRT number-right and proficiency scores are derived from the IRT model and are based on all of the student's responses to the mathematics assessment. That is, the pattern of right and wrong answers, as well as the characteristics of the assessment items themselves, is used to estimate a point on an ability continuum, and this ability estimate, *theta*, then provides the basis for these two types of criterion-referenced scores.

NELS:88-equated IRT number-right and *proficiency probability scores* may be used in a number of ways. Because they are calibrated on the NELS:88 scale, they may be used for cross-sectional intercohort comparisons of students' mathematics achievement in 2004 compared with their counterparts in 1992. The NELS:88-equated number-right scores reflect performance on the whole pool of 81 NELS:88 mathematics items, whereas the proficiency probability scores are criterion-referenced scores that target a specific set of skills. The mean of a proficiency probability score aggregated over a subgroup of students is analogous to an estimate of the percentage of students in the subgroup who have displayed mastery of the particular skill.[10] The proficiency probability scores are particularly useful as measures of gain, because they can be used to relate specific treatments (such as selected coursework) to changes that occur at different points along the score scale. For example, two groups may have similar gains in total scale score points, but for one group, gain may take place at an upper skill level, and for another, at a lower skill level. One would expect to see a relationship between gains in proficiency probability at a particular level and curriculum exposure, such as taking mathematics courses relevant to the skills being mastered.

A.5.2.3 Psychometric Properties of the Tests

Information about the psychometric properties of the test items, the setting of difficulty levels, differential item functioning, and scoring procedures, are provided in the two field test documents (Burns et al. 2003 [NCES 2003-03, chapter 5] and Ingels et al. 2005 [NCES 2006-344, appendix J]). IRT scaling and linking procedures follow the NELS:88 precedent, using a three-parameter IRT model in PARSCALE (Muraki and Bock 1991); the NELS:88 procedure is described in Rock and Pollack (1995b). The same IRT software and procedures were used in the scaling of the Early Childhood Longitudinal Survey (ECLS-K) as detailed in Pollack et al. (2005).

Reliabilities were computed using the variance of the posterior distribution of plausible values for each test-taker's *theta* (ability estimate), compared with the variance of the *thetas* across the whole sample (i.e., error variance versus total variance). The reliability estimates are the proportion of "true variance" (1 minus error variance) divided by total variance (see Samejima 1994 on this procedure).

For the combined base-year and first follow-up tests, the reliability was 0.92 (this reliability is a function of the variance of repeated estimates of the IRT ability parameter [within-variance], compared with the variability of the sample as a whole) (Ingels et al. 2005). This 0.92 reliability applies to all scores derived from the IRT estimation.[11]

The use of IRT-scale scores and the adaptive testing approach used in ELS:2002 limit the concern that gain scores may be unreliable due to floor and ceiling effects.

A.5.2.4 Indicators of Student Motivation at Both Testing Points

One major concern in measuring achievement is whether students are motivated to do their best on low-stakes tests, such as the mathematics assessment in ELS:2002. This concern may be particularly strongly felt with reference to spring-term seniors, who may be in the process of disengaging from high school in anticipation of the transition to postsecondary

[10] On the interpretation of a proportion as a probability, see Fleiss, Levin, and Paik (2003, p. 1).
[11] Imputed test scores were not included in reliability calculations; all test score calculations were based on actual test responses prior to the imputation of missing test scores.

education or the work force, and who may have had their fill of assessments, in the form of such high stakes tests as exit exams and college entrance exams. Although greatest concern may be felt about spring term seniors, concerns about motivation rightly encompass high school sophomores as well.

While there is no single definitive measure of student motivation on the tests, there are several possible indicators of the comprehensiveness and quality of the test data collected. For example, in scoring the 2002 and 2004 tests, the assessment subcontractor examined "pattern marking"[12] and missing responses. In the main, they did not find evidence of pattern marking, or high levels of omitted items. For example, in the ELS:2002 first follow-up with around 11,000 mathematics assessments completed, 17 assessments were discarded for these reasons: 11 test records were deleted because tests were incomplete (fewer than 10 items answered) and 6 more because response patterns indicated lack of motivation to answer questions to the best of the student's ability. In the base year, 10 test records were deleted because tests were incomplete (fewer than 10 items answered). Pattern marking was not observed (perhaps reflecting the fact that the test was in two stages, each stage relatively short).

Given that participation in the survey was voluntary, and that a student could have opted to not participate, or to participate by completing the questionnaire only, the student response rate may also be an indirect indicator of student test-taking motivation. Generally NAEP sees a drop in participation in grade 12, compared to grades 4 and 8. For ELS:2002's predecessor study, NELS:88, lower participation rates were registered in 12th grade as well.[13]

For the ELS:2002 base year, the weighted participation rate was 87 percent. Of the 15,362 participants, 95 percent (weighted) also completed the test. (Some who did not complete the test could not be validly tested for language or disability reasons.)

For the ELS:2002 first follow-up (2004), when most sample members were high school seniors, the overall participation rate increased slightly from the base year to a weighted 89 percent. Some 87 percent (weighted) of questionnaire completers also completed the test.

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[12] An example of "pattern marking" would be responses of "A" for all answers or ABCABCABC through most or all of the test. Patterned responses such as "11111111…" or "12345432123454321…" or "1515151515…" can be identified by a simple algorithm sequentially comparing the difference between each test item and the next one, and calculating the variance of the absolute differences. In the first example given, the inter-item differences are always zero, in the second, always 1 or -1, and in the third, 4 or -4. In each case, the variance of the absolute differences is equal to zero, whereas for four- or five-choice test items, the variance of absolute differences for motivated respondents tends to be close to 1.0. All tests with variances of less than .5 were reviewed and those few with identifiable pattern marking were deleted.

