Learning with graphing calculator (GC):
GC as a cognitive tool

Nataliya Reznichenko

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Abstract

This paper focuses on using technology to support mathematics instruction. Technologies such as computers and calculators are widely used in the teaching of mathematics. The educators advocate the use of these tools to reduce tiresome computations and tasks so that class time can be more effectively used for learning mathematics. The purpose of this review is twofold: (1) to explore the use of computer and calculator as tools for the mathematics teaching and (2) to analyze their effects on the students' achievement in algebra and calculus. This review of literature from 1990-2006 also addresses the implication of curriculum in an age of computer algebra systems.

Introduction

In the digital age, all aspects of life, especially in education, should adapt to progressive technologies. As envisioned by the National Council of Teachers of Mathematics (NCTM, 2000), “electronic technologies [ET]--calculators and computers”--has taken its place in almost all classrooms in schools and colleges across the country (p. 24). Over the past 25 years, researchers have been studying the effectiveness of ET in different areas of education.

The purpose of this review is twofold: (1) to explore the use of computer and calculator as tools for the mathematics teaching and (2) to analyze their effects on the students’ achievement in algebra and calculus. In this paper, the literature from 1990 to 2006 is reviewed. The integration of ET into a traditional educational system and the purpose and objective of the studies in different educational settings is examined. Overall, this literature review explores the findings in the current research on learning mathematics with ET to help identify its effect on students’ academic achievement, particularly, in algebra and calculus.

Statement of the Problem

When calculators were first introduced into classrooms on a daily basis, educators were concerned that their use would develop dependence on calculators and detract or interfere with the learning of mathematical skills (Heid, 1997). Nevertheless, in its position statement on the use of calculators, the NCTM (1998) strongly urges that school districts, teachers, and educators at every level promote calculator usage. Further, in the *Principles and Standards for School Mathematics*, the NCTM (2000) states that ET “are essential tools for teaching, learning, and doing mathematics” (p. 24). Technology influences “the mathematics that is taught and enhances students’ learning” (p. 24). Besides the importance of technology, the NCTM is concerned about possible treatment of ET as a replacement for basic understandings and intuitions. “It [ET] can and should be used to foster those understandings and intuitions” (p. 25). In view of the NCTM’s Technology Principle, there is a need to know how ET is being used for mathematics teaching and what effect it has on the students’ achievement. Particularly, this is important regarding algebra and calculus.

Method of Selecting Relevant Articles/Studies

In order to ascertain a clear understanding of the present uses of computers and calculators in schools, various sources including journal articles and books were reviewed. An on-line computerized search was conducted using Educational Resources Information Center (ERIC). In fact, over 100 research studies may be found on this topic in ERIC between the years of 1990-2006. The areas in which the literature search produced information were**:** (a) technology integration, (b) cognitive tools, (c) computers in education, (d) computers/calculators in mathematics education, (e) computer algebra systems (CAS), and (f) graphing calculators (GC).

Findings and Implications

The literature review of topics relevant to the problem is divided into five sections. The first section considers the technological revolution as a background of the reform in mathematics teaching, and the research on the use of calculators conducted before the 1990s. Technology has been an important part of a change that mathematics education has undergone since the late 1980s. It was found that calculator use during instruction does not hold back computational skills and conceptual understanding. Alleviating the initial fear that traditional skills would not be learned was an important factor in calculator use. It appeared that computers and calculators have been forcing curriculum planners to critically examine the content and methods of teaching school and college mathematics.

The second section presents theoretical aspects of the use of computers and calculators as a cognitive tool by identifying their definitions, classification by functions, and wider classification of computer programs.

The third section discusses the use of computers and calculators in the schools and the effects they have on learning. One of the ways that ET appeared to be used in schools was as a cognitive tool in the development of thinking skills and for problem solving. Using ET in the classroom as a cognitive tool seems most effective.

The forth section concerns the role of computers and calculators as cognitive tools in mathematics education. In mathematics, the role of computers and calculators as cognitive tools is the most useful in the development of thinking skills and for problem solving.

The fifth section discusses studies on the effectiveness of cognitive tools in school and college mathematics. Most studies found that students who used ET developed better understanding of concepts with no significant loss of computational skill.

Technological Revolution as a Background of the Reform in Mathematics Teaching

Technological Revolution

Since the late 1980s, mathematics education has undergone a great change. Technology has been an important part of this change (Heid, 1997). During the technological revolution, educators advocated the use of ET to reduce tiresome computations and tasks so that class time can be more effectively used for learning mathematics. It was assumed that ET could contribute to a broader understanding of mathematical solutions to problems. These were the cornerstone ideas underlying the algebra and calculus reform movement.

