MATHEMATICS
Colorado Annual Standards Review

THE STATE'S PRIME NUMBERS

cde
Colorado Department of Education

A report on the performance of our state's math standards and recommendations for stronger math achievement.

COLORADO STATE BOARD OF EDUCATION

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Special Permission obtained for picture featuring students at Zablocki Elementary, A Milwaukee public school and a National Blue Ribbon School of Excellence.

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COLORADO'S MATH STANDARDS REVIEW

This is the first in a series of annual reviews of the Colorado Model Content Standards. Its purpose is to identify student performance over time on measures of our existing standards, identify ways to affirm and strengthen our standards and more clearly articulate the practices used by Colorado schools to make substantial gain in the achievement of students to the standards.

"Year of Math" Process

The Office of Learning and Results visited, presented and interviewed over 820 mathematics concerned policy, educator, media or university-based individuals. This nine-month series of study and listening, editing and asking was statewide. Research on data points and historical trend data was gathered from state and national resources, university faculty, and department staff including finance, licensure, assessment regional managers, Title I and Information Management Services.

Timeline (2004 – 2005)

August	Built history and examined historical reviews of Colorado Standards and Assessment Frameworks
September	Examined existing Colorado student math performance data
Sept – Dec	Classroom Observations, and Interviews Statewide
Sept – April	Shared data with: State Board of Education 58 sites statewide Higher Education Groups BOCES leadership Public Broadcasting Superintendent and principal state meetings Teacher groups Professional development leaders Colorado Council of Teachers of Mathematics
Sept – April	Identified and surveyed schools making gains or getting better than expected outcomes
Jan – March	Review of Current Research on Math Cognition and Learning
April	Review of Existing Math Resources
April	Compare international and national math standards with student performance
May	Concluding Recommendations
June	Math Summit
July	Board Approval of Standards Changes

NATIONAL OVERVIEW OF K-12 MATHEMATICS

The United States' student performance in mathematics has made slow and uneven advances. Internationally speaking, our nation continues to produce elementary, middle and high school students who rank below most industrialized countries. As Tom Friedman's immediately acclaimed book, *The World is Flat* reveals, our country's global preeminence as a leader in economic, collegiate, medical and market domains is ending with the advent of the Internet. In the past, our country had boundary-based access to intellectual assets. American children's contribution to an emerging math dependent market was not essential, as the competition was small and smart.

With second and third world countries stoking their math and science education systems and realizing the payoff of "lifting" the latest intellectual capitol from a democratic Internet, Americans will soon be startled at the outcome of this practice.

The United States does not have a mandatory national standard for math performance. Our national math standards are merely suggestions as individual states dictate their own bottom line for student performance. Nevertheless, one voluntary national exam, administered every other year for decades does exist: the National Assessment for Educational Progress. While math results for the most recent testing year of 2003 indicate that overall proficiency in math is increasing each year, the results are still surprisingly low. For example, 8th grade math proficiency increased a mere 3% over 2000 test data, and with only 29% of our nation's students scoring at the proficient or above level. The 4th grade results indicate an 8% gain in proficiency over the same period, resulting in only 32% at the proficient or above level.

PISA's (Program for International Student Achievement) 2003 international mathematics assessment given to 15 year old students in 29 countries measures the 'yield' of a nation's education system. PISA is specifically designed to determine overall competencies students have acquired to apply math knowledge and skills to problems with real-world contexts. The PISA ranked the United States 24th out of 29 countries.

TIMSS (The Third International Math and Science Study) 2003 is an international comparison of mathematics achievement in the primary and middle grades. The assessment is based on common aspects of curricula from all participating countries. The United States ranked 12th out of the 16 industrialized nations. In the eighth grade, the US ranked 15th out of the 16 industrialized nations.

COLORADO MATH STANDARDS

Colorado's Model Mathematics Standards were adopted by the Colorado State Board of Education in 1995. The assessment frameworks, which articulate each math topic per grade level that students are expected to, know on the CSAP were built in 1998. The state math test was first issued in 1999.

Math teachers and local districts asked for national, district and local examples of math ideas and articulation by grade level to help in the classroom. Local teachers, committees and vendors contributed in 2000 to this bank known as the voluntary grade level expectations. These choices were accepted as helpful, but were not necessarily closely aligned to the model content standards.

These voluntary grade level *examples* were removed from the state resource bank in 2004 because local and national analyses and practitioners used this as a substitute for a variety of purposes for which it was not intended.

No state math curriculum exists in Colorado, as it is a locally controlled state.

No state math curriculum exists in Colorado, as it is a locally controlled state. Textbooks, curriculum decisions and supplemental resources are decided and purchased at a local district level. Also, at the local level, individual schools and districts determine when a math topic is introduced or offered. Therefore, pre-algebra or geometry classes are present in different grades in different cities. While the state assessment will always ask students a formal geometry question on the tenth grade CSAP, some high schools have made a decision to offer geometry classes in eleventh or twelfth grade.

Various American educational institutions have rated and ranked individual states' standards and performance based on a host of variable conditions. The national teacher union, American Federation of Teachers (AFT) put a premium on the overall *quality* of standards as written. Colorado's standards were given a "*good*" rating and found to be "generally clear and specific and grounded in particular content to meet AFT's common core criterion for math standards." The Fordham Foundation rates math standards based on state curriculum benchmarks. Fordham evaluated and graded Colorado standards by using the outdated grade level expectations, which were never meant to serve as curriculum. As a result, Colorado was issued a "D". *Education Week* evaluates states' math standards based on clarity and alignment. Colorado was given a "B" for its quality math standards, alignment of standards to the state assessment and for rigor.

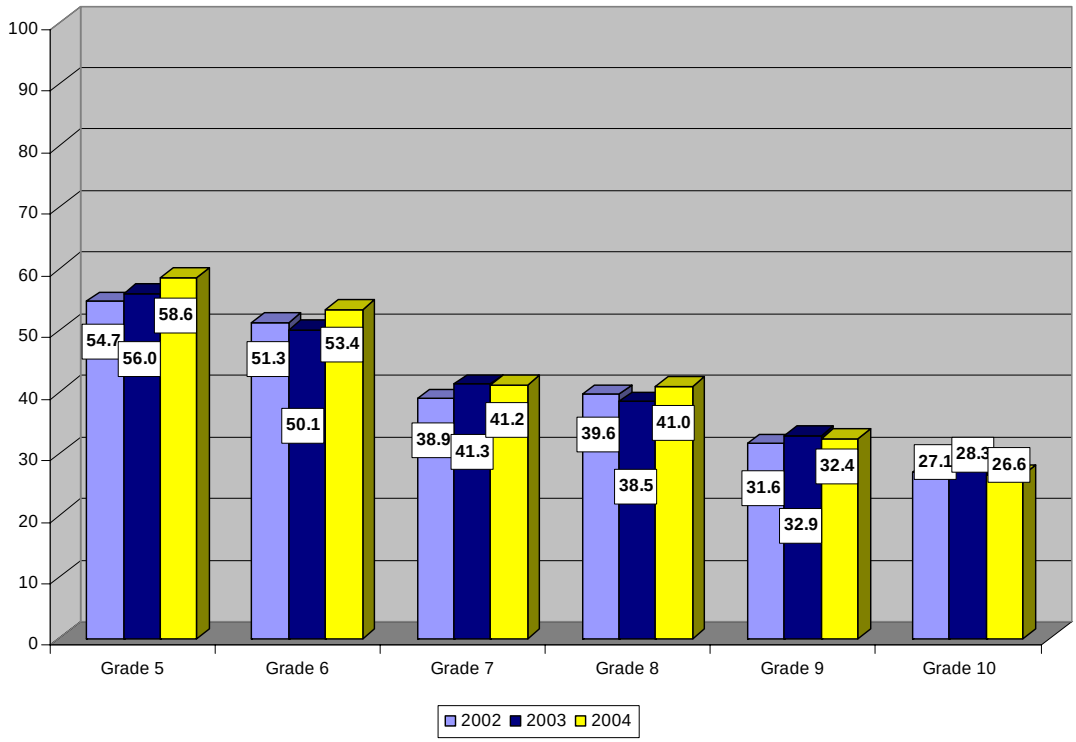
MATH ACHIEVEMENT IN COLORADO

Achievement over Time

Regardless of gender, ethnicity or poverty, students in Colorado are operating below state expectations. Over time, the average percent of students proficient or advanced on the CSAP elementary math assessment are approximately 56%. Middle school students drop to an average 38% proficient or advanced. High school students proficient or advanced amount to a mere 27%. The eleventh grade assessment is the Colorado ACT and while a 36 is a perfect score, Colorado students average a score of 18 on this math assessment. In the last three years, there has been less than 1% gain for any grade on any state math assessment instrument.

Regardless of gender, ethnicity or poverty, students in Colorado are operating below state expectations.