[13] Fully interpreting the senior year decline in test completion in NELS:88 is difficult. There was sample dispersion, and the policy was to test transfer students, though the resources for doing so were limited. In consequence, often a questionnaire might be completed over the telephone and the test sacrificed, despite the student's willingness to be assessed. In contrast, in ELS:2002, transfers were ineligible for the first follow-up test and did not count against the assessment response rate—however, test scores were imputed for all transfers. No test score imputation was undertaken in NELS:88. Because studies such as NELS:88 and ELS:2002 induct their initial samples prior to 12th grade, they may be less affected by a "senioritis" phenomenon, in that students have already committed to the study and may have developed a sense of membership in the panel. Certainly for High School and Beyond (HS&B), the prior longitudinal cohort study that in its sophomore cohort most closely resembles ELS:2002 in design, participation was higher in the modally 12th-grade first follow-up than in the 10th-grade base year (and higher than the 12th-grade participation rate for the HS&B senior cohort that was selected in the same schools in 1980). (The NELS:88 second follow-up assessment data collection is discussed in Ingels, Scott, and Taylor [1998, p. 54]; for the 12th grade experience of HS&B, see Jones et al. [1983] and for ELS:2002 see Ingels et al. [2005]).

Looking specifically at questionnaire completion for senior cohort members who remained in the same school at both points in time—the critical analysis sample for this report—a 97 percent survey participation rate was obtained, with very little difference by subgroup. Race/ethnicity groups, for example, were all at around 97 percent (Ingels et al. 2005 [NCES 2006-344]). If voluntary participation rates are to some degree indicative of student motivation, then there is some evidence that seniors may have taken the assessment seriously.[14] The overall pattern—lack of high numbers of omitted response, lack of "pattern-marking," high test reliability,[15] and high participation rates in both rounds of the study—argues for the credibility and quality of the test data. In short, while lack of motivation for some students surely affected test results in ways that could not be identified and edited out, most test takers answered all or almost all the items, and internal-consistency reliabilities were high for all subgroups examined, both in the field tests and full-scale studies. These are good indications that interpretation of test results in the aggregate should not be significantly compromised by low test-taking motivation.

A.6 Bias Analysis

This analysis is based on 14,710 eligible students who had participated in the base-year interview and the first follow-up interview. Of these, 730 did not participate in the BY mathematics assessment and 650 did not participate in the F1 mathematics assessment. The final analytic sample includes 9,460 respondents, of whom 5,300 have course sequences followed by more than 200 respondents—referred to here as those with designated mathematics course sequences. A bias analysis was conducted to assess the generalizability of the final analysis sample (n = 9,460) and those with designated course sequences (n = 5,300) compared with the target population of sophomores who participated in both the base-year interview and first follow-up interview (n = 14,710). Table A-13 shows the distributions of the student and school characteristics used in this study for each of the three samples, weighted using the panel weight (F1PNLWT).

Using the 5 percentage point threshold as a criterion for meaningful differences, the analytic sample and the analytic sample with designated course sequences differs from the target population in two ways: compared with the target population, the analytic sample and the analytic sample with designated course sequences include a higher proportion of students who are White and a higher proportion of students who expect a bachelor's degree or higher. This is not surprising as the analysis sample excludes students who had transferred, students who were absent on the day of the test administration, and students with incomplete transcripts. This accords with other research which shows that racial/ethnic minorities are more likely to drop out relative to White students (U.S. Department of Education 1999) and that transfer behavior is associated with dropping out (Rumberger and Larson 1998)—both potentially contributing to higher rates of incomplete transcript information among these groups (Ingels et al. 1995). Additionally, compared with the full sophomore panel, the sample of those with designated mathematics course sequences contains more students living with both their father and their mother. Therefore, the analytic sample does not entirely approximate the composition of the full

[14] Note that ELS:2002 sample members were given a cash incentive for participation. The effects of payment on test-taking motivation are unknown. Since test reliabilities were high and incomplete tests and pattern marking did not seem to be a problem, one interpretation might be that students made a reasonable effort, regardless of their reason for doing so.

[15] Imputed test scores were not included in the calculation of reliabilities.

sophomore panel. Despite these differences, it is imperative to have complete transcript information and to have both unimputed mathematics achievement test scores (in the base year and the first follow-up) to accurately answer the research questions posed in this report. A consequence of using the analytic sample is that the findings may not generalize to all students, particularly those who are non-White, those who have educational expectations that do not include college completion, and those who are not living with their mother and father. Readers should keep this caveat in mind when interpreting the results.

Table A-13. Results of bias analysis: sophomore panel members, analytic sample members, and analytic sample members with a designated mathematics course sequence, by student characteristics (weighted): 2002 and 2004

Student characteristic	Sophomore panel	Analytic sample	Analytic sample with designated mathematics course sequences[1]
Sex			
Female	49.6	50.9	50.3
Male	50.4	49.1	49.7
Race/ethnicity			
White	60.4	66.2	66.3
Black	14.3	12.0	12.1
Hispanic	16.1	13.3	13.5
Asian	4.2	4.0	3.7
American Indian	0.1	0.1	0.1
More than one race	4.1	3.9	3.7
Socioeconomic status			
Quartile 1 (low)	24.7	21.1	20.9
Quartile 2	25.1	24.0	23.9
Quartile 3	25.2	26.3	26.5
Quartile 4 (high)	25.1	28.6	28.7
School sector			
Public	92.4	91.5	91.3
Catholic	4.3	5.1	5.2
Other private	3.4	3.4	3.5
Family composition			
Mother and father	57.7	62.2	62.8
Mother or father and guardian	16.1	15.0	14.4
Single parent	21.9	20.0	19.9
Other	4.3	3.2	2.9
Student's educational expectations[2]			
High school or less	7.4	5.0	4.6
Some college	9.7	8.8	8.8
Bachelor's degree or more	68.6	77.1	78.4
Don't know	14.4	9.1	8.3
N	14,710	9,460	5,300

[1] Designated mathematics course sequences are those enrolled by at least 200 students.
[2] Educational expectations of 10th-grade students.
NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. All estimates are weighted using the panel weight (F1PNLWT).
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