The analysis of the role of technology in mathematics education is given by one of the initiators of the reform. According to Heid (1997), the technological revolution has influenced the recent mathematics education reform movement in areas of content, teaching, and learning. She sees growing access to computing technology as “the single most important catalyst” of this reform (p. 5). Also, Heid gave a narrow definition of computing technology as electronic software and hardware without including processes of their work.

Reform in Mathematics Teaching

In algebra, the concept of function was emphasized. The computer tools were used to move students from procedural knowledge to conceptual understanding of functions (Heid, 1997). In calculus, along with reduction of computations and concentration on the concepts, the emphasis was made also on visualizing of the problem. As a result, the reform has produced the shift in pedagogical philosophy from behavioral to constructivist, from independent learning to cooperative, with emphasis on conceptual understanding (Kulik, 2003).

The reform has found its reflection in the NCTM’s (2000) *Principles and Standards for School Mathematics*. NCTM deemphasizes computational skills and encourages the use of calculators and computers. Also, NCTM recommends the use of ET to complete routine computations in order to concentrate on conceptual understanding. ET is also important from the constructivist view because it provides different models of multiple representations that help students to build conceptual connections.

Procedural and conceptual knowledge. Haapasalo and Kadijevich (as cited in Kadijevich & Haapasalo, 2001) define conceptual (abbreviated to C) and procedural (abbreviated to P) knowledge as the following:

> Conceptual knowledge denotes knowledge of and a skillful "drive" along particular networks, the elements of which can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representation forms.
>
> Procedural knowledge denotes dynamic and successful utilization of particular rules, algorithms or procedures within relevant representation form(s). This usually require(s) not only the knowledge of the objects being utilizes, but also the knowledge of the format and syntax for the representational system(s) expressing them. (pp. 156-157)

Furthermore, Kadijevich and Haapasalo (2001) describe the ways of reaching C typically through conscious thinking and P through automated and unconscious steps. Also, P may be demonstrated in a reflective mode of thinking as instrumental understanding. In this case, the students successfully combine two rules without exactly knowing why they work.

Early Research on the Calculator Effectiveness

The early suggestion that school mathematics should shift away from paper-and-pencil calculation to the use of digital calculators was supported in the research literature since the 1980s. Hembree and Dessart (1986) conducted a landmark study on the effects of calculator usage in school mathematics. This meta-analysis integrated the findings of 79 studies on the effects of calculators on student achievement and attitude in pre-college mathematics.

According to Hembree and Dessart (1986), two factors that affected the performance of students in classes taught with and without calculators are the following: (1) the allowance or not of the calculator use during the outcome tests, and (2) types of the tests--on computations (basic operational skill) or on conceptual understanding (problem-solving). Additionally, the researchers found that calculator use during instruction does not hold back computational skills and conceptual understanding. They also found that calculator use in the classroom affected outcome results on computational and conceptual non-calculator tests for average-ability students. As Hembree and Dessart concluded, although the positive effect was negligible, it supported the assumption of the positive effect of calculator use on the cognitive processes.

In 1992, Hembree and Dessart updated their findings with new data that supported or enhanced the conclusions of their earlier study (Hembree & Dessart, 1986). They added a new conclusion about different levels of students' abilities. According to Hembree and Dessart (1992), calculator use during instruction improves outcome results on non-calculator tests for all ability groups of students, not just for average-ability students.

Also, Dunham and Dick (1994) reported on a survey of research on the use of GC in the mathematics classroom. As noted by Dunham and Dick, the majority of the research contained comparisons of students' understanding of specific mathematics topics by students who used GC and those who didn't use GC. Many studies focused on an experimental design where the experimental group used GC or computer-graphing software and the control group used no graphing technology. Research results were mixed. Some researchers reported significant gains by the experimental groups while others reported no significant differences between the groups.

The difficulties in measuring the impact of the GC were discussed. According to Dunham and Dick (1994), these included the difficulty in controlling the curricula differences in the experimental and control groups due to the introduction of the technology. Additionally, the use or non-use of the GC during assessment could negatively impact the mathematical performance of the experimental or control group. In spite of the complications inherent in this kind of research, Dunham and Dick stated, "The early reports from research indicate the GC have the potential to dramatically affect teaching and learning mathematics, particularly in the fundamental areas of functions and graphs" (p. 444).

Today, calculators are widely used in the teaching of mathematics. The opportunity to concentrate on the learning of concepts has opened the door for the wide use of calculators and computer technologies in mathematics classrooms. The reason for the major impact of technology on the reform is its effect on the nature of mathematical thinking and understanding (i.e., cognitive processes) as a cognitive tool (Heid, 1997).