Overview Of Math Proficiency or Above on CSAP Grades 5-10

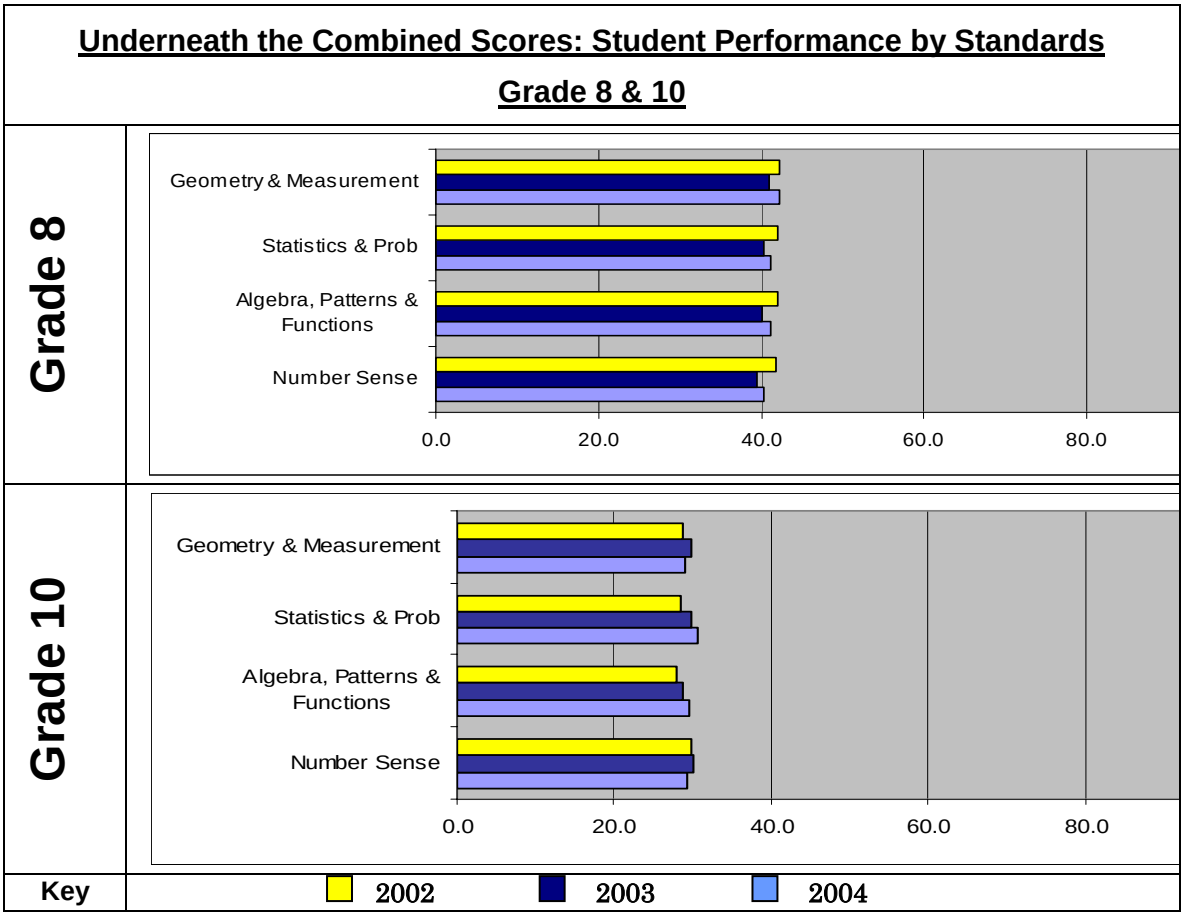


Similarly, National Assessment of Educational Progress (NAEP), a common, state-to-state assessment, indicates that while Colorado ranks with other states moderately well (12th in the nation for 4th grade math performance and 3rd in the nation for 8th grade math performance), these rankings represent in both 4th and 8th grades a 66% failure rate respectively.

Despite conventional wisdom suggesting exaggerated cut points, unrealistic national norms, or a collegiate agenda, math assessments on local, state and international levels are comparable and student performance on different instruments does not vary much beyond an average statistical range. For example, CSAP's 8th grade math results show students at or about 38% proficient or advanced for the past three years. Similarly, the NAEP results reported Colorado eighth graders at 34% proficient or advanced. Other grade level results are also similar to local and national instruments and do not reach divergent conclusions.

With Which Math Standards Do Students Struggle Most?

Each sub-content subject is equally difficult for Colorado's students. Especially at middle and high school, students across regions, demographic categories, gender, and age have a proficiency rate of about 30-40% in geometry, measurement, statistics, probability, algebra, patterns, number functions and number sense. In fact, students are not predominantly fluent in any one math topic.



COLORADO MATHEMATICS EDUCATORS

The Mathematics Teaching Field

There are approximately 3,000 secondary math teachers in Colorado public schools and more than 17,000 such teachers in the elementary grades. While approximately 75% of Colorado's teachers are trained outside of Colorado, it is estimated that Colorado higher education preparation programs produce about 120 formally trained math educators each year for Colorado's classrooms.

The state has an affiliation with the Colorado Council of Teachers of Mathematics, with over 1,400 members. Additionally, CoMath, an online math practitioner support network, boasts of 400 participants. Various other math-specific groups exist, with a more prominent example being approximately 60 teachers who participate in math lesson study activities.

Colorado candidates for initial educator licenses are required to take and pass a content test for endorsement in any content area(s) in which they will teach. The assessment is intended to determine the content knowledge of those candidates seeking licensing and endorsement and is based on what Colorado PK-12 practitioners and content and preparation program faculty have determined a first-year teacher should know and be able to demonstrate.

Two assessments, as adopted by the Colorado State Board of Education (SBE), were deemed by Colorado's educators to be rigorous and validated:

- The National Evaluation Systems (NES) PLACE test (Program for Licensing Assessments for Colorado Educators) is built on Colorado's teacher performance-based standards, which were, in turn, built on the SBE's-adopted Colorado Student Content Standards.
- The Education Testing Service (ETS) provides an optional nationally based testing instrument, the Praxis II, also adopted by the SBE, in five content areas, one of which is an assessment of content-knowledge in Mathematics, for candidates seeking that licensing endorsement. Note: Colorado's Praxis II Math test cut score is the highest such score in the country.

Colorado candidates in teacher preparation programs are required to pass their applicable licensing endorsement content exams *prior* to student teaching. This ensures that all Colorado students have teachers that demonstrate competency in math content, *even* when those teachers are in *pre-licensed student teaching* placements.

By August, 2006, and *now*, for those teaching in Title I programs or schools - educators in Colorado must be fully licensed and endorsed, or "Highly Qualified" (indicating that they have completed 24-semester hours in their content area[s] or have passed their content test[s]) in the subject matter they teach.

Recently, the National Council of Teacher Quality (NCTQ) awarded Colorado an A+ (the only state in the United States to receive this high rating.) NCTQ's recognition was based on:

- The alignment of Colorado teacher performance-based preparation standards with Colorado student content standards;
- Colorado's requirement that all student teachers pass their applicable content test(s), *prior* to direct instruction of students;
- All Colorado teachers meeting all SBE-approved requirements, without waivers, prior to licensing and endorsement.

The Teacher Endorsement Preparation Standards in Math

Colorado math teacher preparation includes and incorporates both nationally recognized math standards and the content knowledge required of Colorado students in the classroom, as identified in Colorado's Student Content Standards. Colorado's teacher endorsement preparation standards in math were adopted, by the State Board of Education, in September, 2003. The Colorado teacher preparation standards are attached (see Appendix C).

The content of Colorado math teacher preparation programs is based on three elements:

1. Nationally-recognized math standards for teachers;
2. Colorado's student math content standards; and
3. The candidate's ability to demonstrate math application and *effectively* instruct students in math.

Colorado math teacher preparation includes both nationally recognized math standards and Colorado Content Model Standards.

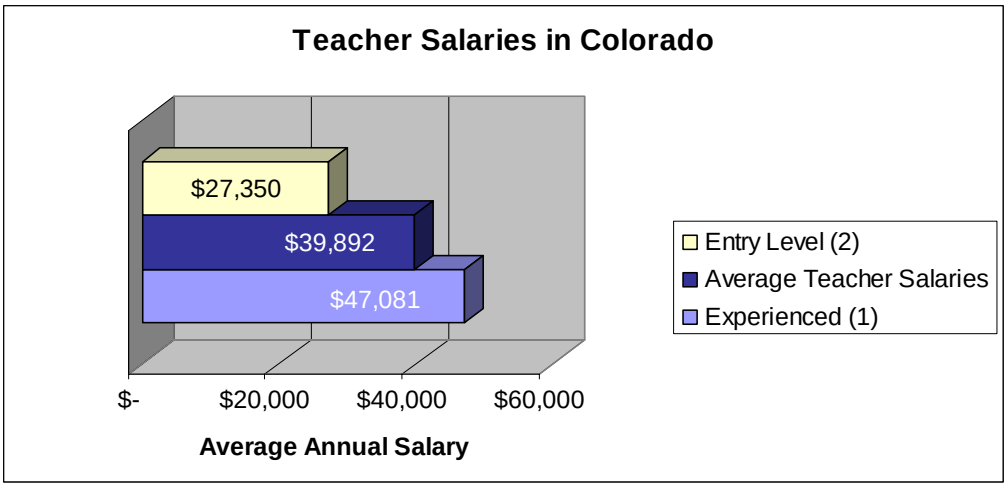
Only a relatively small proportion of Colorado teachers have been prepared under the 2003 State Board of Education-adopted rules. Therefore, correlated student achievement data is not yet available.

All institutions with *new math programs* are required to show how their programs provide their math teaching candidates with the content knowledge required under the 2003 State Board of Education adopted rules. Higher education institutions with *new math programs* must illustrate the effective delivery of that content.

All institutions with *previously approved* math programs are reviewed on a five-year cycle to insure that the content of their math teacher preparation programs matches the content endorsement preparation standards. If their content does not match, the programs are not submitted to the Colorado State Board.

Math Teacher Salaries and the Marketplace

Based on 2003 data [1], the following chart illustrates average Colorado math teacher salaries.



Footnote to Teacher Salaries Graph
1) Experienced Teachers are those with 15 or more years of experience plus a maters' degree.
2) Entry Level Teachers have little experience and no masters' degree.

Colorado teacher salaries have generally been higher in the Metro area and in suburban schools, than those in the Northeast and Southeast regions of Colorado, and in smaller more rural towns, where the lowest salaries are offered.

Nearly three-quarters of Colorado teachers have been trained in out-of-state preparation programs.