A.7 Glossary—Description of Transcript and Survey Variables Used

Each questionnaire and transcript variable employed in analysis in this report is described below (test scores are discussed in section A.5). The topic headings are student and family demographic characteristics, student educational characteristics, and student coursetaking. Some readers may wish to consult the original questionnaires to obtain specific item wording and information about the context in which particular questions were posed. Web-published PDF files containing the base-year and first follow-up questionnaires are available at http://www.nces.ed.gov/surveys/els2002/index.asp. Some readers may desire to have further information about the construction of composite variables (such as socioeconomic status [SES]). The code used to construct these variables can be found in the ECB (Ingels et al. 2005b; NCES 2006-346). For users who would like to consult codebooks of hardcopy frequencies (including both percent and weighted percent) for the variables listed in this glossary, codebooks are also available as an appendix of the base-year to first follow-up data manual (Ingels et al. 2005a; http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2006344). Further information about the transcript component is given in the special addendum to the base-year to first follow-up data file documentation (Bozick et al. 2006).

When the variable is available in the ELS:2002 base-year to first follow-up data file (see the public-use ECB, NCES 2006-346), the variable name appears in parentheses after the bold entry name. ELS:2002 variables used to construct a variable that is not provided in the ELS:2002 base-year data file are named in all capital letters within the descriptive text.

A.7.1 Student and Family Demographic Characteristics

Race/ethnicity (F1RACE): The ELS:2002 race variables reflect new federal standards for collecting race and ethnicity data that allow respondents to mark more than one choice for race. For base-year respondents, information on race/ethnicity was obtained from the base-year student questionnaire when available or from (in order of preference) the sampling roster, the parent questionnaire if the parent respondent was a biological parent, or logical imputation based on other questionnaire items (e.g., surname, native language). The base-year race/ethnicity questions were asked in the first follow-up for newly participating students (i.e., base-year nonrespondents).

The race/ethnicity variable for this report includes six categories: (1) American Indian or Alaska Native, non-Hispanic; (2) Asian or Pacific Islander, including Native Hawaiian, non-Hispanic; (3) Black, including African American, non-Hispanic; (4) Hispanic or Latino; (5) More than one race, non-Hispanic; and (6) White, non-Hispanic.

Sex (F1SEX): For base-year respondents, respondent sex was constructed from the base-year student questionnaire or, where missing, from (in order of preference) the school roster, logical imputation based on first name, or statistical imputation. In the first follow-up, students new to the study were asked whether their sex was male or female.

Socioeconomic status (F1SES1QU): Socioeconomic status exists as both a continuous variable and as a categorical variable based on weighted quartiles. The categorical form of the variable (F1SES1Q) divides SES1 into quartiles based on the weighted marginal distribution. It was recoded to combine the middle two categories of the SES1QU variable. Four categories result: (1) lowest quartile of SES1 (i.e., students below the 25th percentile rank for SES); (2)

lower middle quartile of SES1 (i.e., students whose SES percentile rank was at least 25th and below 50th); (3) upper middle quartile of SES1 (i.e., students whose SES percentile rank was at least 50th and below 75th); and (4) highest quartile of SES1 (i.e., students whose SES percentile rank was at least 75th).

F1SES1 is a NLS-72/HS&B/NELS:88-comparable composite variable constructed from parent questionnaire data when available and from imputation or student substitutions when not. SES is based on five equally weighted, standardized components: father's/guardian's education (F1FATHED), mother's/guardian's education (F1MOTHED), family income (BYINCOME), father's/guardian's occupational prestige score (from F1OCCUFATH), and mother's/guardian's occupational prestige score (from F1OCCUMOTH).

Father's and mother's education were based on parent report when available, otherwise student report, or if still missing, imputed. Income was based on parent questionnaire information or imputed otherwise. The parent questionnaire was the preferred source of data for mother's and father's occupation. Parent questionnaire respondents were asked to describe the father's and mother's occupations and subsequently code each into one of 17 categories. If the respondent provided only text, project staff coded the occupation. In the absence of parent questionnaire occupation data, student-supplied parent occupation text from the base year (for base-year respondents) or first follow-up (for base-year nonrespondents who responded in the first follow-up) was coded by project staff, if possible. Missing occupations were imputed. An occupation prestige value was determined based on the 1961 Duncan socioeconomic index (SEI).[16]

Family composition/configuration (F1FCOMP): F1FCOMP is based on BYFCOMP for base-year respondents and a surrogate for first follow-up new participants. New participants were asked to answer questions about family composition that were asked of parents in the base year. Because family composition can change over time, the variable is only an approximation, in that information was gathered at either of two time points (2002 or 2004) before combining into one measure. The nine response options include (1) Mother and father, (2) Mother and male guardian, (3) Father and female guardian, (4) Two guardians, (5) Mother only, (6) Father only, (7) Female guardian only, (8) Male guardian only, and (9) Lives with student less than half time. These categories were collapsed into four: Mother and father (1), Mother or father and guardian (2 and 3), Single parent—mother or father (5 and 6), and Other (4, 7, 8, and 9).

Educational expectations (BYSTEXP): This variable is taken directly from the student questionnaire when available and imputed otherwise. Students were asked, "As things stand now, how far in school do you think you will get?" The eight response options were (1) Less than high school graduation; (2) High school graduation or GED only; (3) Attend or complete a 2-year school course in a community college or vocational school; (4) Attend college, but not complete a 4-year degree; (5) Graduate from college; (6) Obtain a master's degree or equivalent; (7) Obtain a Ph.D., M.D., or other advanced degree; and (8) Don't know. These levels were collapsed into four categories: High school diploma or less (1 and 2), Some college (3 and 4), College graduate or higher (5, 6 and 7), and Don't Know (8).

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[16] The Duncan SEI is a measure of occupational status based on the income level and educational attainment associated with each occupation (Duncan 1961).