Use of ET as a Cognitive Tool

Definitions, Functions, and Classifications of Cognitive Tools

From the perspective of mathematical thinking and understanding, the concept of "cognitive technologies" or "cognitive tools" is used for the investigation of the impact of technology on mathematics education. In the literature, the terms "cognitive technologies" and "cognitive tools" are used interchangeably.

Definitions of cognitive tools. Tools designed to support cognitive processes are cognitive tools. Many scholars discussed cognitive tools regarding the elements used and the mechanisms worked (this sentence doesn't make sense) (Jonassen, 1996; Kozma, 1987; Heid, 1997). Kozma (1987) defined cognitive tools as computer-based tools that can amplify, extend, or enhance human cognition. Cognitive technologies are defined by Pea as media that help "transcend the limitations of the mind…in thinking, learning, and problem-solving activities" (as cited in Heid, p. 91).

Jonassen (1996) extended the definition of cognitive tools by adding intellectual interaction of cognitive tools and the learners. He defined cognitive tools as "computer-based tools and learning environments that have been adapted or developed to function as intellectual partners with the learner in order to engage and facilitate critical thinking and higher-order learning" (p. 9).

Functions of cognitive tools. Cognitive tools are instruments that can enhance the cognitive powers of learners during their thinking, problem solving, and learning in different ways (Kozma, 1987; Lajoie, 1993; Pea, 1985). Kozma (1987) suggested using computers to facilitate the learning process and assist learners in accomplishing complex cognitive tasks. According to Kozma, the following factors could hinder learning: (a) limited capacity of work memory; (b) difficulty in retrieving needed information from long-term memory; (c) ineffective use of cognitive strategies to restructure information.

Further, Kozma (1987) believed computers could provide help in reducing these factors in the following ways: (a) make large amounts of information immediately available for use to supplement limited short-term memory; (b) enable learners to retrieve prior knowledge and apply it more efficiently in a new situation; (c) allow learners to represent ideas in multiple forms.

Regarding the functions cognitive tools serve, Lajoie (1993) identified their types as the following: (a) support cognitive and meta-cognitive processes; (b) share cognitive load by providing support for lower level cognitive skills so that resources are left for higher order cognitive skills; (c) allow learners to engage in cognitive activities that otherwise would be unreachable for them; (d) allow learners to generate and test hypotheses in the context of problem solving.

Pea (1985) offered her view regarding the role of technology as either an amplifier or a reorganizer. According to Pea, technology that is used to extend the existing curriculum is called an amplifier. In the second role, as a reorganizer, technology is changing the fundamental nature and arrangement of the curriculum. The concepts of amplifier and reorganizer help highlight the environments in which learning occurs and examine the effects of technology on the teaching and learning of mathematics.

Classification of cognitive tools. Cognitive tools differ not only in the purposes they are used for, but also in the structure and features of their interfaces (Heid, 1997). Cognitive tools play substantially different roles in mathematics instruction than in instruction in other fields. Heid listed the cognitive tools and their roles in mathematics education as the following: (a) computer algebra systems (CAS)--enable users to generate symbolic, graphical, and numerical representations and to reason within and among them; (b) micro-worlds and dynamic geometry tools--provide computer worlds in which students can express, develop, and investigate mathematics ideas; (c) technology-based laboratory devices (i.e., calculator-based laboratory devices [CBL] and microcomputer-based laboratory devices [MBL])--give students easy access to collecting and analyzing real-world data; (d) graphing calculators (GC)--give students easy access to computational and graphical results.

Cognitive tools have an ability to generate a larger number and a greater range of examples for students to encounter. They give the learner the opportunity to exceed the limitations of the mind. Cognitive tools have their own niche in the wider categorization of technology.

Types of Programs Used for Computers

The range of possible computer programs that incorporate different learning theories, philosophies, or developments into technology is broad. It varies from early mainframe-based or microcomputer-assisted instruction (CAI) to micro-worlds (e.g. Logo), simulations, hypertext, computer-mediated communication (CMC), and the Internet. Each type of program has individual characteristics, purposes, and different ways to assist student learning (Means, 1994**).** Means classified various types of programs used for computers as the following: (a) tutoring programs (i.e., tutorials and drill-and-practice CAI)--used for direct instruction by providing information, expression, and practice opportunities; (b) exploratory environment programs (i.e., micro-worlds, simulations, and hypertext-based or hypermedia-based learning environments)--used to encourage active student exploration and discovery learning; (c) tool programs (i.e., mathematics tools such as computer algebra tools [CAS] that perform graphing or symbolic manipulation or both, and computer geometry tools that perform construction or visualization of figures)-- also refer to general-purpose technological tools such as word processing, spreadsheets, and data-analysis software that assist in writing, data storage, and data analysis; (d) computer-mediated communication (CMC) media (i.e., e-mail, computer-conferences, computer supported collaborative learning systems [CSCL], and the Internet)--allow groups of teachers and students to communicate and share information electronically to learn and to collaborate across distances. The match between the type of the program and the educational goal is important in the defining of the effectiveness of ET in education.