Nearly three-quarters of Colorado's teachers have been trained in out-of-state programs. Out-of-state applicants for Colorado educator licenses must provide documented evidence of three-years, or more, of demonstrated teaching experience, or pass their applicable content-area tests. To be endorsed, secondary math teachers are required to have 30-hours in mathematics coursework, the equivalent of a math major. As of 2002:

- 42% of Colorado's secondary math teachers have a Masters' degree or higher, while
- 77% of Colorado middle or high school math teachers have more than three years of experience teaching math.

The state educator recruitment web page, www.TeachinColorado.org registered 123 math jobs offered this year, through May, 2005. All regions of the state were currently hiring for math positions. Middle school math teaching positions tallied 33 postings, with 146 active applications for those jobs. There were 64 high school math jobs offered with 134 active applications for those jobs. Other specified math jobs (AP Calculus, Algebra, Algebra II, Geometry and Math Specialist/ Coach, totaled 26). Ninety teachers have applied for those specific subjects. Around 695 teachers were actively seeking employment at the time of this writing, with 3709 actively seeking employment related to math instruction.

What Do We Hear and Observe about Math from the Field

As the long term Colorado student performance data was presented in 58 regional presentations throughout the state and during subsequent interviews and classroom observations, the following questions were posed to the field:

- "What mathematics do you think this represents and how is it different than the math you teach?"
- "Do these results surprise you?"
- "What do you speculate is the cause of these results?"
- "What are some solutions?"

The following themes emerged:

- Across the state, assessment directors, curriculum administrators and teachers told us that students of all ages do not have an automatic fluency with general arithmetic.
- Reporters, editorial boards, parents and local school board members believed the state performance and did not contest these outcomes. Sadly, they saw similar or worse results locally or "at home".
- Math teachers and curriculum specialists reported that the majority of their students are extremely weak in the area of math concepts (understanding patterns, etc)
- Schools at elementary, middle and high schools around the state agreed that students are mostly unable to solve math problems with a degree of proficiency. This was true regardless of age, gender, wealth or ethnic grouping.
- About 10% of the state's educators question the utility of math beyond eighth grade for all students. (Historically, algebra and geometry have been used as gateways for only certain students.)
- Less than one half of the 58 districts visited were firmly clear about math standards and referred instead to the text and quizzes of their own textbook as their goals for math
- When the assessment framework or math topics at grade level were shared, the teachers appreciated the clarity; an overwhelming majority believed that the frameworks were appropriate and reasonable.
- Every city or district discussed the need for stronger teacher development in math and for higher education to align their work to K-12 needs. This was especially true for elementary and middle school teachers.
- There was an honest confounded quality as to how to tackle math deficiencies and where to begin. Without a clear model of math results from which to confidently spring, most conversations trickled back to the need for more money and ending poverty in urban and rural areas.
- There was overall confusion about which students were "stuck", and why and how these students might be engaged.

RESEARCH ON MATH COGNITION AND LEARNING

Mathematicians are forced to resort to written symbols and pictures to describe their world – even to each other. But the symbols are no more that world than the musical notation is music.

Ian Stewart, Nature's Numbers

As with reading, math standards come with an expectation of some prerequisite skills. A student cannot be expected to compare and contrast one tale with another without the "upfront" ability to fluently decode words, recognize vocabulary and make meaning of text. Cognitive and field research, along with student data, show us repeatedly that unless a student is introduced to the prerequisites of written language, reading is not innate nor will it come to a child over time.

In mathematics, the same principles apply. While math is natural and pictorial, like music, it requires a language and symbols in order to be used and shared. Instruction in math, however, is often vague and the essential prerequisites are not understood and accessible for students to repeatedly use and rely upon. As a result, the introduction to *more* math activities, procedures, and inconclusive lessons, slows achievement and stifles an eagerness for math.

Research in Math

The key for advancing Colorado students' math performance may be found in the research findings in the following three distinct areas: 1) four math domains within cognitive development, 2) effective math instruction and 3) math disabilities.

1. **While additional research on *math cognition* and learning is still needed**, reviews of existing research in cognitive psychology and math education indicate consensus on four essential domains in mathematics [2] [3] [4].

Four Essential Math Domains

Explicit Arithmetic Facts and Procedures	Specific Conceptual Models
Robust Reasoning	Applied Problem Solving

2. **Convergent evidence of *effective math instruction* indicates**[5]:

- Frequent classroom assessment is useful
- Ability tracking widens achievement gaps
- How students think about math is important
- Direct instruction helps students learn computational skills and understand math principles
- Students do not have to discover math principles on their own in order to understand them, however, discovery activities can be useful
- Broad achievement gains require coordinated systems at the state, district and school levels

3. **Early research on *math disabilities* indicates**[6]:

- Limited number sense, lack of automaticity with math facts and procedural errors are common in students with math disabilities
- Kindergarten measures of number sense are predictive of math achievement in first and second grades
- A reciprocal relationship exists between conceptual understanding and consistent use of efficient procedures, each helps to develop the other
- Weak automaticity in basic math facts has devastating effects on the ability to solve problems and keep up with comprehension of concepts

Research in mathematics is currently being conducted via the U.S. Department of Education and the National Institute of Child Health & Human Development. Detailed information about this research is available in Appendix C.

COLORADO FINANCIAL RESOURCES FOR MATH INSTRUCTION

Math existing resources

While school and district budget decisions vary, Colorado schools receive from the state up to $165 per student, per year, to purchase equipment, resources, textbooks and supplementary materials for curriculum subject areas. This is over $126 million dollars annually. Most districts use a seven-year cycle of decision-making per subject area. Using a rough average, $80,000 per school is issued per year for these learning resources. Many larger districts obtain overrides to supplement this item of revenue at the local level. Additionally, other districts administer annual gifts, grants and donations to the math instruction agenda. Some of the above resources are also used, instead, to limit class size or alter the number of instructional classes per day. Every district in the state determines how they will spend their dollars to acquire math textbooks, supplementary material, and math appropriate software during their decision-making year. The state does not approve or deny the local district decisions about how to spend this revenue.

While salaries (2003) for math teachers have been addressed, the aggregate local, state and federal investment exceeds $129 million a year for secondary math teacher salaries; elementary mathematics teacher salaries are conservatively estimated at $249 million.

State mathematics initiatives are available in most every district. These funds are a composite of local revenue, state grants and federal initiatives. Examples of these math dollars include:

A Sampling of 2004-2005 State and Federal Math Dollars

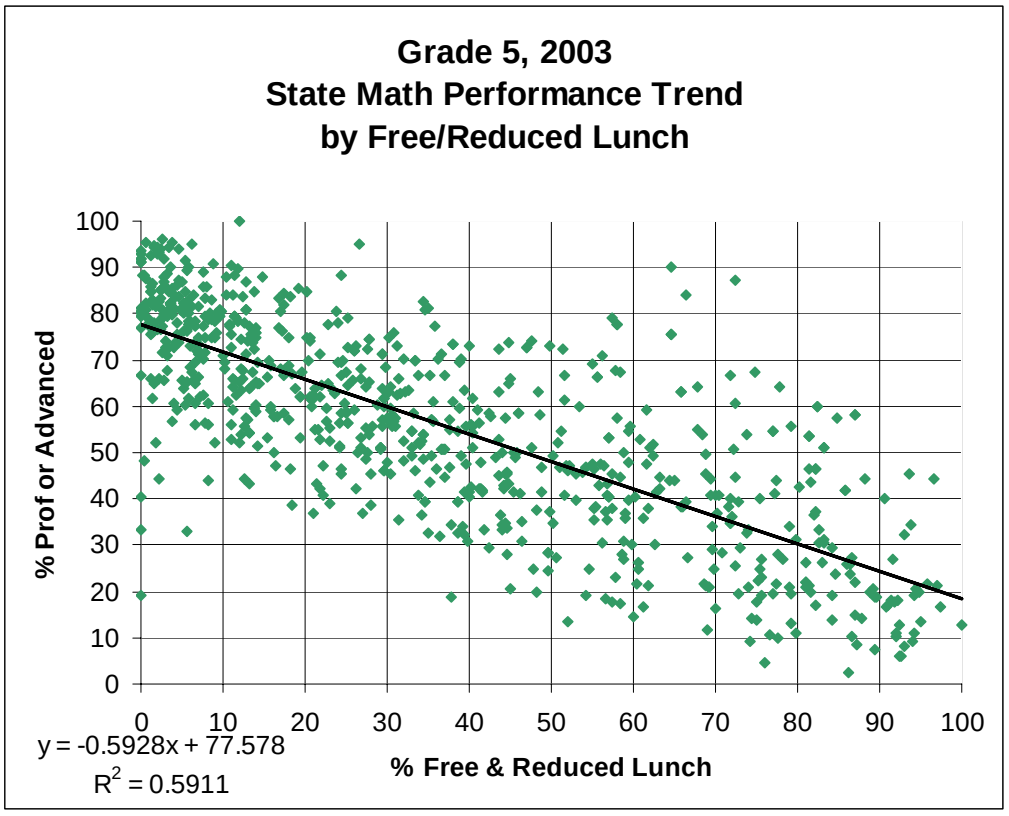
$41,000,000	Average annual Title I and II school and district consolidated math and science resources
$6,000,000	National Science Foundation awards
$1,419,306	Eisenhower Regional Mathematics and Science Consortia Program
$584,846	Trio Upward Bound Math and Science competition
$500,000	More than a third of McREL services to Colorado annually for professional development and research
$328,000	Average annual grant for "MathStar": middle school math & technology
$115,000	State and Regional IDEA "set aside" budget for math initiatives

COLORADO SCHOOLS THAT GET RESULTS

Elementary Schools

Despite the effects often seen by poverty, there do exist elementary schools, which exceed predicted math expectations. There are also elementary schools that make unexpected gains with high achieving students. Both of these schools are of interest to Colorado.