A.7.2 Student Educational Characteristics

School sector (BYSCTRL): This variable indicates the type of school attended by the respondent in the base-year interview. There were no missing data for this variable. The resulting measure includes three school sector categories: public school, Catholic school, other private school.

A.7.3 Mathematics Achievement

See section A.5 for details on the NELS:88-equated IRT-estimated number-right scores and proficiency probability levels.

Coursetaking sequences: This variable was constructed using the student transcript course file. All mathematics courses for which the student earned credit during 2002–03 and 2003–04, the 2 academic years between the 2 mathematics assessments, were classified in one of the following 16 categories using the Classification of Secondary School Courses (CSSC) codes, the hierarchical scheme used to classify and group subject areas and courses:

Category	CSSC	Title
No Mathematics	None	None
Basic Mathematics	270601	Basic Math 1
	270602	Basic Math 2
	270603	Basic Math 3
	270604	Basic Math 4
General Math	270100	Mathematics, Other General
	270101	Mathematics 7
	270102	Mathematics 7, Accelerated
	270103	Mathematics 8
	270106	Mathematics 1
	270107	Mathematics 2
	541001	General Math Skills
	541009	Functional Math Skills, Not For Credit
	562700	Special Education Math
	562701	Resource General Math
	562709	Resource General Math, Not For Credit
Applied Math	110111	Computer Appreciation; Computer Literacy
	110121	Computer Mathematics 1
	110122	Computer Mathematics 2
	110400	Information Sciences and Systems, Other
	110500	Systems Analysis, Other
	119900	Computer and Information Sciences, Other
	270108	Science Mathematics
	270109	Mathematics in the Arts
	270110	Mathematics, Vocational
	270111	Technical Mathematics
	270114	Consumer Mathematics
	270300	Applied Mathematics, Other
	541101	Functional Consumer Math
	541109	Functional Consumer Math, Not For Credit
	541201	Functional Vocational Math
	541209	Functional Vocational Math, Not For Credit
	562711	Resource Vocational Math
	562719	Resource Vocational Math, Not For Credit
	562721	Resource Consumer Math
	562729	Resource Consumer Math, Not For Credit
Pre algebra	270104	Mathematics 8, Accelerated
	270401	Prealgebra
	270402	Algebra 1, Part 1
	270403	Algebra 1, Part 2
Algebra I	270404	Algebra 1
	270421	Mathematics 1, Unified
	270427	Unified Math 1, Part 1
	270428	Unified Math 1, Part 2
	270441	Algebra and Geometry
Geometry	270406	Geometry, Plane
	270407	Geometry, Solid
	270408	Geometry
	270409	Geometry, Informal
	270422	Mathematics 2, Unified
	270425	Geometry, Part 1
	270426	Geometry, Part 2
	270429	Pre-IB Geometry
Algebra II	270405	Algebra 2
	270410	Algebra 3 Algebra II
	270415	Algebra and Analytic Geometry
	270417	Linear Algebra; Matrix Algebra;
Trigonometry	270411	Trigonometry

	270413	Trigonometry and Solid Geometry
	270414	Algebra and Trigonometry
	270430	Pre-IB Algebra 2/ Trigonometry
Other Advanced Math	270112	Mathematics Review; Sat Mathematics; Senior Math
	270200	Actuarial Sciences, Other
	270400	Pure Mathematics, Other
	270412	Analytic Geometry; Geometry, Advanced
	270423	Mathematics 3, Unified
	270424	Mathematics, Independent Study
	270436	Discrete Math
	270437	Finite Math
	279900	Mathematics, Other
Precalculus	270416	Analysis, Introductory
Statistics	270500	Statistics, Other
	270511	Statistics
	270521	Probability
	270531	Probability and Statistics
AP/IB Math (Not Calculus)	270431	IB Math Methods 1
	270432	IB Math Studies 1
	270433	IB Math Studies 2
	270532	AP Statistics
Calculus	270418	Calculus and Analytic Geometry
	270419	Calculus
AP/IB Calculus	270420	Calculus, Advanced Placement;
	270434	IB Math Studies/Calculus
	270435	AP Calculus CD
Other Math	270113	Mathematics Tutoring

Course sequences were then operationalized in terms of a two-course sequence: mathematics course (if any) for which credit was earned in 2002–03 and mathematics course (if any) for which credit was earned in 2003–04. The following sequences were used for this report:

- no mathematics–no mathematics;
- geometry–geometry/no mathematics;
- geometry–algebra II;
- algebra II–no mathematics;
- algebra II–algebra II/trigonometry;
- algebra II–precalculus;
- precalculus–no mathematics;
- precalculus–calculus;
- precalculus–AP/IB calculus; and
- all other patterns.

A.8 Appendix A References

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Appendix B
Standard Error Tables for Estimated Means and Proportions, with Weighted Standard Deviations and Raw Sample Sizes for Means

Table B-1a. Standard errors for table 1 estimates (average mathematics IRT-estimated number-right scores, by selected student characteristics: 2002 and 2004)

Student characteristic	10th grade	12th grade	Change
Total	0.29	0.29	0.08
Sex			
Female	0.33	0.34	0.11
Male	0.34	0.35	0.12
Race/ethnicity			
White	0.26	0.28	0.11
Black	0.58	0.58	0.19
Hispanic	0.60	0.60	0.22
Asian	1.00	1.06	0.35
American Indian	1.96	2.33	0.95
More than one race	0.84	0.84	0.41
Socioeconomic status			
Quartile 1 (low)	0.44	0.43	0.17
Quartile 2	0.40	0.42	0.18
Quartile 3	0.37	0.36	0.14
Quartile 4 (high)	0.36	0.35	0.14
School sector			
Public	0.31	0.31	0.09
Catholic	0.53	0.55	0.19
Other private	0.87	0.93	0.34
Family composition			
Mother and father	0.30	0.30	0.10
Mother or father and guardian	0.48	0.50	0.23
Single parent	0.48	0.49	0.17
Other	0.96	0.93	0.51
Student's educational expectations[1]			
High school or less	0.68	0.73	0.43
Some college	0.58	0.56	0.33
Bachelor's degree or more	0.29	0.30	0.09
Don't know	0.62	0.65	0.27