Effects of ET on Learning

Learning “from” Technology versus Learning “with” Technology

Reeves (1998) summarized and organized the evidence on the effects of using ET for learning into two categories: (1) learning *from* technology and (2) learning *with* technology. According to Reeves, in learning *from* technology, ET serves as a tutor. The student responds to the information, presented by the computer, and in this way learns from the computer. Examples are simple drill-and-practice programs and more comprehensive Integrated Learning Systems (ILS). Further, Reeves states that in learning *with* technology, ET plays as a cognitive tool or exploratory environment. The student uses databases, spreadsheets, expert systems, and communication software to write, analyze data, develop presentations, and conduct research. As several studies demonstrate, the application of ET as a cognitive tool rather than a tutor is very important (Sandholtz, Rinstaff, & Dwyer, 1997; Penuel, Golan, Means, & Korbak, 2000; Indiana State Department of Education, 1990).

Learning “with” Computer: ET in Schools as a Cognitive Tool

Sandholtz et al. (1997) described the Project Apple’s Classrooms of Tomorrow (ACOT) that lasted 10 years. It was sponsored by Apple Computer and provided by researchers from institutions of higher education and not Apple employees. In ACOT classrooms, ET was persistent and available anytime students needed to write, analyze data, develop presentations, and do research. According to Sandholtz et al., their conclusions were the following: (a) increasing higher-order thinking skills far beyond that of expected students’ grade level; (b) enhancing ability to collaborate with peers to develop projects and reports, increasing initiative; (c) increasing teachers’ attitude toward ET.

Penuel et al. (2000) reported that the ACOT findings were reinforced by the following study of the Challenge 2000 Multimedia Project sponsored by SRI International. In the final evaluation of the five-year long project, the students in project and non-project classrooms were required to complete an authentic assessment task. The results were evaluated using a rubric especially developed by SRI to help measure the impact of the use of ET. According to Penuel et al., they found that students in Multimedia Project classrooms consistently out-scored their peers in the non-project classrooms in the following areas: (a) understanding content; (b) adapting their message to their intended audience; (c) applying principles of design in the format and layout of their projects.

Another effort called the Buddy Project supplied students with home computers and modem access to school (Indiana State Department of Education, 1990). According to Indiana State Department of Education, positive effects from this project included an increase in writing skills, better understanding and broader view of math, ability to teach others, and greater problem solving and critical thinking skills. These studies point out how powerful ET can be when it is employed as a tool in the classroom.

Impact of ET on Learning

Means (1994) identifies and explains conditions under which ET could have a positive impact on learning as the following: (a) access--ET enhances learning when students have often access to it in the classroom; (b) integration--ET has greater impact on learning when its use is linked to content standards and integrated into ongoing instructional program; (c) broad-based reform--ET is one tool in an effort to improve learning and it should be planned within the context of the entire school or district planning process; (d) the long term--ET use should be not a one-time event and it requires a long-term effort on the district's part to fund, support and assess their use; (e) professional development--to empower teachers and students to learn *with* computers, there should be ongoing staff development that takes place in large groups, one-on-one, and online; (f) teaching style--teachers should learn new instructional strategies and new roles; (g) balance--together with teaching the facts, teachers should also help students obtain and use the intellectual and workplace skills demanded by the 21st century; (h) vision--principals and superintendents should have a vision of how ET supports learning and teacher productivity.

Cognitive Tools in Mathematics Instruction

The literature review focuses on cognitive tools that make higher-level mathematical activities accessible to students. A broad range of technological tools gives the students the possibility to design and to conduct mathematical investigations.

This literature review identified studies that investigate the ways in which some cognitive tools, such as computer algebra tools, enable students to encounter and investigate mathematical, real, and computer worlds. Computer algebra tools include software that performs graphing or symbolic manipulation or both. Graphing and symbolic calculators are defined as computers because they are programmable and perform many of the same functions as a computer. The tools discussed are computer algebra systems (CAS) and graphing calculators (GC).

Computer Algebra Systems (CAS)

Heid and Edwards (2001) describe CAS as computational utilities that have many linked features such as graphical, numerical, and symbolic manipulation. CAS are descendants of the early symbolic manipulation programs that have been available since the early 1980s. These computer-based mathematics packages performed exact arithmetic calculations and algebraic symbolic manipulation (i.e., simplifying algebraic expressions and solving equations). Later programs such as Derive and

Maple are able to perform complicated tasks in calculus. Some tasks are difficult, if not impossible, using only pencil-and-paper (by hand) methods (Heid, 1988). Starting from the late 1990s, CAS are now available in the user-friendly hand-held calculators' versions of Mathematica or Derive (Heid & Edwards, 2001).