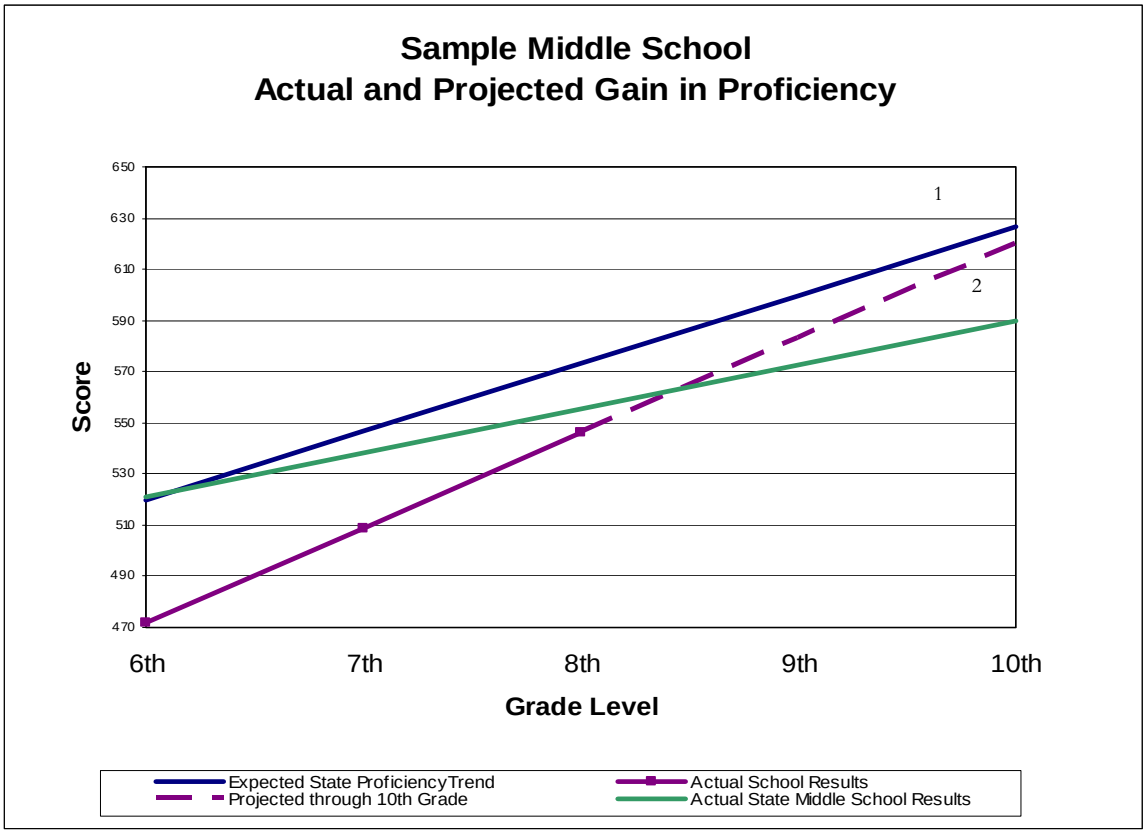
The methodology used to identify schools that get math results below examined the relationship of 5th grade math CSAP scores (2002 – 2004) to the percent of students eligible for free/reduced lunch. Schools that consistently exceeded the free/reduced lunch trend over that period were identified.



Middle and High Schools

Secondary schools, unlike elementary schools, have more years of data. The middle and high schools, which move the same students over time to greater annual math proficiency, are the schools that illustrate the gains that Colorado schools can achieve.

Average gain on the math CSAP scale score was examined for a continuing cohort of middle and high school students. Middle school 6th graders in 2002 who continued in the school as 7th graders in 2003 and 8th graders in 2004 were included. High school 9th graders in 2003 who continued in the school as 10th graders in 2004 were included with their 8th grade scores. Schools making better than the state average gains on math CSAP scale scores were identified.



Footnotes to Sample Secondary School:

1) Expected State Proficiency Trend for Math CSAP: Approximately 26 points of gain per year are necessary to maintain proficiency level.
2) School Results: The state average points gained per year from 2002 through 2004 was 17.2 for middle schools and 9.8 for high schools.

What do Elementary and Secondary "Math-Successful Schools" do to Achieve these Results?

- *Teachers know*
  . . . the standards and the mathematics
- *Quality curriculum*
  . . . supplemented with procedural or conceptual practice
- *Both explicit instruction and inquiry*
  . . . in the four essential math domains (arithmetic, concepts, reasoning and problem-solving)
- *Teacher collaboration*
  . . . focused on student outcomes
- *Student progress monitoring and adjustments*
  . . . made based on data

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

1. Be *clear* about the *math topics* that Colorado students are expected to know at each grade level.

(Your students' math performance is assessed on these math standards and assessment frameworks.)

The Assessment Frameworks are the elements of the state standards that are the exclusive math topics, which make up the state test questions. One of the most apparent differences between classrooms, which have stronger student math performance, and those classrooms that have low student math performance is teacher and student knowledge of these math elements.

Resources:

A) The Colorado Model Content Standards and CSAP Assessment Frameworks for Mathematics: http://www.cde.state.co.us/cdeassess/csap/frameworks/index.htm

B) Math vocabulary list by grade level (draft), which will be used on the state assessment. To be released on www.coloradomath.com/standards_assessment

Guiding Questions

1. How many of your teachers use these math frameworks in their daily instruction?
2. To what degree do your textbooks or activities address these topics and vocabulary words at the proficient level?
3. Are your students introduced to these at the beginning of school or at the first of each lesson?
4. How many of your parents know these as the primary objectives and math vocabulary for their children, for home conversations?
5. How much of your job performance review is based on these math expectations?
6. Are you recruiting, hiring and retaining math teachers who use these math expectations and are effective at moving students to these math topics at proficient levels?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

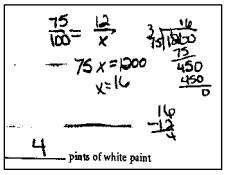
2. Be *clear* about what specific proficient student work looks like in math at each grade level.

(Do you know what "good enough" student work looks like for each math topic?)

The degree to which students can demonstrate what they know is very often underestimated. Do you know what work that is typical of a partially proficient scoring student looks like at each grade level? Half of the reported grades given in Colorado classrooms are far more generous than what the state assessment considers grade level work.

Resources:

C) Examples of high performing and low performing elementary, middle and high school student work: http://www.cde.state.co.us/cdeassess/csap/asrelitems_index.htm



D) Performance Level Descriptors: http://cde.state.co.us/cdeassess/csap/PLD/index.htm

If your daily class questions, homework and quizzes are not graded at these kinds of performance thresholds, you and your students are not on par with the state expectations. If you don't know or don't believe your students can or should do this level of math performance, consider the impact of beliefs and dispositions (see recommendation #3).

Guiding Questions

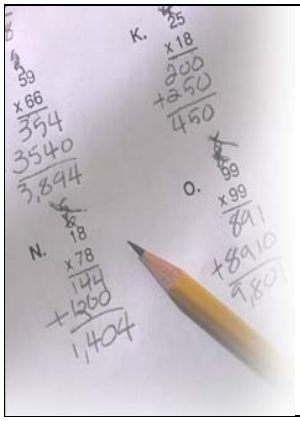
1. How many of your teachers use these math frameworks as the grade level performance thresholds for daily progress monitoring and student grades?
2. How many of your students know these performance expectations?
3. How many of your parents know these performance expectations?
4. Are students entering each new grade level with the necessary prerequisites to perform at grade level?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

3. Check school and *teacher beliefs and dispositions* about how many students can or should know math.

(Your belief in your students' capacity to understand the math affects enormously the chances of students' math success.)

Teaching a math lesson in order to cover the topic was once the job description. Teaching a math lesson with the requirement all students become engaged and proficient in the concepts makes for an entirely different math lesson. Teachers with a productive disposition help students to see mathematics as sensible, useful and doable.



Resources:

E) A selection of Elementary Schools that are exceeding student performance expectations over three years
www.coloradomath.com/recognition.htm

F) A selection of Middle and High Schools that have demonstrated above average student growth over two years
www.coloradomath.com/recognition.htm

Guiding Questions

1. How many of your students believe their teacher is inclined to be excited about math and believe that all of their students can become proficient?
2. Do you have a math department that talks about the math lessons, which engage all student knowledge?
3. To what degree do your teachers have the drive to know which students are confused and follow through on the ways to get them unstuck?
4. To what degree do your teachers know how to extend math lessons beyond what is presented in the curriculum?
5. How do you foster the excitement for teacher (adult) growth and enthusiasm for mathematics?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

4. Ensure that coherent approaches to math instruction are based on *current research* on math cognition and learning.

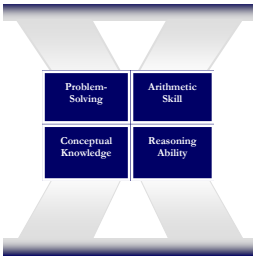
(Current math practice is commonly not grounded in research. Why?)

While additional research on math cognition and learning is still needed to fully understand how math proficiency is developed in children and why students struggle with math, what IS currently known from math research is not commonly seen in school practice.

Resources:

G) A visual depiction of the cognitive framework for math proficiency supported by existing research on math cognition and learning (Appendix A) www.coloradomath.com/4_domains.htm

H) Current Research (Appendix B) www.coloradomath.com/research.htm



Guiding Questions

1. Are your teachers knowledgeable about research findings related to mathematics development and mathematics disabilities?
2. How does your district ensure a coherent and efficient math scope and sequence that includes the essential domains of math so that your students develop the cognitive competencies necessary for math achievement?
3. How are these essential domains intentionally and explicitly included in your math instruction?
4. What messages do your math lessons over time send to your students about habits of mind for math?