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. IRT = Item Response Theory.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Table B-1b. Sample size and standard deviations for table 1 estimates (average mathematics IRT-estimated number-right scores, by selected student characteristics: 2002 and 2004)

Student characteristic	Sample (n)	10th grade	12th grade	Change
Total	9,460	13.44	14.15	6.27
Sex				
Female	4,790	13.12	13.78	5.99
Male	4,670	13.71	14.46	6.54
Race/ethnicity				
White	5,800	12.38	13.13	6.14
Black	1,060	11.47	12.09	5.61
Hispanic	1,200	12.93	13.67	6.98
Asian	900	13.73	14.66	6.79
American Indian	60	11.27	13.26	7.35
More than one race	420	13.15	13.92	6.92
Socioeconomic status				
Quartile 1 (low)	1,900	12.58	13.09	6.53
Quartile 2	2,180	12.55	13.54	6.58
Quartile 3	2,400	12.53	13.00	5.93
Quartile 4 (high)	2,980	11.94	12.26	6.04
School sector				
Public	7,160	13.49	14.18	6.30
Catholic	1,400	11.12	11.45	5.43
Other private	900	11.97	12.30	6.39
Family composition				
Mother and father	6,070	13.13	13.81	6.19
Mother or father and guardian	1,300	12.90	13.68	6.20
Single parent	1,800	13.75	14.31	6.42
Other	290	12.74	13.27	6.80
Student's educational expectations[1]				
High school or less	430	11.38	12.09	7.35
Some college	760	11.80	12.23	7.28
Bachelor's degree or more	7,440	12.76	13.42	6.03
Don’t know	830	13.60	14.38	6.49

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. IRT = Item Response Theory.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Table B-2a. Standard errors for table 2 estimates (average mathematics proficiency probability scores, by selected student characteristics: 2002 and 2004)

Student characteristic	10th grade Level 1	10th grade Level 2	10th grade Level 3	10th grade Level 4	10th grade Level 5	12th grade Level 1	12th grade Level 2	12th grade Level 3	12th grade Level 4	12th grade Level 5	Change Level 1	Change Level 2	Change Level 3	Change Level 4	Change Level 5
Total	0.003	0.008	0.009	0.007	0.001	0.002	0.007	0.008	0.008	0.003	0.002	0.003	0.003	0.003	0.002
Sex															
Female	0.004	0.010	0.011	0.008	0.001	0.002	0.008	0.010	0.010	0.003	0.002	0.004	0.005	0.004	0.002
Male	0.004	0.009	0.011	0.008	0.002	0.002	0.008	0.010	0.010	0.004	0.003	0.004	0.005	0.005	0.003
Race/ethnicity															
White	0.002	0.006	0.008	0.007	0.001	0.001	0.005	0.008	0.009	0.003	0.002	0.003	0.004	0.004	0.003
Black	0.012	0.020	0.017	0.007	0.001	0.007	0.020	0.019	0.011	0.001	0.008	0.009	0.010	0.006	0.002
Hispanic	0.009	0.019	0.019	0.011	0.001	0.006	0.017	0.019	0.015	0.003	0.006	0.011	0.010	0.007	0.002
Asian	0.006	0.019	0.028	0.030	0.010	0.005	0.018	0.025	0.031	0.018	0.005	0.013	0.013	0.014	0.010
American Indian	0.024	0.081	0.079	0.023	#	0.025	0.069	0.072	0.051	0.013	0.029	0.038	0.045	0.037	0.013
More than one race	0.013	0.025	0.029	0.020	0.007	0.008	0.023	0.027	0.024	0.005	0.009	0.014	0.019	0.016	0.007
Socioeconomic status															
Quartile 1 (low)	0.007	0.015	0.014	0.007	0.001	0.004	0.014	0.014	0.010	0.002	0.005	0.008	0.008	0.005	0.001
Quartile 2	0.005	0.012	0.013	0.009	0.001	0.003	0.012	0.013	0.011	0.002	0.004	0.006	0.008	0.006	0.002
Quartile 3	0.005	0.010	0.013	0.009	0.002	0.002	0.009	0.012	0.011	0.003	0.004	0.005	0.008	0.006	0.003
Quartile 4 (high)	0.003	0.008	0.011	0.011	0.003	0.002	0.006	0.009	0.011	0.006	0.002	0.004	0.006	0.007	0.005
School sector															
Public	0.003	0.008	0.010	0.007	0.001	0.002	0.007	0.009	0.009	0.003	0.002	0.003	0.004	0.004	0.002
Catholic	0.003	0.011	0.017	0.018	0.003	0.001	0.008	0.014	0.020	0.007	0.002	0.005	0.009	0.007	0.005
Other private	0.004	0.015	0.024	0.029	0.007	0.003	0.012	0.022	0.031	0.015	0.002	0.009	0.012	0.014	0.010
Family composition															
Mother and father	0.003	0.008	0.009	0.008	0.001	0.002	0.007	0.008	0.009	0.003	0.002	0.004	0.004	0.004	0.002
Mother or father and guardian	0.006	0.014	0.017	0.012	0.002	0.004	0.013	0.017	0.015	0.004	0.005	0.008	0.010	0.008	0.004
Single parent	0.007	0.015	0.015	0.011	0.002	0.004	0.014	0.015	0.012	0.004	0.005	0.006	0.008	0.007	0.003
Other	0.019	0.033	0.032	0.016	0.001	0.015	0.031	0.031	0.020	0.006	0.014	0.020	0.022	0.012	0.006
Student's educational expectations[1]															
High school or less	0.017	0.026	0.019	0.008	0.001	0.012	0.027	0.024	0.013	0.001	0.011	0.018	0.019	0.010	0.001
Some college	0.013	0.021	0.018	0.009	#	0.007	0.019	0.020	0.012	0.001	0.009	0.013	0.014	0.008	0.001
Bachelor's degree or more	0.003	0.008	0.009	0.007	0.001	0.002	0.006	0.008	0.009	0.003	0.002	0.003	0.004	0.004	0.002
Don't know	0.003	0.008	0.009	0.007	0.001	0.002	0.007	0.008	0.008	0.003	0.002	0.003	0.003	0.003	0.002