The use of CAS in secondary school classrooms supports development of deeper conceptual understanding by allowing a linked multiple representation. The reflection of any changes in algebraic, tabular, or graphic models in all representations helps apply algebra to real-world modeling (Heid & Edwards, 2001).

Graphing Calculators (GC)

In mathematics classrooms at all levels, graphing calculators (GC) are becoming common. GC transforms data from either tabular or equation format into graphic representation. The use of GC is specifically recommended by the NCTM in *Curriculum and Evaluation Standards* (NCTM, 1989). NCTM suggests that GC should be used to facilitate student understanding by a multiple representation approach to functions (tables, graphs, symbolic expressions, and real-world modeling).

GC can also improve problem solving (Dick, 1992). According to Dick, the following are some examples of how GC assists with problem solving: (a) it frees up time for instruction by reducing attention to algebraic manipulation; (b) it supplies more tools for problem solving especially useful for students with weaker algebraic skill; (c) using GC, students perceive problem solving differently since they are free from numerical and algebraic computations to concentrate on problem set up and analyzing solutions.

Effectiveness of ET as a Cognitive Tool in Teaching Mathematics

Research on the Use of Computers in School Mathematics

Wenglinsky (1998) summarized findings from a national study of the relationship between different uses of ET and various educational outcomes. He analyzed data from the 1996 National Assessment of Educational Progress (NAEP) in mathematics. According to Wenglinsky, national samples consisted of 6,227 fourth graders and 7,146 eighth graders. Data included information on the frequency of computer use for mathematics in school, access to computers at home and in school, professional development of mathematics teachers in computer use, and the kinds of instructional uses of computers in the schools.

The study found significant difference in the ways in which computers were used, but not in how often they were used (Wenglinsky, 1998). Wenglinsky states that the higher order uses of computers were less in poor, urban, and rural schools, than in non-poor and suburban. The size of the relationship between the various positive uses of technology and academic achievement was small for fourth graders, but significant for eighth graders. According to Wenglinsky, this study found that computers could serve as important tools for improving students' proficiency in mathematics depending on how they are used.

Synthesis Studies on the Effects of Cognitive Tools

The results of the studies published starting from the 1990s have been outlined in this review. After Hembree and Dessart's (1986) study, several meta-analyses on computers and calculators were published (Barton, 2001; Ellington, 2003; King, 1997; Kulik, 2003; Smith, 1996). A decade later, Smith conducted a meta-analysis of over 30 studies completed from 1984 to 1995 on the use of calculators in K-12. According to Smith, students who used calculators for problem solving, computation, and conceptual understanding had significantly higher achievement and attitudes than students who did not use calculators. From eight secondary school comparison studies on use of GC, he concluded that there was no significant difference in achievement between GC group and control group. However, in one study of twelfth graders, the control group outperformed the GC group.

For the same time frame (i.e., 1986--1995), King (1997) conducted a meta-analysis of 30 studies completed on the use of computer-enhanced instruction for different mathematics topics in a college setting. In analyzed studies, computer enhanced instruction included the following: teacher demonstration with a single computer and a classroom display unit, student use of GC or CAS, and student use of computer in a laboratory setting. King reported the following results: (a) ET had statistically significant positive effect on overall student achievement; (b) ET had no significant effect on procedural achievement; (c) for the technology group, the allowance of ET use during the outcome tests had significant positive effect on procedural achievement, while the denial of ET use during the outcome tests had an adverse effect on procedural achievement (though not significant); (d) ET had a slightly negative effect on conceptual achievement when access to it was limited to the classroom and lab only; (e) ET had a positive effect on achievement when used in instruction both as a tool and for demonstration.

During the 1990s, more that 60 studies that investigate the impact of graphing technology on mathematics instruction have emerged (Barton, 2001). Barton conducted her own review of 52 studies investigating the use of GC and CAS in teaching mathematics content (i.e., algebra, trigonometry, and calculus) in different educational settings (i.e., secondary schools and colleges). She used eight studies from Smith's (1996) analysis, 16 studies from King's (1997) analysis, and 28 more studies reported from 1990-2000.

Barton (2001) provides an analysis of the empirical studies regarding overall achievement, conceptual understanding and procedural knowledge. Her review outlines the benefits of the use of technology on student achievement:

> More than two-thirds of the studies compiled for this paper reported better overall achievement for the treatment group (graphing technology and/or CAS) and 75% of the results on measures testing for conceptual understanding favored the treatment group while nearly two-thirds of the results on procedural knowledge indicated no significant difference between the control group and the treatment group. (p. 4)

From the encouraging results of the review, Barton (2001) concludes that these evidences strongly suggest that ET "do assist in increasing conceptual understanding without adversely affecting procedural knowledge" (p. 5).