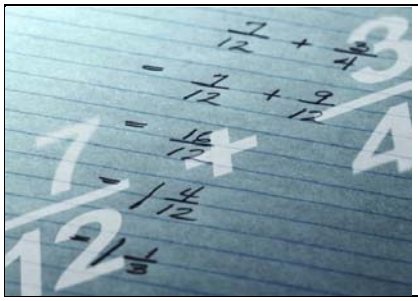
TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

5. Essential Math Domain: *Arithmetic*

Teach students efficient and accurate methods for computing and using computational algorithms and math facts, and provide the necessary practice to build automaticity.

(Why is it important for students to have quick access to math procedures and facts?)

*A solid grounding in arithmetic is a prerequisite to learning advanced mathematics. Procedural fluency is necessary for understanding concepts and solvi*ng problems accurately and efficiently.



Resources:

I) Video Resource available at www.coloradomath.com/instruction.htm demonstrates the arithmetic domain.

Guiding Questions

1. How explicit are you about giving your students these essential math rules in order to help them recognize and trust the ways numbers behave?
2. What are the multiple ways that elementary middle and high school math teachers teach the rules of arithmetic?
3. Do you provide distributed practice for procedures and facts?
4. Do your parents know these rules and tables so that they can support your students' study of the math language?
5. How are these rules incorporated into a coherent and repeated series of expectations for which students take responsibility?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

6. Essential Math Domain: *Concepts*

Teach students the ways that numbers behave and help students build their understanding of math concepts and vocabulary.

(Ask a student a question about why a math rule or table or function exists and too many are flummoxed about how mathematics behaves. Why?)

Students often find the language of math foreign, bizarre, obscure and unpredictable. Too many students separate and memorize tiny math steps without meaning. Math is often a subject of disconnected content and random study. Students need to understand why math operations function the way they do and be able to explain the patterns they find.

Resources:

J) Video Resource available at www.coloradomath.com/instruction.htm demonstrates the conceptual domain.

Guiding Questions

1. How are math concepts and operations taught? Are they made explicit to students?
2. How are your students taught to understand number relationships?
3. Does your math instruction emphasize symbolic notation, diagrams and procedures?
4. Do you explicitly teach math vocabulary in conjunction with concept development?
5. Do your teachers have the depth of knowledge necessary to provide accurate explanations that are comprehensible and useful for students?
6. How do you invite students to discuss and explore math concepts?
7. Are you clear about which concepts are expected to be known at which grades?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

7. Essential Math Domain: *Reasoning* Teach mathematical reasoning skills.

(This cognitive domain builds the capacity for logical thought, reflection, explanation, and justification.)

Math is about using logic to explain and justify a solution to a problem. It is the mental muscle necessary to successfully explore puzzles. It can also extend something known to something not yet known.

Resources:
K) Video Resource available at www.coloradomath.com/instruction.htm demonstrates the reasoning domain.

Guiding Questions

1. How do your questions prompt students to explain and justify logical solutions to problems?
2. How do you address faulty reasoning by students?
3. Do your teachers have the depth of knowledge necessary to respond productively to students' mathematical questions and curiosities?
4. What opportunities do students have to provide logical verbal and written explanations of their reasoning?
5. How often are students asked to apply spatial reasoning to patterns and comparisons?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

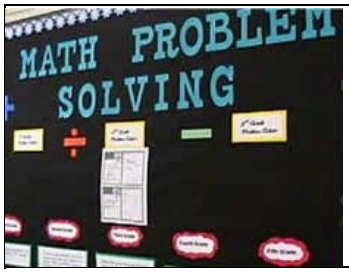
8. Essential Math Domain: *Problem Solving*

Teach students to use knowledge of arithmetic, concepts and reasoning to solve math problems.

(Do you ask students each day to do problem-solving activities or to prove or defend their solutions? Can your students appropriately or strategically apply the solutions to a larger process or need?)

Colorado math teachers insisted that the state assessment not just be a multiple "guess" test. On the CSAP test, greater than 40 % of the value of the exam relies on student ability to defend, write out their thinking behind the answer, or pose possible math approaches.

Math's utility is in its promise to make work easier, build approaches to real need and in expanding the mind to see new solutions. Many of the math performance expectations at the state level are built on asking students to make connections between one method and its application to a new situation.



Resources:
L) Video Resource available at www.coloradomath.com/instruction.htm demonstrates the problem-solving domain.

Guiding Questions

1. Most students are not asked to discuss their thinking about their approach to math solutions. How can you re-construct your style so that students imitate or initiate this transparent discussion about the meta-steps to problem solving?
2. How are you making parents aware of ways to approach math problem solving? Are you providing examples for parents to practice with students at home?
3. Do your teachers have the depth of knowledge necessary to pose mathematical questions and problems that are productive for students' learning?
4. Most teacher questions are posed with less than a three-second delay between question and expected student response. How might you invite prolonged thinking time for students to solve a problem in class?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

9. *Diagnose* what your students understand about the lesson of the day.

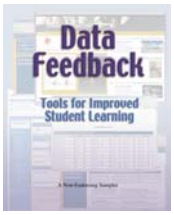
(Do you ask students each day to explain what they think they know after the lesson? Do you summarize the main point of the lesson and ask students to do the same?)

The chief characteristic of a great math teacher includes asking students the best questions:

- *"How might my students have gotten confused about this topic?"*
- *"What assumptions are they making?"*
- *"How can I explore the misperceptions of my students and engage them in doing similar self-questioning for better understanding?"*

Resources:

M) Sample feedback tool for mathematics: Please see *Data Feedback*, a publication by the Colorado Department of Education, available through Interlibrary Loan at: http://cde.carl.org/cgi-bin/cw_cgi?fullRecord+28556+28+2032320393+1+0



N) Lesson study protocols www.coloradomath.com

Guiding Questions

1. Do you screen all students to identify those who have difficulties in math?
2. What, other than your textbook end of chapter questions, do you use to monitor math progress?
3. Does your school facilitate time for teachers to discuss student work and math performance data regularly?
4. Does your district invest in the professional development necessary to grow teachers' capacities to diagnose students' math abilities?
5. How do you grow your teachers' depth of knowledge to interpret and make mathematical judgments about students' questions, solutions, problems and insights in mathematics?

TEN ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

10. *Recruit and develop teachers* with knowledge of math concepts, learning and instruction.

(Your teachers' knowledge of mathematics and math instructional practices significantly impacts your students' math achievement.)

Many elementary and middle school teachers do not feel they have sufficient understanding of the mathematics they teach. While knowledge of mathematics is a critical factor in teaching math, it is not sufficient to guarantee effective instruction.

Resources:

O) Mathematics Educator Standards (Appendix C)
http://www.cde.state.co.us/cdeboard/download/bdregs_301-37.pdf

P) "Mathematics in the 21st Century: What Mathematical Knowledge is Needed for Teaching Mathematics?" Deborah Loewenberg Ball, University of Michigan[7]. Remarks were prepared for the Secretary's Summit on Mathematics, U.S. Department of Education, February 6, 2003. Available at:
http://www.ed.gov/rschstat/research/progs/mathscience/ball.html

Q) Colorado Recruitment web page: www.teachincolorado.com

Guiding Questions

1. What level of mathematics background and training do your teachers have?
2. How are you providing professional development for teachers who need more content knowledge?
3. How are you recruiting teachers with strong math backgrounds to your school?

SUMMARY OF THE 10 ESSENTIAL MATHEMATICS IMPROVEMENT RECOMMENDATIONS

1.	**Be clear about the math topics that Colorado students are expected to know at each grade level.**	The Assessment Frameworks are the elements of the state standards, which are the exclusive math topics that make up the state test questions. One of the most apparent differences between classrooms that have stronger student math performance and those classrooms with low student math performance is teacher and student knowledge of these math elements.
2.	**Be clear about what specific proficient student work looks like in math at each grade level.**	Do you know what work that is typical of a partially proficient student looks like at each grade level? Half of the reported grades given in Colorado classrooms are far more generous than what the state assessment considers grade level work.
3.	**Check school and teacher beliefs and dispositions about how many students can or should know math.**	Teaching a math lesson in order to cover the topic was once the job description. Teaching a math lesson with the requirement all students become engaged and proficient in the concepts makes for an entirely different math lesson. Teachers with a productive disposition help students to see mathematics as sensible, useful and doable.
4.	**Ensure that coherent approaches to math instruction are based on current research on math cognition and learning.**	While additional research on math cognition and learning is still needed to fully understand how math proficiency is developed in children and why students struggle with math, what is currently known from math research is not commonly seen in school practice.
5.	***Essential Math Domain: Arithmetic*** **Teach students efficient and accurate methods for computing using computational algorithms and math facts, and provide the necessary practice to build automaticity.**	A solid grounding in arithmetic is a prerequisite to learning advanced mathematics. Procedural fluency is necessary for understanding concepts and solving problems accurately and efficiently.
6.	***Essential Math Domain: Concepts*** **Teach students the ways that numbers behave and help students build their understanding of math concepts and vocabulary.**	Students often find the language of math foreign, bizarre, obscure and unpredictable. Math is often a subject of disconnected content and random study. Students need to understand why math operations function the way they do and be able to explain the patterns they find.
7.	***Essential Math Domain: Reasoning*** **Teach mathematical reasoning skills.**	Math is about using logic to explain and justify a solution to a problem. It is the mental muscle necessary to successfully explore puzzles. It can also extend something known to something not yet known.
8.	***Essential Math Domain: Problem Solving*** **Teach students to use knowledge of arithmetic, concepts and reasoning to solve math problems.**	Many of the math performance expectations at the state level are built on asking students to make connections between one method and its application to a new situation.
9.	**Diagnose what your students understand about the lesson of the day.**	The chief characteristic of a great math teacher includes the ability to ask questions.
10.	**Recruit and develop teachers with knowledge of math concepts, learning and instruction.**	Many elementary and middle school teachers do not feel they have sufficient understanding of the mathematics they teach. While knowledge of mathematics is a critical factor in teaching math, it is not sufficient to guarantee effective instruction.