† Not applicable.
# Rounds to zero.
[1] Educational expectations of 10th-grade students.
NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Table B-2b. Sample sizes and standard deviations for table 2 estimates (average mathematics proficiency probability scores, by selected student characteristics: 2002 and 2004)

Student characteristic	Sample (n)	10th grade Level 1	Level 2	Level 3	Level 4	Level 5	12th grade Level 1	Level 2	Level 3	Level 4	Level 5	Change Level 1	Level 2	Level 3	Level 4	Level 5
Total	9,460	0.168	0.390	0.457	0.353	0.074	0.111	0.357	0.443	0.414	0.150	0.122	0.220	0.300	0.239	0.122
Sex																
Female	4,790	0.169	0.397	0.457	0.336	0.052	0.109	0.365	0.447	0.401	0.119	0.118	0.219	0.295	0.233	0.100
Male	4,670	0.168	0.381	0.456	0.368	0.091	0.114	0.349	0.438	0.425	0.176	0.126	0.221	0.306	0.246	0.142
Race/ethnicity																
White	5,800	0.125	0.331	0.438	0.371	0.076	0.082	0.299	0.404	0.421	0.163	0.093	0.190	0.297	0.252	0.134
Black	1,060	0.242	0.430	0.374	0.174	0.028	0.156	0.428	0.434	0.256	0.042	0.181	0.249	0.287	0.165	0.048
Hispanic	1,200	0.224	0.440	0.419	0.258	0.039	0.159	0.423	0.455	0.339	0.084	0.156	0.291	0.318	0.206	0.073
Asian	900	0.132	0.331	0.425	0.409	0.157	0.095	0.309	0.397	0.435	0.255	0.112	0.232	0.279	0.261	0.179
American Indian	60	0.157	0.450	0.435	0.150	†	0.150	0.439	0.446	0.291	0.090	0.187	0.267	0.338	0.217	0.090
More than one race	420	0.198	0.392	0.453	0.325	0.085	0.123	0.362	0.451	0.405	0.107	0.136	0.248	0.338	0.245	0.111
Socioeconomic status																
Quartile 1 (low)	1,900	0.227	0.440	0.423	0.235	0.027	0.152	0.424	0.453	0.314	0.057	0.160	0.267	0.307	0.198	0.049
Quartile 2	2,180	0.178	0.409	0.449	0.291	0.028	0.127	0.387	0.461	0.370	0.086	0.133	0.248	0.314	0.221	0.076
Quartile 3	2,400	0.150	0.354	0.450	0.349	0.061	0.087	0.320	0.418	0.412	0.132	0.116	0.200	0.307	0.237	0.116
Quartile 4 (high)	2,980	0.095	0.273	0.396	0.401	0.118	0.064	0.228	0.338	0.413	0.222	0.070	0.165	0.276	0.273	0.177
School sector																
Public	7,160	0.174	0.396	0.458	0.348	0.115	0.115	0.365	0.448	0.410	0.144	0.126	0.224	0.301	0.235	0.118
Catholic	1,400	0.077	0.276	0.409	0.374	0.035	0.035	0.218	0.338	0.412	0.171	0.056	0.174	0.293	0.271	0.137
Other private	900	0.089	0.278	0.401	0.394	0.064	0.064	0.223	0.332	0.415	0.236	0.060	0.190	0.286	0.271	0.185
Family composition																
Mother and father	6,070	0.150	0.363	0.450	0.369	0.097	0.097	0.327	0.423	0.423	0.164	0.102	0.212	0.298	0.247	0.133
Mother or father and guardian	1,300	0.176	0.393	0.454	0.324	0.117	0.117	0.368	0.452	0.392	0.123	0.136	0.226	0.303	0.226	0.101
Single parent	1,800	0.197	0.431	0.454	0.316	0.129	0.129	0.402	0.463	0.387	0.123	0.151	0.234	0.306	0.227	0.106
Other	290	0.236	0.439	0.432	0.232	0.182	0.182	0.428	0.455	0.289	0.093	0.178	0.258	0.301	0.192	0.081
Student's educational expectations[1]																
High school or less	430	0.287	0.417	0.318	0.159	0.212	0.212	0.438	0.389	0.227	0.036	0.203	0.288	0.319	0.164	0.026
Some college	760	0.242	0.435	0.401	0.197	0.157	0.157	0.421	0.443	0.270	0.034	0.187	0.278	0.340	0.186	0.032
Bachelor's degree or more	7,440	0.133	0.351	0.446	0.367	0.087	0.087	0.314	0.415	0.421	0.163	0.098	0.202	0.295	0.247	0.134
Don’t know	830	0.195	0.425	0.446	0.319	0.068	0.125	0.403	0.464	0.384	0.133	0.148	0.253	0.295	0.231	0.101

† Not applicable.

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native.

Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Table B-3. Standard errors for table 3 estimates (weighted percentage and unweighted frequency of students taking mathematics course sequences: 2002 and 2004)

Course sequence	Weighted percentage	Unweighted frequency
Algebra II–no mathematics	0.55	†
Geometry–geometry/no mathematics	0.47	†
Algebra II–precalculus	0.45	†
Algebra II–algebra II/trigonometry	0.44	†
Precalculus–AP/IB calculus	0.40	†
No mathematics–no mathematics	0.40	†
Geometry–algebra II	0.38	†
Precalculus–no mathematics	0.30	†
Precalculus–calculus	0.27	†
All other patterns	1.05	†

† Not applicable.