Nikolaou (2001) conducted a meta-analysis of 24 empirical studies on the effects of hand-held calculator use on mathematics achievement and problem-solving abilities of secondary school students from 1987 to 1999. During extraction and synthesis of the main findings, Nikolaou calculated a total of 103 effect sizes from 15 studies on mathematics achievement and nine studies on problem-solving abilities. The results of this meta-analysis showed that 92 percent of calculated effect sizes were positive while 8 percent were negative. An overall average effect size was .4961 with a standard deviation of .9291.

Based on the positive results of the study, Nikolaou (2001) concluded about the clarity of calculator inclusion in mathematics instruction. Also, he made some recommendations on how ET should be used and to what extent at different school levels. According to Nikolaou, in the elementary school, the use of ET should be very limited. The students should not develop a misunderstanding about learning mathematics as only process of pressing the appropriate buttons on ET. With progress to the middle high school, students should be gradually exposed to the capabilities of ET. Also, Nikolaou states the latter should not be viewed as a panacea for the problems with achievement in mathematics education. ET should be utilized as tools for enhancement in the learning of the various mathematical concepts.

Ellington (2003) conducted a meta-analysis of findings from 54 research studies on students' achievement and attitude in pre-college mathematics classes published from 1983-2002. When calculators (including GC) were used in instruction and assessment, Ellington found improvement of the skills with the following results for the weighted mean effect size: (a) operational--non-calculators g= .17, calculators g= .38; (b) computational--non-calculators g= .03, calculators g= .43; (c) conceptual (i.e., skills necessary to understand mathematical concepts)--non-calculators g= .05, calculators g= .44; (d) problem-solving (i.e., the number of problems attempted)--non-calculators g= .16, calculators g= .33; (e) selectivity (i.e., the ability to select an appropriate problem-solving strategy)--non-calculators g= .30, calculators g= .20.

Regarding student attitudes, Ellington (2003) also found improvement in the following constructs: (a) attitude toward mathematics g= .32; (b) self-concept in mathematics g= .05; (c) attitude toward use of calculators in mathematics g= .09.

As a part of a bigger project, Kulik (2003) conducted a literature review of 12 empirical research studies on the effectiveness of computer and calculator tools applications in colleges and universities. From evaluation of the studies in algebra and calculus, he reported average effect size unusually high.

> In the typical study, computer and calculator use raised student scores on tests of conceptual understanding a total of 0.88 standard deviations. This means that students who used computers and graphing calculators while studying algebra and calculus scored 0.88 standard deviation units higher on conceptual tests than did students in the control group. If control group students scored at the 50th percentile on a conceptual test, scores of students using computers or calculators would be at the 80th percentile or above. Evaluations of educational innovations rarely report average effect sizes this high. (p. 39)

Kulik (2003) also provided general statistical data illustrating that during the last four decades ET was increasingly helpful for improving learning in college courses.

> The median effect size in studies of computer-based college teaching was –0.13 in 5 evaluation studies published during the 1960s, 0.22 in 85 studies published during the 1970s, 0.35 in 35 studies published during the 1980s, and 0.46 in 44 studies published during the 1990s. (p. viii)

In conclusion, computer/calculator-based instructions are an important part in many successful college courses. Also, in general, research indicates large positive effects of ET on student achievement in conceptual understanding. Additionally, the studies found that ET does not decrease student achievement in computational skills and attitudes toward mathematics.

Empirical Studies on Cognitive Tools

Many empirical studies on the effectiveness of ET were conducted. In some studies ET was used as a supplement to traditional instruction (TI), in others ET was implemented in curriculum (O'Callaghan, 1998; O'Callaghan & Kirshner 1994; Hollar & Norwood, 1999).

The studies reviewed in this section differ by the following criteria: (a) type of tool--CAS (Mayes, 1995; O'Callaghan, 1998; O'Callaghan & Kirshner, 1994), GC (Hollar & Norwood, 1999; Quesada & Maxwell, 1994; Ruthven, 1990; Guttenberger, 1992; Avalos, 1994); (b) mathematics context--school algebra (Hollar & Norwood, 1999; Avalos, 1994), college algebra (Mayes, 1995; O'Callaghan, 1998; O'Callaghan & Kirshner, 1994), pre-calculus and trigonometry (Quesada & Maxwell, 1994; Ruthven, 1990; Guttenberger, 1992), calculus (Heid, 1988; Palmiter, 1991; Lauten, Graham, & Ferrini-Mundy, 1994; Rubin & Nemirovsky, 1991; Judson, 1990; Smith, 1994); (c) educational setting--school (Ruthven, 1990; Guttenberger, 1992; Avalos, 1994; Ganguli, 1990; Yerushalmy, 1991), college (Mayes, 1995; O'Callaghan, 1998; O'Callaghan & Kirshner, 1994; Hollar & Norwood, 1999; Quesada & Maxwell, 1994); (d) developing software tools (Ganguli, 1990; Yerushalmy, 1991); (e) testing problem-solving ability (Jackson, Berger, & Edwards, 1992; Mayes, 1992).