SEVEN STATEWIDE MATH SUPPORT SYSTEMS FOR SCHOOLS

Local classrooms and schools are not alone in raising student math performance, but they are the most direct and intimate point of contact with students. No Governor, lawmaker, state board member or state agency will ever be responsible for good or bad student math performance.

Seven support systems do exist to put a forced focus on the very best math practices for our math educators. This more precise focus has not always been applied, but could be a significant difference, especially to any school or district that does not always have local resource, or time to enhance math instruction beyond usual practice.

Below are possible recommendations of how each professionally supportive network might extend their own reach to effectively disseminate what we know about:

- *The Colorado Math Standards*
- *The **prerequisite** Math Essentials (Arithmetic, Concepts, Reasoning, Problem solving)*
- *The Habits of Math- Successful Schools*

1. Math Support System: Math Related Associations or Networks

a. Capitalize on the benefits of currently issued math newsletters to thousands of Colorado math educators and even support a more advanced circulation, which may additionally indicate the best examples of mathematics problems and teacher /student solutions. These quarterlies could include a series that focuses on precise math topics that feature frank discussions about the potential for students to get "stuck" or confused.

b. Provide more names of regional math teachers, professional mathematicians, serious math users and collegiate mathematics teachers who can coach, discuss how to introduce math essential domains, build provocative math problems and pose interesting math questions.

c. Post the names of teachers who are excellent at diagnosing and discussing specific pedagogy barriers that are appearing in classrooms and with specific students. Tap into the data pool of McREL and other networks for research and video images of benchmarked examples.

d. Subsidize the annual CCTM math conference from a variety of partners. This enhanced version may include "webinars", more math software exhibitions, a larger university presence or even more opportunities for

e. seasonal and regional math teachers to talk productively at an adult level about engaging and inviting mathematics problems and insights.

f. Financially contribute more to the very successful CCTM school- level projects, which "seed" intense and personal math professional development.

2. Math Support System: Teacher Networks and Associations

a. Continue their substantive support for teaching and learning outreach with schools and teaching professionals in math.

b. These dedicated teachers also may appreciate the electronic dissemination of the math assessment frameworks and math vocabulary lists, video clips of teaching and learning examples of the math domains and names of their colleagues who are being recognized for math achievement in their region.

c. Continue to recognize the significant contribution from CEA to co-sponsor Summer Standards and Assessment Summits.

3. Math Support System: CDE (Colorado Department of Education)

a. Develop a Colorado Math Webpage with resources for school districts.

b. Offer, once a year, CTB/McGraw Hill scoring feedback "webinars" to teachers regarding specific observations about annual student math performance.

c. Electronically issue annual, specific examples, by grade level, of student performance benchmark reminders and math anchor papers.

d. Post annually the names of schools that move student math performance in significant ways.

e. Leverage all federal and state dollars issued by CDE around much more precise expectations for math practice.

f. Continue to recognize the significant contribution from CDE to co-sponsor Summer Standards and Assessment Summits.

4. Math Support System : Professional Development Community

a. Capitalize on their universal understanding of these members for professional learning communities and for pre- and post- needs assessments. Focus these conversations and data collections **even more** on the four (pre-requisite) math domains.

b. Continue to offer professional development in math content, math assessment and math instruction based on both the findings of math research and student needs assessment to support math teachers seeking certificated renewal in mathematics.

SEVEN MATH SUPPORT SYSTEMS

c. Exploit even stronger partnerships with local community colleges and universities in order to build an intentional and long-term math mentor program for new math teachers and emerging math leaders.

5. Math Support System: Leadership Associations and District Administrators

a. Raise awareness and celebrate the critical role both administrators and teachers play in improving math performance. Examine in print and visit schools and districts called out as improving math achievement. Note the synergy and success of an intentional leadership agenda in math.

b. Promote teacher supervision and evaluation practices aligned with the essential domains of math instruction **and** the habits of classrooms with consistent math success.

c. Continue to recognize the significant contribution from CASE to co-sponsor Summer Standards and Assessment Summits.

6. Math Support System: Local School Boards

a. Promote the explicit inclusion of the four essential domains of math and math assessment frameworks in district curriculum and textbook decisions.

b. Consider the findings of this math report in math teacher and administrator hiring's.

c. Electronically disseminate to teachers in your district information about the characteristics of schools that get positive results in mathematics.

d. Continue to recognize the significant contribution from CASB to co-sponsor Summer Standards and Assessment Summits.

7. Math Support System: Higher Education

a. Reinforce the 2003 amended Colorado Teacher Performance Standards, through recognition of teacher preparation programs that have demonstrated effective incorporation of Colorado Standards as the keystone of their programs.

b. Ensure that teacher preparation programs include the state's math assessment frameworks in their fieldwork and student teaching applications.

c. Publicize documented success stories, with regard to the correlation between math preparation and candidate success with student achievement.

SEVEN MATH SUPPORT SYSTEMS

d. Encourage the financial and intellectual support for both new and veteran teachers of mathematics through courses offered by math departments and schools of education that develop:

- Math content knowledge in the essential domains of math
- Knowledge of math standards and assessment frameworks
- Ability to accurately explain and explicitly teach arithmetic, concepts, reasoning, and problem solving
- Ability to assess, interpret and make judgments about students' questions, solutions, problems and insights

CONCLUDING STATEMENTS

The profile of Colorado's student math performance stands half achieved. While the state standards are nearly identical in topic to other states, the language of Colorado's math benchmarks is not always clear and the awareness of Colorado teachers regarding the grade-by-grade assessment frameworks is not pervasive. The addition of third and fourth grade math tests this 2005 will put a new energy into local schools to ask for and examine specific math goals and grade-by-grade student math expectations.

Colorado ranks within the top 20 in the nation for math achievement results. According to NAEP results, however, this reflects only a 34 % proficiency rate in both fourth and eighth grades, respectively. However, state assessments indicate a decline in proficiency rates as students move through the grades. These report less than a 55% proficiency rate at 5th grade over time, a 41% proficiency in the middle grades and a 28% success rate for tenth graders. Information collected from teachers and administrators indicated that ninth and tenth graders' inability to perform arithmetic and apply problem-solving strategies were the primary contributing factors.

National research talks about a "natural calling" children have for abstractly seeing math in their mind, an impulse to sort objects or the delight in seeing a repeated pattern. Classroom mathematics exist to apply and codify the vocabulary, concepts, math facts, reasoning ability and problem solving into a language, in order to formally develop deeper math proficiency and use.

The prerequisites in math, like in reading, open doors to strategic fluency and the understanding necessary to more advanced thinking in mathematics.

The prerequisites in math, like in reading, open doors to strategic fluency and the understanding necessary to more advanced thinking in mathematics. It is not so much the cut points or some "national advanced math" agenda that reveal secondary math students inability to do well on either criterion or normed tests. It is this shaky foundation in basic **arithmetic** and the **concepts** they are built upon that stifle the ability and confidence to attempt more math. Unformed **reasoning** skills and **problem solving** tactics also separate the successful student from the one who fails or dislikes mathematics. **These cognitive domains build a capacity for logical thought, reflection, explanation, and justification**.

Math is about using logic to explain and substantiate a solution to a problem. It is the mental muscle necessary to successfully explore puzzles. It can also extend something known to something not yet known. Secondary mathematics must demand that these foundations and strategic and procedural fluencies are illustrated and practiced on a regular basis.

CONCLUDING STATEMENTS

Students need to understand why math operations function the way they do and be able to explain the patterns they find. Ask a student a question about why a math rule, table, or function exists and too many are flummoxed by how mathematics operates. Students separate and memorize tiny math steps without meaning. Often they find the language of math obscure and unpredictable. Too many of Colorado's teachers and administrators express a difficulty with math themselves and are not convinced that it is essential to have students practice beyond eighth grade.

Unfortunately, lessons are too often introduced out of context and end inconclusively. The mathematics research that does exist is not often enough a part of conventional school practice. Instead, poorly written mathematics textbooks dominate the Colorado mathematics curriculum.

Nevertheless, there are Colorado schools, despite poverty or other barriers, consistently making gains with students and exceeding state expectations of proficiency growth. Despite differences in textbooks, demographics or years of teachers' experience, **math 'success-schools' possess five common qualities:**

First, the math teachers know the math standards *and* the mathematics content. In some cases, there is professional development in the mathematics, itself. In other cases, the math is inherent in the teacher education and preparation.

Second, math curricula are supplemented with either procedural or conceptual practice. Depending on the text utilized, this extra effort to participate with the math in ways not addressed by the textbook or curricula is noted as especially helpful for student learning.

Third, explicit teaching and inquiry in the four math domains is constantly referenced. Students at elementary and secondary levels demonstrate and handle arithmetic, conceptual understanding, reasoning activities and problem solving.

Fourth, teacher focus is on student outcomes and the united intention to help students obtain proficiency in math, not just *address* math daily.

Finally, in every case of strong math gain, these schools all refer to the power of monitoring student progress, asking students questions and making adjustments based on feedback. This practice is strategically useful for students, and considered a form of professional growth for the faculty that regularly discuss these small steps of individual student math progress and reversals.