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample and are ordered hierarchically. AP/IB calculus = Advanced Placement/International Baccalaureate calculus.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Table B-4. Standard errors for table 4 estimates (percentage of students taking mathematics course sequences, by selected student characteristics: 2002 and 2004)

Student characteristic	Course sequence									
	No mathematics–no mathematics	Geometry–geometry/no mathematics	Geometry–algebra II	Algebra II–no mathematics	Algebra II–algebra II/trigonometry	Algebra II–pre-calculus	Pre-calculus–no mathematics	Pre-calculus–calculus	Pre-calculus–AP/IB calculus	All other patterns
Total	0.40	0.47	0.38	0.55	0.44	0.45	0.30	0.27	0.40	1.05
Sex										
Female	0.44	0.52	0.49	0.67	0.59	0.56	0.42	0.29	0.48	1.25
Male	0.53	0.58	0.46	0.71	0.47	0.52	0.37	0.36	0.52	1.20
Race/ethnicity										
White	0.49	0.53	0.38	0.66	0.52	0.53	0.39	0.37	0.54	1.25
Black	0.87	1.08	1.42	1.65	1.34	1.06	0.56	0.22	0.57	2.59
Hispanic	0.81	1.29	1.21	1.32	0.68	0.93	0.79	0.22	0.73	2.12
Asian	0.71	1.38	0.75	1.95	0.77	1.28	0.41	0.52	2.17	3.02
American Indian	6.77	3.23	2.79	3.04	1.30	3.39	†	†	2.66	6.92
More than one race	1.50	1.82	1.28	2.02	1.50	1.23	1.60	0.71	1.09	3.12
Socioeconomic status										
Quartile 1 (low)	0.86	1.09	0.72	0.99	0.69	0.65	0.38	0.16	0.56	1.58
Quartile 2	0.70	0.89	0.60	0.96	0.67	0.60	0.47	0.26	0.44	1.58
Quartile 3	0.69	0.75	0.63	0.99	0.63	0.70	0.52	0.45	0.59	1.66
Quartile 4 (high)	0.38	0.47	0.53	0.79	0.71	0.84	0.57	0.52	0.82	1.58
School sector										
Public	0.44	0.51	0.41	0.59	0.45	0.47	0.33	0.28	0.42	1.12
Catholic	0.38	0.50	0.98	1.59	2.04	1.86	0.77	1.26	1.36	3.77
Other private	0.55	0.79	0.77	1.55	3.06	2.41	1.42	2.09	1.73	4.65
Family composition										
Mother and father	0.47	0.55	0.40	0.71	0.47	0.53	0.41	0.34	0.51	1.24
Mother or father and guardian	0.81	0.86	0.75	1.26	0.83	0.98	0.55	0.27	0.56	1.89
Single parent	0.71	0.91	0.84	1.07	0.80	0.69	0.46	0.41	0.58	1.65
Other	1.81	2.05	1.37	2.15	1.72	1.44	1.09	0.53	1.41	3.77

See notes at end of table.

Table B-4. Standard errors for table 4 estimates (percentage of students taking mathematics course sequences, by selected student characteristics: 2002–2004)—Continued

Student characteristic	Course sequence									
	No mathematics–no mathematics	Geometry–geometry/no mathematics	Geometry–algebra II	Algebra II–no mathematics	Algebra II–algebra II/trigonometry	Algebra II–pre-calculus	Pre-calculus–no mathematics	Pre-calculus–calculus	Pre-calculus–AP/IB calculus	All other patterns
Student's educational expectations[1]										
High school or less	2.38	2.02	1.44	1.65	0.99	0.40	0.59	0.27	0.08	2.88
Some college	1.83	1.78	1.02	1.58	0.67	0.55	0.43	0.10	0.34	2.53
Bachelor's degree or more	0.33	0.47	0.41	0.58	0.52	0.55	0.38	0.33	0.48	1.15
Don't know	1.22	1.11	1.01	1.49	0.56	0.74	0.56	0.46	1.08	2.13

† Not applicable.

[1] Educational expectations of 10th-grade students.

NOTE: Black includes African American, Hispanic includes Latino, Asian includes Native Hawaiian or Other Pacific Islander, and American Indian includes Alaska Native. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. The nine course sequences listed refer to those followed by more than 3 percent of the sample and are ordered hierarchically.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Table B-5a. Standard errors for table 5 estimates (average mathematics IRT-estimated number-right scores, by mathematics course sequences: 2002 and 2004)

Course sequence	10th grade	12th grade	Change
Total	0.29	0.29	0.08
No mathematics–no mathematics	0.83	0.81	0.37
Geometry–geometry/no mathematics	0.62	0.58	0.34
Geometry–algebra II	0.75	0.84	0.35
Algebra II–no mathematics	0.47	0.48	0.22
Algebra II–algebra II/trigonometry	0.60	0.61	0.30
Algebra II–precalculus	0.58	0.58	0.27
Precalculus–no mathematics	0.64	0.61	0.36
Precalculus–calculus	0.68	0.67	0.40
Precalculus–AP/IB calculus	0.47	0.38	0.27
All other patterns	0.40	0.41	0.12

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample and are ordered hierarchically. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. IRT = Item Response Theory.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Table B-5b. Sample sizes and standard deviations for table 5 estimates (average mathematics IRT-estimated number-right scores, by mathematics course sequences: 2002 and 2004)

Course sequence	Sample (n)	10th grade	12th grade	Change
Total	9,460	13.44	14.15	6.27
No mathematics–no mathematics	480	13.15	13.02	7.00
Geometry–geometry/no mathematics	650	11.00	10.64	6.67
Geometry–algebra II	420	9.55	10.17	5.94
Algebra II–no mathematics	1,160	10.64	10.75	6.22
Algebra II–algebra II/trigonometry	680	9.74	9.40	5.93
Algebra II–precalculus	730	9.93	9.62	5.90
Precalculus–no mathematics	330	9.22	8.23	5.66
Precalculus–calculus	240	7.28	6.98	4.85
Precalculus–AP/IB calculus	610	7.95	6.03	5.56
All other patterns	4,160	14.32	15.08	6.12

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample and are ordered hierarchically. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. IRT = Item Response Theory.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."