To connect the evidences from research to the reform in mathematics education, in this literature review, the studies are grouped under course titles--algebra and calculus. Within these titles, there are subtitles for different types of tools --complete CAS and portable CAS--graphing calculators (GC).

Algebra

Effects of complete CAS. Several studies were conducted with complete CAS (Mayes, 1995; O'Callaghan, 1998; O'Callaghan & Kirshner, 1994). In college algebra, Mayes conducted a study, where an experimental group was treated with CAS Derive for demonstration in the classroom and for hands-on activities in computer laboratories. The control group was taught without CAS. For the experimental group, the study showed a much higher performance on conceptual tests and the results of almost an equal score on computational tests.

In another college algebra study, a computer-intensive algebra (CIA) curriculum was implemented. Student achievement in the experimental group, taught by the researcher, was compared with student achievement in two traditional algebra (TA) curriculum groups, taught by the researcher and another teacher (O'Callaghan, 1998; O'Callaghan & Kirshner, 1994). The results of the posttest of the CIA group compared to TA groups indicated development of much clearer and deeper conceptual understanding of functions on the O'Callaghan's functional test and higher achievement on traditional computational skills test. According to O'Callaghan, regarding attitude toward mathematics, CAI students showed an improvement in their attitudes.

Effects of GC. More studies in the college setting show that students in classes with GC scored higher on final examinations than students in comparable classes without calculators (Hollar & Norwood, 1999; Quesada & Maxwell, 1994). In intermediate algebra in the university setting, Hollar and Norwood have extended O'Callaghan's (1998) CIA study by using his component competencies (i.e., modeling, interpreting, translating, and reifying) and the process-object framework to investigate the effects of a graphing approach curriculum implemented GC TI-82. In a balanced design, two instructors were teaching both experimental and control classes. Two experimental classes were treated with GC enriched curriculum; two control classes used TA curriculum without GC. According to Hollar and Norwood, the students in experimental classes significantly outperformed on the conceptual test compared to TA classes. The MANOVA revealed an overall significant treatment effect at the α =.01. Students' scores did not differ on the computational test. Researchers did not find significant difference in attitude toward mathematics between groups, but results in the experimental group were a little higher.

In a pre-calculus course in the college setting, Quesada and Maxwell (1994) treated five sections in the experimental group with GC together with a textbook written for their use. Eight sections in the control group used scientific calculators with a regular textbook. The experimental group that used GC on the final exam had outperformed the control group that used scientific calculators on the final exam.

Several studies show a positive effect of ET on student achievement and class environment in school settings (Ruthven, 1990; Guttenberger, 1992; Avalos, 1994). The use of GC in pre-calculus in the school setting was studied by Ruthven. The researcher found that an ability to symbolize graphs algebraically was better in the control group that used GC compared to the control group without calculators. There was no difference in ability of verbal interpretation of graphs between groups.

Guttenberger (1992) found that in trigonometry high school students who used a computer graphing tool had higher scores on exams and higher retention scores when compared with students who did not use this tool. In middle school, Avalos (1994) exposed children without algebra experience to algebraic symbols and concepts using a programmable GC. This study found that the calculators created an environment in which the students could solve problems and explore concepts.

Calculus

Studies with positive result. The following studies in calculus show higher development of skills and understanding of concepts with the use of ET (Heid, 1988; Palmiter, 1991; Lauten et al., 1994; Rubin & Nemirovsky, 1991). Heid conducted a landmark study of college calculus students in a computer-intensive class who used various algebra tools for non-symbolic representations like graphs, tables, and applications. In this experimental course computer tools such as MuMath (a symbol-manipulation program that computes limits, derivatives, and integrals) were used to provide multiple representations of problems and to facilitate computational skills. The control group was taught in the calculus course with emphasis on traditional skill in symbolic manipulation. Results showed that the experimental group had a better understanding of calculus concepts and similar calculation skills compared to the control group.

In a calculus course for engineering in a university setting, Palmiter (1991) investigated the effect of CAS Macsyma on concept and skill acquisition in a randomly assigned experimental group that was treated for five weeks versus a control group that covered material in 10 weeks. The score of the experimental group on both the conceptual test and calculus computational exam was significantly higher when compared with the control group.