CONCLUDING STATEMENTS

Colorado teachers and school districts, in collaboration with state supports such as higher education, web resources and professional associations, may use these findings to accelerate student performance. Colorado math standards are similar to those represented in other states and on international assessments. They align to our suggested national math standards and our state's math assessment is comparable to the topics and outcome expectations on local, national and international tests. Preliminary content validity and alignment studies on our state math assessment for the United States Department of Education indicate that our state has a math assessment that is proportional and parallel to our standards and math content that is fitting to what our math educators identified as grade level appropriate. Therefore, no changes to our state math standards are recommended at this time.

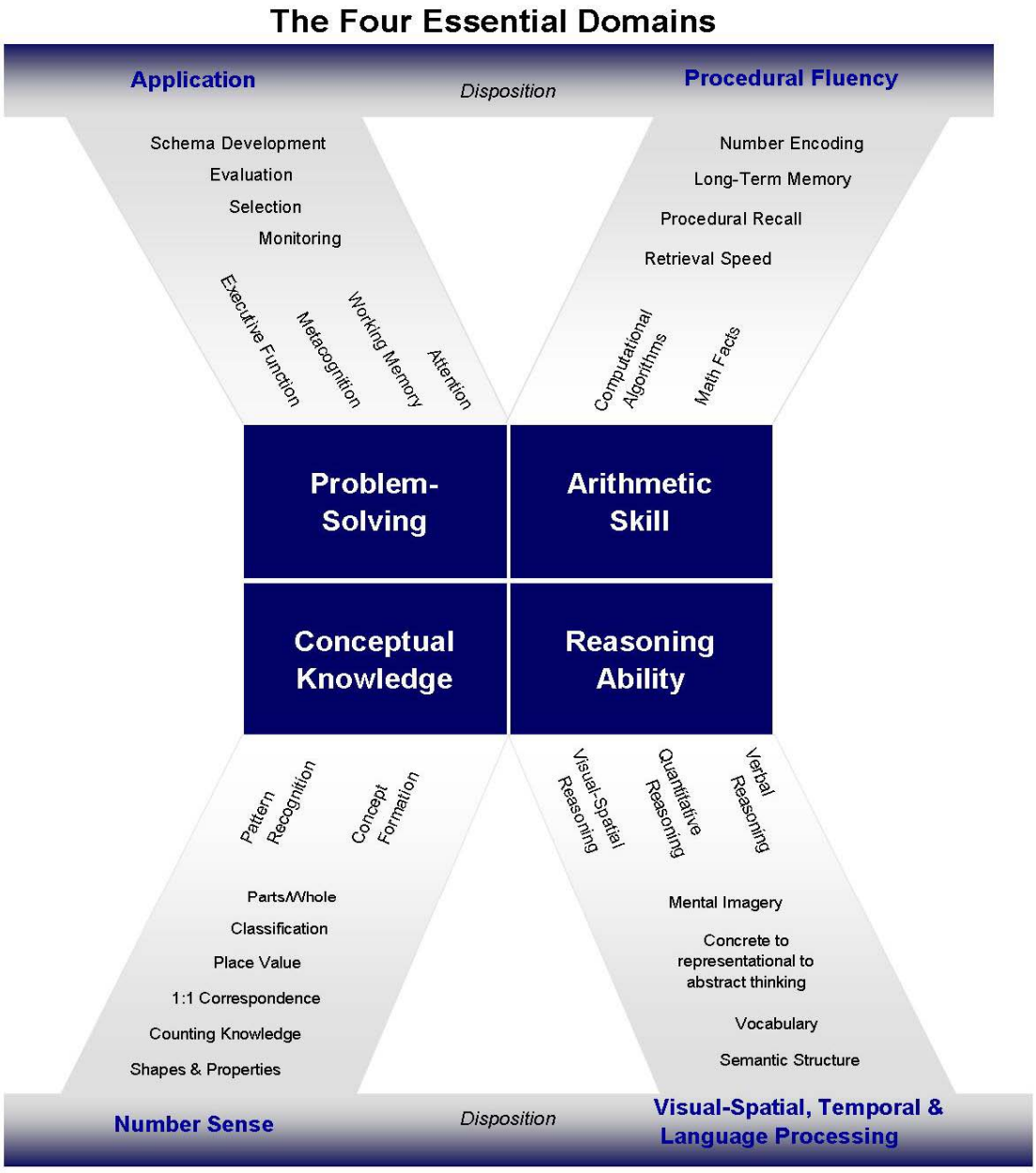
Without changing the math standards or the state math assessment, the state can clarify the math benchmarks for teachers in the classroom. Additionally, the state can provide each teacher a grade-to-grade articulation of the assessment frameworks. There also exist K-12 instructional sequences to better guide schools and districts about *optional* ways to unfold more effective and robust math curriculum.

The significance of this report is a call to examine the essential pre-requisites students apparently require in order to excel in mathematics. That is, the clarification and definition of the four essential domains in mathematics that students require for success. These four domains matter. Schools successful in mathematics mentally exercise students in these four areas early and often, even in high school.

A statewide priority dedicated to mathematics achievement will require federal, state and local partnerships. It will require a sharpened focus to grants, education awards, teacher education, professional development, and evaluation.

Most importantly, this investigation into Colorado's math standards provides valuable insight into the successful habits of schools making math gains with their students. Instituting these habits in classrooms across the state will yield demonstrable student math achievement.

APPENDIX A – COGNITIVE FRAMEWORK OF MATHEMATICAL PROFICIENCY



Sources: Adding it Up; Berninger & Richards; Geary; Levine; Van de Walle

Note: The center boxes contain the Four Domains of Mathematics supported by current research consensus. The shaded areas indicate contributing factors and functions currently under investigation. Future research findings should be used to guide further definition and elaboration of this model.

APPENDIX B – CURRENT RESEARCH

U.S. Department of Education

Mathematics and Science Initiative

http://www.ed.gov/rschstat/research/progs/mathscience/index.html

The Secretary's Mathematics and Science Initiative was launched in February 2003 to:

- Engage the public in recognizing the need for better mathematics and science education for all children.
- Initiate a campaign to recruit, prepare, train, and retain teachers with strong backgrounds in mathematics and science.
- Develop a research base to improve knowledge of what boosts student learning in mathematics and science.

Institute for Educational Sciences (IES)

National Center for Education Research

The mission of the National Center for Education Research is to: 1) sponsor sustained research that will contribute to the solution of education problems and lead to the accumulation of knowledge about education to ensure that all children have access to a high-quality education, improve teaching and learning and student academic achievement, close the achievement gap, and improve access and opportunity for postsecondary education; 2) support research syntheses and promote the use of scientifically valid research findings to improve education policy and practice; 3) Promote quality through the use of accepted practices of scientific inquiry to gain knowledge about the validity of education theories, practices, or conditions; and 4) Promote scientifically valid research findings that can provide the basis for improving academic instruction and lifelong learning.

Cognition and Student Learning Research Program

http://www.ed.gov/programs/edresearch/awards.html#cognition

The purpose of the Cognition research program is to improve student learning by bringing recent advances in cognitive science to bear on significant problems in education. The long-term outcome of this program is to develop approaches to instruction that are based on principles of learning and information processing gained from cognitive science and to provide evidence of their usefulness in education settings.

Mathematics and Science Education Research Program

http://www.ed.gov/programs/edresearch/awards.html#math

Education research has yet to make a substantial contribution to improving science and mathematics achievement across the spectrum of the American student population. Basic research in such areas as the study of cognition have improved the understanding of how people learn in certain areas germane to learning in mathematics and science, but the existing research knowledge has yet to be developed into instructional strategies and curricular programs that show demonstrably more successful results and are then widely adopted by schools.

The Institute's research program in Mathematics and Science Education is designed to encourage efforts to turn research toward the improvement of educational practice. Researchers receiving grants from the Institute under this research program contribute to the realization of three goals: (a) developing new interventions and approaches to mathematics and science education that will eventually result in improving mathematics and science achievement; (b) establishing the efficacy of existing interventions and approaches to mathematics and science education with small efficacy or replication trials; and (c) providing evidence on the effectiveness of mathematics and science interventions taken to scale. The long-term outcome of this program will be an array of tools and strategies (e.g., curricula, programs) that have been demonstrated to be effective for improving mathematics and science learning and achievement.

APPENDIX B – CURRENT RESEARCH

National Institutes of Health

National Institute of Child Health & Human Development

Mathematics and Science Cognition and Learning – Development and Disorders

http://www.nichd.nih.gov/about/crmc/cdb/math.htm

This Program encourages both basic and intervention research in all aspects of mathematical thinking and problem solving, as well as in scientific reasoning, learning, and discovery. An important priority within this Program is the investigation of individual differences that may moderate achievement in math and science. Of particular interest is the delineation of skill sets needed to attain proficiency in these domains, the means to address the kinds of learning difficulties that frequently emerge in each of these areas, and the development of effective instructional methods for mitigating these difficulties.

Mathematical Cognition and Learning

This area of programmatic emphasis seeks to support research on the normal development of mathematical proficiency, including both conceptual and procedural knowledge. Specific domains of interest include, but are not limited to: basic numerical representations and processing, arithmetic comprehension and procedural skills, proficiency with fractions and other types of rational numbers, algebraic problem solving, geometric thinking, concepts of probability and chance, and measurement concepts and skills. Longitudinal studies of the development of mathematical proficiency are especially welcomed, beginning either in the preschool period or in the early grades.