Table B-6a. Standard errors for table 6 estimates (average mathematics proficiency probability scores, by mathematics course sequences: 2002 and 2004)

Course sequence	10th grade					12th grade					Change				
	Level 1	Level 2	Level 3	Level 4	Level 5	Level 1	Level 2	Level 3	Level 4	Level 5	Level 1	Level 2	Level 3	Level 4	Level 5
Total	0.003	0.008	0.009	0.007	0.001	0.002	0.007	0.008	0.008	0.003	0.002	0.003	0.003	0.003	0.002
No mathematics–no mathematics	0.014	0.028	0.027	0.015	0.001	0.010	0.026	0.027	0.017	0.002	0.010	0.013	0.016	0.010	0.002
Geometry–geometry/no mathematics	0.011	0.024	0.020	0.008	#	0.006	0.023	0.022	0.010	#	0.009	0.013	0.015	0.008	#
Geometry–algebra II	0.010	0.031	0.027	0.010	#	0.004	0.031	0.035	0.018	0.001	0.008	0.017	0.021	0.013	0.001
Algebra II–no mathematics	0.005	0.015	0.018	0.011	0.001	0.003	0.013	0.018	0.014	0.001	0.004	0.008	0.011	0.008	0.001
Algebra II–algebra II/trigonometry	0.004	0.017	0.026	0.015	0.001	0.001	0.013	0.022	0.023	0.001	0.004	0.012	0.017	0.014	0.001
Algebra II–precalculus	0.005	0.016	0.022	0.017	0.001	0.001	0.010	0.018	0.023	0.004	0.004	0.010	0.012	0.014	0.004
Precalculus–no mathematics	0.006	0.011	0.023	0.026	0.008	0.001	0.007	0.016	0.028	0.007	0.005	0.008	0.016	0.019	0.009
Precalculus–calculus	#	0.003	0.019	0.034	0.007	#	0.001	0.008	0.026	0.023	#	0.003	0.015	0.022	0.020
Precalculus–AP/IB calculus	#	0.004	0.010	0.018	0.009	#	#	0.002	0.011	0.019	#	0.004	0.009	0.012	0.016
	0.005	0.011	0.012	0.009	0.002	0.003	0.010	0.012	0.011	0.003	0.003	0.004	0.005	0.005	0.002
All other patterns	0.003	0.008	0.009	0.007	0.001	0.002	0.007	0.008	0.008	0.003	0.002	0.003	0.003	0.003	0.002

† Not applicable.

Rounds to zero.

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample and are ordered hierarchically. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), “Base Year, 2002” and “First Follow-up, 2004.”

Table B-6b. Sample sizes and standard deviations for table 6 estimates (average mathematics proficiency probability scores, by mathematics course sequences: 2002 and 2004)

Course sequence	Sample (n)	10th grade Level 1	Level 2	Level 3	Level 4	Level 5	12th grade Level 1	Level 2	Level 3	Level 4	Level 5	Change Level 1	Level 2	Level 3	Level 4	Level 5
Total	9,460															
No mathematics–no mathematics	480	0.227	0.437	0.435	0.278	0.031	0.173	0.431	0.451	0.296	0.053	0.166	0.233	0.322	0.201	0.036
Geometry–geometry/no mathematics	650	0.206	0.421	0.398	0.185	0.008	0.116	0.409	0.434	0.210	0.005	0.172	0.284	0.328	0.149	0.007
Geometry–algebra II	420	0.139	0.392	0.382	0.163	0.002	0.067	0.356	0.442	0.259	0.019	0.123	0.273	0.353	0.197	0.018
Algebra II–no mathematics	1,160	0.127	0.367	0.431	0.254	0.018	0.085	0.325	0.425	0.323	0.018	0.102	0.222	0.345	0.214	0.023
Algebra II–algebra II/ trigonometry	680	0.090	0.308	0.435	0.261	0.017	0.020	0.214	0.357	0.368	0.026	0.085	0.225	0.351	0.257	0.026
Algebra II–precalculus	730	0.086	0.275	0.398	0.318	0.020	0.015	0.177	0.308	0.388	0.091	0.079	0.206	0.291	0.288	0.087
Precalculus–no mathematics	330	0.093	0.184	0.323	0.352	0.091	0.013	0.103	0.231	0.364	0.096	0.084	0.140	0.231	0.263	0.110
Precalculus–calculus	240	0.002	0.037	0.216	0.353	0.075	†	0.009	0.096	0.275	0.229	0.002	0.035	0.174	0.275	0.206
Precalculus–AP/IB calculus	610	0.013	0.090	0.188	0.325	0.154	†	†	0.044	0.189	0.310	0.013	0.089	0.175	0.255	0.271
All other patterns	4,160	0.194	0.414	0.464	0.359	0.085	0.134	0.388	0.460	0.418	0.149	0.131	0.219	0.279	0.232	0.114

† Not applicable.

NOTE: The nine course sequences listed refer to those followed by more than 3 percent of the sample and are ordered hierarchically. AP/IB calculus = Advanced Placement/International Baccalaureate calculus. Proficiency Levels are: Level 1—simple arithmetical operations on whole numbers, such as simple arithmetic expressions involving multiplication or division of integers; Level 2—simple operations with decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents; Level 3—simple problem solving, requiring the understanding of low-level mathematical concepts, such as simplifying an algebraic expression or comparing the length of line segments illustrated in a diagram; Level 4—understanding of intermediate-level mathematical concepts and/or multistep solutions to word problems such as drawing an inference based on an algebraic expression or inequality; and Level 5—complex multistep word problems and/or advanced mathematics material such as a two-step problem requiring evaluation of functions.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Education Longitudinal Study of 2002 (ELS:2002), "Base Year, 2002" and "First Follow-up, 2004."