In a case study of college calculus students, Lauten et al., (1994) found that, although understanding of calculus concepts increased with use of the GC, students did not totally operate with the graphing calculators in problem-solving situations. Rubin and Nemirovsky (1991) designed three different computer-enhanced environments and tested their effects on student's learning of calculus concepts. While there were some differences in the problem-solving uses of these tools, they all contributed to student development of calculus concepts.

Studies with less positive results. Some calculus studies found less positive results (Judson, 1990; Smith, 1994). For example**,** Judson used Maple software in **a** calculus class and found no significant difference from a control class in achievement. However, it was noted that the Maple class had higher motivation, interest, and class participation.

Smith (1994) used Derive in two calculus classes. One class was required to interactively use the software for problems, and the other only saw it used by the instructor for demonstration problems. She found no significant difference between the two groups, either in achievement or in attitude measures.

Software Tools

In addition, there are studies that tested developed software tools (Ganguli, 1990; Yerushalmy, 1991). Ganguli tested developed dynamic graphing programs that were used in college algebra classes for graphing demonstration. Although there was no difference between experimental and control groups on the posttest, the experimental group did score significantly higher on the comprehensive final exam two weeks later.

With seventh graders, Yerushalmy (1991) tested experimental symbolic and graphing algebra software. The four groups were treated with different versions. Based on this qualitative study the researcher suggested that, because of extensive work with multiple representations, the group with the graph version might have had a deeper understanding of the concepts.

Problem-Solving Ability

In some studies, researchers were testing students' problem-solving ability (Jackson et al., 1992; Mayes, 1992). The primary purpose of the study conducted by Jackson, et al. was to infuse problem-solving activities requiring higher order thinking into inner-city classrooms. The researchers investigated the use of computers with a graphing application program to teach principles of the design and interpretation of graphs to a population of students with little or no prior knowledge of graphs or data analysis. According to Jackson, et al., it differed from most earlier work in the field of computer-assisted graphing in three major ways: (1) the researchers focused broadly on a wide variety of types of graphs, used in several different subject matter contexts; (2) they conducted research into problem-solving processes in an unmodified whole-classroom environment; (3) they supplemented participant observations on students cognition with extensive behavioral sequences from data that was gathered automatically and unobtrusively by the computer.

The most surprising findings identified by Jackson et al., (1992) were the examples of how intelligent (but mostly mathematically inexperienced) students independently reinvented some important abstract concepts, such as interpolation and the ordinal treatment of numeric variables. They stated that most cases involved the creative use of sorted column or pie graphs when the problem was designed to call for line or scatter graphs.

Mayes (1992) conducted a study to determine if the use of the computer as a tool in guided-discovery learning episodes would enhance the mathematical problem-solving ability of secondary school students. The 189 students involved in the 10-week study were from three high schools of an urban mid-western school district with a student population of 4,102, which included both rural and urban backgrounds. The 82 students from a second-year algebra class, which received only the problem-solving treatment, formed the PS group. The 107 students from a second-year algebra class, which received both the problem-solving treatment and the computer treatment, formed the computer problem-solving (CPS) group.

Mayes (1992) found that the results of the between subjects effects indicated that the school attended was a significant factor in test performance. A significant difference occurred in the three-way interaction of the treatment group, mathematics quality score, and the difference contrast. According to Mayes, this difference supported the hypothesis that there was an improvement in problem-solving ability due to treatments if the quality score in mathematics was taken into account. The results of the study also indicated that the number of mathematics courses and the level of achievement in those courses should be considered when choosing an environment for the instruction of problem solving. Also, Mayes states that students of the low-mathematics achievement level performed better in a non-computer problem-solving approach. Students at this level may be overwhelmed by the joint problem-solving and computer treatment due to lower initial mathematics knowledge.

Overall, it may be concluded from the reviewed literature that the experiments typically lasted a minimum of one term. Also, most studies found that students that used ET developed better understanding of concepts with no significant loss of computational skill.

Another conclusion can be done about the role of GC as a tool. GC can be used in the following different ways as a tool: (a) for the symbolic manipulation or graphical display of mathematical functions and equations; (b) for the collection, examination and analysis of data; (c) to foster collaborative learning and teach students to work as a team; (d) to aid in solving realistic problems that enables the student to concentrate on problem aspects and interpretation rather than computational aspects; (e) to discover, visualize, or investigate mathematical theories.

Summary

In conclusion, ET such as calculators and computers create opportunities for worthwhile activities. ET is an enhancer that puts students in an active role and teachers in a facilitator role. Students perceive problem solving differently when they are free from numerical and algebraic computations to concentrate on problem set up and analysis of solutions. This literature review shows that, in secondary school and in college, algebra and calculus students learn mathematics more in-depth with the use of ET. Important concepts such as functions, modeling, and problem solving are better understood when ET is utilized.

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