Mathematical Learning Disabilities

Studies in this area are encouraged that are aimed at delineating the nature and extent of specific learning disabilities in mathematics, including diagnosis, classification, etiology, prevention, and treatment. Subject populations of interest include children with idiopathic math learning disabilities, co-morbid math and reading disabilities, and children with neurodevelopmental disorders for whom deficient math performance represents one of the primary cognitive sequelae. Epidemiological longitudinal studies are needed to generate an accurate estimate of the prevalence of specific learning disabilities in mathematics. Of particular importance are the effects of poverty on the failure to develop mathematical proficiency, and the identification of risk and protective factors within these contexts.

The National Academies

National Research Council
Center for Education

http://www7.nationalacademies.org/cfe/

The Center's mission is to promote evidence-based policy analysis that is both responsive and anticipatory: responsive to government's and other stakeholders' program and research interests; and anticipatory of long-term challenges, opportunities, and needs that affect the future of education research and policy priorities.

Mathematical Sciences Education Board

http://www7.nationalacademies.org/mseb/

The Mathematical Sciences Education Board (MSEB) current mission is to provide national leadership and guidance for policies, programs, and practices supporting the improvement of mathematics education at all levels and for all members of our society. We are actively pursuing several initiatives that focus on the learning, instruction, and assessment of mathematics; equity in mathematics; attracting and retaining students in mathematics majors and in mathematically intensive careers; capacity building and professionalization of mathematics education; evidence of effectiveness in mathematics education; and the public perception of mathematics, mathematics learning, and mathematics teaching.

APPENDIX C – COLORADO TEACHER PREPARATION STANDARDS

8.14 **Mathematics Education.** To be endorsed in mathematics, an applicant shall hold a bachelor's or higher degree from a four-year accepted institution of higher education; have completed an approved teacher preparation program; an approved program in mathematics; and have demonstrated the competenciesspecified below:

(1) **The mathematics educator is knowledgeable about the history of mathematics, and is able to:**

(a) articulate, to students, and effectively instruct about the developmental contributions of mathematical systems to and from diverse cultures and societies.

(b) effectively demonstrate, to students, number systems, number theory, and algebraic structures to include, but not be limited to:

(i) number sense, including mental mathematics, estimation, and reasonableness of results.

(ii) basic number theory, the role of algorithms, and alternative computational algorithms.

(iii) the theory and applications of abstract and linear algebra.

(c) effectively demonstrate, to students, and instruct about functions, to include, but not be limited to:

(i) precise mathematical language and symbolism.

(ii) recognition of functions as a unifying concept in mathematics.

(iii) polynomial, rational, algebraic, and transcendental functions and their applications.

(iv) a variety of representations of functions: tabular, graphical, symbolic, verbal, and how to utilize, compare, and contrast these representations.

(v) the distinction between use of continuous and discrete approaches in the solution of mathematical problems.

(d) effectively demonstrate, to students, and instruct about geometry and measurement, to include, but not be limited to:

(i) a solid foundation in the basics of Euclidean geometry in two and three dimensions.

(ii) geometric investigations through two- and three-dimensional physical models, drawings, and computer and calculator graphics.

(iii) synthetic, coordinate, transformational, and vector geometry, with an emphasis on problem-solving.

(iv) geometric reasoning, both formal and informal, using the van hiele hierarchy

(v) the role of the axiomatic method and proof.

(vi) the role of non-Euclidean geometries.

(vii) connections of geometry to other math disciplines, especially between algebra and geometry.

(viii) attributes of measurement including length, area, volume, capacity, time, temperature, angles, weight, and mass.

(ix) standard and non-standard units of measurement, including both metric and U.S. customary.

(x) derivations of the formulas for the perimeter, area, and volume of common figures.

(xi) indirect measurement and its applications. 8.14 (1) (d) (xii) dimensional analysis.

(e) effectively demonstrate, to students, and instruct about statistics and probability, to include, but not be limited to:

(i) descriptive and inferential statistics and probability from both experimental and theoretical viewpoints.

(ii) discrete and continuous probability distributions and use of such distributions to make inferences about probability.

(iii) collecting, organizing, analyzing, and interpreting data, and effective communication of the results.

(iv) exploratory data analysis including measures of central tendency, measures of variation, and general distributions.

(v) multiple representations of data including histograms and box plots.

(vi) confidence intervals, hypothesis testing, correlation, and regression.

(vii) simulation, as a problem-solving technique for making decisions.

APPENDIX C – COLORADO TEACHER PREPARATION STANDARDS

(viii) empirical probability, using both hands-on and computer simulations.
(ix) geometric probability
(x) potential misuses of statistics and common misconceptions surrounding probability

(f) effectively demonstrate, to students, and instruct about mathematical models which occur in the physical or biological sciences, and in areas related to population dynamics, economics, or scheduling problems, including, but not limited to, such concerns as traffic flow.

(g) effectively instruct students regarding calculus and analysis, to include, but not be limited to: conceptual knowledge of limit, infinity, continuity, differentiation, and integration; applications of calculus in the sciences and business; modeling and solving problems involving rates of change and optimization; and sequences and series.

(h) effectively instruct students regarding discrete mathematics to include, but not be limited to: symbolic logic, induction and recursion, relations, equivalence relations and functions, introduction to graph theory and modeling applications, difference equations, linear programming, and introduction to combinatorics.

(2) **The mathematics educator is able to effectively demonstrate, to students, and instruct about:**

(a) approaches to problem-solving, which utilize mathematical content, in identifying, analyzing, formulating, and solving problems that occur in mathematical processes and everyday situations.

(b) the utilization of mathematical ideas, verbally and in writing, using both everyday language and mathematical terminology.

(c) the utilization of verbal and written discourse, between teacher and students, and among students, to develop and extend students' mathematical understanding.

(d) the construction and evaluation of mathematical conjectures and arguments, to validate one's own mathematical thinking.

(e) independent study in mathematics.

(f) the use of mathematics in studying patterns and relationships.

(g) the interrelationships within mathematics; how to connect concrete, pictorial, and abstract representations; connections between mathematics and other disciplines, and real world situations, through the selection of appropriate applications from fields such as, but not limited to, natural sciences, social sciences, business, and engineering, and is able to:

(h) utilize a wide variety of resource materials, including, but not limited to: manipulative materials, graphing calculators, computers, and other technologies, as tools in learning and for the application(s) of mathematics.

(i) utilize assessment data to monitor students' acquisition of mathematical skills and abilities, and in the process of determining appropriate delivery of instruction, based on identified student need; and to select appropriate mathematical tasks to reinforce and promote students' development of mathematical concepts and skills.

(j) create an engaging and effective environment in which all students develop mathematically, in order to participate more fully in a technologically-based society.

(k) create an environment in which reflection, uncertainty, and inquiry are incorporated in the learning of mathematics skills, abilities, and concepts

(l) apply appropriate knowledge of current research in the teaching and learning of mathematics, and incorporate national, state, and local guidelines, related to mathematics instruction.

(3) **The mathematics educator shall consistently seek out professional development in the field of mathematics which can provide enhanced knowledge, skills, and abilities in the content area and participate in professional organizations, as appropriate and relevant to the field.**

(4) **Field experiences: the mathematics educator has completed the field experience**.

FOOTNOTES

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3) Berninger, V. & Richards, T. (2000). Building a Computing Brain Neurologically. In *Brain Literacy for Educators and Pyschologists.* San Diego: Academic Press.

4) Donovan & Bransford, (Eds), National Academy Press (2005). *How Students Learn History, Mathematics, and Science in the Classroom.* Washington DC: National Academy Press.

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6) Gersten, R. (1999). Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities. *The Journal of Special Education, 44*, 18-28. http://www.ldonline.org/ld_indepth/math_skills/gersten_dyscalculia.html

7) Ball, D. (2003). *Mathematics in the 21st century: What mathematical knowledge is needed for teaching mathematics?* US Department of Education. Retrieved 05/2005. from http://www.ed.gov/rschstat/research/progs/mathscience/ball.html

8) Geary, D. (1994). *Children's Mathematical Development: Research and Practical Applications.* Washington DC: American Psychological Association.

9) Levine, M. (1998). *Developmental Variation and Learning Disorders.* Cambridge: Educators Publishing Service.

10) Van de Walle, J. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally (5th ed.).* Boston: Pearson.

RESOURCES

A. The Colorado Model Content Standards and CSAP Assessment Frameworks for Mathematics.
http://www.cde.state.co.us/cdeassess/ csap/frameworks/index.htm

B. Mathematics Vocabulary by Grade Level.
www.coloradomath.com/standards_assessment

C. Examples of High & Low Quality Elementary, Middle School and High School Student Work. http://www.cde.state.co.us/cdeassess/ csap/asrelitems_index.htm

D. Anchor Papers.
http://www.cde.state.co.us/cdeassess/csap/PLD/

E. A Selection of Elementary Schools Exceeding Student Performance Over Three Years.
www.coloradomath.com/recognition.htm

F. A Selection of Middle and High Schools that have Demonstrated Above Average Student Growth over Two Years.
www.coloradomath.com/recognition.htm

G. Cognitive Framework of Mathematical Proficiency.
www.coloradomath.com/4_domains.htm

H. Current Research in Mathematics.
www.coloradomath.com/research.htm

I. "Data Feedback: Tools for Improved Student Learning." Office of Learning and Results, Colorado Department of Education, 2004. Sample "Benchmark" provided by Edison.
http://cde.carl.org/cgi-bin/cw_cgi?fullRecord+28556+28+2032320393+1+0

J. Lesson Study Protocols.
www.coloradomath.com/instruction

K. "Mathematics in the 21st Century: What Mathematical Knowledge is Needed for Teaching Mathematics?" Deborah Loewenberg Ball, University of Michigan. Remarks prepared for the Secretary's Summit on Mathematics, U.S. Department of Education, February 6, 2003.s Available at:
http://www.ed.gov/rschstat/research/progs/mathscience/ball.html

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